

THE CHARACTERIZATION OF INTEGRAL CURVES IN GALILEAN SPACE

AYKUT TOPLAMA¹, MUSTAFA DEDE¹

Manuscript received: 20.10.2025; Accepted paper: 10.06.2026;

Published online: 30.09.2026.

Abstract. *The theory of curves is one of the central research areas in differential geometry. Depending on their geometric and analytical properties, curves are studied and characterized within various classes. In this paper, we investigate integral curves as a particular class of curves in the Galilean space and establish their characterizations. Moreover, several illustrative examples are provided to demonstrate and visualize this particular type of curve.*

Keywords: Galilean Space, Curves, Non-Euclidean Geometry, Integral Curve.

Mathematics Subject Classification: 53A35.

1. INTRODUCTION

The theory of curves, which holds an important place in differential geometry, is characterized by a mathematical object commonly referred to as the Frenet frame [1]. The Frenet frame defines an orthonormal system consisting of tangent, normal, and binormal vectors constructed on a space curve, thereby helping to understand the spatial motion of an object [2,3]. On the other hand, curve theory itself is divided into many special curve types, such as Smarandache [4-7], Tzitzeica [8-10], Helix [11], Salkowski [12-14], and helical extension curves [15]. In addition to these special curves, integral curves also form one of these classes [16-20]. Moreover, the characterizations of special curves constructed through different frames can be found in [21-29].

The Galilean space, belonging to the Cayley–Klein family of geometries, provides a wide perspective for characterizing non-Euclidean curves. Galilean space represents the transition from special relativity to classical mechanics as a limiting case of pseudo-Euclidean space; Galilean space is currently attracting increasing interest due to its relationship with fundamental concepts such as velocity, momentum, and energy [30-32]. Furthermore, this space has been extended to a 4-dimensional curved structure, and the 4-dimensional structure of spherical curves, a special class of curves, has been characterized [33]. In addition, characterizations of special curves such as the Mannheim curve [34], the Osculating curve [35], and the Bertrand curve [36,37] have been constructed. Moreover, the characterizations of special curves constructed through different frames can be found in the Galilean space [38-44].

This article presents a new characterization of integral curves, a special class of space curves in Galilean space; it also expresses the curvature and torsion functions of these curves. The proposed characterization is supported by various examples, and it is also shown that

¹ University of Kilis, Faculty of Kilisli Muallim Rifat Education, Department of Mathematics, 79000 Kilis, Turkey. E-mail: aykut.toplama@kilis.edu.tr; mustafadede@kilis.edu.tr.

integral curves are plane curves. This result is also demonstrated through different graphical representations.

2. PRELIMINARIES

In this section, basic definitions and theorems are expressed in Galilean space. Galilean space is one of the real Cayley-Klein spaces derived from the projective metric with signature $(0,0,+,+)$ [39]. The absolute structure of the Galilean geometry is defined by the ordered triple $\{\omega, f, I\}$, where ω denotes the ideal (absolute) plane, f is a line lying on ω , and I represent the fixed elliptic involution acting on the points of f . In the Galilean space, a plane is called a Euclidean plane if it contains the absolute straight-line f , and an isotropic plane otherwise; it is said to be an isotropic plane. More explicitly, planes obtained by fixing the first coordinate in the affine coordinate system (x, y, z) , e.i., $x = \text{constant}$, together with the absolute plane ω , belong to the class of Euclidean planes, whereas all remaining planes are classified as isotropic. This classification can also be formulated in terms of vectors: let $u = (u_1, u_2, u_3)$ be a vector in the Galilean space. If $u_1 \neq 0$, then u is called a non-isotropic vector, while $u_1 = 0$, characterizes an isotropic vector. The geometry of Galilean space can be found in more detail in [32,44]. Let $u = (u_1, u_2, u_3)$ and $v = (v_1, v_2, v_3)$ be two non-isotropic vectors in the Galilean space; the scalar product is defined by

$$\langle u, v \rangle = u_1 v_1, \|u\| = |u_1|.$$

If both vectors are isotropic, i.e., of the form $p = (0, p_2, p_3)$ and $q = (0, q_2, q_3)$, then the scalar product is defined by

$$\langle p, q \rangle_1 = p_2 q_2 + p_3 q_3, \|p\|_1 = \sqrt{(p_2)^2 + (p_3)^2}.$$

In addition, let $u = (1, u_2, u_3)$ and $v = (1, v_2, v_3)$ be two non-isotropic vectors in the Galilean space; the angle measure between u and v is defined as

$$\|u - v\|_1 = \sqrt{(v_2 - u_2)^2 + (v_3 - u_3)^2}.$$

On the other hand, the angle measure between two isotropic vectors $p = (0, p_2, p_3)$ and $q = (0, q_2, q_3)$ is expressed as

$$\cos \theta = \frac{p_2 q_2 + p_3 q_3}{\sqrt{(p_2)^2 + (p_3)^2} \sqrt{(q_2)^2 + (q_3)^2}}.$$

Moreover, let $u = (u_1, u_2, u_3)$ and $v = (v_1, v_2, v_3)$ be two non-isotropic vectors in the Galilean space $\mathbb{G}_3$ [39], the vector product is defined as

$$u \wedge_G v = \begin{vmatrix} 0 & e_2 & e_3 \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = (0, -u_1 v_3 + u_3 v_1, u_1 v_2 - u_2 v_1).$$

Furthermore, observe that every isotropic vector is perpendicular to any non-isotropic vector. Therefore, the vector product of two isotropic vectors vanishes in Galilean space [39].

