

ON Λ AND V SETS WITH IDEALS OF THE RING $(2^\xi, \Delta, \cap)$ MONOJ KUMAR DAS¹, CHHAPIKUL MIAH², SHYAMAPADA MODAK¹

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Abstract. Definitions of Λ and V sets were introduced in different ways in the literature. Further study of these set combines with generalized open sets, gives several properties of them. But their extensions, studied with the help of algebraic ideals of the ring $(2^\xi, \Delta, \cap)$, are a new idea. These ideals rigorously characterize the same sets.

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1. INTRODUCTION AND PRELIMINARIES

The opposite versions of interior and closure operators are introduced with the help of Λ -sets [1] and V -sets [1]. For this introduction, the kernel of a set plays a remarkable part. These Λ and V sets not only induced through open sets as well as closed sets but also introduced through generalized open sets [2-6]. Here, we considered the generalized open sets through the operator symbol.

Let ξ be a nonempty set and τ be a topology on ξ . A subset A of a $\mathbb{T}S$ (where $\mathbb{T}S$ denotes the topological space throughout the paper) (ξ, τ) is said to be k -open if $A \subseteq k(A)$, k is the composition of the interior operator (denoted as o) and the closure operator (denoted as δ). For example, if $k = o$, then A is open, and if $k = o\delta \cup \delta o$, then A is called b -open [2]. For other values of k , we may consider semi-open sets [4], preopen sets [5], and β -open sets [8] (or semi-preopen sets [3]), etc. In this paper, we only consider four generalized open sets and the open set together; that is k may take the value $k \in \{o, o\delta, \delta o, o\delta o, o\delta \cup \delta o, \delta o\delta\}$.

Let A be a subset of a $\mathbb{T}S$ (ξ, τ) . Define, $A^{\Lambda_k} = \bigcap \{O \subseteq \xi : O \supseteq A, O \in \mathcal{O}^k\}$, and $A^{V_k} = \bigcup \{F \subseteq \xi : F \subseteq A, F^c \in \mathcal{O}^k\}$, where $\mathcal{O}^k$ is the collection of all k -open sets in the $\mathbb{T}S$ (ξ, τ) and F^c is the complement of F . That is, if $k = o\delta$, then $\mathcal{O}^{o\delta}$ is the collection of all semi-open sets in (ξ, τ) . Before starting the next section, we mentioned that, $A \subseteq o(A)$ (respectively, $A \subseteq o\delta(A)$, $A \subseteq \delta o(A)$, $A \subseteq o\delta(A) \cup \delta o(A)$, $A \subseteq \delta o\delta(A)$) stands for open (respectively, semi-open, preopen, b -open, β -open, or semi-pre open). If we do not mention k , then k is always belongs to $\{o, o\delta, \delta o, o\delta o, o\delta \cup \delta o, \delta o\delta\}$.

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2. Λ_k AND V_k -SETS

Through this section, we have explored various properties and examples of Λ_k and V_k -sets. A subset $A \subseteq \xi$ said to be a Λ_k (respectively, V_k) set in the $\mathbb{T}S(\xi, \tau)$ if $A^{\Lambda_k} = A$ (respectively, $A^{V_k} = A$). It is obvious that $A^{V_k} \subseteq A \subseteq A^{\Lambda_k}$. The collection of all Λ_k (respectively, V_k) sets in a $\mathbb{T}S(\xi, \tau)$ is denoted as $\mathcal{O}^{\Lambda_k}$ (respectively, $\mathcal{O}^{V_k}$). For various values of k , we get different properties of $\mathcal{O}^{\Lambda_k}$ (see [2-5, 7]). But $\mathcal{O}^{\Lambda_k}$ (or $\mathcal{O}^{V_k}$) have some common properties.

Proposition 1.

- (i) The sets $\emptyset$ and ξ are members of $\mathcal{O}^{\Lambda_k}$ and $\mathcal{O}^{V_k}$.
- (ii) $\mathcal{O}^{\Lambda_k}$ (respectively, $\mathcal{O}^{V_k}$) closed under arbitrary unions.
- (iii) $\mathcal{O}^{\Lambda_k}$ (respectively, $\mathcal{O}^{V_k}$) closed under arbitrary intersections.
- (iv) A subset B of ξ is a Λ_k set if and only if B^c (complement of B) is a V_k set.
- (v) $(\xi \setminus B)^{\Lambda_k} = \xi \setminus B^{V_k}$, where $B \subseteq \xi$.
- (vi) If a subset B of ξ is k -clopen, then B is Λ_k as well as V_k set.

Proof: Proofs are obvious from [9] and hence omitted.

From (i), (ii), (iii) of Proposition 1, $\mathcal{O}^{\Lambda_k}$ (respectively, $\mathcal{O}^{V_k}$) constitutes an Alexandroff topology [9-10] and hence a topology.

A $\mathbb{T}S(\xi, \tau)$ is called an Alexandroff topological space if τ is closed under arbitrary intersections.

$\mathcal{O}^{\Lambda_k}$ (respectively, $\mathcal{O}^{V_k}$) may or may not be a discrete topology.

Example 1. Consider $\mathbb{R}$ with cofinite topology and $A \subseteq \mathbb{R}$, $x \in \mathbb{R} \setminus A$. Then, $U_x = \mathbb{R} \setminus \{x\}$ is an open set containing A . Also, $\bigcap_{x \in \mathbb{R} \setminus A} U_x = A$. Thus, $A = \ker(A)$. Hence, A is a Λ_o -set. Since A is an arbitrary subset of $\mathbb{R}$, thus every subset of $\mathbb{R}$ is a Λ_o -set.

Thus, we get the discrete topology on $\mathbb{R}$. But it is not obvious that every topology induced through $\mathcal{O}^{\Lambda_k}$ (respectively, $\mathcal{O}^{V_k}$) on $\mathbb{R}$ give the discrete topology.

Example 2.

(i) Consider the topology $\tau = \{\emptyset, \mathbb{R}, \mathbb{Q}, \mathbb{R} \setminus \mathbb{Q}\}$ on $\mathbb{R}$. Then $\mathcal{O}^{\Lambda_o} = \mathcal{O}^{V_o}$, but they do not constitute a discrete topology.

(ii) Let $\xi = \{a, b, c\}$, $\tau = \{\emptyset, \xi, \{a\}, \{b, c\}\}$ and $\mathbb{C}(\tau)$ (collection of all closed sets) = $\{\emptyset, \xi, \{a\}, \{b, c\}\}$. Suppose $k = o$. Then, $\{a\}^{\Lambda_o} = \{a\}$, $\{b\}^{\Lambda_o} = \{b, c\}$, $\{c\}^{\Lambda_o} = \{b, c\}$, $\{a, b\}^{\Lambda_o} = \xi$, $\{a, c\}^{\Lambda_o} = \xi$. Λ_o -sets are: $\{a\}, \{b, c\}, \emptyset, \xi$. Also, $\{a\}^{V_o} = \{a\}$, $\{b\}^{V_o} = \emptyset$, $\{c\}^{V_o} = \emptyset$, $\{a, b\}^{V_o} = \emptyset$, $\{b, c\}^{V_o} = \{b, c\}$, $\{a, c\}^{V_o} = \emptyset$. V_o -sets are: $\{a\}, \{b, c\}, \emptyset, \xi$. Then, the space is not a discrete topological space.

We have that $\mathcal{O}^{\Lambda_k}$ (respectively, $\mathcal{O}^{V_k}$) is not closed under the complementation property. Therefore, $\mathcal{O}^{\Lambda_k}$ (respectively, $\mathcal{O}^{V_k}$) will not be a σ -algebra [11] when $\mathcal{O}^{\Lambda_k}$ (respectively, $\mathcal{O}^{V_k}$) is closed under the complementation property. Further, the collections $\mathcal{O}^{\Lambda_k}$ and $\mathcal{O}^{V_k}$ are not equal, but they are always formed algebras [11].

It is interesting that $\mathcal{O}^{\Lambda_k} \cup \mathcal{O}^{V_k}$ does not constitute a σ -algebra as well. Thus, one can called the collection $\mathcal{O}^{\Lambda_k}$ (respectively, $\mathcal{O}^{V_k}$) by pre σ -algebra.

Example 3. Consider $\mathbb{R}$, endowed with topology $\tau = \{\emptyset, \mathbb{R}, \mathbb{Q}\}$ where $\mathbb{Q}$ denotes the set of rationals. Then, $\mathbb{C}(\tau) = \{\emptyset, \mathbb{R}, \mathbb{R} \setminus \mathbb{Q}\}$. Then, $\mathbb{Q}^{\Lambda_o} = \mathbb{Q}$, but $\mathbb{Q}^{V_o} = \emptyset$. Therefore, $\mathcal{O}^{\Lambda_o} \neq \mathcal{O}^{V_o}$.

