

ORIGINAL PAPER

STUDY OF MATHEMATICAL MODEL RELATED TO VISCOUS VELOCITIES IN INCOMPRESSIBLE FLUIDS

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Abstract. *In the present model, it is assumed that the modified form of radial velocity as proposed by the Baker-Sterling model introduces a central low-pressure zone into the radial distribution and an exponential function into the axial distribution of the radial velocity. Sinclair [24] reported that a central low-pressure zone is a vital characteristic for the formation and persistence of atmospheric vortices such as dust devils and tornadoes. Furthermore, the inclusion of the exponential function in this innovative model results in a smooth and realistic velocity transition. The fluid motion is considered steady, incompressible, viscous, and axisymmetric to suitably represent atmospheric vortices. The radial, azimuthal, and axial velocities and pressure are formulated. All velocity components are plotted and discussed for various values of the central low-pressure zone and heights. Consequently, the axial velocity reaches its maximum away from the periphery of the low-pressure zone due to its presence. Additionally, as a result of the modifications to the radial velocity, the azimuthal velocity in the current model disappears at a finite radial distance rather than at an asymptotic distance, which aligns more closely with real vortex observations. Moreover, the radial pressure is also plotted and discussed. It is to be noted that two parameters, pressure and azimuthal velocity, are displaying viscous effects.*

Keywords: Atmospheric vortex; error function; low-pressure zone; viscous flow; azimuthal velocity.

Mathematics Subject Classification: MSC2020.

1. INTRODUCTION

Dust devils are small, localised vortex structures frequently observed in arid and semi-arid regions where dry and heated ground generates rising air currents. These vortices are formed when hot air near the surface quickly ascends through cooler, denser air above, creating instability. As the air rises, it rotates to form a swirling column that can lift dust and debris, rendering the vortex visible. Notably, dust devils have also been observed on Mars, where the reduced gravity and atmospheric pressure have resulted in significantly larger vortices.

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Studying dust devils is essential to understand their formation, dynamics, and effects. Numerous analytical, numerical, and empirical models of air vortices exist. Rankine [1] provided a simplified model assuming solid body rotation in the core, accompanied by a potential vortex outside the core, based on a specific form of azimuthal velocity. However, the absence of radial and axial velocity components renders this model impractical. Similar to the Rankine model [1], many other models lack either radial or axial velocity components (Taylor [2], Lamb [3], etc.). Furthermore, various models of air vortices involve several idealizations. In the models proposed by Rott [4] and Long [5], the radial velocity is dependent solely on radial coordinates. Kieu and Zhang [6] have developed a vortex model employing a particular viscosity formulation. Several steady-state vortex models have been established by researchers including Rankine [1], Taylor [2], Lamb [3], Rott [4], Long [5], Kieu and Zhang [6], and Burgers [7] etc., which serve to validate the observed experimental data and numerical simulation. However, Makarieva et al. [8] have constructed more detailed models.

Vyas et al. [9] have provided a viscous solution to the Navier-Stokes equations. Furthermore, Vyas and Majdalani [10] presented an inviscid solution to the Navier-Stokes equations. Both models have employed a cylindrical coordinate system. Xu and Hangan [11] introduced an inviscid flow vortex model based on a modified Rankine model [1]. An exact solution of the reduced Navier-Stokes equations was offered by the Burgers [7] and Rott [4] models. In harmony with the Burgers-Rott model, the Sullivan model [12] is also an exact solution of the reduced Navier-Stokes equations. However, the main drawback of both the Burgers and Sullivan models is that axial velocity and pressure increase without bound as height increases. Moreover, additional models proposed by Wood and Brown [13], Kim and Matsui [14], Gillmeier et al. [15], etc. have exhibited unbounded velocities.

To comprehend the actual structures of the atmospheric vortex flow field, a fundamental mathematical model is required that minimises the error in the observed data and facilitates the explanation of the velocity and pressure fields. Researchers such as Bluestein et al. [16], Wurman et al. [17], Lewellen et al. [18], Bluestein et al. [19], Bluestein et al. [20], Davies-Jones and Wood [21], etc., used tornado data in their research, which is obtained from a safe distance using Doppler radar; however, the data gathered through this method is predominantly limited to tornado genesis.

Baker and Sterling [22] have proposed a vortex model that reflects the pressure and flow characteristics of single-celled and two-celled vortices. The present study analyzes only the solution for the single-cell vortex with radial inflow and axial updraft to simplify comparisons with previously discussed models. According to Ying and Chang [23], a tornado is a colossal vortex column featuring a central low-pressure zone. By investigating the profiles of temperature, pressure, and wind velocity inside dust devils, Sinclair [24] confirmed the existence of a warm, low-pressure core.

In the current model, we consider a modified form of radial velocity presented by the Baker and Sterling model [1], whereby a central low-pressure zone in the radial distribution has been introduced. Additionally, the radial velocity as outlined by the Baker and Sterling model [1] also needs a modification in the axial distribution through the introduction of an exponential function, as taken from the work of Zhang et al. [25].

In Section 2 [REDUCED EQUATIONS], the reduced Navier-Stokes equations and the continuity equation for steady, viscous, incompressible, and axisymmetric flow are presented. In Section 3 [VORTEX MODEL], a novel analytical vortex model is developed to describe viscous fluid motion with a central low-pressure zone. The primary results are examined in Section 4 [RESULTS], and conclusions are drawn in the last section of this research.

2. REDUCED EQUATIONS

Let us consider the steady, viscous, incompressible, and axisymmetric flow to model the dust devil phenomenon. This reduces the Navier-Stokes equations in the three-dimensional cylindrical coordinate system (r', ϕ', z') , where r' , ϕ' and z' represent the radial, azimuthal, and axial directions, respectively, as

$$u' \frac{\partial u'}{\partial r'} + w' \frac{\partial u'}{\partial z'} - \frac{v'^2}{r'} = -\frac{1}{\rho} \frac{\partial p'}{\partial r'} + \nu \left(\frac{\partial^2 u'}{\partial r'^2} + \frac{1}{r'} \frac{\partial u'}{\partial r'} + \frac{\partial^2 u'}{\partial z'^2} - \frac{u'}{r'^2} \right), \quad (1)$$

$$u' \frac{\partial v'}{\partial r'} + w' \frac{\partial v'}{\partial z'} + \frac{u'v'}{r'} = \nu \left(\frac{\partial^2 v'}{\partial r'^2} + \frac{1}{r'} \frac{\partial v'}{\partial r'} + \frac{\partial^2 v'}{\partial z'^2} - \frac{v'}{r'^2} \right) \quad (2)$$

$$u' \frac{\partial w'}{\partial r'} + w' \frac{\partial w'}{\partial z'} = -\frac{1}{\rho} \frac{\partial p'}{\partial z'} + \nu \left(\frac{\partial^2 w'}{\partial r'^2} + \frac{1}{r'} \frac{\partial w'}{\partial r'} + \frac{\partial^2 w'}{\partial z'^2} \right) \quad (3)$$

where u' , v' and w' represent the radial, azimuthal, and axial velocities, respectively, p' denotes the pressure, ν is the kinematic viscosity, and ρ stands for the vortex air density.

The equation of continuity for an incompressible fluid is expressed as

$$\frac{\partial u'}{\partial r'} + \frac{u'}{r'} + \frac{\partial w'}{\partial z'} = 0. \quad (4)$$

To maintain dynamical similarity, the Equations (1)-(4) are non-dimensionalise using the dimensionless variables as follows

$$r = \frac{r'}{r_m}, \quad z = \frac{z'}{z_m}, \quad u = \frac{u'}{u_m}, \quad v = \frac{v'}{u_m}, \quad w = \frac{w'}{u_m}, \quad p = \frac{p'}{\rho u_m^2}, \quad (5)$$

where u_m represents the maximum radial velocity, r_m and z_m denote the radius and height for maximum radial velocity, respectively.