In Galilean space $\mathbb{G}_3$, for an admissible curve of the form

$$\beta(s) = (s, \beta_2(s), \beta_3(s)),$$

The associated Frenet frame is defined as

$$T(s) = (1, \beta_2'(s), \beta_3'(s)), N(s) = \frac{1}{\kappa} (0, \beta_2''(s), \beta_3''(s)), B(s) = \frac{1}{\kappa} (0, -\beta_3''(s), \beta_2''(s))$$

where the tangent vector $T(s)$ is non-isotropic, the normal $N(s)$ with binormal vectors $B(s)$ are isotropic. The curvature and the torsion are given by, respectively

$$\kappa(s) = \sqrt{(\beta_2''(s))^2 + (\beta_3''(s))^2},$$

$$\tau(s) = \frac{1}{\kappa^2} \det(\beta'(s), \beta''(s), \beta'''(s)).$$

The Frenet equations can be expressed in matrix form as

$$\begin{bmatrix} T'(s) \\ N'(s) \\ B'(s) \end{bmatrix} = \begin{bmatrix} 0 & \kappa(s) & 0 \\ 0 & 0 & \tau(s) \\ 0 & -\tau(s) & 0 \end{bmatrix} \begin{bmatrix} T(s) \\ N(s) \\ B(s) \end{bmatrix}.$$

We can also note that the symbolic vector product of the vectors $\{T, N, B\}$ are defined by

$$T \wedge_G N = B, N \wedge_G B = 0, T \wedge_G B = -N$$

in Galilean space $\mathbb{G}_3$ [39].

In the Galilean space $\mathbb{G}_3$, for a non-admissible curve given by

$$\beta(s) = (0, \beta_2(s), \beta_3(s)),$$

the associated Frenet frame is defined by

$$T(s) = (0, \beta_2'(s), \beta_3'(s)), N(s) = (0, -\beta_3'(s), \beta_2'(s)), B(s) = (0, 0, 0).$$

Moreover, the curvature is obtained as $\kappa(s) = \|T'(s)\|_1$ and the torsion function vanishes because the curve lies in an isotropic plane in the Galilean space $\mathbb{G}_3$ [32,39]. The distinction between admissible and non-admissible curves is determined by the character of their tangent vectors: a curve is called admissible when its tangent vector is non-isotropic, whereas it is called non-admissible when its tangent vector is isotropic.

3. THE CHARACTERIZATION OF INTEGRAL CURVES IN GALILEAN SPACE

In this section, we introduce a new class of curves in Galilean space called integral curves. Additionally, the characterizations of these special curves are constructed. Furthermore, several examples are presented to illustrate their geometric representations. This special class of curves is divided into two categories: principal normal integral curves and binormal integral curves.

3.1. THE CHARACTERIZATION OF PRINCIPAL NORMAL INTEGRAL CURVES IN GALILEAN SPACE

Definition 3.1.1. Let $\beta(s)$ be an admissible curve in three-dimensional Galilean space; the curve obtained by integrating the principal normal vector field associated with $\beta(s)$ is called a principal normal integral curve $\gamma(s)$, and it can be expressed

$$\gamma(s) = \int N_{\beta}(s) ds \quad (3.1.1)$$

Theorem 3.1.2. Let $\beta(s)$ be an admissible curve in three-dimensional Galilean space; let $\{T_{\beta}, N_{\beta}, B_{\beta}\}$ denote the Frenet frame of the curve $\beta(s)$ in the Galilean space. The principal normal integral curve $\gamma(s)$, generated by the principal normal vector $N_{\beta}(s)$ of $\beta(s)$, has Frenet vector fields given by

$$T_{\gamma}(s) = N_{\beta}(s), N_{\gamma}(s) = B_{\beta}(s), B_{\gamma}(s) = (0,0,0).$$

where $T_{\gamma}(s)$, $N_{\gamma}(s)$ and $B_{\gamma}(s)$ represent the tangent, principal normal, and binormal vector fields of the principal normal integral curve $\gamma(s)$, respectively. The curvature and torsion functions of this curve are given by

$$\kappa_{\gamma}(s) = \tau_{\gamma}(s), \tau_{\gamma}(s) = 0.$$

Proof 3.2.1: Based on the definition of the principal normal integral curve given in (3.1.1), the first, second, and third-order derivatives of this curve are obtained as, respectively,

$$\begin{aligned} \gamma'(s) &= N_{\beta}(s), \\ \gamma''(s) &= N'_{\beta}(s) = \tau_{\beta}(s)B_{\beta}(s), \\ \gamma'''(s) &= (\tau(s)B_{\beta}(s))' = -\tau'_{\beta}(s)N_{\beta}(s) + \tau_{\beta}(s)B'_{\beta}(s). \end{aligned}$$

