

A STANCU-TYPE GENERALIZATION OF KING'S OPERATORS

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Abstract. In this paper, we introduce a generalization of the King operator and study some shape-preserving properties.

Keywords: King's operators; Bernstein-Stancu operators; shape-preserving property.

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1. INTRODUCTION

J. P. King defined in [1] a class of operators which preserve the function x^2 :

$$V_n(f, x) = \sum_{j=0}^n p_{n,j}(v_n(x)) f\left(\frac{j}{n}\right), \quad p_{n,j}(v_n(x)) = \binom{n}{j} v_n(x)^j (1 - v_n(x))^{n-j}, \quad (1)$$

$f \in C[0,1]$, $x \in [0,1]$, where $v_n \in C[0,1]$, $n \in N$, are given by

$$v_n(x) = \frac{-1}{2(n-1)} + \sqrt{\frac{n}{n-1}x^2 + \frac{1}{4(n-1)^2}}, \quad n \in N, n \geq 2.$$

The following generalized Bernstein operators were introduced by D. D. Stancu in [2], [3].

$$S_{n,r,s}(f, x) = \sum_{j=0}^{n-rs} p_{n-rs,j}(x) \sum_{i=0}^s p_{s,i}(x) f\left(\frac{j+ir}{n}\right), \quad (2)$$

$f \in C[0,1]$, $x \in [0,1]$, where $n \in N$, $r, s \in N_0 = N \cup \{0\}$ such that $rs < n$.

Bernstein's operators are obtained for $s = 0$ or $s = 1$, $r = 0$ or $s = 1$, $r = 1$. We consider the Stancu-King operators given by

$$V_{n,r,s}(f, x) = \sum_{j=0}^{n-rs} p_{n-rs,j}(v_{n,r,s}(x)) \sum_{i=0}^s p_{s,i}(v_{n,r,s}(x)) f\left(\frac{j+ir}{n}\right), \quad (3)$$

$f \in C[0,1]$, $x \in [0,1]$, $v_{n,r,s} \in C[0,1]$ with $v_{n,r,s}(x) \in [0,1]$ for all $x \in [0,1]$, where $n \in N$, $r, s \in N_0 = N \cup \{0\}$ such that $rs < n$.

Every operator $V_{n,r,s}$ is a positive linear operator on $C[0,1]$. The values of these operators on the test functions $e_k(x) = x^k$, $k = 0, 1, 2$, are:

$$V_{n,r,s}(e_0, x) = 1,$$

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$$V_{n,r,s}(e_1, x) = v_{n,r,s}(x),$$

$$V_{n,r,s}(e_2, x) = \frac{n(n-1)-rs(r-1)}{n^2} v_{n,r,s}(x)^2 + \frac{n+rs(r-1)}{n^2} v_{n,r,s}(x).$$

Therefore, according to Korovkin's theorem, we have

$$\lim_{n \rightarrow \infty} V_{n,r,s}(f) = f \text{ uniformly on } [0,1] \text{ iff } \lim_{n \rightarrow \infty} v_{n,r,s} = e_1 \text{ uniformly on } [0,1].$$

If we impose condition $V_{n,r,s}(e_2, x) = x^2$, $x \in [0,1]$, we obtain

$$v_{1,r,s}(x) = x^2,$$

$$v_{n,r,s}(x) = \frac{-n + rs(r-1)}{2[n(n-1) - rs(r-1)]}$$

$$+ \sqrt{\frac{n^2}{n(n-1) - rs(r-1)} x^2 + \left(\frac{n + rs(r-1)}{2[n(n-1) - rs(r-1)]} \right)^2}, n \geq 2.$$

The function $v_{n,r,s}$ is increasing on the interval $[0,1]$ and $v_{n,r,s}(0) = 0$, $v_{n,r,s}(1) = 1$. Since the sequence of functions $(v_{n,r,s})_n$ is uniformly convergent to e_1 , we have that

$$\lim_{n \rightarrow \infty} V_{n,r,s}(f) = f \text{ uniformly on } [0,1], \text{ for each } f \in C[0,1].$$

King's operators are obtained for $s = 0$ or $s = 1$, $r = 0$ or $s = 1$, $r = 1$. In the following, we study the monotonicity and convexity properties of these operators, and we determine the Lipschitz constant for $V_{n,r,s}f$ if $f \in Lip_M \alpha$.