With the help of Equation (5), Equations (1)-(4) are transformed into the following equations as

$$u \frac{\partial u}{\partial r} + kw \frac{\partial u}{\partial z} - \frac{v^2}{r} = -\frac{\partial p}{\partial r} + \frac{1}{Re} \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + k^2 \frac{\partial^2 u}{\partial z^2} - \frac{u}{r^2} \right), \quad (6)$$

$$u \frac{\partial v}{\partial r} + kw \frac{\partial v}{\partial z} + \frac{uv}{r} = \frac{1}{Re} \left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} + k^2 \frac{\partial^2 v}{\partial z^2} - \frac{v}{r^2} \right), \quad (7)$$

$$u \frac{\partial w}{\partial r} + kw \frac{\partial w}{\partial z} = -k \frac{\partial p}{\partial z} + \frac{1}{Re} \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + k^2 \frac{\partial^2 w}{\partial z^2} \right), \quad (8)$$

$$\frac{\partial u}{\partial r} + \frac{u}{r} + k \frac{\partial w}{\partial z} = 0, \quad (9)$$

where $k = \frac{r_m}{z_m}$ represents the ratio of radial to axial length scales and $Re = \frac{u_m r_m}{\nu}$ signifies the Reynolds number. Since the height of a dust devil is greater than its radius, k should lie between 0 and 1.

3. VORTEX MODEL

In this study, the axial distribution of the radial velocity in the Baker and Sterling model [22] is modified by introducing an exponential function while considering the boundary condition $u = 0$ at $z = 0$. The motivation for introducing this exponential function is drawn from the work of Zhang et al. [25]. By modifying the axial distribution with an exponential function, the Baker and Sterling model [1] satisfies the boundary conditions more naturally and provides a smooth and realistic transition in velocity profiles. Furthermore, the central low-pressure zone is a key characteristic for forming dust devils, as indicated by Sinclair [24], Maddox et al. [26], and others. Consequently, we consider a central low-pressure cylindrical zone of radius R_c . Thus, the radial distribution of the radial velocity is also modified by introducing a parameter $a_0 = R_c/r_m$ (the radius of the central low-pressure zone in a non-dimensionalized system) into the Baker and Sterling model [22]. These modifications enhance the model's accuracy in representing the structure of the vortex.

Thus, the modified radial velocity is expressed as

$$u(r, z) = \frac{-2(r - a_0)z \exp\left(\frac{1 - z^2}{2}\right)}{\{1 + (r - a_0)^2\}}. \quad (10)$$

By solving Equation (9) with the help of Equation (10), we obtain the axial velocity as

$$w(r, z) = \frac{-2[a_0\{1 - (r - a_0)^2\} + 2(r - a_0)] \exp\left(\frac{1 - z^2}{2}\right)}{kr\{1 + (r - a_0)^2\}^2} + f(r), \quad (11)$$

where $f(r)$ represents an arbitrary function of r . This arbitrary function is determined by applying the boundary condition $w = 0$ at $z = 0$ to Equation (11). Upon solving, we find

$$f(r) = \frac{2[a_0\{1 - (r - a_0)^2\} + 2(r - a_0)] \exp\left(\frac{1}{2}\right)}{kr\{1 + (r - a_0)^2\}^2}. \quad (12)$$

Then, using Equation (11), the axial velocity can be formulated as

$$w(r, z) = \frac{2\sqrt{e}[a_0\{1 - (r - a_0)^2\} + 2(r - a_0)] \left\{1 - \exp\left(-\frac{z^2}{2}\right)\right\}}{kr\{1 + (r - a_0)^2\}^2}, \quad (13)$$

where $e \approx 2.7$ is a constant.

By substituting $u(r, z)$ from Equation (10) and $w(r, z)$ from Equation (13) into Equation (6), we arrive at

$$\begin{aligned}
\frac{v^2}{r} &= \frac{4e(r-a_0)\{1-(r-a_0)^2\}z^2\exp(-z^2)}{\{1+(r-a_0)^2\}^3} \\
&- \frac{4e(r-a_0)[a_0\{1-(r-a_0)^2\}+2(r-a_0)](1-z^2)\left\{\exp\left(-\frac{z^2}{2}\right)-\exp(-z^2)\right\}}{r\{1+(r-a_0)^2\}^3} + \frac{\partial p}{\partial r} \\
&- \frac{1}{Re} \left[\frac{4\sqrt{e}(r-a_0)\{3-(r-a_0)^2\}z\exp\left(-\frac{z^2}{2}\right)}{\{1+(r-a_0)^2\}^3} - \frac{2\sqrt{e}\{1-(r-a_0)^2\}z\exp\left(-\frac{z^2}{2}\right)}{r\{1+(r-a_0)^2\}^2} \right. \\
&\quad \left. + \frac{2k^2\sqrt{e}(r-a_0)z(3-z^2)\exp\left(-\frac{z^2}{2}\right)}{\{1+(r-a_0)^2\}} + \frac{2\sqrt{e}(r-a_0)z\exp\left(-\frac{z^2}{2}\right)}{r^2\{1+(r-a_0)^2\}} \right] \quad (14)
\end{aligned}$$

Substituting $u(r, z)$ from Equation (10) and $w(r, z)$ from Equation (13) into Equation (8), we obtain

$$\begin{aligned}
k \frac{\partial p}{\partial z} &= \frac{4e}{k} f_1(r) z \left\{ \exp\left(-\frac{z^2}{2}\right) - \exp(-z^2) \right\} - \frac{4e}{k} f_2(r) z \left\{ \exp\left(-\frac{z^2}{2}\right) - \exp(-z^2) \right\} \\
&\quad + \frac{2\sqrt{e}}{k Re} \left[f_3(r) \left\{ 1 - \exp\left(-\frac{z^2}{2}\right) \right\} + f_4(r) \left\{ 1 - \exp\left(-\frac{z^2}{2}\right) \right\} \right. \\
&\quad \left. + k^2 f_5(r) (1 - z^2) \exp\left(-\frac{z^2}{2}\right) \right] \quad (15)
\end{aligned}$$

where the functions $f_1(r)$, $f_2(r)$, $f_3(r)$, $f_4(r)$, and $f_5(r)$ are defined in APPENDIX A.

Integrating Equation (15), with respect to z , we derive

$$\begin{aligned}
kp(r, z) &= \frac{2e}{k} f_1(r) \left\{ 1 - \exp\left(-\frac{z^2}{2}\right) \right\}^2 - \frac{2e}{k} f_2(r) \left\{ 1 - \exp\left(-\frac{z^2}{2}\right) \right\}^2 \\
&\quad + \frac{2\sqrt{e}}{k Re} \left[f_3(r) \left\{ z - \sqrt{\frac{\pi}{2}} \operatorname{erf}\left(\frac{z}{\sqrt{2}}\right) \right\} + f_4(r) \left\{ z - \sqrt{\frac{\pi}{2}} \operatorname{erf}\left(\frac{z}{\sqrt{2}}\right) \right\} \right. \\
&\quad \left. + k^2 f_5(r) z \exp\left(-\frac{z^2}{2}\right) \right] + g(r), \quad (16)
\end{aligned}$$

where $g(r)$ is any arbitrary function of r and the error function is defined as

$$\operatorname{erf}(s) = \frac{2}{\sqrt{\pi}} \int_0^s \exp(-\theta^2) d\theta.$$

Partially differentiating Equation (16), with respect to r , we obtain

$$\begin{aligned}
k \frac{\partial p}{\partial r} &= \frac{2e}{k} g_1(r) \left\{ 1 - \exp\left(-\frac{z^2}{2}\right) \right\}^2 - \frac{2e}{k} g_2(r) \left\{ 1 - \exp\left(-\frac{z^2}{2}\right) \right\}^2 \\
&\quad + \frac{2\sqrt{e}}{k Re} \left[g_3(r) \left\{ z - \sqrt{\frac{\pi}{2}} \operatorname{erf}\left(\frac{z}{\sqrt{2}}\right) \right\} + g_4(r) \left\{ z - \sqrt{\frac{\pi}{2}} \operatorname{erf}\left(\frac{z}{\sqrt{2}}\right) \right\} \right] \quad (17)
\end{aligned}$$

$$+k^2g(r)z\exp\left(-\frac{z^2}{2}\right)\left] + \frac{dg(r)}{dr},\right.$$

where the functions $g_1(r)$, $g_2(r)$, $g_3(r)$, $g_4(r)$, and $g_5(r)$ are defined in APPENDIX B.