Again, if $i \in \mathbb{R} \setminus \mathbb{Q}$ and $r \in \mathbb{Q}$, then $\{i, r\} \notin \mathcal{O}^{\Lambda_o}$ and $\{i, r\} \notin \mathcal{O}^{V_o}$. Therefore, $\mathcal{O}^{\Lambda_o}$ and $\mathcal{O}^{V_o}$ do not constitute discrete topologies. Again, $\mathcal{O}^{\Lambda_o} = \{\emptyset, \mathbb{R}, \mathbb{Q}\}$ and $\mathcal{O}^{V_o} = \{\emptyset, \mathbb{R}, \mathbb{R} \setminus \mathbb{Q}\}$ do not form σ -algebra.

Now for the question, does the collection $\mathcal{O}^{\Lambda_k}$ (respectively, $\mathcal{O}^{V_k}$) form a ring under the operations symmetric difference Δ and intersection $\cap$? Obviously, they do not satisfy the closure property with respect to Δ and it is followed by the following example.

Example 4. Recall the Example 3, and $(\mathbb{R} \Delta \mathbb{Q})^{\Lambda_o} = (\mathbb{R} \setminus \mathbb{Q})^{\Lambda_o} = \mathbb{R}$. Thus, $(\mathbb{R} \Delta \mathbb{Q}) \notin \mathcal{O}^{\Lambda_o}$ and $\mathcal{O}^{\Lambda_o}$ does not form a ring.

Theorem 1. For a $\mathbb{T}S (\xi, \tau)$, the following properties are equivalent:

- (i) (ξ, τ) is $k-T_1$;
- (ii) Every subset of ξ is a Λ_k -set;
- (iii) Every subset of ξ is a V_k -set.

where $k-T_1$ is defined as the similar fashion with T_1 topology. That is, a topological space (ξ, τ) is said to be a $k-T_1$ space if and only if each singleton set is k -closed in (ξ, τ) . The reader should not understand that the collection of Λ_k (respectively, V_k) sets always constitutes a discrete topological space. For example.

Consider $\mathbb{R}$ endowed with the topology $\tau = \{\emptyset, \mathbb{R}\} \cup \{(-\frac{1}{n}, \frac{1}{n}) : n \in \mathbb{N}\}$. Then, $\{0\}$ is a Λ_o set, and it is not an open set either. Further, $(4, 5)$ is a Λ_o set. Therefore, $\mathbb{R} \setminus \{0\}$ is a V_o set but $(-\infty, 4) \cup [5, \infty)$ is not V_o set, whereas $\mathbb{R} \setminus \{0\}$ is not a closed set in $(\mathbb{R}, \tau)$. We are also mentioning here that for this example, $\mathcal{O}^o \neq \mathcal{O}^{\Lambda_o} \neq \mathcal{O}^{V_o} \neq \mathcal{O}^k$. It is further mentioned that $\mathcal{O}^{\Lambda_o} \cup \mathcal{O}^{V_o}$ is not a discrete space. Because: $(-5, -4)$ is neither a Λ_o -set nor a V_o -set.

This is a typical example for an infinite space. To obtain the finer collection of $\mathcal{O}^{\Lambda_k}$ (respectively, $\mathcal{O}^{V_k}$), we consider further characterizations of $\mathcal{O}^{\Lambda_k}$ (respectively, $\mathcal{O}^{V_k}$)-sets.

Theorem 2. Let (ξ, τ) be a $\mathbb{T}S$. Then, $\emptyset \neq A \in (\mathcal{O}^{\Lambda_k} \cap \mathcal{O}^{V_k})$ if and only if there exists a nonempty k -open set U and nonempty k -closed set F such that $F \subseteq A \subseteq U$.

Proof: Given that $A \in (\mathcal{O}^{\Lambda_k} \cap \mathcal{O}^{V_k})$. Then, $A^{\Lambda_k} = A = A^{V_k}$. Thus,

$$\bigcap \{U : A \subseteq U \in \mathcal{O}^{\Lambda_k}\} = A = \bigcup \{F : F \subseteq A, F \in \mathbb{C}(\mathcal{O}^{\Lambda_k})\}$$

where $\mathbb{C}(\mathcal{O}^{\Lambda_k})$ is a collection of those sets whose complement is Λ_k set). This implies, $F \subseteq A \subseteq U$.

Conversely, suppose that the condition holds. Now, $\bigcup_{F \subseteq A} F \subseteq \bigcup_{F \subseteq A} A \subseteq \bigcup_{F \subseteq A} U$, where $A \in \mathbb{C}(\mathcal{O}^{\Lambda_k})$. Then, $A^{V_k} \subseteq A \subseteq U$. Again,

$$A^{V_k} \subseteq A \subseteq \bigcap_{A \subseteq U} U \Rightarrow A^{V_k} \subseteq A \subseteq A^{\Lambda_k} \Rightarrow A = A^{\Lambda_k} = A^{V_k} \Rightarrow A \in (\mathcal{O}^{\Lambda_k} \cap \mathcal{O}^{V_k}).$$

Corollary 1. Let (ξ, τ) be a $\mathbb{T}$ S. Then, $A \in (\mathcal{O}^{\Lambda_k} \cap \mathcal{O}^{V_k})$ if and only if A is Λ_k -clopen set.

Starting from the topology on a set, we reached two finer collections $\mathcal{O}^{\Lambda_k}$ and $\mathcal{O}^{V_k}$. Now, we are investigating another extension method.

We define local closeness in Λ_k as well as V_k sets. A subset A of ξ is said to be a $\Lambda_k - V_k$ locally closed if A can be decomposed as $A = B \cap C$, where $B \in \mathcal{O}^{\Lambda_k}$ and $C \in \mathcal{O}^{V_k}$. Collection of all $\Lambda_k - V_k$ locally closed sets in a topological space (ξ, τ) is denoted as $\mathcal{O}^{\Lambda_k - V_k}$.

Proposition 2. Let (ξ, τ) be a $\mathbb{T}$ S. Then,

- (i) $\emptyset, \xi \in \mathcal{O}^{\Lambda_k - V_k}$.
- (ii) $\mathcal{O}^{\Lambda_k} \subseteq \mathcal{O}^{\Lambda_k - V_k}$.
- (iii) $\mathcal{O}^{V_k} \subseteq \mathcal{O}^{\Lambda_k - V_k}$.
- (iv) $\mathcal{O}^{\Lambda_k - V_k}$ closed under arbitrary intersections.
- (v) for $\{O_i : i \in L \text{ (index set)}\} \subseteq \mathcal{O}^{\Lambda_k - V_k}$, $\bigcup_{i \in L} O_i$ is a subset of a locally closed set.

Proof: The proofs are obvious and hence omitted.

Thus, we have an extension of the topology τ and have the relation, $\tau \subseteq \mathcal{O}^{\Lambda_k}$ (respectively, $\mathcal{O}^{V_k} \subseteq \mathcal{O}^{\Lambda_k - V_k}$). But $\mathcal{O}^{\Lambda_k - V_k}$ fails to hold Alexandroff topology as well as topology. Note that $\mathcal{O}^{\Lambda_k - V_k}$ is not a σ -algebra.

To avoid this situation, one can define decomposition by: A subset A of ξ is said to be $\Lambda_k - V_k$ direct sum of B, C if $A = B \cup C$, where $B \in \mathcal{O}^{\Lambda_k}$ and $C \in \mathcal{O}^{V_k}$. Collection of all direct-sum subsets of a $\mathbb{T}$ S (ξ, τ) is denoted as $dir\mathcal{O}^{\Lambda_k - V_k}$. Then, we have $\emptyset, \xi \in dir\mathcal{O}^{\Lambda_k - V_k}$, $dir\mathcal{O}^{\Lambda_k - V_k}$ closed under arbitrary unions but fails to hold the arbitrary intersections property. Therefore, we conclude that $dir\mathcal{O}^{\Lambda_k - V_k}$ does not form an Alexandroff topology as well as a topology.

