

EINSTEIN–DEBYE MODELING OF HEAT CAPACITY IN KANE-BAND NARROW-GAP SEMICONDUCTORS OVER A WIDE TEMPERATURE RANGE

ZAFER DOĞAN^{1*}, TURAL MEHMETOĞLU², ABDULLAH HAKAN YAVUZ¹

Manuscript received: 19.08.2026; Accepted paper: 04.09.2026;

Published online: 30.09.2026.

Abstract. In this study, we investigate the temperature dependence of the heat capacities of Kane-type semiconductors using an exact Einstein–Debye approximation over a wide range of parameters. The Einstein–Debye method provides accurate results across different temperature regimes, from cryogenic to room temperature and beyond. The proposed heat capacity formulation is validated against numerical calculations and demonstrates excellent agreement with experimental and theoretical data for various Kane-type semiconductors, including InSb, InAs, and GaSb compounds. We analyze the influence of key material parameters, including the energy gap and Debye temperature, on the heat capacity behavior. This work provides a practical analytical tool for predicting thermal properties in Kane-type semiconductors and is particularly valuable for device modeling and thermal management applications in narrow-gap semiconductor technologies.

Keywords: Kane semiconductors; heat capacities; Einstein-Debye approximation; temperature dependence; nonparabolic dispersion.

1. INTRODUCTION

Kane-type semiconductors are a class of narrow-gap semiconductor materials characterized by their nonparabolic energy-momentum dispersion relation, first described by Kane [1]. These materials exhibit unique electronic band structures where the conduction and valence band coupling is strong near the band extrema, resulting in significant deviations from the simple parabolic band approximation used in wide-gap semiconductors [2,3]. The Kane model describes the $k \cdot p$ coupling between valence and conduction bands within an effective multiband framework, while incorporating remote band influences through second-order perturbative corrections [4]. The Kane-type nonparabolic dispersion relation provides an effective framework for describing narrow-gap semiconductors. It has been employed both in hydrodynamic subband transport models for semiconductor electrons [5] and in obtaining exact solutions for position-dependent band-gap structures under external non-uniform electric fields [6]. Kane-type semiconductors play a crucial role in modern electronics due to their unique transport properties. Indium antimonide (InSb) exhibits one of the highest room-temperature electron mobilities among conventional semiconductors ($\sim 78,000 \text{ cm}^2/(\text{V}\cdot\text{s})$), making it highly attractive for high-frequency devices, magnetic field sensors, and infrared detection systems [7-9]. The electronic properties of III–V narrow-gap semiconductors are

¹ Tokat Gaziosmanpaşa University, Faculty of Engineering and Architecture, Department of Electrical and Electronics Engineering, 60150 Tokat, Türkiye. E-mail: hakan.yavuz@gop.edu.tr.

² Tokat Gaziosmanpaşa University, Graduate Education Institute, Department of Electrical and Electronics Engineering, 60150 Tokat, Türkiye. E-mail: turalmehmetoglu@yahoo.co.uk.

* Corresponding author: zafer.dogan@gop.edu.tr.

governed by strong conduction–valence band coupling, leading to pronounced band non-parabolicity and energy-dependent effective masses, as comprehensively summarized by Vurgaftman et al. [10]. Recent advances have also demonstrated the potential of narrow-gap semiconductor devices for high-speed, low-power electronics and monolithic integration with silicon photonics platforms in mid-infrared detection systems [11–15].

Understanding the thermal properties of semiconductors, particularly the temperature dependence of heat capacity, is essential for device design and optimization [16,17]. Heat capacity directly reflects the thermal energy storage capability and plays a fundamental role in thermal management, energy conversion efficiency, and device reliability [18]. In semiconductor devices, thermal effects significantly influence carrier transport, optical properties, and overall performance [19]. The temperature-dependent heat capacity governs heat dissipation rates and temperature gradient development within device active regions [20]. For narrow-gap semiconductors in infrared detection and thermal imaging applications, precise knowledge of thermal properties is essential, as these devices often require cryogenic cooling for optimal performance [21,22]. Additionally, in thermoelectric applications, the temperature dependence of the electronic band structure and thermopower plays a critical role in determining the figure of merit and energy conversion efficiency [23,24]. Despite the technological importance of Kane-type semiconductors, their thermal properties remain relatively understudied compared to their electronic and optical characteristics [25,26]. The heat capacity of semiconductors originates from both lattice vibrations (phonons) and electronic excitations. While the electronic contribution can become appreciable in narrow-gap materials due to thermal carrier excitation across small band gaps [18,19], the lattice contribution typically dominates the thermal properties across most temperature ranges [18, 19]. Traditional Debye and Einstein models are successful for wide-gap semiconductors and insulators. However, they often prove inadequate for compound and narrow-gap semiconductors over broad temperature ranges, as demonstrated for I–III–VI₂ chalcopyrite compounds [29] and the narrow-gap semiconductor Mg₂Pb [30]. Moreover, experimental heat capacity determination remains challenging due to material quality limitations, surface effects, and the requirement for extended temperature range measurements. Consequently, accurate analytical descriptions of the temperature-dependent heat capacity of Kane-type semiconductors that account for their unique properties remain scarce in the literature.