Thus, the Frenet frame of the principal normal integral curve is found to be

$$\begin{aligned} T_{\gamma}(s) &= \gamma'(s) = N_{\beta}(s), N_{\gamma}(s) = \frac{\gamma''(s)}{\|\gamma''(s)\|} = B_{\beta}(s), \\ B_{\gamma}(s) &= T_{\gamma}(s) \wedge N_{\gamma}(s) = (0,0,0). \end{aligned}$$

Considering the above equations, the curvature functions of the principal normal integral curve $\gamma(s)$ are obtained as follows:

$$\kappa_\gamma(s) = \|\gamma''(s)\| = \|\tau_\beta(s)B_\beta(s)\| = |\tau_\beta(s)|,$$

$$\tau_\gamma(s) = \frac{1}{\kappa_\gamma^2(s)} \langle \gamma'(s) \wedge_G \gamma''(s), \gamma'''(s) \rangle_1 = 0,$$

where $\gamma'(s) \wedge_G \gamma''(s) = (0, 0, 0)$.

3.2. THE CHARACTERIZATION OF BINORMAL INTEGRAL CURVES IN GALILEAN SPACE

Definition 3.2.1. Let $\beta(s)$ be an admissible curve in three-dimensional Galilean space, the curve obtained by integrating the binormal vector field associated with $\beta(s)$ is called a binormal integral curve $\gamma(s)$, and it can be expressed

$$\gamma(s) = \int B_\beta(s) ds \quad (3.2.1)$$

Theorem 3.2.2. Let $\beta(s)$ be an admissible curve in three-dimensional Galilean space; let $\{T_\beta, N_\beta, B_\beta\}$ denote the Frenet frame of the curve $\beta(s)$ in the Galilean space. The binormal integral curve $\gamma(s)$, generated by the binormal vector $B_\beta(s)$ of $\beta(s)$, has Frenet vector fields given by

$$T_\gamma(s) = B_\beta(s), N_\gamma(s) = -N_\beta(s), B_\gamma(s) = (0, 0, 0).$$

where $T_\gamma(s)$, $N_\gamma(s)$ and $B_\gamma(s)$ represent the tangent, principal normal, and binormal vector fields of the binormal integral curve $\gamma(s)$, respectively. The curvature and torsion functions of this curve are given by

$$\kappa_\gamma(s) = \tau_\beta(s), \tau_\gamma(s) = 0.$$

Proof 3.2.2: Based on the definition of the binormal integral curve given in (3.2.1), the first, second, and third-order derivatives of this curve are obtained as, respectively,

$$\begin{aligned} \gamma'(s) &= B_\beta(s), \\ \gamma''(s) &= B_\beta'(s) = -\tau_\beta(s)N_\beta(s) \\ \gamma'''(s) &= (-\tau_\beta(s)N_\beta(s))' = -\tau_\beta'(s)N_\beta(s) - \tau_\beta^2(s)B_\beta(s). \end{aligned}$$

Thus, the Frenet frame of the binormal integral curve is found to be

$$\begin{aligned} T_\gamma(s) &= \gamma'(s) = B_\beta(s), N_\gamma(s) = \frac{\gamma''(s)}{\|\gamma''(s)\|} = -N_\beta(s), \\ B_\gamma(s) &= T_\gamma(s) \wedge_G N_\gamma(s) = (0, 0, 0). \end{aligned}$$

Considering the above equations, the curvature functions of the binormal integral curve $\gamma(s)$ are obtained as follows:

$$\kappa_\gamma(s) = \|\gamma''(s)\| = \|-\tau_\beta(s)N_\beta(s)\| = |\tau_\beta(s)|,$$

$$\tau_{\gamma}(s) = \frac{1}{\kappa_{\gamma}^2(s)} \langle \gamma'(s) \wedge_G \gamma''(s), \gamma'''(s) \rangle_1 = 0,$$

where $\gamma'(s) \wedge_G \gamma''(s) = (0, 0, 0)$.

Corollary: As can be seen from the above theorems, the integral curve of any curve in Galilean space is a planar curve due to the torsion vanishing.