Again, the following operators may give the further extension: define $\mathfrak{O}^{\Lambda_k}, I^{V_k} : 2^\xi \rightarrow 2^\xi$ by $\mathfrak{O}^{\Lambda_k}(A) = \bigcap \{U : A \subseteq U, U \in \mathcal{O}^{\Lambda_k}\}$ and $I^{V_k}(A) = \bigcup \{F : A \supseteq F, F \in \mathcal{O}^{V_k}\}$. Then, the following hold:

Proposition 3. For any subset A of a topological space (ξ, τ) and for $\{A_\lambda : \lambda \in L\} \subseteq 2^\xi$,

- (i) $A \subseteq \mathfrak{U}^{\Lambda_k}(A)$.
- (ii) $\mathfrak{U}^{\Lambda_k}(A^c) = (I^{V_k}(A))^c$.
- (iii) $\mathfrak{U}^{\Lambda_k}(\emptyset) = \emptyset$.
- (iv) $\mathfrak{U}^{\Lambda_k}(\mathfrak{U}^{\Lambda_k}(A)) = \mathfrak{U}^{\Lambda_k}(A)$.
- (v) $\mathfrak{U}^{\Lambda_k}(A) \subseteq \mathfrak{U}^{\Lambda_k}(B)$, when $A \subseteq B$.
- (vi) $\mathfrak{U}^{\Lambda_k}(A) = A$, when $A \in \mathcal{O}^{\Lambda_k}$.
- (vii) $I^{V_k}(A) = A$, when $A \in \mathcal{O}^{V_k}$.
- (viii) $(\bigcup_{\lambda \in L} \mathfrak{U}^{\Lambda_k}(A_\lambda)) = \mathfrak{U}^{\Lambda_k}(\bigcup_{\lambda \in L} A_\lambda)$.

Proof: The proof of this proposition will be similar to Proposition 4.1 of [9].

Theorem 3. $\mathfrak{U}^{\Lambda_k}$ is a Kuratowski closure operator on ξ .

Suppose τ^{Λ_k} is the topology on ξ induced by $\mathfrak{U}^{\Lambda_k}$ and $\tau^{\Lambda_k^c} = \{A : A \subseteq \xi, \mathfrak{U}^{\Lambda_k}(\xi \setminus A) = \xi \setminus A\}$. We consider a family $\{A : A \subseteq \xi, \mathfrak{U}^{\Lambda_k}(A) = A\}$ which are the collection of closed sets in (ξ, τ^{Λ_k}) . We know that $\tau^{\Lambda_k^c}$ is also formed a topology on ξ and satisfying the following.

Proposition 4. Let (ξ, τ) be a topological space. Then,

- (i) $\tau^{\Lambda_k} = \{A : A \subseteq \xi, I^{V_k}(A) = A\}$.
- (ii) $\mathcal{O}^{\Lambda_k} = \tau^{\Lambda_k^c}$.
- (iii) $\mathcal{O}^{V_k} = \tau^{\Lambda_k^c}$.
- (iv) for $\{A^c : A \in \mathcal{O}^{\Lambda_k}\} = \tau^{\Lambda_k}$, $\mathcal{O}^{\Lambda_k} \subseteq \mathcal{O}^k$.
- (v) for $\mathcal{O}^{\Lambda_k} \subseteq \mathcal{O}^k$, $\tau^{\Lambda_k} = \{A : A \subseteq \xi, A = A^{V_k}\}$.
- (vi) for $\mathcal{O}^{\Lambda_k} \subseteq \mathcal{O}^{k^c}$ (collection of k -closed sets), $\mathcal{O}^{V_k} = \tau^{\Lambda_k}$.

For the next extension of Λ_k and V_k sets, we take the help of ideals in the upcoming section.

3. LOCAL FUNCTIONS ON $\mathcal{O}^{\Lambda_k}$ AND $\mathcal{O}^{V_k}$

In this section, we are fitting a model to discuss the neighbouring points of a set via $\mathcal{O}^{\Lambda_k}$ and $\mathcal{O}^{V_k}$ collections. The tool ideal will be the parameter of these models. A collection $\mathbb{I}$ of subsets of ξ , satisfying hereditary and finite additivity properties, then $\mathbb{I}$ is mentioned as an ideal on ξ and it was first introduced by Kuratowski [12]. This ideal is nothing but the ideal of the ring $(2^\xi, \Delta, \cap)$ where Δ stands for the symmetric difference of two subsets of ξ i.e., for $A, B \in 2^\xi$, $A \Delta B = (A \setminus B) \cup (B \setminus A)$. It is followed by the following lemma:

Lemma 1. Kuratowski's ideal on ξ is synonymous with the ideal of the ring $(2^\xi, \Delta, \cap)$.

Proof: Let $\mathbb{I}$ be an ideal of the ring $(2^\xi, \Delta, \cap)$. Let $A \subseteq B \in \mathbb{I}$. As $A \in 2^\xi$ and $B \in \mathbb{I} \Rightarrow A \cap B = A \in \mathbb{I}$. Therefore, obeys the hereditary property.

Secondly, suppose that $A, B \in \mathbb{I}$. Now, for $A \cup B = (A \Delta B) \Delta (A \cap B)$, $A \cup B \in \mathbb{I}$. Thus, $\mathbb{I}$ is closed under finite additivity.

Conversely, suppose that $\mathbb{I}$ is a Kuratowski's ideal on ξ . Suppose $A \in \mathbb{I}$ and $B \in 2^\xi$. Then, by the hereditary property of $\mathbb{I}$, $A \cap B \in \mathbb{I}$. By finite additivity, $(A \setminus B) \cup (B \setminus A)$ i.e., $A \Delta B \in \mathbb{I}$. Conclude that $\mathbb{I}$ is an ideal of the ring $(2^\xi, \Delta, \cap)$.

Thus, the limit points of a set can be formulated via ideals of the ring $(2^\xi, \Delta, \cap)$. Let A be a subset of ξ . Action of the ideal $\mathbb{I}$ on the set A considered as $A^{\Delta_{\mathbb{I}_k}}(\mathcal{O}^{\Delta_k})$ (or simply $A^{\Delta_{\mathbb{I}_k}} = \{x \in \xi : A \cap O_x \notin \mathbb{I}\}$, where $O_x \in \mathcal{O}^{\Delta_k}(x)$ (the collection of all Δ_k sets containing x) and $A^{\nabla_{\mathbb{I}_k}}(\mathcal{O}^{\nabla_k})$ (or simply $A^{\nabla_{\mathbb{I}_k}} = \{x \in \xi : A \cap W_x \notin \mathbb{I}\}$, where $W_x \in \mathcal{O}^{\nabla_k}(x)$ (the collection of all ∇_k sets containing x)).

Theorem 4. Let (ξ, τ) be a TTS and $\mathbb{I}$ is an ideal of the ring $(2^\xi, \Delta, \cap)$. Then,

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|--|--|
| (i) $\emptyset^{\Delta_{\mathbb{I}_k}} = \emptyset$ | (i) $\emptyset^{\nabla_{\mathbb{I}_k}} = \emptyset$ |
| (ii) $A_1^{\Delta_{\mathbb{I}_k}} \subseteq A_2^{\Delta_{\mathbb{I}_k}}$, when $A_1 \subseteq A_2$. | (ii) $A_1^{\nabla_{\mathbb{I}_k}} \subseteq A_2^{\nabla_{\mathbb{I}_k}}$, when $A_1 \subseteq A_2$ |
| (iii) $((A_1)^{\Delta_{\mathbb{I}_k}})^{\Delta_{\mathbb{I}_k}} \subseteq A_1^{\Delta_{\mathbb{I}_k}}$ | (iii) $((A_1)^{\nabla_{\mathbb{I}_k}})^{\nabla_{\mathbb{I}_k}} \subseteq A_1^{\nabla_{\mathbb{I}_k}}$ |
| (iv) $(A_1 \cup A_2)^{\Delta_{\mathbb{I}_k}} = A_1^{\Delta_{\mathbb{I}_k}} \cup A_2^{\Delta_{\mathbb{I}_k}}$ | (iv) $(A_1 \cup A_2)^{\nabla_{\mathbb{I}_k}} = A_1^{\nabla_{\mathbb{I}_k}} \cup A_2^{\nabla_{\mathbb{I}_k}}$ |
| (v) for $\mathbf{I} \in \mathbb{I}$, $\mathbf{I}^{\Delta_{\mathbb{I}_k}} = \emptyset$ and | (v) for $\mathbf{I} \in \mathbb{I}$, $\mathbf{I}^{\nabla_{\mathbb{I}_k}} = \emptyset$ and |
| $(A_1 \setminus \mathbf{I})^{\Delta_{\mathbb{I}_k}} = A_1^{\Delta_{\mathbb{I}_k}} = (A_1 \cup \mathbf{I})^{\Delta_{\mathbb{I}_k}}$ | $(A_1 \setminus \mathbf{I})^{\nabla_{\mathbb{I}_k}} = A_1^{\nabla_{\mathbb{I}_k}} = (A_1 \cup \mathbf{I})^{\nabla_{\mathbb{I}_k}}$ |

Proof:

(iv) Suppose, $x \in (A_1 \cup A_2)^{\Delta_{\mathbb{I}_k}}$. But $x \notin A_1^{\Delta_{\mathbb{I}_k}} \cup A_2^{\Delta_{\mathbb{I}_k}}$. Then, there exist $O'_x, O''_x \in \mathcal{O}^{\Delta_k}(x)$ such that $A_1 \cap O'_x \in \mathbb{I}$ and $A_2 \cap O''_x \in \mathbb{I}$. This implies that $(A_1 \cap O'_x) \cup (A_2 \cap O''_x) \in \mathbb{I}$, and for $(A_1 \cup A_2) \cap (O'_x \cap O''_x) \subseteq (A_1 \cap O'_x) \cup (A_2 \cap O''_x) \in \mathbb{I}$, $x \notin (A_1 \cup A_2)^{\Delta_{\mathbb{I}_k}}$, a contradiction.