2. MAIN RESULTS

Lemma 1. For $f \in C[0,1]$, $0 \leq x < y \leq 1$ and $\lambda \in [0,1]$, we have the following representation

$$V_{n,r,s}(f, (1-\lambda)x + \lambda y)$$

$$= \sum_{k_1+l_1=0}^{n-rs} \sum_{k_2+l_2=0}^s p_{n-rs, k_1, l_1} (v_{n,r,s}(x), v_{n,r,s}(y) - v_{n,r,s}(x))$$

$$\cdot p_{s, k_2, l_2} (v_{n,r,s}(x), v_{n,r,s}(y) - v_{n,r,s}(x))$$

$$\cdot \sum_{m_1=0}^{l_1} p_{l_1, m_1} \left(\frac{v_{n,r,s}((1-\lambda)x + \lambda y) - v_{n,r,s}(x)}{v_{n,r,s}(y) - v_{n,r,s}(x)} \right)$$

$$\cdot \sum_{m_2=0}^{l_2} p_{l_2, m_2} \left(\frac{v_{n,r,s}((1-\lambda)x + \lambda y) - v_{n,r,s}(x)}{v_{n,r,s}(y) - v_{n,r,s}(x)} \right)$$

$$\cdot f \left(\frac{k_1 + m_1 + r(k_2 + m_2)}{n} \right).$$
(4)

We specify that

$$p_{n,k,l}(u, v) = \frac{n!}{k!l!(n-k-l)!} u^k v^l (1-u-v)^{n-k-l}$$

are the two-variable Bernstein basis polynomials, $u, v \geq 0, u + v \leq 1$.

Proof: The representation is obtained using the method in [4].

$$\begin{aligned}
 & V_{n,r,s}(f, (1-\lambda)x + \lambda y) \\
 &= \sum_{j=0}^{n-rs} \binom{n-rs}{j} [v_{n,r,s}(x) + v_{n,r,s}((1-\lambda)x + \lambda y) - v_{n,r,s}(x)]^j \\
 &\quad \cdot [1 - v_{n,r,s}(y) + v_{n,r,s}(y) - v_{n,r,s}((1-\lambda)x + \lambda y)]^{n-rs-j} \\
 &\quad \cdot \sum_{i=0}^s \binom{s}{i} [v_{n,r,s}(x) + v_{n,r,s}((1-\lambda)x + \lambda y) - v_{n,r,s}(x)]^i \\
 &\quad \cdot [1 - v_{n,r,s}(y) + v_{n,r,s}(y) - v_{n,r,s}((1-\lambda)x + \lambda y)]^{s-i} f\left(\frac{j+ir}{n}\right) \\
 &= \sum_{j=0}^{n-rs} \sum_{i=0}^s \binom{n-rs}{j} \binom{s}{i} \sum_{k_1=0}^j \binom{j}{k_1} v_{n,r,s}(x)^{k_1} \\
 &\quad \cdot [v_{n,r,s}((1-\lambda)x + \lambda y) - v_{n,r,s}(x)]^{j-k_1} \\
 &\quad \cdot \sum_{p_1=0}^{n-rs-j} \binom{n-rs-j}{p_1} (1 - v_{n,r,s}(y))^{n-rs-j-p_1} \\
 &\quad \cdot [v_{n,r,s}(y) - v_{n,r,s}((1-\lambda)x + \lambda y)]^{p_1} \\
 &\quad \cdot \sum_{k_2=0}^i \binom{i}{k_2} v_{n,r,s}(x)^{k_2} [v_{n,r,s}((1-\lambda)x + \lambda y) - v_{n,r,s}(x)]^{i-k_2} \\
 &\quad \cdot \sum_{p_2=0}^{s-i} \binom{s-i}{p_2} [1 - v_{n,r,s}(y)]^{s-i-p_2} [v_{n,r,s}(y) - v_{n,r,s}((1-\lambda)x + \lambda y)]^{p_2} f\left(\frac{j+ir}{n}\right).
 \end{aligned}$$