Several experimental and theoretical studies have addressed the temperature-dependent heat capacity of III–V narrow-gap semiconductors, providing benchmark data against which analytical models can be validated. At cryogenic temperatures, Cetas et al. [44] measured the specific heats of GaAs, GaSb, InAs, and InSb between 1 and 30 K with high precision, revealing a free-carrier contribution linear in temperature superimposed on the lattice term. Piesbergen [45] extended these low-temperature calorimetric measurements to III–V compounds including AlSb, GaAs, GaSb, InP, InAs, and InSb over the 12–273 K range, establishing reference Debye temperatures widely used in subsequent modeling. At higher temperatures, Cai and Wei [46] reported the temperature dependence of the specific heat, thermal expansion coefficient, thermal diffusivity, and thermal conductivity of InSb up to 800 K, relevant to phase-change and infrared-detector applications. Theoretically, Baloglu et al. [47] used *ab initio* density functional theory within the quasi-harmonic approximation to obtain temperature-dependent elastic and thermal properties of GaSb and InSb, complementing the empirical Debye–Grüneisen description adopted in the present work. These studies collectively show that, despite the wealth of experimental and first-principles data available at specific temperature intervals, a unified analytical formulation capable of reproducing the heat capacity of Kane-type semiconductors continuously from cryogenic temperatures up to the melting point remains lacking. The present study addresses this gap by

developing such a formulation without relying on a temperature-dependent Debye parameter or computationally expensive first-principles calculations.

The Einstein-Debye approximation provides a theoretical framework for deriving accurate analytical expressions for heat capacity over wide parameter ranges and has been successfully applied to solids with complex lattice structures [31,38]. This framework, developed in Ref. [38], incorporates characteristic temperatures for acoustic and optical phonons into the total internal energy expression. The Einstein-Debye approximation has demonstrated excellent agreement with experimental data from cryogenic to elevated temperatures for various materials, including compound semiconductors [32-35]. The approach simultaneously describes the low-temperature regime dominated by acoustic phonons and high-temperature contributions from optical phonon modes [36,37], enabling reliable thermal property predictions without extensive numerical computations or detailed phonon density of states knowledge.

The original contribution of the present study can be summarized as follows: (i) a closed-form Einstein–Debye analytical formulation is applied, for the first time, to Kane-type narrow-gap semiconductors InSb, InAs, and GaSb. The proposed formulation reproduces the heat capacity over the entire temperature range, from cryogenic temperatures to the melting point, using a single set of temperature-independent Debye and Einstein temperatures. (ii) the systematic correlation identified between the fundamental energy gap and the adopted Einstein temperature is used to link the Kane band nonparabolicity to the lattice dynamics governing the heat capacity, providing a physical justification, beyond simple curve-fitting, for classifying these materials as Kane-type semiconductors within a thermal context; and (iii) the isobaric heat capacity was calculated using the proposed analytical approach without requiring the explicit definition of a temperature-dependent Debye temperature. This approach provides a simpler, physically transparent, and computationally efficient alternative for predicting thermal properties in device modeling and thermal management applications.

In the present work, we employ the Einstein-Debye approximation to evaluate the heat capacity of Kane-type semiconductors. We focus on the lattice contribution, which dominates the thermal properties of these materials across the investigated temperature range. The electronic contribution to heat capacity, while potentially significant at very high temperatures, remains below 2-3% of the total heat capacity at room temperature for the narrow-gap semiconductors considered here (InSb, InAs, and GaSb) and is therefore neglected in this analysis. We validate our analytical formulation through comparison with available experimental data for representative Kane-type semiconductors. The results show that the Einstein-Debye approximation provides accurate predictions across different temperature regimes while remaining computationally efficient, making it a practical tool for predicting thermal properties in device modeling and thermal management applications.