The operator $Cl^{\Delta_{\mathbb{I}_k}}$ (respectively, $Cl^{\nabla_{\mathbb{I}_k}}$) defined as $Cl^{\Delta_{\mathbb{I}_k}}(A_1) = A_1 \cup A_1^{\Delta_{\mathbb{I}_k}}$ (respectively, $Cl^{\nabla_{\mathbb{I}_k}}(A_1) = A_1 \cup A_1^{\nabla_{\mathbb{I}_k}}$) satisfies the following:

- | | |
|---|---|
| (i) $Cl^{\Delta_{\mathbb{I}_k}}(\emptyset) = \emptyset$ | (i) $Cl^{\nabla_{\mathbb{I}_k}}(\emptyset) = \emptyset$ |
| (ii) $Cl^{\Delta_{\mathbb{I}_k}}(Cl^{\Delta_{\mathbb{I}_k}}(A)) \subseteq Cl^{\Delta_{\mathbb{I}_k}}(A)$, for $A \subseteq \xi$. | (ii) $Cl^{\nabla_{\mathbb{I}_k}}(Cl^{\nabla_{\mathbb{I}_k}}(A)) \subseteq Cl^{\nabla_{\mathbb{I}_k}}(A)$, for $A \subseteq \xi$ |
| (iii) $Cl^{\Delta_{\mathbb{I}_k}}(A_1 \cup A_2) = Cl^{\Delta_{\mathbb{I}_k}}(A_1) \cup Cl^{\Delta_{\mathbb{I}_k}}(A_2)$,
for $A_1, A_2 \subseteq \xi$ | (iii) $Cl^{\nabla_{\mathbb{I}_k}}(A_1 \cup A_2) = Cl^{\nabla_{\mathbb{I}_k}}(A_1) \cup Cl^{\nabla_{\mathbb{I}_k}}(A_2)$,
for $A_1, A_2 \subseteq \xi$ |

Thus, $\{A \subseteq \xi : Cl^{\Delta_{\mathbb{I}_k}}(\xi \setminus A) = \xi \setminus A\}$ and $\{A \subseteq \xi : Cl^{\nabla_{\mathbb{I}_k}}(\xi \setminus A) = \xi \setminus A\}$ form topologies on ξ but not form Alexandroff topologies, because:

$$\{A \subseteq \xi : Cl^{\Delta_{I_k}}(\xi \setminus A) = \xi \setminus A\} \text{ and } \{A \subseteq \xi : Cl^{\nabla_{I_k}}(\xi \setminus A) = \xi \setminus A\}$$

contain $\emptyset$, ξ and are closed under arbitrary unions, but not closed under arbitrary intersections. For example, we are considering the following:

Example 4. Let $i \in \mathbb{R} \setminus \mathbb{Q}$ and consider the topology $\tau = \{\emptyset, \mathbb{R}, \mathbb{Q} \cup \{i\}, \mathbb{R} \setminus \mathbb{Q} \cup \{i\}\}$ on the real line $\mathbb{R}$. Here, $\mathbb{Q}$ denotes the set of all rationals. Suppose the ideal $\mathbb{I} = \{\emptyset, \{i\}\}$ plays on the topological space $(\mathbb{R}, \tau)$. Then, $\mathcal{O}^{\Lambda_o} = \{\emptyset, \mathbb{R}, \mathbb{Q} \cup \{i\}, \mathbb{R} \setminus \mathbb{Q} \cup \{i\}\} = \mathcal{O}^{V_o}$. Then, $(\mathbb{R} \setminus \mathbb{Q})^{\Delta_{I_o}} = \mathbb{R} = (\mathbb{R} \setminus \mathbb{Q})^{\nabla_{I_o}}$ and for each $r \in \mathbb{R} \setminus \mathbb{Q} \cup \{i\}$, $\{r\}^{\Delta_{I_o}} = \mathbb{R} \setminus \mathbb{Q} \cup \{i\} = \{r\}^{\nabla_{I_o}}$, $\{i\}^{\Delta_{I_o}} = \emptyset = \{i\}^{\nabla_{I_o}}$. Therefore, $(\mathbb{R} \setminus \mathbb{Q})^{\Delta_{I_o}} = \mathbb{R} = (\mathbb{R} \setminus \mathbb{Q})^{\nabla_{I_o}} \neq \bigcup_{r \in \mathbb{R} \setminus \mathbb{Q}} \{r\}^{\Delta_{I_o}} = \mathbb{R} \setminus \mathbb{Q} \cup \{i\} = \bigcup_{r \in \mathbb{R} \setminus \mathbb{Q}} \{r\}^{\nabla_{I_o}}$.

These two topologies are called $^*\Lambda_k$ and *V_k topologies respectively and they are denoted as $\tau^{*\Lambda_k}$ and τ^{*V_k} respectively.

Lemma 2. Let (ξ, τ) be a TS and $\mathbb{I}$ be an ideal of the ring $(2^\xi, \Delta, \cap)$. Then, $\mathbb{B}_{\Lambda_k} = \{O \setminus \mathbf{I} : O \in \mathcal{O}^{\Lambda_k}, \mathbf{I} \in \mathbb{I}\}$ and $\mathbb{B}_{V_k} = \{O \setminus \mathbf{I} : O \in \mathcal{O}^{V_k}, \mathbf{I} \in \mathbb{I}\}$ are bases for the topologies $^*\Lambda_k$ and *V_k respectively.

Proof: For each member x of ξ , we can obviously get an element from $\mathbb{B}_{\Lambda_k}$ which contains x . Let $t \in B_1 \cap B_2$, where $B_1, B_2 \in \mathbb{B}_{\Lambda_k}$. Then we choose $B_1 = O_1 \setminus \mathbf{I}_1$ and $B_2 = O_2 \setminus \mathbf{I}_2$. Consider $B_3 = B_1 \cap B_2 = (O_1 \setminus \mathbf{I}_1) \cap (O_2 \setminus \mathbf{I}_2) = O_1 \cap O_2 \setminus \mathbf{I}_1 \cup \mathbf{I}_2 \in \mathbb{B}_{\Lambda_k}$. Thus, $\mathbb{B}_{\Lambda_k}$ is a basis.

Now, we shall show that $\mathbb{B}_{\Lambda_k}$ is a basis for the topology $^*\Lambda_k$ on ξ . Suppose A is a $^*\Lambda_k$ closed set. Then, $Cl^{\Delta_{I_k}}(A) = A \cup A^{\Delta_{I_k}} = A \Rightarrow A^{\Delta_{I_k}} \subseteq A$. Now, suppose O is a $^*\Lambda_k$ open set. Then, $\xi \setminus O$ is a $^*\Lambda_k$ closed set $\Rightarrow (\xi \setminus O)^{\Delta_{I_k}} \subseteq (\xi \setminus O) \Rightarrow O \subseteq \xi \setminus (\xi \setminus O)^{\Delta_{I_k}}$. Thus, for $x \in O$, $x \notin (\xi \setminus O)^{\Delta_{I_k}}$. Then, there exists a $O' \in \mathcal{O}^{\Delta_{I_k}}(x)$ such that $O' \cap (\xi \setminus O) \in \mathbb{I}$. Put $\mathbf{I} = O' \cap (\xi \setminus O)$ and we have $x \in O' \setminus \mathbf{I} \subseteq O$. Hence, $\mathbb{B}_{\Lambda_k}$ is a basis for the $^*\Lambda_k$ topology on ξ .