We reverse the summation order and change the index $j - k_1 + p_1 = l_1, i - k_2 + p_2 = l_2$.

$$\begin{aligned}
 & V_{n,r,s}(f, (1-\lambda)x + \lambda y) \\
 &= \sum_{k_1=0}^{n-rs} \sum_{j=k_1}^{n-rs} \sum_{l_1=j-k_1}^{n-rs-k_1} \sum_{k_2=0}^s \sum_{i=k_2}^s \sum_{l_2=i-k_2}^{s-k_2} \binom{n-rs}{j} \binom{j}{k_1} \binom{n-rs-j}{l_1-j+k_1} \binom{s}{i} \binom{i}{k_2} \binom{s-i}{l_2-i+k_2} \\
 &\quad \cdot v_{n,r,s}(x)^{k_1+k_2} [1 - v_{n,r,s}(y)]^{n-rs-k_1-l_1+s-k_2-l_2} [v_{n,r,s}((1-\lambda)x + \lambda y) - v_{n,r,s}(x)]^{i+j-k_1-k_2} \\
 &\quad \cdot [v_{n,r,s}(y) - v_{n,r,s}((1-\lambda)x + \lambda y)]^{l_1-j+k_1+l_2-i+k_2} f\left(\frac{j+ir}{n}\right).
 \end{aligned}$$

We reverse the summation order and change the index $j - k_1 = m_1, i - k_2 = m_2$ and we obtain the following representation:

$$\begin{aligned}
 & V_{n,r,s}(f, (1-\lambda)x + \lambda y) \\
 &= \sum_{k_1+l_1=0}^{n-rs} \sum_{m_1=0}^{l_1} \frac{(n-rs)!}{k_1!l_1!(n-rs-k_1-l_1)!} v_{n,r,s}(x)^{k_1} [v_{n,r,s}(y) - v_{n,r,s}(x)]^{l_1} [1 - \\
 &\quad v_{n,r,s}(y)]^{n-rs-k_1-l_1} \\
 &\quad \cdot \sum_{k_2+l_2=0}^s \sum_{m_2=0}^{l_2} \frac{s!}{k_2!l_2!(s-k_2-l_2)!} v_{n,r,s}(x)^{k_2} [v_{n,r,s}(y) - v_{n,r,s}(x)]^{l_2} [1 - v_{n,r,s}(y)]^{s-k_2-l_2} \\
 &\quad \cdot \binom{l_1}{m_1} \left[\frac{v_{n,r,s}((1-\lambda)x + \lambda y) - v_{n,r,s}(x)}{v_{n,r,s}(y) - v_{n,r,s}(x)} \right]^{m_1} \cdot \left[1 - \frac{v_{n,r,s}((1-\lambda)x + \lambda y) - v_{n,r,s}(x)}{v_{n,r,s}(y) - v_{n,r,s}(x)} \right]^{l_1-m_1} \\
 &\quad \cdot \binom{l_2}{m_2} \left[\frac{v_{n,r,s}((1-\lambda)x + \lambda y) - v_{n,r,s}(x)}{v_{n,r,s}(y) - v_{n,r,s}(x)} \right]^{m_2} \cdot \left[1 - \frac{v_{n,r,s}((1-\lambda)x + \lambda y) - v_{n,r,s}(x)}{v_{n,r,s}(y) - v_{n,r,s}(x)} \right]^{l_2-m_2}
 \end{aligned}$$

$$\begin{aligned}
& \cdot f\left(\frac{k_1+m_1+r(k_2+m_2)}{n}\right). \\
& = \sum_{k_1+l_1=0}^{n-rs} \sum_{k_2+l_2=0}^s p_{n-rs,k_1,l_1} (v_{n,r,s}(x), v_{n,r,s}(y) - v_{n,r,s}(x)) \\
& \quad \cdot p_{s,k_2,l_2} (v_{n,r,s}(x), v_{n,r,s}(y) - v_{n,r,s}(x)) \\
& \quad \cdot \sum_{m_1=0}^{l_1} p_{l_1,m_1} \left(\frac{v_{n,r,s}((1-\lambda)x + \lambda y) - v_{n,r,s}(x)}{v_{n,r,s}(y) - v_{n,r,s}(x)} \right) \\
& \quad \cdot \sum_{m_2=0}^{l_2} p_{l_2,m_2} \left(\frac{v_{n,r,s}((1-\lambda)x + \lambda y) - v_{n,r,s}(x)}{v_{n,r,s}(y) - v_{n,r,s}(x)} \right) \\
& \quad \cdot f\left(\frac{k_1+m_1+r(k_2+m_2)}{n}\right).
\end{aligned}$$

Theorem 1. If f is a non-decreasing function, then $V_{n,r,s}f$ is a non-decreasing function.