2. FORMULAS FOR THE SPECIFIC HEAT CAPACITY AT CONSTANT VOLUME AND CONSTANT PRESSURE WITHIN THE EINSTEIN–DEBYE APPROACH

In Kane-type semiconductors, the nonparabolic energy–momentum dispersion relation modifies both the electronic density of states and the temperature dependence of the fundamental energy gap [1,4]. The energy gap itself exhibits well-known temperature dependence, commonly described by the Varshni relation [27]. This temperature dependence arises from two physical mechanisms: (i) the thermal expansion of the lattice, which shifts the relative positions of the conduction and valence band edges, and (ii) the renormalization of the band gap through electron–phonon interactions, which become stronger as the phonon population increases with temperature [27,28]. Both mechanisms are therefore directly linked

to the lattice dynamics that govern the heat capacity investigated in the present work. While the lattice heat capacity remains the dominant contribution across most of the temperature range investigated here, the Kane band structure influences the thermal properties of narrow-gap semiconductors through the following physically distinct pathway: the strong $k \cdot p$ coupling in Kane-type materials directly affects the optical phonon branch frequencies near the Γ -point through the electron–phonon renormalization of the interatomic force constants [28,41]. This connection is evidenced by the systematic correlation observed in the present work between the fundamental energy gap and the Einstein temperature adopted for each material: InSb ($E_g \approx 0.17$ eV, $\theta_E = 138$ K), InAs ($E_g \approx 0.36$ eV, $\theta_E = 169$ K), and GaSb ($E_g \approx 0.73$ eV, $\theta_E = 184$ K) — the three compounds show a monotonic increase in θ_E with increasing E_g . This trend is consistent with the strengthening of the interatomic restoring forces as the Kane nonparabolicity weakens with increasing gap, which in turn raises the characteristic optical phonon frequencies and hence the Einstein temperature. A similar correlation between phonon energies and band gap has been reported for III–V semiconductors in the context of electron–phonon coupling studies [41]. Accordingly, the Einstein temperature, as employed in the present Einstein–Debye formulation, implicitly encodes the influence of the Kane band structure on the lattice dynamics of these narrow-gap semiconductors.

In seminal works [38,39], an Einstein–Debye approximation was introduced that simultaneously accounts for both the optical and acoustic branches of the phonon spectrum. Under these assumptions, the total internal energy of a solid may be written as the sum of the optical and acoustic phonon energy contributions in the following form [38]:

$$E(T) = 3N_A k_B T \left[D_n \left(\frac{\theta_D}{T} \right) + (s-1) \frac{\theta_E}{T} \left(e^{\frac{\theta_E}{T}} - 1 \right)^{-1} \right]. \quad (1)$$

Based on the proposed approximation, the expressions for the isochoric and isobaric specific heat capacities are given, respectively, as follows:

$$C_V(T) = 3N_A k_B M(T, \theta_D, \theta_E), \quad (2)$$

and

$$C_P(T) = 3N_A k_B [M(T, \theta_D, \theta_E) + Q(T, \theta_D, \theta_E, \gamma_{ac}, \gamma_{op})]. \quad (3)$$

The quantities $M(T, \theta_D, \theta_E)$ and $Q(T, \theta_D, \theta_E, \gamma_{ac}, \gamma_{op})$, appearing in Equations (2) and (3), are defined, respectively, through the following relations:

$$M(T, \theta_D, \theta_E) = L_V(T, \theta_D) + (s-1)A(T, \theta_E), \quad (4)$$

$$\begin{aligned} Q(T, \theta_D, \theta_E, \gamma_{ac}, \gamma_{op}) = & [\gamma_{ac} L_V(T, \theta_D) + (s-1)\gamma_{op} \\ & \times A(T, \theta_E)]^2 \left\{ \gamma_{ac} \left[\frac{3\theta_D}{8T} (\gamma_{ac} + 1) + (1 - 3\gamma_{ac}) D_n \left(1, \frac{\theta_D}{T} \right) \right. \right. \\ & \left. \left. + 3\gamma_{ac} \frac{\theta_D}{T} (e^{\theta_D/T} - 1)^{-1} \right] + (s-1)\gamma_{op} \left[\frac{\theta_E}{2T} (\gamma_{op} + 1) \right. \right. \\ & \left. \left. + \frac{\theta_E}{T} (e^{\theta_E/T} - 1)^{-1} \left(1 + \gamma_{op} + \gamma_{op} \frac{\theta_E}{T} e^{\theta_E/T} (e^{\theta_E/T} - 1)^{-1} \right) \right] \right\}^{-2}. \end{aligned} \quad (5)$$

In the Equations (1)–(3), T is the absolute temperature, N_A is the number of Bravais lattice points in one mole, k_B is the Boltzmann constant, θ_D is the Debye temperature, θ_E is the Einstein temperature, s is the number of atoms in the base of Bravais lattice points, γ_{ac} is the acoustic Grüneisen parameter, γ_{op} is the optic Grüneisen parameter. The quantities