Substituting $\frac{\partial p}{\partial r}$ from Equation (17) into Equation (14), we obtain

$$\begin{aligned} \frac{v^2}{r} = & \frac{4e(r-a_0)\{1-(r-a_0)^2\}z^2\exp(-z^2)}{\{1+(r-a_0)^2\}^3} \\ & - \frac{4e(r-a_0)[a_0\{1-(r-a_0)^2\}+2(r-a_0)](1-z^2)\left\{\exp\left(-\frac{z^2}{2}\right)-\exp(-z^2)\right\}}{r\{1+(r-a_0)^2\}^3} \\ & - \frac{1}{Re} \left[\frac{4\sqrt{e}(r-a_0)\{3-(r-a_0)^2\}z\exp\left(-\frac{z^2}{2}\right)}{\{1+(r-a_0)^2\}^3} - \frac{2\sqrt{e}\{1-(r-a_0)^2\}z\exp\left(-\frac{z^2}{2}\right)}{r\{1+(r-a_0)^2\}^2} \right. \\ & \left. + \frac{2k^2\sqrt{e}(r-a_0)z(3-z^2)\exp\left(-\frac{z^2}{2}\right)}{\{1+(r-a_0)^2\}} + \frac{2\sqrt{e}(r-a_0)z\exp\left(-\frac{z^2}{2}\right)}{r^2\{1+(r-a_0)^2\}} \right] \\ & + \frac{2e}{k^2}g_1(r)z\left\{1-\exp\left(-\frac{z^2}{2}\right)\right\}^2 \\ & - \frac{2e}{k^2}g_2(r)\left\{1-\exp\left(-\frac{z^2}{2}\right)\right\}^2 \\ & + \frac{2\sqrt{e}}{k^2Re}\left[g_3(r)\left\{z-\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{z}{\sqrt{2}}\right)\right\}+g_4(r)\left\{z-\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{z}{\sqrt{2}}\right)\right\}\right. \\ & \left.+k^2g_5(r)z\exp\left(-\frac{z^2}{2}\right)\right]+\frac{dg(r)}{dr}. \end{aligned} \quad (18)$$

Applying the boundary condition $v = 0$ at $z = 0$ to equation (18), we obtain

$$\frac{dg(r)}{dr} = 0. \quad (19)$$

From Equation (19), we find that $g(r)$ is a constant function, i.e.,

$$g(r) = C. \quad (20)$$

Substituting $g(r)$ from Equation (20) into Equation (18), we derive the azimuthal velocity as

$$\begin{aligned} v(r, z) = & \left[\frac{4er(r-a_0)\{1-(r-a_0)^2\}z^2\exp(-z^2)}{\{1+(r-a_0)^2\}^3} \right. \\ & \left. - \frac{4e(r-a_0)[a_0\{1-(r-a_0)^2\}+2(r-a_0)](1-z^2)\left\{\exp\left(-\frac{z^2}{2}\right)-\exp(-z^2)\right\}}{\{1+(r-a_0)^2\}^3} \right] \end{aligned} \quad (21)$$

$$\begin{aligned}
& -\frac{r}{Re} \left[\frac{4\sqrt{e}(r-a_0)\{3-(r-a_0)^2\}z\exp\left(-\frac{z^2}{2}\right)}{\{1+(r-a_0)^2\}^3} - \frac{2\sqrt{e}\{1-(r-a_0)^2\}z\exp\left(-\frac{z^2}{2}\right)}{r\{1+(r-a_0)^2\}^2} \right. \\
& \quad \left. + \frac{2k^2\sqrt{e}(r-a_0)z(3-z^2)\exp\left(-\frac{z^2}{2}\right)}{\{1+(r-a_0)^2\}} + \frac{2\sqrt{e}(r-a_0)z\exp\left(-\frac{z^2}{2}\right)}{r^2\{1+(r-a_0)^2\}} \right] \\
& \quad + \frac{2er}{k^2} g_1(r)z \left\{ 1 - \exp\left(-\frac{z^2}{2}\right) \right\}^2 \\
& \quad - \frac{2er}{k^2} g_2(r) \left\{ 1 - \exp\left(-\frac{z^2}{2}\right) \right\}^2 \\
& \quad + \frac{2\sqrt{e}r}{k^2 Re} \left[g_3(r) \left\{ z - \sqrt{\frac{\pi}{2}} \operatorname{erf}\left(\frac{z}{\sqrt{2}}\right) \right\} + g_4(r) \left\{ z - \sqrt{\frac{\pi}{2}} \operatorname{erf}\left(\frac{z}{\sqrt{2}}\right) \right\} \right. \\
& \quad \left. + k^2 g_5(r)z\exp\left(-\frac{z^2}{2}\right) \right] \Bigg|^\frac{1}{2}.
\end{aligned}$$

Again, from Equation (16), the pressure is obtained as

$$\begin{aligned}
k\{p(r, z) - p(a_0, 0)\} = & \frac{2e}{k} f_1(r)z \left\{ 1 - \exp\left(-\frac{z^2}{2}\right) \right\}^2 - \frac{2e}{k} f_2(r) \left\{ 1 - \exp\left(-\frac{z^2}{2}\right) \right\}^2 \\
& + \frac{2\sqrt{e}}{k Re} \left[f_3(r) \left\{ z - \sqrt{\frac{\pi}{2}} \operatorname{erf}\left(\frac{z}{\sqrt{2}}\right) \right\} + f_4(r) \left\{ z - \sqrt{\frac{\pi}{2}} \operatorname{erf}\left(\frac{z}{\sqrt{2}}\right) \right\} \right. \\
& \left. + k^2 f_5(r)z\exp\left(-\frac{z^2}{2}\right) \right]. \quad (22)
\end{aligned}$$

The complete calculation of the pressure can be found in APPENDIX C.

4. RESULTS AND DISCUSSION

In this model, it is assumed that the modified form of radial velocity as presented by the Baker and Sterling model [22] is used, and the axial velocity is subsequently evaluated using the continuity equation (9). Furthermore, the azimuthal velocity is derived for a viscous fluid using the radial component (6) and the axial component (8) of the reduced and non-dimensionalised Navier-Stokes equations. The pressure is also assessed in this model.

4.1. RADIAL VELOCITY

Fig. 1 illustrates the radial profile of non-dimensionalised radial velocity as given by equation (10) for different values of a_0 , the radius of the central low-pressure zone, and various heights. The graphs indicate that the magnitude of radial velocity increases with height when $z < 1$ and decreases with height when $z \geq 1$ (see Figs. 1a-f). Additionally, it is noted that the radial velocity is zero at a radial distance of $r = a_0$, and then increases radially, reaching its maximum at $r = a_0 + 1$ and $z = 1$, but asymptotically vanishes at large radii (see

Figs. 1a-f). The dashed axial line indicates the boundary of the central low-pressure zone (see Figs. 1a-f). Furthermore, the radial velocity curves are situated below the x-axis, indicating that the flow is directed inward.