Note that $\tau \subseteq \mathcal{O}^{\Lambda_k} \subseteq \mathcal{O}^{*\Lambda_k}$ (collection of all open sets of $^*\Lambda_k$ topology) and $\tau \subseteq \mathcal{O}^{V_k} \subseteq \mathcal{O}^{*V_k}$ (collection of all open sets of *V_k topology). For further comparison, we recall the basis of the * -topology τ^* [13], $\mathbf{B}(\mathbb{I}, \tau) = \{V \setminus \mathbf{I} : V \in \tau, \mathbf{I} \in \mathbb{I}\}$. We observed that $\tau^* \subseteq \mathcal{O}^{*\Lambda_k}$ and $\tau^* \subseteq \mathcal{O}^{*V_k}$. Further extension through Δ_{I_k} and ∇_{I_k} operators using the facts of [14-15] may be possible, and these extensions give the α sets [6] of $^*\Lambda_k$ and *V_k respectively. To do this, one can use the author Njåstad's expression of α sets, $O \setminus N$ where $O \in \mathcal{O}^{*\Lambda_k}$ (respectively, $\mathcal{O}^{*V_k}$) and N is the nowhere dense set [16] in $(\xi, \mathcal{O}^{*\Lambda_k})$ (respectively,

$\mathcal{O}^{*V_\alpha})$ & this expression is equal to $(O \setminus N) \subseteq \text{Int}_{*\Lambda_k} \text{Cl}_{*\Lambda_k}(\xi \setminus (\xi \setminus (O \setminus N))^{\Delta_{\mathbb{I}_k}})$ (respectively, $\text{Int}_{*V_k} \text{Cl}_{*V_k}(\xi \setminus (\xi \setminus (O \setminus N))^{\nabla_{\mathbb{I}_k}})$).

Lemma 3. The ideal $\mathbb{I}$ of the ring $(2^\xi, \Delta, \cap)$ is playing on the $\mathbb{TS}(\xi, \tau)$ through $G \subseteq \xi$ by:

- (i) $G^{\Delta_{\mathbb{I}_k}}$ is $*\Lambda_k$ closed. (i) $G^{\nabla_{\mathbb{I}_k}}$ is $*V_k$ closed
(ii) for $\xi = \xi^{\Delta_{\mathbb{I}_k}}$ and $G \in \mathcal{O}^{*\Lambda_k}$, (ii) for $\xi = \xi^{\nabla_{\mathbb{I}_k}}$ and $G \in \mathcal{O}^{*V_k}$,
 $\text{Cl}^{\Delta_{\mathbb{I}_k}}(G) = \text{Cl}_{\Lambda_k}(G)$. $\text{Cl}^{\nabla_{\mathbb{I}_k}}(G) = \text{Cl}_{V_k}(G)$.

Lemma 4. The ideal $\mathbb{I}$ of the ring $(2^\xi, \Delta, \cap)$ can be made the following equivalent conditions on the $\mathbb{TS}(\xi, \tau)$:

- (i) $\{\emptyset\} = \mathbb{I} \cap \mathcal{O}^{\Lambda_k}$ (i) $\{\emptyset\} = \mathbb{I} \cap \mathcal{O}^{V_k}$
(ii) $\xi = \xi^{\Delta_{\mathbb{I}_k}}$ (ii) $\xi = \xi^{\nabla_{\mathbb{I}_k}}$
(iii) for any $O \in \mathcal{O}^{\Lambda_k}$, $O^{\Delta_{\mathbb{I}_k}} = \text{Cl}_{\Lambda_k}(O)$. (iii) for any $O \in \mathcal{O}^{V_k}$, $O^{\nabla_{\mathbb{I}_k}} = \text{Cl}_{V_k}(O)$

Proof:

(i) $\Rightarrow$ (ii) From definition, $\xi^{\Delta_{\mathbb{I}_k}} \subseteq \xi$.

For reverse inclusion let $p \in \xi$. If possible, let $p \notin \xi^{\Delta_{\mathbb{I}_k}}$. Then, $\exists U \in \mathcal{O}^{\Lambda_k(p)}$, collection of all k sets containing p such that $U \cap \xi \in \mathbb{I}$. This implies $U \in \mathbb{I}$, a contradiction. Hence, $\xi \subseteq \xi^{\Delta_{\mathbb{I}_k}}$. Thus, $\xi = \xi^{\Delta_{\mathbb{I}_k}}$.

(ii) $\Rightarrow$ (i) Assume that $\xi = \xi^{\Delta_{\mathbb{I}_k}}$ holds. If possible, let $\emptyset \neq V \in \mathbb{I} \cap \mathcal{O}^{\Lambda_k}$ and $p \in V$. Then $V \cap \xi \subseteq V \in \mathbb{I} \cap \mathcal{O}^{\Lambda_k}$. This implies $V \cap \xi \in \mathbb{I} \cap \mathcal{O}^{\Lambda_k}$ and hence $V \cap \xi \in \mathbb{I}$. Thus $p \notin \xi^{\Delta_{\mathbb{I}_k}}$, a contradiction. Thus, $\{\emptyset\} = \mathbb{I} \cap \mathcal{O}^{\Lambda_k}$.

(i) $\Rightarrow$ (iii) Assume that $\{\emptyset\} = \mathbb{I} \cap \mathcal{O}^{\Lambda_k}$.

Let $p \in \mathcal{O}^{\Lambda_k}$. Then for all $U \in \mathcal{O}^{\Lambda_k(p)}$ such that $U \cap O \notin \mathbb{I}$. This implies $U \cap O \neq \emptyset$ and hence $p \in \text{Cl}_k(O)$. Thus, $O^{\Delta_{\mathbb{I}_k}} \subseteq \text{Cl}_k(O)$. Also, take $p \in \text{Cl}_k(O)$. Then, for all $O_p \in \mathcal{O}^{\Lambda_k(p)}$ such that $O_p \cap O \neq \emptyset$. This implies, $O_p \cap O \notin \mathbb{I}$ for all $O_p \in \mathcal{O}^{\Lambda_k(p)}$ such that $O_p \cap O \neq \emptyset$ as $\{\emptyset\} = \mathbb{I} \cap \mathcal{O}^{\Lambda_k}$. Thus, $p \in O^{\Delta_{\mathbb{I}_k}}$ and hence $\text{Cl}_k(O) \subseteq O^{\Delta_{\mathbb{I}_k}}$. Therefore, for any $O \in \mathcal{O}^{\Lambda_k}$, $O^{\Delta_{\mathbb{I}_k}} = \text{Cl}_k(O)$.

(iii) $\Rightarrow$ (i) Assume for any $O \in \mathcal{O}^{\Lambda_k}$, $O^{\Delta_{\mathbb{I}_k}} = \text{Cl}_k(O)$. If possible, let, $\emptyset \neq V \in \mathbb{I} \cap \mathcal{O}^{\Lambda_k}$. Then, $V \in \mathbb{I}$ and $V \in \mathcal{O}^{\Lambda_k}$. Thus, $V^{\Delta_{\mathbb{I}_k}} = \text{Cl}_k(V)$ and $V^{\Delta_{\mathbb{I}_k}} = \emptyset$. But, $\text{Cl}_k(V) \neq \emptyset$, a contradiction. Thus, $\{\emptyset\} = \mathbb{I} \cap \mathcal{O}^{\Lambda_k}$.

Lemma 5. Let (ξ, τ) be a $\mathbb{TS}$. Then, the ideal $\mathbb{I}$ of the ring $(2^\xi, \Delta, \cap)$ can be obtained the following more ideals on the ring $(2^\xi, \Delta, \cap)$:

- (i) $\mathbb{I}_{nd}^{\Lambda_k} = \{A \subseteq \xi : \text{Int}_{\Lambda_k}(\text{Cl}_{\Lambda_k}(A)) = \emptyset\}$.

$$(ii) \mathbb{I}_{nd}^{V_k} = \{A \subseteq \xi : Int_{V_k}(Cl_{V_k}(A)) = \emptyset\}.$$

Proof: (i) It is obvious that $A_1 \subseteq A_2 \in \mathbb{I}_{nd}^\Lambda \Rightarrow A_1 \in \mathbb{I}_{nd}^\Lambda$.

Suppose $Int_{\Lambda_k}(Cl_{\Lambda_k}(A_1)) = \emptyset$ and $Int_{\Lambda_k}(Cl_{\Lambda_k}(A_2)) = \emptyset$. Let $O = Int_{\Lambda_k}(Cl_{\Lambda_k}(A_1 \cup A_2))$.