$L_V(T, \theta_D)$ and $A(T, \theta_E)$, appearing in Equations (4) and (5), are defined, respectively, through the following relations:

$$A(T, \theta_E) = \left(\frac{\theta_E}{2T}\right)^2 \left[\sinh\left(\frac{\theta_E}{2T}\right) \right]^{-2}, \quad (6)$$

$$L_V(T, \theta_D) = (n+1)D_n\left(1, \frac{\theta_D}{T}\right) - \frac{\theta_D}{T} \frac{n}{e^{\frac{\theta_D}{T}} - 1} \quad (7)$$

Here, the parameter n typically takes values between 3 and 5 for metals and compounds. An effective alternative approach for the analytical evaluation of the isobaric specific heat capacity is provided by the Nernst–Lindemann approximation, which expresses the isobaric heat capacity in terms of the isochoric heat capacity and is applicable over a wide temperature range, as given by the following relation [40]:

$$C_P(T) = C_V(T) \left(1 + \frac{A_0 T}{T_m} C_V(T)\right), \quad (8)$$

Here, $A_0 = 5.1 \times 10^{-3} J^{-1} K mol$ and T_m denotes the melting temperature. As can be seen from Equations (1) – (8), the expressions for the heat capacities reduce to calculations involving n -dimensional Debye functions, which provide more accurate and consistent results and are defined as follows:

$$D_n(\beta, x) = \frac{n}{x^n} \int_0^x \frac{t^n}{(e^t - 1)^\beta} dt, \quad (9)$$

Here, n may assume both integer and non-integer values, and within the Einstein–Debye approximation the parameter β is fixed at $\beta = 1$. All calculations in the present work were performed for $n = 3$. The Debye function plays a fundamental role in describing the thermal properties of solids, particularly in the low-temperature regime. By accounting for the continuous spectrum of phonon excitations, it provides a more realistic description of the temperature dependence of specific heat compared to classical models. The Debye function enables systematic treatment of acoustic phonon contributions and establishes a direct connection between thermal behavior and characteristic parameters such as the Debye temperature. As a result, it serves as a reliable theoretical tool for modeling heat capacity in semiconductors and complex crystalline materials.

3. NUMERICAL RESULTS AND DISCUSSION

III–V compound semiconductors are widely used in high-performance optoelectronic applications because of their favorable intrinsic properties, including direct band gaps, high electron mobility, and comparatively low exciton binding energies. Among these materials, InSb, InAs, and GaSb are particularly important for infrared optoelectronic devices, including IR sensors, where they are frequently subjected to significant temperature gradients during both fabrication processes and operational conditions.

In this study, a comprehensive and systematic evaluation of the temperature-dependent heat capacities of InSb, InAs, and GaSb has been carried out within the framework of the Einstein–Debye approximation. Since the Einstein–Debye approach explicitly accounts for the contributions of both acoustic and optical phonons to the heat capacity, it enables a more accurate and physically consistent evaluation of thermodynamic properties. In recent years, this combined framework has gained increasing importance, as it provides an improved description of lattice dynamics over a wide temperature range, thereby enhancing the

reliability of thermodynamic calculations in advanced semiconductor materials. Throughout this work, all calculations are carried out with the acoustic and optic Grüneisen parameters taken as $\gamma_{ac} = \gamma_{op} = 1$.

Figs. 1-6 present the temperature-dependent C_P and C_V heat capacities of InSb, InAs, and GaSb over a wide temperature range. The calculated numerical results are systematically compared with the available experimental and theoretical data reported in Refs. [41,42].

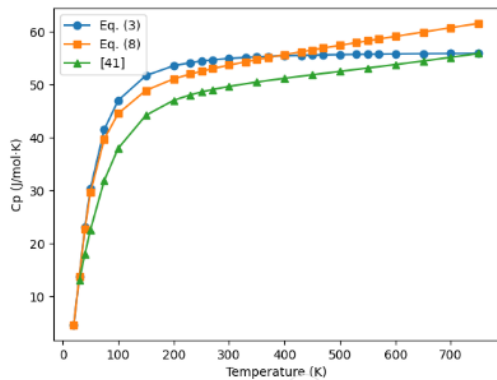


Figure 1. Variation C_P ($\frac{J}{mol.K}$) heat capacity of InSb as a function of temperature ($\theta_D = 160K$, $\theta_E = 138K$, $T_m=800K$).

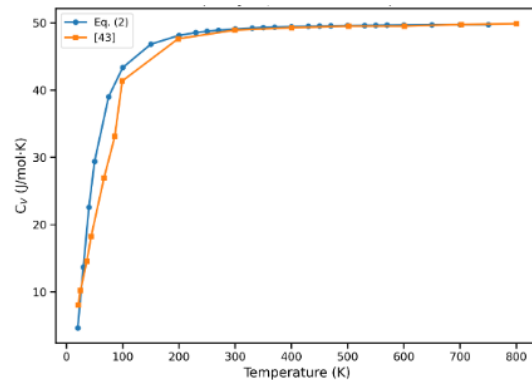


Figure 2. Variation C_V heat capacity of InSb as a function of temperature ($\theta_D = 160K$, $\theta_E = 138K$, $T_m=800K$).