Proof: Let $0 \leq x < y \leq 1$. From (4), we have

$$\begin{aligned}
& V_{n,r,s}(f, y) - V_{n,r,s}(f, x) \\
& = \sum_{k_1+l_1=0}^{n-rs} \sum_{k_2+l_2=0}^s p_{n-rs,k_1,l_1} (v_{n,r,s}(x), v_{n,r,s}(y) - v_{n,r,s}(x)) \\
& \quad \cdot p_{s,k_2,l_2} (v_{n,r,s}(x), v_{n,r,s}(y) - v_{n,r,s}(x)) \\
& \quad \cdot \left[f\left(\frac{k_1+l_1+r(k_2+l_2)}{n}\right) - f\left(\frac{k_1+rk_2}{n}\right) \right] \geq 0.
\end{aligned}$$

Remark 1. If f is a convex function, then $V_{n,r,s}f$ is not necessarily a convex function. If the operators $V_{n,r,s}$ preserved convexity, and therefore concavity, then they should reproduce polynomials of degree 1, which does not happen.

Theorem 2. If f is a non-decreasing convex function, then $V_{n,r,s}f$ is a convex function.

Proof: Let $0 \leq x < y \leq 1$ and $\lambda \in (0,1)$. From (4), we have

$$\begin{aligned}
& (1-\lambda)V_{n,r,s}(f, x) + \lambda V_{n,r,s}(f, y) - V_{n,r,s}(f, (1-\lambda)x + \lambda y) \\
& = \sum_{k_1+l_1=0}^{n-rs} \sum_{k_2+l_2=0}^s p_{n-rs,k_1,l_1} (v_{n,r,s}(x), v_{n,r,s}(y) - v_{n,r,s}(x)) \\
& \quad \cdot p_{s,k_2,l_2} (v_{n,r,s}(x), v_{n,r,s}(y) - v_{n,r,s}(x)) \\
& \quad \cdot \left[(1-\lambda)f\left(\frac{k_1+rk_2}{n}\right) + \lambda f\left(\frac{k_1+l_1+r(k_2+l_2)}{n}\right) \right. \\
& \quad \left. - \sum_{m_1=0}^{l_1} p_{l_1,m_1} \left(\frac{v_{n,r,s}((1-\lambda)x + \lambda y) - v_{n,r,s}(x)}{v_{n,r,s}(y) - v_{n,r,s}(x)} \right) \right. \\
& \quad \left. \cdot \sum_{m_2=0}^{l_2} p_{l_2,m_2} \left(\frac{v_{n,r,s}((1-\lambda)x + \lambda y) - v_{n,r,s}(x)}{v_{n,r,s}(y) - v_{n,r,s}(x)} \right) f\left(\frac{k_1+m_1+r(k_2+m_2)}{n}\right) \right] \\
& = \sum_{k_1+l_1=0}^{n-rs} \sum_{k_2+l_2=0}^s p_{n-rs,k_1,l_1} (v_{n,r,s}(x), v_{n,r,s}(y) - v_{n,r,s}(x)) \\
& \quad \cdot p_{s,k_2,l_2} (v_{n,r,s}(x), v_{n,r,s}(y) - v_{n,r,s}(x)) \\
& \quad \cdot \left[f\left(\frac{k_1+rk_2}{n}\right) + \lambda \left(f\left(\frac{k_1+l_1+r(k_2+l_2)}{n}\right) - f\left(\frac{k_1+rk_2}{n}\right) \right) \right. \\
& \quad \left. - \sum_{m_1=0}^{l_1} p_{l_1,m_1} \left(\frac{v_{n,r,s}((1-\lambda)x + \lambda y) - v_{n,r,s}(x)}{v_{n,r,s}(y) - v_{n,r,s}(x)} \right) \right. \\
& \quad \left. \cdot \sum_{m_2=0}^{l_2} p_{l_2,m_2} \left(\frac{v_{n,r,s}((1-\lambda)x + \lambda y) - v_{n,r,s}(x)}{v_{n,r,s}(y) - v_{n,r,s}(x)} \right) f\left(\frac{k_1+m_1+r(k_2+m_2)}{n}\right) \right].
\end{aligned}$$