Example: Consider an admissible curve in three-dimensional Galilean space,

$$\begin{aligned} \beta: I &\rightarrow \mathbb{E}^3 \\ s &\rightarrow \beta(s) = (s, 5 \cos s, 5 \sin s) \end{aligned}$$

then the first-, second-, and third-order derivatives of this curve are obtained

$$\begin{aligned} \beta'(s) &= (1, -5 \sin s, 5 \cos s), \\ \beta''(s) &= (0, -5 \cos s, -5 \sin s) \\ \beta'''(s) &= (0, 5 \sin s, -5 \cos s). \end{aligned}$$

Thus, the Frenet frame of this curve is found to be

$$\begin{aligned} T(s) &= (1, -5 \sin s, 5 \cos s), \\ N(s) &= (0, -\cos s, -\sin s), \\ B(s) &= (0, \sin s, -\cos s). \end{aligned}$$

Additionally, the vector product of the first derivative and the second derivative of $\beta(s)$ is found

$$\beta'(s) \wedge_G \beta''(s) = \begin{vmatrix} 0 & e_2 & e_2 \\ 1 & -5 \sin s & 5 \cos s \\ 0 & -5 \cos s & -5 \sin s \end{vmatrix} = (0, 5 \sin s, -5 \cos s)$$

Thus, the curvature and torsion functions of the curve are obtained as follows:

$$\begin{aligned} \kappa(s) &= \sqrt{(\beta_2''(s))^2 + (\beta_3''(s))^2} = 5, \\ \tau(s) &= \frac{\langle \beta'(s) \wedge \beta''(s), \beta'''(s) \rangle_1}{\kappa^2} = 1. \end{aligned}$$

Thus, from (3.1.1), the principal normal integral curve of the $\beta(s)$ curve is calculated as

$$\gamma(s) = \int N(s) ds = (0, -\sin s, \cos s)$$

In Galilean space, the Frenet frame for this obtained $\gamma(s)$ curve is given as

$$\begin{aligned} T_{\gamma}(s) &= (0, -\cos s, -\sin s), \\ N_{\gamma}(s) &= (0, \sin s, -\cos s), \\ B_{\gamma}(s) &= (0, 0, 0). \end{aligned}$$

The curvature and torsion functions of the curve are found

$$\kappa_{\gamma}(s) = \sqrt{(\gamma_2''(s))^2 + (\gamma_3''(s))^2} = 1,$$

$$\tau_{\gamma}(s) = \frac{1}{\kappa^2} \det(\beta'(s), \beta''(s), \beta'''(s)) = 0.$$

In Fig. 1, a graphical representation of the curve $\beta(s)$ and its curvature function is presented. Also, a graphical representation of the obtained principal normal integral curve is provided, as well as the curvature function.