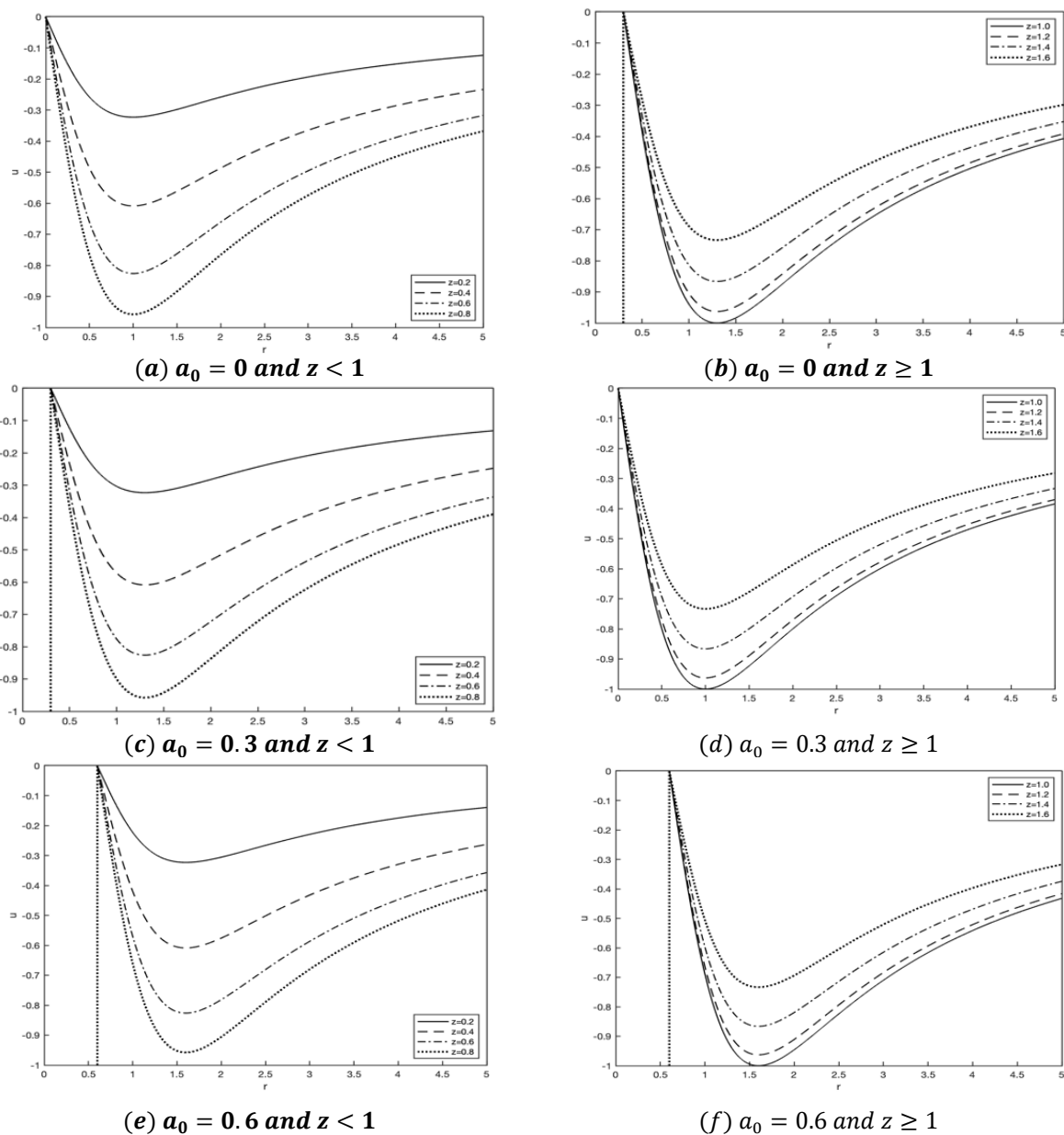


Figure 1. Plots of radial velocity u versus r .

4.2. AXIAL VELOCITY

Fig. 2 presents the radial profile of non-dimensionalised axial velocity as given in equation (13) for different values of a_0 , various heights, and $k = 0.2$. The graphs reveal that when $a_0 = 0$, the maximum axial velocity occurs at the central axis (see Fig. 2(a)), and when a_0 is non-zero, the axial velocity increases radially and reaches its maximum at a certain radial distance from the boundary of the cylindrical low-pressure zone, vanishing at a finite radial distance (see Figs. 2b-c). It is noteworthy that when a_0 is non-zero, the axial velocity attains its maximum at some radial distance from the periphery of the low-pressure zone

rather than on the cylindrical boundary. The plots clearly indicate that as the radius of the central low-pressure zone increases, the magnitude of the maximum axial velocity decreases (see Figs. 2a-c). Furthermore, it is observed that the axial velocity increases in magnitude with height (see Figs. 2a-c). Moreover, the axial velocity curves are positioned above the x-axis, indicating that the flow is directed upward.

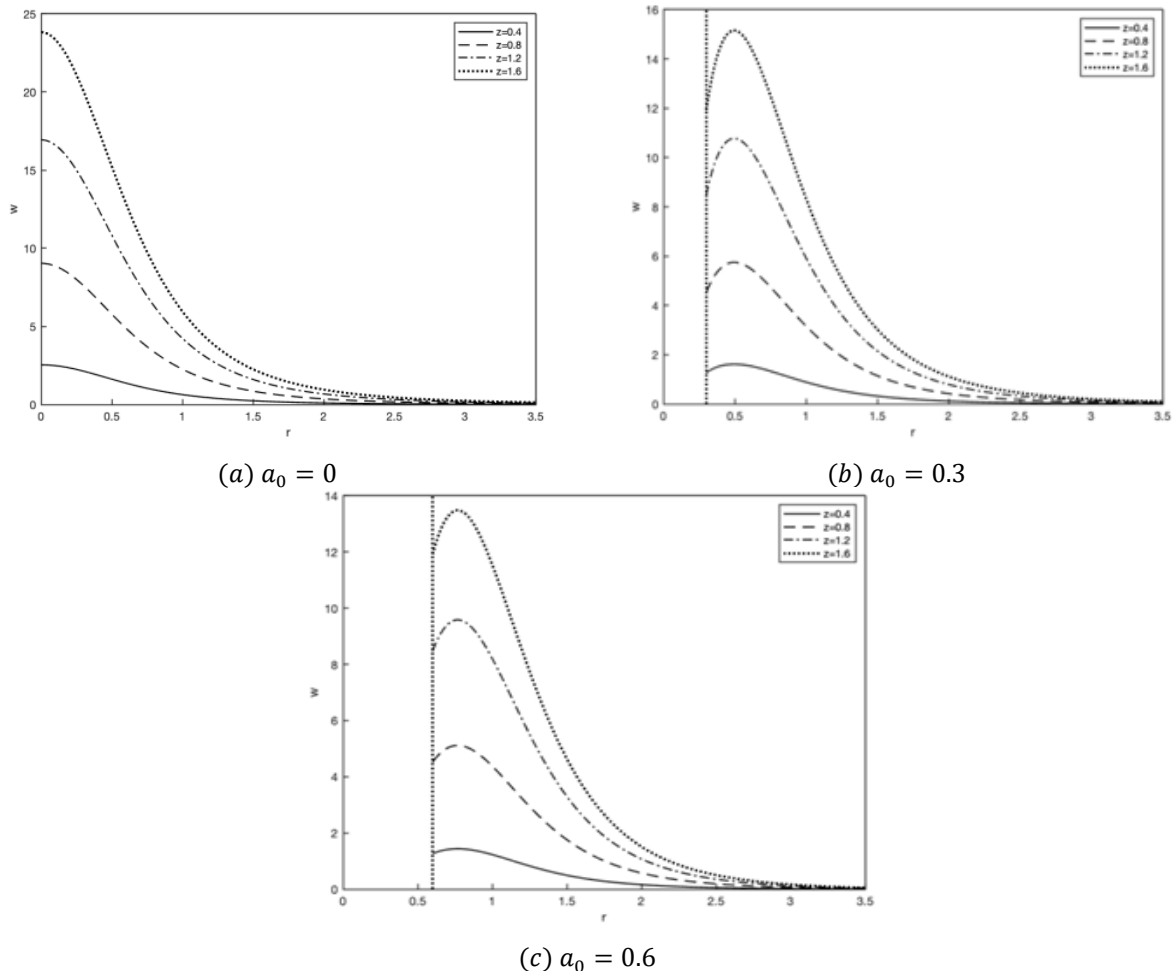


Figure 2. Plots of axial velocity w versus r .