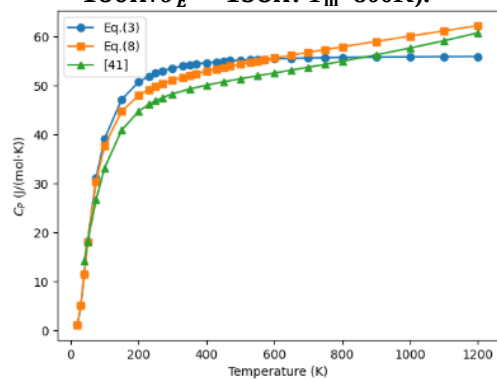


Figure 3. Variation C_P ($\frac{J}{mol.K}$) heat capacity of InAs as a function of temperature ($\theta_D = 280K$, $\theta_E = 169K$, $T_m=1215K$).

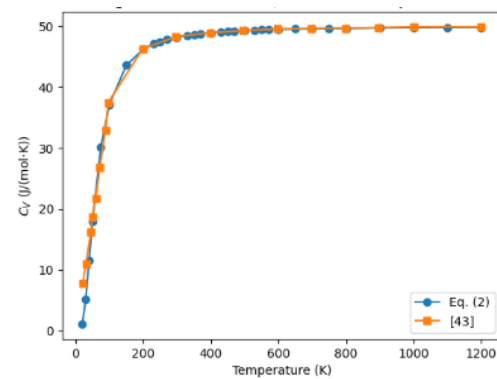


Figure 4. Variation C_V ($\frac{J}{mol.K}$) heat capacity of InAs as a function of temperature ($\theta_D = 280K$, $\theta_E = 169K$, $T_m=1215K$).

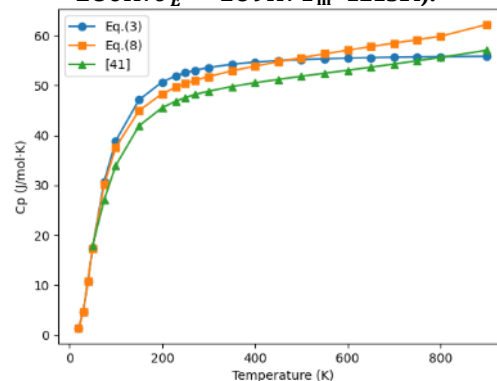


Figure 5. Variation C_P ($\frac{J}{mol.K}$) heat capacity of GaSb as a function of temperature ($\theta_D = 266K$, $\theta_E = 184K$, $T_m=985K$).

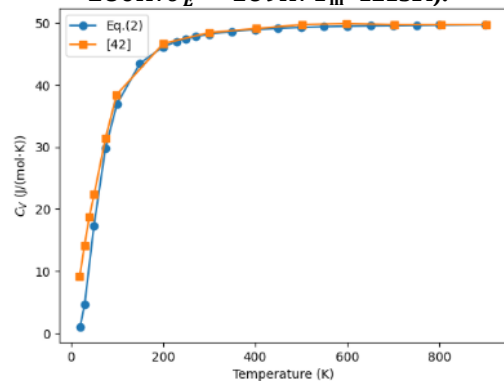


Figure 6. Variation C_V ($\frac{J}{mol.K}$) heat capacity of GaSb as a function of temperature ($\theta_D = 266K$, $\theta_E = 184K$, $T_m=985K$).

In the study of Pässler [41], the heat capacity calculations were performed by introducing a temperature-dependent Debye temperature, $\theta_D(T)$, to reproduce experimental data more accurately. The present results further clarify the role of the Debye temperature in

modeling lattice heat capacity. Such a treatment effectively compensates for deviations from the original harmonic Debye approximation. In contrast, in the present work, the C_p values of InSb, InAs, and GaSb, presented in Figs. 1, 3, and 5, were calculated within the Einstein–Debye framework using an average constant Debye and Einstein’s temperatures. The comparison with Pässler’s results shows that the Einstein–Debye approach provides comparable and physically consistent predictions over the entire temperature range up to the melting temperature. These findings suggest that, although the introduction of $\theta_D(T)$ may improve numerical agreement in specific temperature intervals, the Einstein–Debye approach offers a more general and physically transparent description without requiring an explicitly temperature-dependent Debye parameter. Fig. 2, 4, and 6 illustrate the temperature dependence of the C_V heat capacities of InSb, InAs, and GaSb, respectively. The obtained results were compared with the data reported in [42,43], where the thermodynamic properties were calculated using the density functional approach. The comparison indicates that the C_V values calculated within the Einstein–Debye framework are in good agreement with the corresponding literature results over the considered temperature range. This consistency further supports the reliability of the present model and demonstrates that the Einstein–Debye approach provides physically meaningful predictions comparable to those obtained from more computationally intensive first-principles methods. Consequently, the model performs reliably across a broad temperature range while preserving conceptual simplicity.