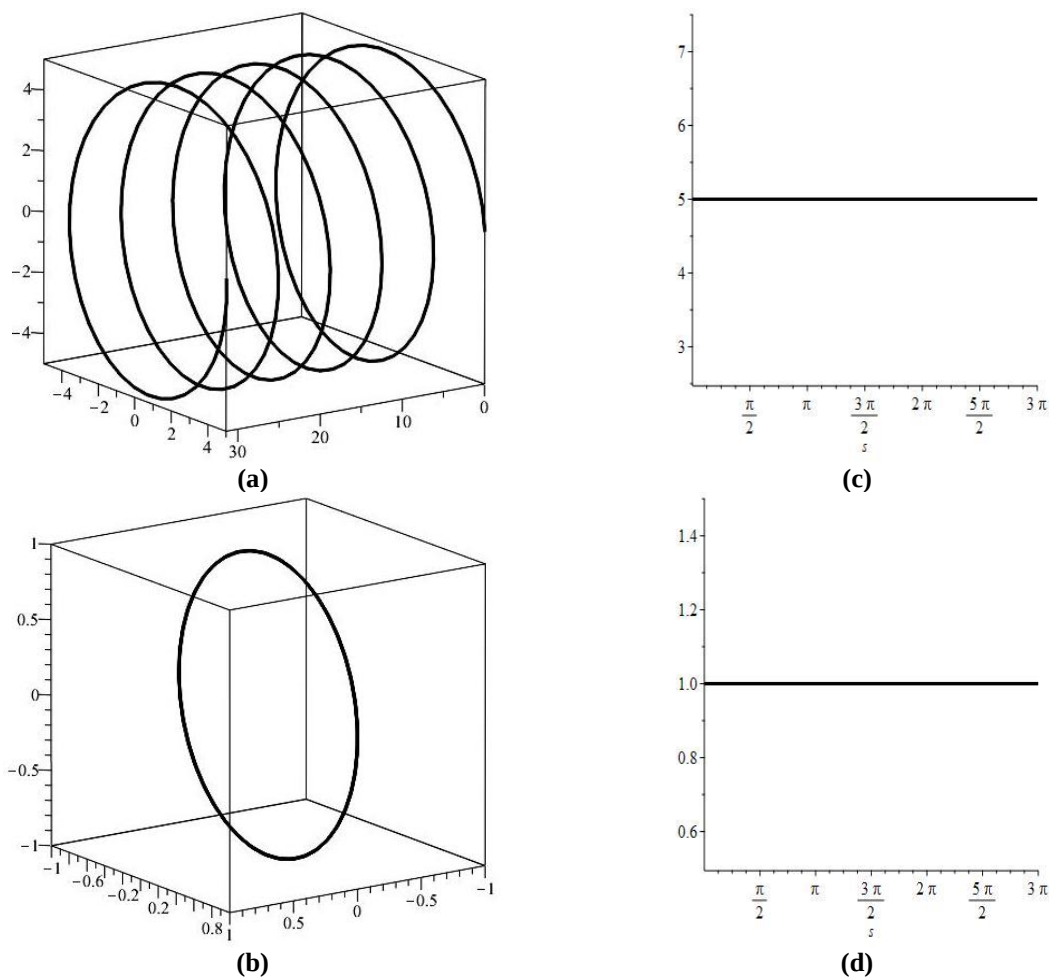


Figure 1. (a) The graphical representation of the curve $\beta(s)$; (b) the graphical representation of the principal normal integral curve $\gamma(s)$; (c) the graphical representation of the curvature function of curve $\beta(s)$; (d) the graphical representation of the curvature function of the principal normal integral curve $\gamma(s)$.

Example: Consider an admissible curve in three-dimensional Galilean space,

$$\beta: I \rightarrow \mathbb{E}^3$$

$$s \rightarrow \beta(s) = (s, 3 \sin(2s), 5 \cos(2s))$$

Then, in Galilean space, the curvature and torsion functions of the curve $\beta(s)$ are found, respectively

$$\kappa(s) = 4\sqrt{16(\cos(2s))^2 + 9},$$

$$\tau(s) = \frac{-30}{16(\cos(2s))^2 + 9}.$$

The principal normal integral curve generated by the curve $\beta(s)$ is given by

$$\gamma(s) = \left(0, \frac{3 \operatorname{arcsinh}\left(\frac{4 \cos(2s)}{3}\right)}{8}, \frac{-5 \operatorname{arcsin}\left(\frac{4 \sin(2s)}{5}\right)}{8} \right).$$

Thus, in Galilean space, the Frenet frame for this obtained $\gamma(s)$ curve is given by

$$T_\gamma(s) = \left(0, \frac{-3 \sin(2s)}{\sqrt{16(\cos(2s))^2 + 9}}, \frac{-5 \cos(2s)}{\sqrt{16(\cos(2s))^2 + 9}} \right),$$

$$N_\gamma(s) = \left(0, \frac{5 \cos(2s)}{\sqrt{16(\cos(2s))^2 + 9}}, \frac{-3 \sin(2s)}{\sqrt{16(\cos(2s))^2 + 9}} \right),$$

$$B_\gamma(s) = (0, 0, 0).$$

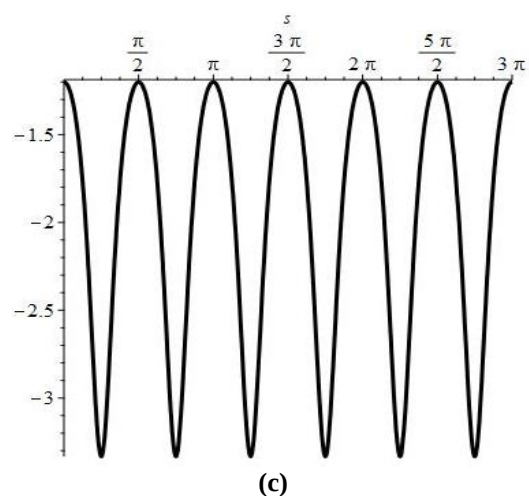
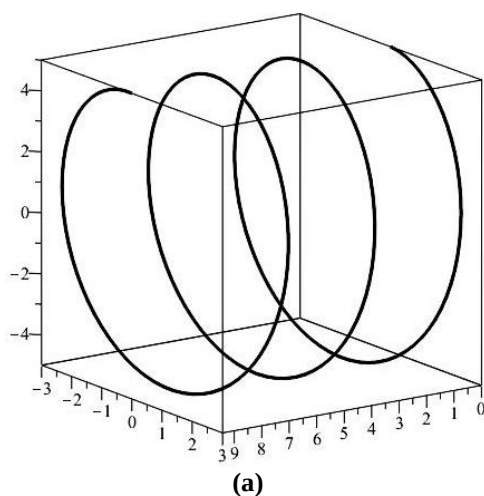
In addition, the curvature and torsion functions of the curve $\gamma(s)$ are computed as

$$\kappa_\gamma(s) = \frac{30}{16(\cos(2s))^2 + 9}$$

$$\tau_\gamma(s) = 0.$$

respectively.

In Fig. 2, a graphical representation of the curve $\beta(s)$ and its torsion function is presented. Also, a graphical representation of the obtained principal normal integral curve is provided, as well as the curvature function.