4.3. AZIMUTHAL VELOCITY

Fig. 3 illustrates the radial profile of non-dimensionalized azimuthal velocity as given in equation (21) for different values of a_0 , various heights, and $k = 0.2$. The air vortices exhibit extremely high Reynolds numbers, lying in the range of 1000-10000, and due to these high Reynolds numbers, the impact of viscosity is expected to be relatively minor. In this study, Re is considered as $Re = 1500$. It is observed that when $a_0 = 0$, the azimuthal velocity is zero on the central axis and subsequently increases rapidly with radius, reaching its maximum at some radial distance before decreasing at a slower rate, vanishing at a finite radial distance and remaining zero for large radii (see Fig. 3a). Additionally, when a_0 is non-zero, the rotation commences somewhat away from the periphery of the low-pressure zone and increases rapidly with radius, attaining its maximum value at a certain radial distance before decreasing radially and vanishing at a finite radial distance, and remaining zero for large radii (see Figs. 3b-c). Moreover, in this model, the azimuthal velocity vanishes much

sooner at a specific radial distance as compared to the earlier model, Baker and Sterling [1], where it vanishes asymptotically in the radial direction. Further, it is observed that the azimuthal velocity vanishes quickly at low altitudes and takes longer time to vanish as the altitude increases (see Figs. 3a-c). Furthermore, it can be observed that the magnitude of maximum azimuthal velocity increases with height (see Figs. 3a-c).

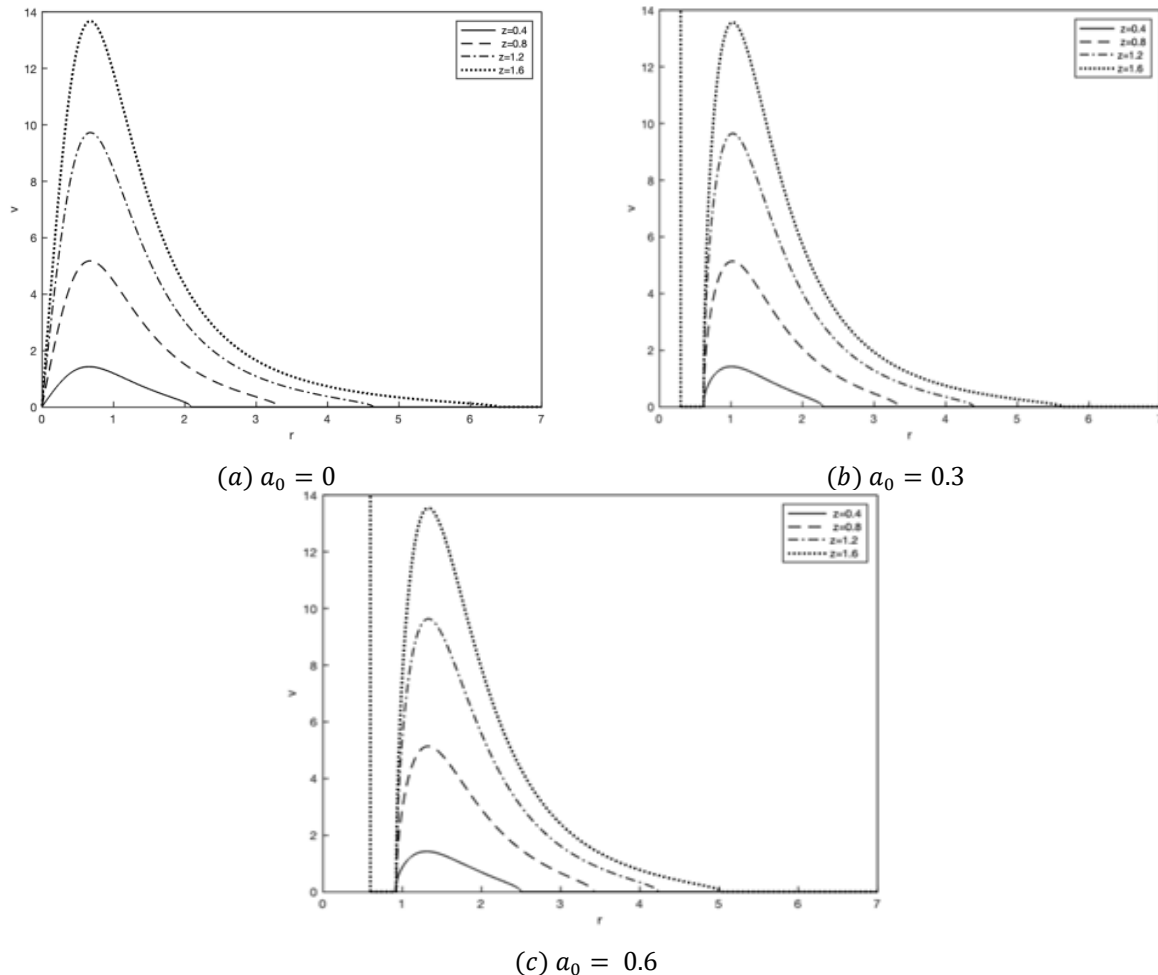


Figure 3. Plots of azimuthal velocity v versus r .

Fig. 4 compares the azimuthal velocity of the current model with that of the Baker and Sterling model [1]. The azimuthal velocity for the Baker and Sterling model is plotted with a swirl ratio (S) of one. From the graph, it is observed that the azimuthal velocity of the current model increases rapidly in comparison to the Baker and Sterling model [1]. Additionally, the maximum magnitude of the azimuthal velocity in the current model exceeds that of the Baker and Sterling model [1]. The graph further illustrates that the azimuthal velocity of the current model diminishes at a finite radial distance and remains zero at larger radii, while the azimuthal velocity of the Baker and Sterling model [1] approaches zero asymptotically at large radii.

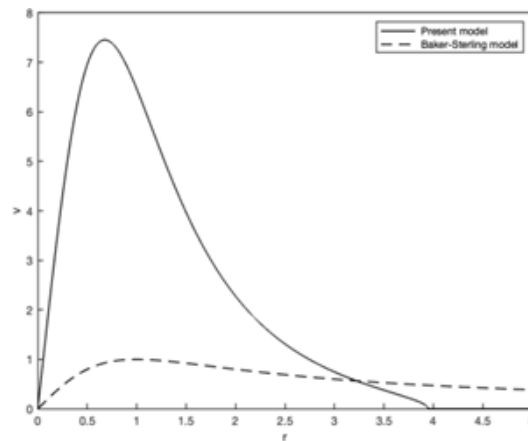
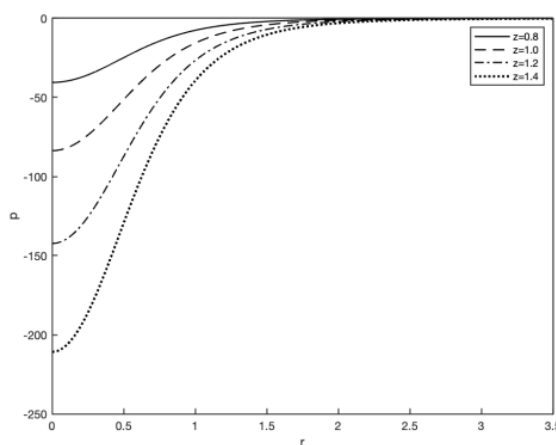
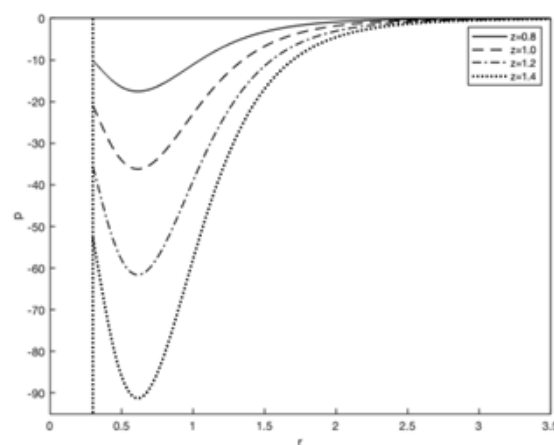
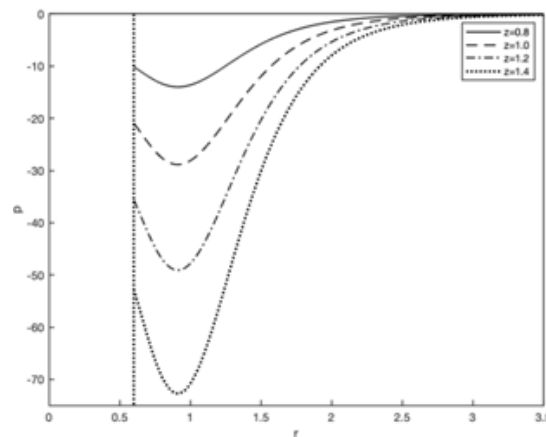
(a) $a_0 = 0$ and $z = 1$