As shown in the figures, the proposed Einstein–Debye–based approach agrees well with literature data across the entire investigated temperature range. Notably, the consistency of the results demonstrates that the present method remains valid and reliable from low temperatures up to temperatures approaching the melting points of each semiconductor material. The results substantiate that the proposed model yields highly accurate and physically meaningful heat-capacity predictions over the entire operational and processing temperature regimes of InSb, InAs, and GaSb, thereby establishing its efficacy for thermal analysis and device-related applications of these III–V compound semiconductors.

The present analytical framework is therefore physically consistent with the Kane model: although the heat capacity calculation itself is based on a lattice-dynamical description, the material-specific characteristic temperatures (θ_D, θ_E) that enter the Einstein–Debye expressions are themselves a consequence of the underlying Kane band structure. This connection justifies the designation of InSb, InAs, and GaSb as Kane-type semiconductors in the context of thermal property analysis and validates the use of the Einstein–Debye framework as an effective, physically transparent tool for the thermal characterization of narrow-gap materials.

4. CONCLUSION

In conclusion, the present results show very good agreement with experimental values, despite the well-known challenges of precise heat-capacity measurements across wide temperature intervals. The obtained data are expected to serve as a useful reference for evaluating the thermal behavior of III–V semiconductors under varying temperature conditions. In particular, the quantified temperature-induced variations in heat capacities reported in this work provide valuable insights for the thermal design, performance optimization, and reliability assessment of optoelectronic devices operating under non-isothermal environments.

REFERENCES

- [1] Kane, E.O., Band structure of indium antimonide, *Journal of Physics and Chemistry of Solids*, **1**(4), 249, 1957. [https://doi.org/10.1016/0022-3697\(57\)90013-6](https://doi.org/10.1016/0022-3697(57)90013-6)
- [2] Gritsyuk, P.M., Shtivel'man, K.Y., Lipnik, A.A., Generalized Kane model of the band spectrum of diamond-like semiconductors. Secular determinant and dispersion equations, *Soviet Physics Journal*, **34**(1), 22, 1991. <https://doi.org/10.1007/BF00914116>
- [3] Gritsyuk, P.M., Shtivel'man, K.Y., Chopik, A.V., Generalized Kane model for the band spectrum of diamond-like semiconductors. Nonparabolicity and anisotropy of the dispersion relations, *Russian Physics Journal*, **35**(12), 1087, 1992. <https://doi.org/10.1007/BF00559109>
- [4] Whalley, L.A., Frost, J.M., Morgan, B.J., Walsh, A., Impact of nonparabolic electronic band structure on the optical and transport properties of photovoltaic materials, *Physical Review B*, **99**(8), 085207, 2019. <https://doi.org/10.1103/PhysRevB.99.085207>
- [5] Mascali, G., Romano, V., A non parabolic hydrodynamical subband model for semiconductors based on the maximum entropy principle, *Mathematical and Computer Modelling*, **55**(3-4), 1003, 2012. <https://doi.org/10.1016/j.mcm.2011.09.026>
- [6] Hashimzade, F.M., Babayev, A.M., Artunc, E., Cakmak, S., The exact solutions of Kane equations which have position-dependence band gap in an external non-uniform electric field, *Physica E: Low-dimensional Systems and Nanostructures*, **28**(4), 355, 2005. <https://doi.org/10.1016/j.physe.2005.04.009>
- [7] Kono, J., Leotin, J. (Eds.), *Narrow Gap Semiconductors: Proceedings of the 12th International Conference on Narrow Gap Semiconductors*, CRC Press, Boca Raton, 2006. <https://doi.org/10.1201/9781482269215>
- [8] Chu, J., Sher, A., *Physics and Properties of Narrow Gap Semiconductors*, Springer, New York, 2008. <https://doi.org/10.1007/978-0-387-74801-6>
- [9] Smith, R.A., *Semiconductors*, 2nd ed., Cambridge University Press, London, 1978.
- [10] Vurgaftman, I., Meyer, J.R., Ram-Mohan, L.R., Band parameters for III–V compound semiconductors and their alloys, *Journal of Applied Physics*, **89**(11), 5815, 2001. <https://doi.org/10.1063/1.1368156>
- [11] Ashley, T., Elliott, C.T., Operation and properties of narrow-gap semiconductor devices near room temperature using non-equilibrium techniques, *Semiconductor Science and Technology*, **6**(12C), C99, 1991. <https://doi.org/10.1088/0268-1242/6/12C/020>
- [12] Capper, P. (Ed.), *Properties of Narrow Gap Cadmium-based Compounds*, INSPEC/IEE, London, 1994.
- [13] Yeh, N.T., Chiu, P.C., Chyi, J.I., Ren, F., Pearton, S.J., Sb-based semiconductors for low power electronics, *J. Mater. Chem. C*, **1**(31), 4616, 2013. <https://doi.org/10.1039/c3tc30585f>
- [14] Jia, B.W., Tan, K.H., Loke, W.K., Wicaksono, S., Yoon, S.F., Monolithic integration of InSb photodetector on silicon for mid-infrared silicon photonics, *ACS Photonics*, **5**(4), 1512, 2018. <https://doi.org/10.1021/acsphotonics.7b01546>
- [15] Delli, E., Letka, V., Hodgson, P.D., Repiso, E., Hayton, J.P., Craig, A.P., Lu, Q., Beanland, R., Krier, A., Marshall, A.R.J., Carrington, P.J., Mid-infrared InAs/InAsSb superlattice nBn photodetector monolithically integrated onto silicon, *ACS Photonics*, **6**(2), 538, 2019. <https://doi.org/10.1021/acsphotonics.8b01550>
- [16] Koroleva, O.N., Mazhukin, A.V., Determination of thermal conductivity and heat capacity of silicon electron gas, *Mathematica Montisnigri*, **40**, 99, 2017.