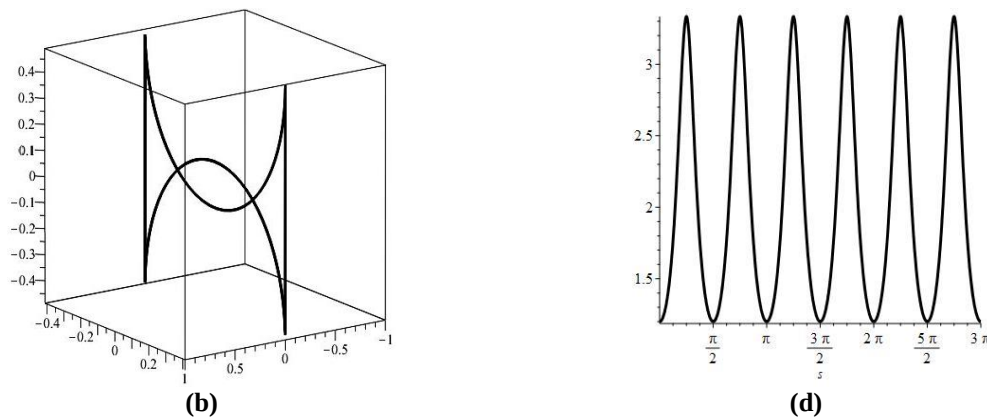


Figure 2. (a) The graphical representation of the curve $\beta(s)$; (b) the graphical representation of the principal normal integral curve $\gamma(s)$; (c) the graphical representation of the torsion function of curve $\beta(s)$; (d) the graphical representation of the curvature function of the principal normal integral curve $\gamma(s)$.

Example: Consider a curve to be an admissible curve in three-dimensional Galilean space,

$$\beta: I \rightarrow \mathbb{E}^3$$

$$s \rightarrow \beta(s) = (s, 3 \sin(2s), 5 \cos(3s))$$

then, in Fig. 3, a graphical representation of the curve $\beta(s)$ and its torsion function is presented. Also, a graphical representation of the obtained principal normal integral curve is provided, as well as the curvature function.

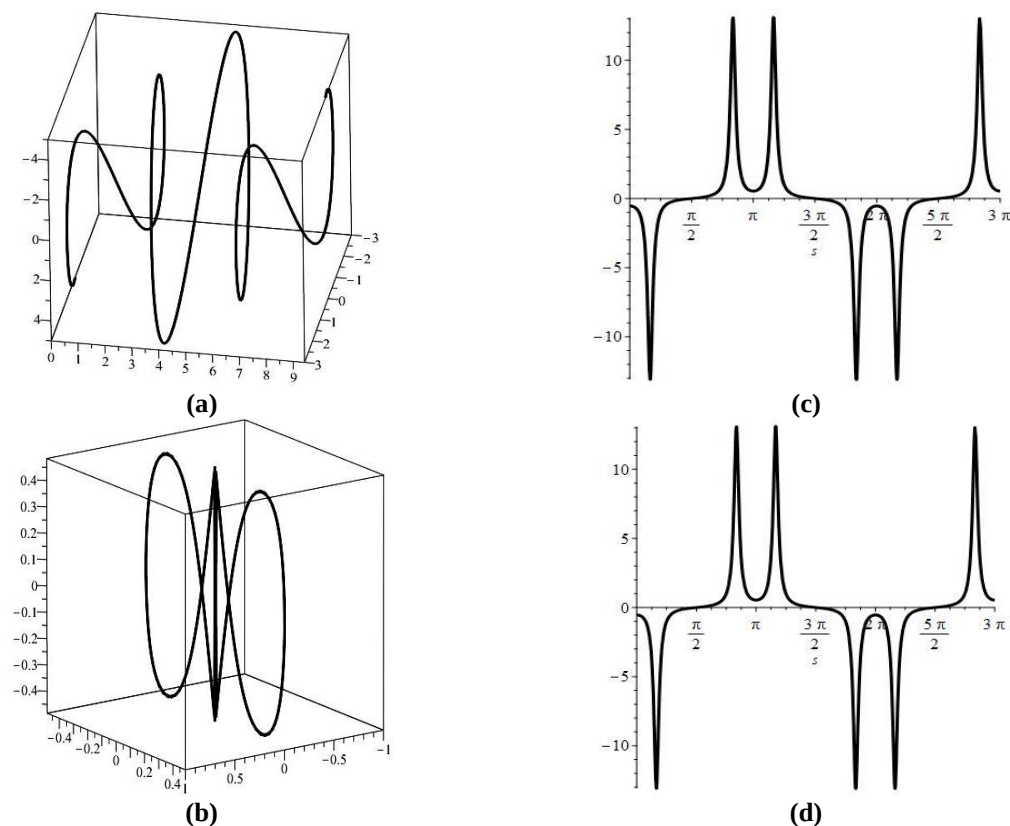


Figure 3. (a) The graphical representation of the curve $\beta(s)$; (b) the graphical representation of the principal normal integral curve $\gamma(s)$; (c) the graphical representation of the torsion function of curve $\beta(s)$; (d) the graphical representation of the curvature function of the principal normal integral curve $\gamma(s)$.

4. CONCLUSION

A new characterization has been presented for the principal normal integral curve and the binormal integral curve as a special class of curves in Galilean space. Based on this characterization, it has been shown that the curvature of the principal normal integral curve of the examined curve is equal to the absolute value of the curve's torsion. In addition, the torsion of this special curve has vanished, i.e., the curve is a plane curve. Furthermore, this result has been supported through various examples and graphical representations.