Figure 4. Comparison between the azimuthal velocity of the present model and the Baker and Sterling model [22].

4.4. RADIAL PRESSURE

Fig. 5 displays the radial profile of the non-dimensionalized radial pressure difference given in equation (22) for different values of a_0 , various heights, $Re = 1500$, and $k = 0.2$. From the graphs, it is observed that when $a_0 = 0$, the pressure difference is minimal on the central axis, then increases radially and disappears at a finite radial distance (see Fig. 5a). It is further observed that when a_0 is non-zero, the pressure difference is minimal slightly away from the edge of the low-pressure zone (see Figs. 5b-c). The graphs show that for non-zero values of a_0 , the pressure difference retains some finite value at the boundary of the low-pressure zone, decreases radially, reaches its minimum at a certain radial distance, then increases radially and disappears at a finite radial distance from the cylindrical low-pressure zone. Moreover, the magnitude of the pressure difference increases with height (see Figs. 5a-c).

(a) $a_0 = 0$ (b) $a_0 = 0.3$



(c) $a_0 = 0.6$

Figure 5. Plots of pressure difference versus r .

5. CONCLUSIONS

This study presents an axisymmetric model of steady, viscous, and incompressible flow. In this model, a modified form of the radial velocity is adopted from the Baker and Sterling model [22] by incorporating an exponential function and the radius of the central low-pressure zone. Unlike the said model, the present model accounts for the flow of the viscous fluid.

The radial velocity analysis demonstrates that the flow is directed inward throughout the vortex region, as indicated by the negative radial velocity values. The magnitude of the radial velocity increases with height when $z < 1$ and decreases when $z \geq 1$, showing that the inflow is stronger at lower altitudes and weakens with height. The radial velocity is zero at $r = a_0$, increases radially outward to reach its maximum near $r = a_0 + 1$ and $z = 1$, and vanishes asymptotically at large radial distances, indicating the outer limit of the vortex influence. Overall, the results confirm that the radial velocity distribution is sensitive to both the radius of the central low-pressure zone and height, with the strongest inward motion occurring near the boundary of the low-pressure zone at lower heights.

The analysis of the radial profiles of non-dimensional axial velocity indicates that the axial velocity distribution is significantly influenced by the parameter a_0 and the vertical height z . When $a_0 = 0$, the maximum axial velocity occurs along the central axis, which indicates a purely axis-centred upward flow. However, for non-zero values of a_0 , the maximum axial velocity shifts radially outward from the centre, peaking near the boundary of the low-pressure zone before vanishing at larger radial distances. As a_0 increases, the magnitude of the maximum axial velocity decreases, demonstrating that a stronger radial spread of the vortex weakens the upward motion. Additionally, the axial velocity increases with height, suggesting that the vertical flow strengthens in the upper region of the vortex. All axial velocity profiles remain positive, confirming that the flow is directed upward throughout the vortex structure.

The radial profile of azimuthal velocity exhibits characteristic behaviour that varies significantly with the parameter a_0 and height z . When $a_0 = 0$, the velocity field displays classical vortex characteristics with zero azimuthal velocity at the central axis, followed by a rapid radial increase to a maximum value, and subsequent gradual decay to zero at a radial

distance and remaining zero for large radii. This represents a symmetric rotational flow pattern centred on the axis. When a_0 assumes non-zero values, the rotational flow initiates at a radial location displaced from the periphery of the low-pressure zone rather than at the centre. The velocity increases rapidly with radius, reaches a peak at a radial distance, and then diminishes to zero at a finite radial distance. This demonstrates that the parameter a_0 effectively shifts the centre of rotation away from the central axis. A notable distinction from the Baker and Sterling model [22] is that the azimuthal velocity in the present model vanishes at a specific finite radial distance rather than decaying asymptotically. This provides a more localized vortex structure with clearly defined spatial boundaries. Vertical stratification plays a crucial role in the velocity distribution. At lower altitudes, the azimuthal velocity decays more rapidly with radius, while at higher altitudes, the velocity persists over larger radial distances before vanishing. Furthermore, the magnitude of maximum azimuthal velocity increases monotonically with height, indicating stronger rotational intensity at elevated positions.

It is concluded that the radial pressure distribution exhibits distinct behaviour depending on the value of the parameter a_0 and height z . When $a_0 = 0$, the pressure difference is minimal at the central axis and increases monotonically with radial distance before vanishing at a finite radial distance. This represents the simplest case with the symmetric pressure distribution about the axis. For non-zero values of a_0 , the pressure difference becomes more complex. The minimum pressure difference shifts away from the boundary of the low-pressure zone, and the pressure profile displays a characteristic pattern where it maintains a finite value at the boundary of the low-pressure zone, decreases radially to reach a minimum at some radial distance, then increases again before dissipating. It is also concluded that the magnitude of the radial pressure difference increases with height, indicating vertical stratification in the pressure field. Higher altitudes experience more noticeable pressure changes, which have significant implications for understanding the three-dimensional structure of the flow field around the cylindrical low-pressure zone. Furthermore, the pressure difference eventually disappears at a finite radial distance from the low-pressure zone in all cases examined, confirming that the influence of the low-pressure region is limited and does not extend infinitely.

The present model is formulated based on the assumptions of steady, axisymmetric, viscous, and incompressible flow. As a result, turbulence, compressibility, thermal buoyancy, and surface roughness are excluded from consideration. The model offers qualitative insight into vortex structure but lacks direct validation against observational or experimental data. Future research should incorporate these physical effects and compare analytical predictions with field measurements, laboratory experiments, or high-resolution numerical simulations to enhance the model's applicability to realistic atmospheric vortices.