We have

$$f\left(\frac{k_1+l_1+r(k_2+l_2)}{n}\right) - f\left(\frac{k_1+rk_2}{n}\right) \geq 0.$$

Since $v_{n,r,s}''(x) > 0$ for all $x \in [0,1]$, it follows that $v_{n,r,s}$ is a (strictly) convex function, which implies that

$$\frac{v_{n,r,s}((1-\lambda)x+\lambda y) - v_{n,r,s}(x)}{v_{n,r,s}(y) - v_{n,r,s}(x)} \leq \lambda.$$

It results that

$$\begin{aligned} & (1-\lambda)V_{n,r,s}(f, x) + \lambda V_{n,r,s}(f, y) - V_{n,r,s}(f, (1-\lambda)x + \lambda y) \\ & \geq \sum_{k_1+l_1=0}^{n-rs} \sum_{k_2+l_2=0}^s p_{n-rs, k_1, l_1} (v_{n,r,s}(x), v_{n,r,s}(y) - v_{n,r,s}(x)) \\ & \quad \cdot p_{s, k_2, l_2} (v_{n,r,s}(x), v_{n,r,s}(y) - v_{n,r,s}(x)) \\ & \quad \cdot \left[\left(1 - \frac{v_{n,r,s}((1-\lambda)x+\lambda y) - v_{n,r,s}(x)}{v_{n,r,s}(y) - v_{n,r,s}(x)} \right) f\left(\frac{k_1+rk_2}{n}\right) \right. \\ & \quad \left. + \frac{v_{n,r,s}((1-\lambda)x+\lambda y) - v_{n,r,s}(x)}{v_{n,r,s}(y) - v_{n,r,s}(x)} f\left(\frac{k_1+l_1+r(k_2+l_2)}{n}\right) \right. \\ & \quad \left. - \sum_{m_1=0}^{l_1} p_{l_1, m_1} \left(\frac{v_{n,r,s}((1-\lambda)x+\lambda y) - v_{n,r,s}(x)}{v_{n,r,s}(y) - v_{n,r,s}(x)} \right) \right. \\ & \quad \left. \cdot \sum_{m_2=0}^{l_2} p_{l_2, m_2} \left(\frac{v_{n,r,s}((1-\lambda)x+\lambda y) - v_{n,r,s}(x)}{v_{n,r,s}(y) - v_{n,r,s}(x)} \right) f\left(\frac{k_1+m_1+r(k_2+m_2)}{n}\right) \right] \\ & = \sum_{k_1+l_1=0}^{n-rs} \sum_{k_2+l_2=0, l_1+rl_2 \neq 0}^s p_{n-rs, k_1, l_1} (v_{n,r,s}(x), v_{n,r,s}(y) - v_{n,r,s}(x)) \\ & \quad \cdot p_{s, k_2, l_2} (v_{n,r,s}(x), v_{n,r,s}(y) - v_{n,r,s}(x)) \\ & \quad \cdot \sum_{m_1=0}^{l_1} \sum_{m_2=0}^{l_2} p_{l_1, m_1} \left(\frac{v_{n,r,s}((1-\lambda)x+\lambda y) - v_{n,r,s}(x)}{v_{n,r,s}(y) - v_{n,r,s}(x)} \right) \\ & \quad \cdot p_{l_2, m_2} \left(\frac{v_{n,r,s}((1-\lambda)x+\lambda y) - v_{n,r,s}(x)}{v_{n,r,s}(y) - v_{n,r,s}(x)} \right) \\ & \quad \cdot \left[\left(1 - \frac{m_1+rm_2}{l_1+rl_2} \right) f\left(\frac{k_1+rk_2}{n}\right) + \frac{m_1+rm_2}{l_1+rl_2} f\left(\frac{k_1+l_1+r(k_2+l_2)}{n}\right) - f\left(\frac{k_1+m_1+r(k_2+m_2)}{n}\right) \right]. \end{aligned}$$