- [17] Koroleva, O.N., Mazhukin, A.V., Mazhukin, V.I., Modeling of silicon characteristics in the semiconductor-metal phase transition region, *Mathematica Montisnigri*, **41**, 73, 2018.
- [18] Kittel, C., *Introduction to Solid State Physics*, 8th ed., Wiley, New York, 2005.
- [19] Ashcroft, N.W., Mermin, N.D., *Solid State Physics*, Holt-Saunders, Philadelphia, 1976.
- [20] Wagner, M., *Thermal Conductivity in Semiconductors*, Institute for Microelectronics, TU Wien, Vienna, 2007.
- [21] Rogalski, A., Adamiec, K., Rutkowski, J., *Narrow-Gap Semiconductor Photodiodes*, SPIE Press, Bellingham, 2000.
- [22] Tennant, W.E., Rule 07 Revisited: Still a good heuristic predictor of p/n HgCdTe photodiode performance?, *Journal of Electronic Materials*, **39**(7), 1030, 2010. <https://doi.org/10.1007/s11664-010-1084-9>
- [23] Tomczak, J.M., Haule, K., Kotliar, G., Thermopower of the Correlated Narrow Gap Semiconductor FeSi and Comparison to RuSi. In: *NATO Science for Peace and Security Series B: Physics and Biophysics*, Springer, Dordrecht, pp. 43-58, 2013. https://doi.org/10.1007/978-94-007-4984-9_4
- [24] Cao, J., Querales-Flores, J.D., Fahy, S., Savić, I., Thermally induced band gap increase and high thermoelectric figure of merit of n-type PbTe, *Materials Today Physics*, **12**, 100172, 2020. <https://doi.org/10.1016/j.mtphys.2019.100172>
- [25] Bouarissa, N., Aourag, H., The temperature dependence of the band gaps in narrow-gap semiconductors, *Infrared Physics and Technology*, **38**(3), 153, 1997. [https://doi.org/10.1016/S1350-4495\(97\)00007-8](https://doi.org/10.1016/S1350-4495(97)00007-8)
- [26] Seeger, K., *Semiconductor Physics: An Introduction*, 9th ed., Springer, Berlin, 2004.
- [27] Varshni, Y. P., Temperature dependence of the energy gap in semiconductors, *Physica*, **34**(1), 149, 1967. [https://doi.org/10.1016/0031-8914\(67\)90062-6](https://doi.org/10.1016/0031-8914(67)90062-6)
- [28] Yu, P.Y., Cardona, M., *Fundamentals of Semiconductors: Physics and Materials Properties*, 4th ed., Springer, Berlin, 2010. <https://doi.org/10.1007/978-3-642-00710-1>
- [29] Bachmann, K. J., Hsu, F. S. L., Thiel, F. A., Kasper, H. M., Debye temperature and standard entropies and enthalpies of compound semiconductors of the type I-III-VI₂, *Journal of Electronic Materials*, **6**(4), 431, 1977. <https://doi.org/10.1007/BF02660497>
- [30] Schwartz, R.G., Shanks, H., Gerstein, B.C., Thermal study of II–IV semiconductors: Heat capacity and thermodynamic functions of Mg₂Pb from 5–300°K, *Journal of Solid State Chemistry*, **3**(4), 533, 1971. [https://doi.org/10.1016/0022-4596\(71\)90099-5](https://doi.org/10.1016/0022-4596(71)90099-5)
- [31] Askerov, B.M., Cankurtaran, M., Isobaric specific heat and thermal expansion of solids in the Debye approximation, *Physica Status Solidi (B)*, **185**(2), 341. <https://doi.org/10.1002/pssb.2221850204>
- [32] Doğan, Z., Mehmetoğlu, T., Accurate Calculations of the Heat Capacities of Pure Metals Using the Einstein–Debye Approximation, *Journal of Engineering Physics and Thermophysics*, **92**(6), 1620, 2019. <https://doi.org/10.1007/s10891-019-02082-7>
- [33] Mehmetoglu, T., Use of Einstein-Debye method in the analytical and semi-empirical analysis of isobaric heat capacity and thermal conductivity of nuclear materials, *Journal of Nuclear Materials*, **527**, 151827, 2019. <https://doi.org/10.1016/j.jnucmat.2019.151827>
- [34] Dogan, Z., Mehmetoglu, T., A New Unified Analytical Expression for Entropy and its Application to Power Electronic Semiconductors, *Journal of Electronic Materials*, **54**(3), 2334, 2025. <https://doi.org/10.1007/s11664-024-11655-z>
- [35] Dogan, Z., Mehmetoglu, T., Analytical investigation of thermodynamic properties of power electronic semiconductor materials, *Journal of Computational Electronics*, **23**(3), 507, 2024. <https://doi.org/10.1007/s10825-024-02167-4>