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APPENDIX A

The functions $f_1(r)$, $f_2(r)$, $f_3(r)$, $f_4(r)$, and $f_5(r)$ are given by

$$f_1(r) = \frac{(r - a_0)}{r^2\{1 + (r - a_0)^2\}^4} [3a_0(r - a_0)^4 + (2a_0^2 - 8)(r - a_0)^3 - 12a_0(r - a_0)^2 - 6a_0^2(r - a_0) + a_0],$$

$$f_2(r) = \frac{1}{r^2\{1 + (r - a_0)^2\}^4} [a_0\{1 - (r - a_0)^2\} + 2(r - a_0)]^2,$$

$$f_3(r) = \frac{1}{r^4\{1 + (r - a_0)^2\}^6} [r^2\{1 + (r - a_0)^2\}^3\{12a_0(r - a_0)^3 + (6a_0^2 - 24)(r - a_0)^2 - 24a_0(r - a_0)$$

$$- 6a_0^2\} - [6r^2(r - a_0)\{1 + (r - a_0)^2\}^2 + 2r\{1 + (r - a_0)^2\}^3]\{3a_0(r - a_0)^4 + (2a_0^2 - 8)(r - a_0)^3 - 12a_0(r - a_0)^2 - 6a_0^2(r - a_0)\} + a_0],$$

$$f_4(r) = \frac{1}{r^3\{1 + (r - a_0)^2\}^3} [3a_0(r - a_0)^4 + (2a_0^2 - 8)(r - a_0)^3 - 12a_0(r - a_0)^2 - 6a_0^2(r - a_0) + a_0],$$

$$f_5(r) = \frac{1}{r\{1 + (r - a_0)^2\}^2} [a_0\{1 - (r - a_0)^2\} + 2(r - a_0)].$$

APPENDIX B

The functions $g_1(r)$, $g_2(r)$, $g_3(r)$, $g_4(r)$, and $g_5(r)$ are given by

$$\begin{aligned}
 g_1(r) &= \frac{1}{r^4\{1 + (r - a_0)^2\}^8} \left[-(r - a_0)[8r^2(r - a_0)\{1 + (r - a_0)^2\}^3 \right. \\
 &\quad + 2r\{1 + (r - a_0)^2\}^4]\{3a_0(r - a_0)^4 \\
 &\quad + (2a_0^2 - 8)(r - a_0)^3 - 12a_0(r - a_0)^2 - 6a_0^2(r - a_0)a_0\} \\
 &\quad + r^2\{1 + (r - a_0)^2\}^4 \\
 &\quad [3a_0(r - a_0)^4 + (2a_0^2 - 8)(r - a_0)^3 - 12a_0(r - a_0)^2 \\
 &\quad - 6a_0^2(r - a_0) + a_0 \\
 &\quad + (r - a_0)\{12a_0(r - a_0)^3 + (6a_0^2 - 24)(r - a_0)^2 \\
 &\quad - 24a_0(r - a_0) - 6a_0^2\}]], \\
 g_2(r) &= \frac{1}{r^4\{1 + (r - a_0)^2\}^8} \left[2r^2\{1 + (r - a_0)^2\}^4\{2a_0(r - a_0) - 2\}[a_0\{(r - a_0)^2 - 1\} \right. \\
 &\quad - 2(r - a_0)] \\
 &\quad - [a_0\{(r - a_0)^2 - 1\} - 2(r - a_0)]^2[8r^2(r - a_0)\{1 \\
 &\quad + (r - a_0)^2\}^3 \\
 &\quad + 2r\{1 + (r - a_0)^2\}^4], \\
 g_3(r) &= \frac{1}{r^8\{1 + (r - a_0)^2\}^{12}} \left[r^4\{1 + (r - a_0)^2\}^6[6r^2(r - a_0)\{1 \right. \\
 &\quad + (r - a_0)^2\}^2\{12a_0(r - a_0)^3 \\
 &\quad + (6a_0^2 - 24)(r - a_0)^2 \\
 &\quad - 24a_0(r - a_0) - 6a_0^2\} + \{1 \\
 &\quad + (r - a_0)^2\}^3[r^2\{36a_0(r - a_0)^2 \\
 &\quad + (12a_0^2 - 48)(r - a_0) \\
 &\quad - 24a_0\} + 2r\{12a_0(r - a_0)^3 + (6a_0^2 - 24)(r - a_0)^2 \\
 &\quad - 24a_0(r - a_0) - 6a_0^2\}] \\
 &\quad - [6r^2(r - a_0)\{1 + (r - a_0)^2\}^2 + 2r\{1 \\
 &\quad + (r - a_0)^2\}^3]\{12a_0(r - a_0)^3 \\
 &\quad + (6a_0^2 - 24)(r - a_0)^2 - 24a_0(r - a_0) - 6a_0^2\} \\
 &\quad - \{12r(r - a_0)\{1 + (r - a_0)^2\}^2 \\
 &\quad + 6(r - a_0)[4r^2(r - a_0)\{1 + (r - a_0)^2\} + 2r\{1 \\
 &\quad + (r - a_0)^2\}^2] \\
 &\quad + 6r^2\{1 + (r - a_0)^2\}^2 + 2\{1 + (r - a_0)^2\}^3\}\{3a_0(r - a_0)^4 \\
 &\quad + (2a_0^2 - 8)(r - a_0)^3 \\
 &\quad - 12a_0(r - a_0)^2 - 6a_0^2(r - a_0) + a_0\}] - [12r^4(r - a_0)\{1 \\
 &\quad + (r - a_0)^2\}^5 \\
 &\quad + 4r^3\{1 + (r - a_0)^2\}^6][r^2\{1 + (r - a_0)^2\}^3\{12a_0(r - a_0)^3 \\
 &\quad + (6a_0^2 - 24)(r - a_0)^2 \\
 &\quad - 24a_0(r - a_0) - 6a_0^2\} - [6r^2(r - a_0)\{1 + (r - a_0)^2\}^2 \\
 &\quad + 2r\{1 + (r - a_0)^2\}^3] \\
 &\quad \{3a_0(r - a_0)^4 + (2a_0^2 - 8)(r - a_0)^3 - 12a_0(r - a_0)^2 \\
 &\quad - 6a_0^2(r - a_0) + a_0\}]], \\
 g_4(r) &= \frac{1}{r^6\{1 + (r - a_0)^2\}^6} \left[r^3\{1 + (r - a_0)^2\}^3\{12a_0(r - a_0)^3 \right. \\
 &\quad + (6a_0^2 - 24)(r - a_0)^2 - 24a_0(r - a_0) \\
 &\quad - 6a_0^2\} - [6r^3(r - a_0)\{1 + (r - a_0)^2\}^2 \\
 &\quad + 3r^2\{1 + (r - a_0)^2\}^3]\{3a_0(r - a_0)^4 \\
 &\quad + (2a_0^2 - 8)(r - a_0)^3 - 12a_0(r - a_0)^2 - 6a_0^2(r - a_0) \\
 &\quad + a_0\}], \\
 g_5(r) &= \frac{1}{r^2\{1 + (r - a_0)^2\}^3} \left[3a_0(r - a_0)^4 + (2a_0^2 - 8)(r - a_0)^3 - 12a_0(r - a_0)^2 \right. \\
 &\quad - 6a_0^2(r - a_0) + a_0].
 \end{aligned}$$

APPENDIX C

Substituting Equation (20) into Equation (16), we get

$$kp(r, z) = \frac{2e}{k} f_1(r) z \left\{ 1 - \exp\left(-\frac{z^2}{2}\right) \right\}^2 - \frac{2e}{k} f_2(r) \left\{ 1 - \exp\left(-\frac{z^2}{2}\right) \right\}^2 \\ + \frac{2\sqrt{e}}{kRe} \left[f_3(r) \left\{ z - \sqrt{\frac{\pi}{2}} \operatorname{erf}\left(\frac{z}{\sqrt{2}}\right) \right\} \right. \\ \left. + f_4(r) \left\{ z - \sqrt{\frac{\pi}{2}} \operatorname{erf}\left(\frac{z}{\sqrt{2}}\right) \right\} + k^2 f_5(r) z \exp\left(-\frac{z^2}{2}\right) \right] + C.$$

Assuming that $p(r, z)$ is finite when $r = a_0$, $z = 0$, and using this assumption in Equation (23), we get $C = kp(a_0, 0)$. Now, substituting the value of C into Equation (23), we get

$$k\{p(r, z) - p(a_0, 0)\} = \frac{2e}{k} f_1(r) z \left\{ 1 - \exp\left(-\frac{z^2}{2}\right) \right\}^2 - \frac{2e}{k} f_2(r) \left\{ 1 - \exp\left(-\frac{z^2}{2}\right) \right\}^2 \\ + \frac{2\sqrt{e}}{kRe} \left[f_3(r) \left\{ z - \sqrt{\frac{\pi}{2}} \operatorname{erf}\left(\frac{z}{\sqrt{2}}\right) \right\} + f_4(r) \left\{ z - \sqrt{\frac{\pi}{2}} \operatorname{erf}\left(\frac{z}{\sqrt{2}}\right) \right\} \right. \\ \left. + k^2 f_5(r) z \exp\left(-\frac{z^2}{2}\right) \right].$$