Claim: $O = \emptyset$

Given that $O \subseteq Cl_{\Lambda_k}(A_1 \cup A_2) = Cl_{\Lambda_k}(A_1) \cup Cl_{\Lambda_k}(A_2)$. Now $O \cap \xi \setminus Cl_{\Lambda_k}(A_2) \subseteq Cl_{\Lambda_k}(A_1) \cup Cl_{\Lambda_k}(A_2) \cap \xi \setminus Cl_{\Lambda_k}(A_2) = Cl_{\Lambda_k}(A_1) \cap \xi \setminus Cl_{\Lambda_k}(A_2) \subseteq Cl_{\Lambda_k}(A_1)$
 $\Rightarrow Int_{\Lambda_k}(O \cap \xi \setminus Cl_{\Lambda_k}(A_2)) \subseteq Int_{\Lambda_k}(Cl_{\Lambda_k}(A_1)) = \emptyset \Rightarrow Int_{\Lambda_k} O \cap Int_{\Lambda_k}(\xi \setminus Cl_{\Lambda_k}(A_2)) \subseteq \emptyset$
 $\Rightarrow O \cap \xi \setminus Cl_{\Lambda_k}(A_2) \subseteq \emptyset \Rightarrow O \subseteq \xi \setminus (\xi \setminus Cl_{\Lambda_k}(A_2)) = Cl_{\Lambda_k}(A_2) \Rightarrow Int_{\Lambda_k} O \subseteq Int_{\Lambda_k}(Cl_{\Lambda_k}(A_2))$
 $= \emptyset \Rightarrow O = \emptyset \Rightarrow A_1 \cup A_2 \in \mathbb{I}_{nd}^\Lambda$.

Corollary 2. Let (ξ, τ) be a $\mathbb{T}S$. Then, the ideal $\mathbb{I}$ of the ring $(2^\xi, \Delta, \cap)$ has the following roles for obtaining α set:

$$(i) (\mathcal{O}^{\Lambda_k})^\alpha = \{O \setminus N : O \in \mathcal{O}^{\Lambda_k}, N \in \mathbb{I}_{nd}^{\Lambda_k}\}.$$

$$(ii) (\mathcal{O}^{V_k})^\alpha = \{O \setminus N : O \in \mathcal{O}^{V_k}, N \in \mathbb{I}_{nd}^{V_k}\}.$$

For the ideal $\mathbb{I}$ of the ring $(2^\xi, \Delta, \cap)$ with a $\mathbb{T}S$ (ξ, τ) , $\mathbb{I}_{nd}^k \cap \mathcal{O}^{\Lambda_k} = \emptyset$ and $\mathbb{I}_{nd}^k \cap \mathcal{O}^{V_k} = \emptyset$ hold.

4. DYNAMICAL SYSTEM WITH RESPECT TO Λ_k AND V_k SETS

Let (ξ, τ) (or simply ξ) be a $\mathbb{T}S$. A continuous function $h: \xi \rightarrow \xi$ together with the space ξ is called the Dynamical system [17]. This continuity with respect to τ does not mean that h continuous with respect to $\mathcal{O}^{\Lambda_k}$ topology (respectively, $\mathcal{O}^{V_k}$ topology) and $\mathcal{O}^{*\Lambda_k}$ (respectively, $\mathcal{O}^{*V_k}$) topology. So, it is important that, dynamical system depends upon topology. That is if (ξ, h) is a dynamical system with respect to the topology τ , then it is not necessarily a dynamical system with respect to $\mathcal{O}^{\Lambda_k}$ topology denoted as $(\xi, h)^{\mathcal{O}^{\Lambda_k}}$.

For example, we consider the map $h: \mathbb{R} \rightarrow \mathbb{R}$ defined as $h(x) = \begin{cases} 1, & \text{if } x \in \mathbb{Q}; \\ 0, & \text{if } x \in \mathbb{R} \setminus \mathbb{Q}. \end{cases}$

Usual topology $\mathbb{R}_u$ acts on $\mathbb{R}$ is not a dynamical system. But, h is a dynamical system when $\mathbb{R}_u^{\Lambda_k}$ plays on $\mathbb{R}$.

Dynamical system and topological transitivity (see [18-19]) are to much related to the dense set (see [16]). In this connection dense set with respect to the original topology is not certainly equal to the dense set with respect to Λ_k sets related topology. For example, for the usual topology on the real line $\mathbb{R}$, the set $\{0\}$ is the intersection of the sets

$\{(-\frac{1}{n}, \frac{1}{n}) : n \in \mathbb{N}\}$, that is $\{0\}$ is the Λ_0 set, but it is not an open set and does not contain any non-empty interior. So, there is no scope for equality of collection of dense set with respect to usual topology on $\mathbb{R}$ with the collection of dense sets with respect to the collection of Λ_0 sets of the usual topology. However, it is worthy that, collection of dense sets with respect to the collection of Λ_k sets of a topological space (ξ, τ) is the subcollection of the collection of dense sets with respect to the τ topology.

Let A be a subset of a $\mathbb{TS}(\xi, \tau)$. A point $p \in A$ is called a Λ_k (respectively, V_k)-isolated point of A if $\exists O \in \mathcal{O}^{\Lambda_k}$ (respectively, $\mathcal{O}^{V_k}$) such that $O \cap A = \{p\}$.

A $\mathbb{TS}(\xi, \tau)$ is called Λ_k (respectively, V_k)-perfect if it does not have Λ_k -isolated point or equivalently cardinality of every non-empty Λ_k (respectively, V_k)-set is infinite.

Let (ξ, τ) be a $\mathbb{TS}$. A function $h: \xi \rightarrow \xi$ is called Λ_k (respectively, V_k) orbit-transitive if $\exists$ a point x in ξ such that $Cl_{\Lambda_k}(\mathbb{O}[x, h])$ (respectively, $Cl_{V_k}(\mathbb{O}[x, h])$) = ξ . where $\mathbb{O}[x, h] = \{x, h(x), h^2(x), \dots\}$.

Let (ξ, τ) be a $\mathbb{TS}$. A function $h: \xi \rightarrow \xi$ is called strictly Λ_k (respectively, V_k) orbit-transitive if $\exists$ a point x in ξ such that $Cl_{\Lambda_k}(\mathbb{O}[h(x), h])$ (respectively, $Cl_{V_k}(\mathbb{O}[h(x), h])$) = ξ .

Let (ξ, τ) be a $\mathbb{TS}$. A function $h: \xi \rightarrow \xi$ is called Λ_k -irresolute (respectively, V_k -irresolute) if and only if for every $U \in \mathcal{O}^{\Lambda_k}$ (respectively, $U \in \mathcal{O}^{V_k}$), $h^{-1}(U) \in \mathcal{O}^{\Lambda_k}$ (respectively, $h^{-1}(U) \in \mathcal{O}^{V_k}$).

In a dynamical system (ξ, h) , a point y is called a ω -limit (respectively, ω - $\overline{\text{limit}}$) point of x under h [6, 18, 19] if for any $n \in \mathbb{N}$ and for any open subset U of ξ such that $y \in U$, $\exists$ a positive integer $m \geq n$ such that $h^m(x) \in U$ (respectively, $Cl(U)$). The set of all ω -limit (respectively, ω - $\overline{\text{limit}}$) points of x under h , is denoted as $\omega(x, h)$ (respectively, $Cl(\omega(x, h))$). It is obvious that $\omega(x, h) \subseteq Cl(\omega(x, h))$.

Definition 1. [9] Let (ξ, h) be a Dynamical system. For any subsets A_1 and A_2 of ξ , define the hitting times subsets of $\mathbb{Z}$:

- (i) $N_h(A_1, A_2) = \{p \in \mathbb{Z} : A_1 \cap h^{-p}(A_2) \neq \emptyset\}$.
- (ii) $n_h(A_1, A_2) = \{p \in \mathbb{Z}_+ : A_1 \cap h^{-p}(A_2) \neq \emptyset\}$.

A Dynamical system (ξ, h) is called $\mathbb{Z}_k$ -transitive, if for every pair of non-empty $U, V \in \mathcal{O}^{\Lambda_k}$, the subset $N_f(U, V) \neq \emptyset$ and the Dynamical system (ξ, h) is called $\mathbb{Z}_{+k}$ -transitive, if, for each pair of non-empty $U, V \in \mathcal{O}^{\Lambda_k}$, the subset $n_h(U, V) \neq \emptyset$.