- [36] Hermann, R.P., Grandjean, F., Long, G.J., Einstein oscillators that impede thermal transport, *American Journal of Physics*, **73**(2), 110, 2005. <https://doi.org/10.1119/1.1783899>
- [37] Cope, E.J., Bustamante, J., Johnson, Z.M., Lancaster, A., Gurunathan, R., George, J., Agne, M.T., Heat capacity estimation of complex materials for energy technologies, *Joule*, **9**(8), 102054, 2025. <https://doi.org/10.1016/j.joule.2025.102054>
- [38] Cankurtaran, M., Askerov, B.M., Equation of state, isobaric specific heat, and thermal expansion of solids with polyatomic basis in the Einstein-Debye approximation, *Phys. Status Solidi B*, **194**(2), 499, 1996. <https://doi.org/10.1002/pssb.2221940207>
- [39] Askerov, B. M., Figarova, S. R., *Thermodynamics Gibbs Method and Statistical Physics of Electron Gases*, Springer, Heidelberg, 2009.
- [40] Nernst, W., Lindemann, F.A., Specific heat and quantum theory, *Zeitschrift für Elektrochemie und angewandte physikalische Chemie*, **17**(20), 817, 1911.
- [41] Pässler, R., Non-Debye heat capacity formula refined and applied to GaP, GaAs, GaSb, InP, InAs, and InSb, *AIP Advances*, **3**(8), 082108, 2013. <https://doi.org/10.1063/1.4818273>
- [42] Ahmad, B.M., Abulla, H.T., Rammoo, M.N.S., Theoretical calculations of the properties of the binary compound semiconductor GaSb, *Semiconductor Physics, Quantum Electronics and Optoelectronics*, **27**(4), 427, 2024. <https://doi.org/10.15407/spqeo27.04.427>
- [43] Hachelafi, K., Amrani, B., Hassan, F.E.H., Hiadsi, S., Theoretical study of InAs, InSb and their alloys InAs_xSb_{1-x}, *Advances in Condensed Matter Physics*, **2009**, 376610, 2009.
- [44] Cetas, T.C., Tilford, C.R., Swenson, C.A., Specific heats of Cu, GaAs, GaSb, InAs, and InSb from 1 to 30 K, *Physical Review Journals Archive*, **174**, 835, 1968. <https://doi.org/10.1103/PhysRev.174.835>
- [45] Piesbergen, U., Die durchschnittlichen Atomwärmen der A(III)B(V)-Halbleiter AlSb, GaAs, GaSb, InP, InAs, InSb und die Atomwärmen des Elements Germanium zwischen 12 und 273 K, *Zeitschrift für Naturforschung A*, **18**(2), 141, 1963.
- [46] Cai, X., Wei, J., Temperature dependence of the thermal properties of InSb materials used in data storage, *Journal of Applied Physics*, **114**(8), 083507, 2013. <https://doi.org/10.1063/1.4819224>
- [47] Baloğlu, E. C., Ustunel, H., Dal, H., Temperature-dependent thermoelastic properties of GaSb and InSb semiconductors: Identification through ab initio DFT simulations, *Physica B: Condensed Matter*, **643**, 414135, 2022. <https://doi.org/10.1016/j.physb.2022.414135>