REFERENCES

- [1] Rankine, W.J.M., *A Manual of Applied Physics*, Charles Griffin and Company, London, United Kingdom, 1882. https://scholar.google.com/scholar?hl=en&as_sdt=0%2C5&q=W.+J.+M.+Rankine%2C+A+Manual+of+Applied+Physics&btnG=
- [2] Taylor, G.I., Phenomena Connected with Turbulence in the Lower Atmosphere, *Proceedings of the Royal Society of London. Series A*, **94**(658), 137, 1918. <https://doi.org/10.1098/rspa.1918.0002>
- [3] Lamb, H., *Hydrodynamics*, Cambridge Mathematical Library, Cambridge University Press, Cambridge, 1993.
- [4] Rott, N., On the Viscous Core of a Line Vortex, *Journal of Applied Mathematics and Physics*, **9**(5), 543, 1958. <https://doi.org/10.1007/BF02424773>
- [5] Long, R.R., A Vortex in an Infinite Viscous Fluid, *Journal of Fluid Mechanics*, **11**(4), 611, 1961. <https://doi.org/10.1017/S0022112061000767>
- [6] Kieu, C.Q., Zhang, D.L., An Analytical Model for the Rapid Intensification of Tropical Cyclones, *Quarterly Journal of the Royal Meteorological Society*, **135**(642), 1336, 2009. <https://doi.org/10.1002/qj.433>
- [7] Burgers, J.M., A Mathematical Model Illustrating the Theory of Turbulence, *Advances in Applied Mechanics*, **1**, 171, 1948. [https://doi.org/10.1016/S0065-2156\(08\)70100-5](https://doi.org/10.1016/S0065-2156(08)70100-5)

- [8] Makarieva, A.M., Gorshkov, V.G., Nefiodov, A.V., Condensational Theory of Stationary Tornadoes, *Physics Letters A*, **375**(24), 2259, 2011. <https://doi.org/10.1016/j.physleta.2011.04.023>
- [9] Vyas, A.B., Majdalani, J., Chiaverini, M.J., The Bidirectional Vortex. Part 2: Viscous Core Corrections, Proceedings of the 39th AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit, Paper No. AIAA 2003-5053, 2003. <https://doi.org/10.2514/6.2003-5053>
- [10] Vyas, A.B., Majdalani, J., Exact Solution of the Bidirectional Vortex, *AIAA Journal*, **44**(10), 2208, 2006. <https://doi.org/10.2514/1.14872>
- [11] Xu, Z., Hangan, H., An Inviscid Solution for Modeling of Tornado-Like Vortices, *Journal of Applied Mechanics*, **76**(3), 031011, 2009. <https://doi.org/10.1115/1.3063632>
- [12] Sullivan, R.D., A Two-Cell Vortex Solution of the Navier-Stokes Equations, *Journal of the Aerospace Sciences*, **26**(11), 767, 1959. <https://doi.org/10.2514/8.8303>
- [13] Wood, V.T., Brown, R.A., Simulated Tornadoic Vortex Signatures of Tornado-Like Vortices Having One- and Two-Celled Structures, *Journal of Applied Meteorology and Climatology*, **50**(11), 2338, 2011. <https://doi.org/10.1175/JAMC-D-11-0118.1>
- [14] Kim, Y.C., Matsui, M., Analytical and Empirical Models of Tornado Vortices: A Comparative Study, *Journal of Wind Engineering and Industrial Aerodynamics*, **171**, 230, 2017. <https://doi.org/10.1016/j.jweia.2017.10.009>
- [15] Gillmeier, S., Sterling, M., Hemida, H., Baker, C.J., A Reflection on Analytical Tornado-Like Vortex Flow Field Models, *Journal of Wind Engineering and Industrial Aerodynamics*, **174**, 10, 2018. <https://doi.org/10.1016/j.jweia.2017.12.017>
- [16] Bluestein, H.B., Ladue, J.G., Stein, H., Speheger, D., Unruh, W.F., Doppler Radar Wind Spectra of Supercell Tornadoes, *Monthly Weather Review*, **121**(8), 2200, 1993. [https://doi.org/10.1175/1520-0493\(1993\)121<2200:DRWSOS>2.0.CO;2](https://doi.org/10.1175/1520-0493(1993)121<2200:DRWSOS>2.0.CO;2)
- [17] Wurman, J., Straka, J.M., Rasmussen, E.N., Fine-Scale Doppler Radar Observations of Tornadoes, *Science*, **272**(5269), 1774, 1996. <https://doi.org/10.1126/science.272.5269.1774>
- [18] Lewellen, W.S., Lewellen, D.C., Sykes, R.I., Large-Eddy Simulation of a Tornado's Interaction with the Surface, *Journal of the Atmospheric Sciences*, **54**(5), 581, 1997. [https://doi.org/10.1175/1520-0469\(1997\)054<0581:LESOAT>2.0.CO;2](https://doi.org/10.1175/1520-0469(1997)054<0581:LESOAT>2.0.CO;2)
- [19] Bluestein, H.B., Lee, W.C., Bell, M., Weiss, C.C., Pazmany, A.L., Mobile Doppler Radar Observations of a Tornado in a Supercell near Bassett, Nebraska, on 5 June 1999. Part II: Tornado-Vortex Structure, *Monthly Weather Review*, **131**(12), 2968, 2003. [https://doi.org/10.1175/1520-0493\(2003\)131<2968:MDROOA>2.0.CO;2](https://doi.org/10.1175/1520-0493(2003)131<2968:MDROOA>2.0.CO;2)
- [20] Bluestein, H.B., A Review of Ground-Based, Mobile, W-Band Doppler-Radar Observations of Tornadoes and Dust Devils, *Dynamics of Atmospheres and Oceans*, **40**(3), 163, 2005. <https://doi.org/10.1016/j.dynatmoce.2005.03.004>
- [21] Davies-Jones, R.P., Wood, V.T., Simulated Doppler Velocity Signatures of Evolving Tornado-Like Vortices, *Journal of Atmospheric and Oceanic Technology*, **23**(8), 1029, 2006. <https://doi.org/10.1175/JTECH1903.1>
- [22] Baker, C.J., Sterling, M., Modelling Wind Fields and Debris Flight in Tornadoes, *Journal of Wind Engineering and Industrial Aerodynamics*, **168**, 312, 2017. <https://doi.org/10.1016/j.jweia.2017.06.017>
- [23] Ying, S.J., Chang, C.C., Exploratory Model Study of Tornado-Like Vortex Dynamics, *Journal of the Atmospheric Sciences*, **27**(1), 3, 1970. [https://doi.org/10.1175/1520-0469\(1970\)027<0003:EMSOTL>2.0.CO;2](https://doi.org/10.1175/1520-0469(1970)027<0003:EMSOTL>2.0.CO;2)
- [24] Sinclair, P.C., The Lower Structure of Dust Devils, *Journal of the Atmospheric Sciences*, **30**(8), 1599, 1973. [https://doi.org/10.1175/1520-0469\(1973\)030<1599:TLSODD>2.0.CO;2](https://doi.org/10.1175/1520-0469(1973)030<1599:TLSODD>2.0.CO;2)

- [25] Zhang, H., Wang, H., Xu, Z., Khoo, B.C., A Novel Three-Dimensional Analytical Tornado Model Constructed Based on Force Balance Analysis, *Physics of Fluids*, **35**(6), 065133, 2023. <https://doi.org/10.1063/5.0156170>
- [26] Maddox, R.A., Gilmore, M.S., Doswell III, C.A., Johns, R.H., Crisp, C.A., Burgess, D.W., Hart, J.A., Piltz, S.F., Meteorological Analyses of the Tri-State Tornado Event of March 1925, *E-Journal of Severe Storms Meteorology*, **8**(1), 1, 2013. <https://doi.org/10.55599/ejssm.v8i1.46>