REFERENCES

- [1] Stoker, J.J., *Differential geometry*, New York, Copyright John Wiley & Sons, 1989.
- [2] Farouki, R. T., Giannelli, C., Sampoli, M. L., Sestini, A., Rotation-minimizing osculating frames, *Aided Geometric Design*, **31**(1), 27, 2014. <https://doi.org/10.1016/j.cagd.2013.11.003>
- [3] Do Carmo, M.P., *Differential Geometry of Curves and Surfaces*, Prentice Hall, Englewood Cliffs, NJ, 1976.
- [4] Aliç, S., Yılmaz, B., Smarandache curves according to alternative frame in E^3 , *Journal of Universal Mathematics*, **4**(2), 140, 2021. <https://doi.org/10.33773/jum.956862>
- [5] Eren, K., Ersoy, S., On characterization of Smarandache curves constructed by modified orthogonal frame, *Mathematical Sciences and Applications E-Notes*, **12**(3), 101, 2024. <https://doi.org/10.36753/mathenot.1409228>
- [6] Senyurt, S., Altun, Y., Smarandache Curves of the Evolute Curve According to Sabban Frame, *Communications in Advanced Mathematical Sciences*, **3**(1), 1, 2020. <https://doi.org/10.33434/cams.594690>
- [7] Senyurt, S., Ayvaci, K.H., Canli, D., Smarandache curves according to Flc-frame in Euclidean 3-space, *Fundamentals of Contemporary Mathematical Sciences*, **4**(1), 16, 2023. <https://doi.org/10.54974/fcmathsci.1142404>
- [8] Guven, I., Yilmaz, B., Has, A., Tzitzeica Curves According to Alternative Moving Frame, *In Advanced Mathematics for the Modeling and Solution of Challenging Problems in Engineering*, Cham: Springer Nature Switzerland, **582**, 101, 2025. https://doi.org/10.1007/978-3-031-82342-8_8
- [9] Eren, K., Ersoy, S., Characterizations of Tzitzeica curves using Bishop frames, *Mathematical Methods in the Applied Sciences*, **45**(18), 12046, 2022. <https://doi.org/10.1002/mma.7483>
- [10] Senyurt, S., Eren, K., Ayvaci, K.H., Characterizations of tzitzeica curves using Flc frame, *Sigma Journal of Engineering and Natural Sciences*, **42**(1), 37, 2024. <https://izlik.org/JA36DE44BZ>
- [11] Yılmaz, B., Has, A., New approach to slant helix, *International Electronic Journal of Geometry*, **12**(1), 111, 2019. <https://doi.org/10.36890/iejg.545879>
- [12] Çakmak, A., Sahin, V., Characterizations of adjoint curves according to alternative moving frame, *Fundamental Journal of Mathematics and Applications*, **5**(1), 42, 2022. <https://doi.org/10.33401/fujma.1001730>
- [13] Mazlum, S.G., Bishop Frames of Salkowski Curves in E^3 , *Bitlis Eren Üniversitesi Fen Bilimleri Dergisi*, **13**(1), 79, 2024. <https://doi.org/10.17798/bitlisfen.1345438>