Hence, for $l \geq 0$, we have $l \in N_h(A_1, A_2)$ if and only if $l \in n_h(A_1, A_2)$ for $l \leq 0$, we have $l \in N_h(A_1, A_2)$ if and only if $-l \in n_h(A_1, A_2)$. Thus, $N_h(A_1, A_2) = n_h(A_1, A_2) \cup -n_h(A_1, A_2)$.

Lemma 6. Let (ξ, h) be a Dynamical system such that ξ has no Λ_k quasi-isolated points.

Then, for any $n \in \mathbb{N}$, a point $x \in \xi$ is a Λ_k -transitive point of $\hat{h}$ if and only if $\hat{h}^n(x)$ is a Λ_k -transitive point of $\hat{h}$.

Proposition 5. Let (ξ, h) be a Dynamical system. If $\hat{h}$ is Λ_k -irresolute on ξ has no Λ_k quasi-isolated points, then $\hat{h}$ is ω_{Λ_k} -transitive if and only if $\hat{h}$ is strictly Λ_k orbit-transitive.

Proof: Sufficient part: Let $\hat{h}$ be strictly k orbit-transitive. Then, $\exists x \in \xi$ such that $Cl_{\Lambda_k}(\mathbb{O}(\hat{h}(x), \hat{h})) = \xi$.

Assume $\hat{h}$ is Λ_k -irresolute, then for any $n \in \mathbb{N}$, $\hat{h}^n$ is Λ_k -irresolute. Consider any non-empty member U containing x of $\mathcal{O}^k$, then $\mathbb{O}[\hat{h}(x), \hat{h}] \cap U \neq \emptyset$. Then, $\exists m \in \mathbb{N}$ such that $\hat{h}^m(x) \in U$ and $\exists$ a member U_1 containing x of $\mathcal{O}^k$ such that $\hat{h}^m(U_1) \subseteq U$. Also, by strictly k orbit-transitive of $\hat{h}$, there exists $n_1 \in \mathbb{N}$ such that $\hat{h}^{n_1} \in U_1$. Taking, $m_1 = m + n_1$. Then, $\{\hat{h}^m(x), \hat{h}^{m_1}(x)\} \subseteq U$. Continuing this process, we get a sequence of positive integers $m < m_1 < m_2 < m_3 < \dots$ such that $\{\hat{h}^m(x), \hat{h}^{m_1}(x), \hat{h}^{m_2}(x), \dots\} \subseteq U$. This implies, $x \in \omega_k(x, \hat{h})$. Since, $\hat{h}$ is Λ_k -irresolute, then $\hat{h}(\omega_k(x, \hat{h})) \subseteq \omega_{\Lambda_k}(x, \hat{h})$ and hence $\mathbb{O}[x, \hat{h}] \subseteq \omega_{\Lambda_k}(x, \hat{h})$. Also, since $\xi \setminus \omega_{\Lambda_k}(x, \hat{h}) \in \mathcal{O}^k(\xi)$, then $\omega_{\Lambda_k}(x, \hat{h}) \supset Cl_{\Lambda_k} \mathbb{O}[x, \hat{h}] \supset Cl_{\Lambda_k} \mathbb{O}[f(x), \hat{h}] = \xi$. Thus, $\hat{h}$ is ω_{Λ_k} -transitive.

Assume ξ has no k quasi-isolated points, then by Lemma 6 all the points in the orbit $\mathbb{O}[x, \hat{h}]$ are k transitive points of $\hat{h}$. Then, for any $y \in \xi$, any member U containing y of $\mathcal{O}^k$, there exists $m \geq n$ such that $\hat{h}^m(x) \in U$. This implies that $y \in \omega_{\Lambda_k}(x, \hat{h})$. Hence $\omega_{\Lambda_k}(x, \hat{h}) = \xi$. Thus, $\hat{h}$ is ω_{Λ_k} -transitive. This completes the proof.

Note 1. All of the above facts are true for the collections $\mathcal{O}^{V_k}$ of the topological space (ξ, τ) .

5. ω -LIMIT POINTS VS CLUSTER POINTS OF FILTERS

A filter is a dual concept of an ideal. That is a filter $\mathbb{F}$ is a collection of subsets of ξ such that $\mathbb{I} = \{A \subseteq \xi : \xi \setminus A \in \mathbb{F}\}$ is a non trivial proper ideal of the ring $(2^\xi, \Delta, \cap)$.

Let $\mathbb{F}$ be a filter on the set ξ . Then, a subfamily β of $\mathbb{F}$ is said to be a base [20] for $\mathbb{F}$ if for any $A \in \mathbb{F}$, there exists $B \in \beta$ such that $B \subseteq A$.

Let (ξ, τ) be a TS and let $\mathbb{F}$ be a filter on ξ . A point x of ξ is said to be a limit [12] of $\mathbb{F}$ with respect to τ (or $\mathbb{F}$ is said to converge to x with respect to τ) if every neighbourhood of x belongs to $\mathbb{F}$.

In a Dynamical system (ξ, h) and for $x \in \xi$, we consider $A_n = \{h^m(x) : m \geq n\}$. Then, $\mathbb{F}_{xh} = \{A \subseteq \xi : A \supseteq A_n \text{ for some } n \in \mathbb{N}\}$ is a filter on ξ .

It is obvious that $\lambda(\mathbb{F}_{xh}) = \omega(x, h)$ (denotes the collection of all ω limits of x under h) $\subseteq \partial(\mathbb{F}_{xh})$ (where $\partial(\mathbb{F}_{xh})$ and $\lambda(\mathbb{F}_{xh})$ denote the set of all cluster points of $\mathbb{F}_{xh}$ and the set of all limits of $\mathbb{F}_{xh}$ respectively). But converse relation need not hold in general.

Example 5. Consider the subspace $\mathbb{N}$ of the usual $\mathbb{T}\mathbb{S}$ $\mathbb{R}$. Define $h : \mathbb{N} \rightarrow \mathbb{N}$ by $h(x) = x^2$. Let $m = 2$ and $x = 2$, then $A_2 = \{16, 256, \dots\}$. The filter $\mathbb{F}_{2h}$ does not converge and $\omega(2, h) = \emptyset$. But $16 \in \partial(\mathbb{F}_{2h})$.

However, if the filter $\mathbb{F}_{xh}$ is an ultrafilter, then the three sets of the above relation coincide. One can define the limits and cluster point of a filter on a Alexandroff $\mathbb{T}\mathbb{S}$. In this situation, we can discuss these on the Alexandroff $\mathbb{T}\mathbb{S}$ $\mathcal{O}^{\Lambda_k}$ and $\mathcal{O}^{V_k}$ induced from a certain $\mathbb{T}\mathbb{S}$ (ξ, τ) .

Thus, for a Dynamical system (ξ, h) , $\lambda_k(\mathbb{F}_{xh}) = \omega_k(x, h) \subseteq \partial_k(\mathbb{F}_{xh})$. These sets will be the same when the filter $\mathbb{F}_{xh}$ will be the ultrafilter.

Now we shall discuss a few words about the proof of $\lambda(\mathbb{F}_{xh}) = \omega(x, h)$. Suppose $y \in \omega(x, h)$. Then, for $y \in U \in \mathcal{O}(y)$, there exists $m \in \mathbb{N}$ such that $h^m(x) \in U$. Hence $U \in \mathbb{F}_{xh}$ for $m = p$. On the other hand, if $t \in \lambda(\mathbb{F}_{xh})$, then $\mathcal{O}(t) \subseteq \mathbb{F}_{xh}$. Thus, $\exists$ some $r \in \mathbb{N}$ such that $O_t \subseteq A_n$ for $n \geq r$. Hence, $t \in \omega(x, h)$.

Theorem 5. In a dynamical system (ξ, h) , $t \in \lambda(\mathbb{F}_{xh}) \Rightarrow h(t) \in \lambda(h_*(\mathbb{F}_{xh}))$.

Proof: Let $h(\mathbb{F}_{xh})$ be a base for some filter $\mathbb{F}_1$ on ξ and let V be any open set containing $h(t)$. Thus, $h(t) \in V$ where $V \in \tau$. This implies, $t \in h^{-1}(V)$. Thus, $h^{-1}(V)$ is an open set containing t as h is continuous map. So, $h^{-1}(V) \in \mathbb{F}_{xh}$ as $t \in \lambda(\mathbb{F}_{xh})$ and hence $V \in h(\mathbb{F}_{xh})$. Thus, $V \in \mathbb{F}_1$ since $h(\mathbb{F}_{xh})$ is a base of filter $\mathbb{F}_1$. This implies, $h(t) \in \lambda(h_*(\mathbb{F}_{xh}))$.