- [14] Mazlum, S.G., Senyurt, S., Bektas, M., Salkowski curves and their modified orthogonal frames in E^3 , *Journal of New Theory*, **(40)**, 12, 2022. <https://doi.org/10.53570/jnt.1140546>
- [15] Dede, M., Helical extension curve of a space curve, *Mediterranean Journal of Mathematics*, **18**(1), 6, 2021. <https://doi.org/10.1007/s00009-020-01625-0>
- [16] Güven, I.A., Some integral curves with a new frame, *Open Mathematics*, **18**(1), 1332, 2020. <https://doi.org/10.1515/math-2020-0078>
- [17] Elsharkawy, A., Baizeed, H., Some integral curves according to quasi-frame in Euclidean 3-space, *Scientific African*, **27**, e02583, 2025. <https://doi.org/10.1016/j.sciaf.2025.e02583>
- [18] Yeneroğlu, M., Duyan, A., Associated Curve According to Bishop Frame in 4-Dimensional Euclidean Space, *Journal of Science and Arts*, **24**(1), 105, 2024. <https://doi.org/10.46939/J.Sci.Arts-24.1-a09>
- [19] Döldül, B.U., On some new frames along a space curve and integral curves with Darboux q-vector fields in E^3 , *AIMS Mathematics*, **9**(7), 17871, 2024. <https://doi.org/10.3934/math.2024869>
- [20] Arıkan, M., Nurkan, S.K., Adjoint curve according to modified orthogonal frame with torsion in 3-space, *Uşak Üniversitesi Fen Ve Doğa Bilimleri Dergisi*, **4**(2), 54, 2020. <https://doi.org/10.47137/usufedbid.798189>
- [21] Kaya, O., Önder, M., New partner curves in the Euclidean 3-space, *International Journal of Geometry*, **6**(2), 41, 2017. <https://ijgeometry.com/wp-content/uploads/2017/10/41-50.pdf>
- [22] Azak, A.Z., Involute-evolute curves according to modified orthogonal frame, *Journal of Science and Arts*, **21**(2), 385, 2021. <https://doi.org/10.46939/J.Sci.Arts-21.2-a06>
- [23] Bukcu, B., Karacan, M.K., Spherical curves with modified orthogonal frame, *Journal of New Results in Science*, **5**(10), 60, 2016. <https://izlik.org/JA87EB28RG>
- [24] Lone, M.S., Es, H., Karacan, M.K., Bükcü, B., Mannheim curves with modified orthogonal frame in Euclidean 3-space, *Turkish Journal of Mathematics*, **43**(2), 648, 2019. <https://doi.org/10.3906/mat-1807-177>
- [25] Yakut, A.T., Kızılay, A., On the curve evolution with a new modified orthogonal Saban frame, *AIMS Mathematics*, **9**(10), 29573, 2024. <https://doi.org/10.3934/math.20241432>
- [26] Barros, M., General helices and a theorem of Lancret, *Proceedings of the American Mathematical Society*, **125**(5), 1503, 1997. <https://www.jstor.org/stable/2162098>
- [27] Deshmukh, S., Chen, B.Y., Alshammari, S. H., On rectifying curves in Euclidean 3-space, *Turkish Journal of Mathematics*, **42**(2), 609, 2018. <https://doi.org/10.3906/mat-1701-52>
- [28] Chen, B.Y., When does the position vector of a space curve always lie in its rectifying plane? *The American Mathematical Monthly*, **110**(2), 147, 2003. <https://doi.org/10.2307/3647775>
- [29] Chen, B., Dillen, F., Rectifying curves as centrodes and extremal curves, *Bulletin-Institute of Mathematics Academia Sinica*, **33**(2), 77, 2005.
- [30] Struik, D.J., *Lectures on classical differential geometry*, Dover, New York, 1988.
- [31] Fock, V., *The theory of space, time and gravitation*, 2nd Ed, England: Pergamon Press, 1964.
- [32] Yaglom, I.M., *A simple non-Euclidean geometry and its physical basis*. Springer -Verlag New York Inc., 2012.
- [33] Yılmaz, S., Construction of the Frenet-Serret frame of a curve in 4D Galilean space and some applications, *International Journal of the Physical Sciences*, **5**(8), 1284, 2010.
- [34] Bektaş, M., Ergüt, M., Öğrenmiş, A.O., *Special curves of 4D Galilean space*, arXiv preprint arXiv:1111.0419, 2011.

- [35] Yoon, D.W., Lee, J.W., Lee, C.W., Osculating curves in the Galilean 4-space, *International Journal of Pure and Applied Mathematics*, **100**(4), 497, 2015. <http://dx.doi.org/10.12732/ijpam.v100i4.9>
- [36] Öztekin, H., Special Bertrand curves in 4D Galilean space, *Mathematical Problems in Engineering*, **2014**(1), 318458, 2014. <https://doi.org/10.1155/2014/318458>
- [37] Ögrenmis, A.O., Öztekin, H., Ergüt, M., Bertrand curves in Galilean space and their characterizations, *Kragujevac Journal of Mathematics*, **32**(32), 139, 2009.
- [38] Ogrenmis, A.O., Ergut, M., Bektas, M., On the helices in the Galilean space G_3 , *Iranian Journal of Science and Technology*, Transaction A, **31**(42), A2, 2007. <https://doi.org/10.22099/ijsts.2007.2327>
- [39] Divjak, B., Milin-Šipuš, Z., Special curves on ruled surfaces in Galilean and pseudo-Galilean spaces, *Acta Mathematica Hungarica*, **98**(3), 203, 2002. <https://doi.org/10.1023/A:1022821824927>
- [40] Saad, M.K., Spacelike and timelike admissible Smarandache curves in pseudo-Galilean space, *Journal of the Egyptian Mathematical Society*, **24**(3), 416, 2016. <http://dx.doi.org/10.1016/j.joems.2015.09.001>
- [41] Yilmaz, B., Metin, S., Gok, I., Yayli, Y., Harmonic curvature functions of some special curves in Galilean 3-space, *Honam Mathematical Journal*, **41**(2), 301, 2019. <https://doi.org/10.5831/hmj.2019.41.2.301>
- [42] Elzawy, M., Mosa, S., A novel method for Mannheim curves in the Galilean 3-space, *AIMS Mathematics*, **9**(11), 31239, 2024. <https://doi.org/10.3934/math.20241506>
- [43] Divjak, B., Curves in pseudo-Galilean geometry, *Annales Universitatis Scientiarum Budapestinensis de Rolando Eötvös Nominatae*, Sectio Mathematica, **41**, 117, 1998.
- [44] Röschel, O., *Die Geometrie des Galileischen Raumes*, Habilitationsschrift, Leoben, 1984.