We know that every filter is contained in an ultrafilter [12].

Theorem 6. [20] The following equivalent statements hold for a $\mathbb{T}\mathbb{S}$ (ξ, τ) :

- (i) ξ is compact;
- (ii) Every filter on ξ has a cluster point in ξ ;
- (iii) Every filter on ξ has a convergent subfilter.

Theorem 7. Let (ξ, h) be a dynamical system where ξ is a compact space. Then,

- (i) the filter $\mathbb{F}_{xh}$ has a convergent subfilter.
- (ii) for ultrafilter $\mathbb{U}_{xh}$ containing the filter $\mathbb{F}_{xh}$, $\partial(\mathbb{U}_{xh}) \neq \emptyset$.

$$(iii) \omega(x, \hbar) = \lambda(\mathbb{F}_{x\hbar}) \subseteq \lambda(\mathbb{U}_{x\hbar}) = \partial(\mathbb{U}_{x\hbar}).$$

Proof: The proof is straightforward and hence omitted.

In a dynamical system $(\xi, \hbar)$ with an ideal $\mathbb{I}$ on ξ , a point y is called a $\star^\omega$ -limit (respectively, $(\Delta_{\mathbb{I}_k})^\omega$ -limit) point of x under $\hbar$ if for any $n \in \mathbb{N}$ and for any $U \in \xi$ such that $y \in U$, $\exists$ a positive integer $m \geq n$ such that $\hbar^m(x) \in U^*(\mathbb{I})$ (respectively, $\hbar^m(x) \in U^{\Delta_{\mathbb{I}_k}}$). The set of all $\star^\omega$ -limit (respectively, $(\Delta_{\mathbb{I}_k})^\omega$ -limit) points of x under $\hbar$, is denoted as $\star^\omega(x, \hbar)$ (respectively, $(\Delta_{\mathbb{I}_k})^\omega(x, \hbar)$).

It is obvious that $\star^\omega(x, \hbar) \subseteq (\Delta_{\mathbb{I}_o})^\omega(x, \hbar) \subseteq (\Delta_{\mathbb{I}_{o\delta}})^\omega(x, \hbar)$ (respectively, $(\Delta_{\mathbb{I}_{\delta o}})^\omega(x, \hbar) \subseteq (\Delta_{\mathbb{I}_{o\delta \cup \delta o}})^\omega(x, \hbar) \subseteq (\Delta_{\mathbb{I}_{\delta o \delta}})^\omega(x, \hbar)$).

Proposition 4. In a dynamical system $(\xi, \hbar)$ with an ideal $\mathbb{I}$ of the ring $(2^\xi, \Delta, \cap)$, then the following hold:

- (i) $\star^\omega(x, \hbar) \subseteq Cl(\omega(x, \hbar))$;
- (ii) $\star^\omega(x, \hbar) \subseteq \xi^{*\mathbb{I}}$ if $\star^\omega(x, \hbar) = \omega(x, \hbar)$;
- (iii) $(\Delta_{\mathbb{I}_k})^\omega(x, \hbar) \subseteq \xi^{\Delta_{\mathbb{I}_k}}$ if $(\Delta_{\mathbb{I}_k})^\omega(x, \hbar) = \omega_{\Lambda_k}(x, \hbar)$ where $\Lambda_k \in \mathcal{O}^{\Lambda_k}$.

Proof:

(i) Let $y \in \star^\omega(x, \hbar)$. Then, for any $n \in \mathbb{N}$ and for any open subset U of ξ with $y \in U$, $\exists$ a positive integer $m \geq n$ such that $\hbar^m(x) \in U^*(\mathbb{I})$. Also $U^*(\mathbb{I}) \subseteq Cl(U)$. This implies, $\hbar^m(x) \in Cl(U)$ and hence $y \in Cl(\omega(x, \hbar))$. Hence the result.

(ii) If possible, let $\exists y \in \star^\omega(x, \hbar)$ such that $y \notin \xi^*$. Since $y \notin \xi^*$, then $\exists y \in O_y \in \tau(y)$ such that $O_y \cap \xi \in \mathbb{I}$. This implies, $O_y \in \mathbb{I}$.

Also, since $y \in \star^\omega(x, \hbar) = \omega(x, \hbar)$, then for any $n \in \mathbb{N}$, $\exists m \geq n$ such that $\hbar^m(x) \in O_y^*(\mathbb{I})$ and $\hbar^m(x) \in O_y$. This implies, $O_y \cap O_y \notin \mathbb{I}$, a contradiction. This proves the result.

(iii) Similar to (ii).

4. CONCLUSIONS

Join study of algebraic ideals with dynamical system, filters and Λ and V sets are made through this article. The idea is new; therefore, there are many scopes for further study of this concept, like: (i) study of Λ and V sets in front of different types of ideals like maximal ideals, prime ideals; (ii) further study of dynamical systems with the help of several ideals; (iii) study of different limits of a filter through several ideals of $(2^\xi, \Delta, \cap)$; (iv) making several topologies through several ideals of $(2^\xi, \Delta, \cap)$ to discuss the localization property of a set; (v) several examples in the real line help to discuss various counterparts.

REFERENCES

- [1] Maki, H., Generalized Λ -Sets and the Associated Closure Operator, *The Special Issue in Commemoration of Prof. Kazusada IkEDA's Retirement* **1**, 139, 1986.
- [2] Andrijević, D., On b -Open Sets, *Matematički Vesnik*, **48**, 59, 1996.
- [3] Andrijević, D., Semi-preopen sets, *Matematički Vesnik*, **38**, 24, 1986.
- [4] Levine, N., Semi-open sets and semi-continuous mappings in topological spaces, *The American Mathematical Monthly*, **70**, 36, 1963.
- [5] Mashhour, A. S., Abd El-monsef, M. E., El-Deeb, N., On Precontinuous and Weak Precontinuous Mappings, *Proceedings of the Mathematical and Physical Society of Egypt*, **53**, 47, 1982.
- [6] Njåstad, O., On Some Classes of Nearly Open Sets, *Pacific Journal of Mathematics*, **15**(3), 961, 1965.
- [7] Abd El-Monsef, M. E., El-Deeb, S. N., Mahmoud, R. A., β -Open Sets and β -Continuous Mappings, *Bulletin of the Faculty of Science, Assiut University*, **12**, 77, 1983.
- [8] Caldas, M., Jafari, S., Noiri, T., On Λb -sets and the associated topology $\tau_{\Lambda b}$, *Acta Mathematica Hungarica*, **110**(4), 337, 2006. <https://doi.org/10.1007/S10474-006-0028-6>
- [9] Akin, E., Carlson, J. D., Conceptions of topological transitivity, *Topology and its Applications*, **159**, 2815, 2012. <https://doi.org/10.1016/j.topol.2012.04.016>
- [10] Arenas, F. G., Alexandroff spaces, *Acta Mathematica Universitatis Comenianae*, **68**(1), 17, 1999.
- [11] Srivastava, S. M., *A Course on Borel Sets*, Springer, New York, NY, 1998. <https://doi.org/10.1007/b98956>
- [12] Kuratowski, K., *Topology* **1**, Academic Press, 1966.
- [13] Hashimoto, H., On some local properties on spaces, *Mathematics Japonica*, **11**, 127, 1952.
- [14] Modak, S., Some New Topologies on Ideal Topological Spaces, *Proceedings of the National Academy of Sciences, India Section A: Physical Sciences*, **82**(3), 233, 2012. <https://www.tandfonline.com/doi/full/10.1080/16583655.2018.1489585>
- [15] Modak, S., Bandyopadhyay, C., A Note on ψ -Operator, *Bulletin of the Malaysian Mathematical Sciences Society*, **30** (1), 43, 2007.
- [16] Willard, S., *General Topology*, Addison-Wesley Publishing Company, 1970.
- [17] Birkhoff, G. D., *Dynamical Systems*, American Mathematical Society, Providence, RI, 1927.
- [18] Kolyada, S., Snoha, L., Some aspects of topological transitivity — a survey, *Grazer Mathematische Berichte*, **334**, 3, 1997.
- [19] Mai, J., Sun, W., Transitivity of maps of general topological spaces, *Topology and its Applications*, **157**, 946, 2010. <https://doi.org/10.1016/j.topol.2009.12.011>
- [20] Joshi, K. D., *Introduction to General Topology*, Revised Edition, Wiley Eastern Limited, 1984.