For $l_1 + rl_2 \neq 0$ we have

$$\frac{k_1+m_1+r(k_2+m_2)}{n} = \left(1 - \frac{m_1+rm_2}{l_1+rl_2} \right) \frac{k_1+rk_2}{n} + \frac{m_1+rm_2}{l_1+rl_2} \cdot \frac{k_1+l_1+r(k_2+l_2)}{n}.$$

Since f is a convex function it results

$$\begin{aligned} & f\left(\frac{k_1+m_1+r(k_2+m_2)}{n}\right) \\ & \leq \left(1 - \frac{m_1+rm_2}{l_1+rl_2} \right) f\left(\frac{k_1+rk_2}{n}\right) + \frac{m_1+rm_2}{l_1+rl_2} f\left(\frac{k_1+l_1+r(k_2+l_2)}{n}\right) \end{aligned}$$

from where

$$(1-\lambda)V_{n,r,s}(f, x) + \lambda V_{n,r,s}(f, y) - V_{n,r,s}(f, (1-\lambda)x + \lambda y) \geq 0.$$

So, $V_{n,r,s}f$ is a convex function. We denote by $Lip_M \alpha$ the class of Lipschitz continuous functions on $[0,1]$ with exponent $0 < \alpha \leq 1$ and the Lipschitz constant $M > 0$ i.e., the set of all real-valued continuous functions f defined on $[0,1]$ that verify the condition $|f(x) - f(y)| \leq M \cdot |x - y|^\alpha, x, y \in [0,1]$.

Theorem 3. If $f \in Lip_M \alpha$, then $V_{n,r,s}f \in Lip_{M\left(\frac{2n^2}{n(2n-1)-rs(r-1)}\right)} \alpha$.

Proof: Let $0 \leq x < y \leq 1$. From (4), we have

$$\begin{aligned} & |V_{n,r,s}(f, y) - V_{n,r,s}(f, x)| \\ & \leq \sum_{k_1+l_1=0}^{n-rs} \sum_{k_2+l_2=0}^s p_{n-rs, k_1, l_1} (v_{n,r,s}(x), v_{n,r,s}(y) - v_{n,r,s}(x)) \\ & \quad \cdot p_{s, k_2, l_2} (v_{n,r,s}(x), v_{n,r,s}(y) - v_{n,r,s}(x)) \\ & \quad \cdot \left| f\left(\frac{k_1+l_1+r(k_2+l_2)}{n}\right) - f\left(\frac{k_1+rk_2}{n}\right) \right|. \end{aligned}$$

Because $f \in Lip_M \alpha$, it results that $\left| f\left(\frac{k_1+l_1+r(k_2+l_2)}{n}\right) - f\left(\frac{k_1+rk_2}{n}\right) \right| \leq M \left(\frac{l_1+rl_2}{n}\right)^\alpha$ so we obtain

$$\begin{aligned} & |V_{n,r,s}(f, y) - V_{n,r,s}(f, x)| \\ & \leq M \sum_{k_1+l_1=0}^{n-rs} \sum_{k_2+l_2=0}^s p_{n-rs, k_1, l_1} (v_{n,r,s}(x), v_{n,r,s}(y) - v_{n,r,s}(x)) \\ & \quad \cdot p_{s, k_2, l_2} (v_{n,r,s}(x), v_{n,r,s}(y) - v_{n,r,s}(x)) \left(\frac{l_1+rl_2}{n}\right)^\alpha \\ & \leq M \left(\sum_{k_1+l_1=0}^{n-rs} \sum_{k_2+l_2=0}^s p_{n-rs, k_1, l_1} (v_{n,r,s}(x), v_{n,r,s}(y) - v_{n,r,s}(x)) \right. \\ & \quad \cdot p_{s, k_2, l_2} (v_{n,r,s}(x), v_{n,r,s}(y) - v_{n,r,s}(x)) \left.\right) \frac{l_1+rl_2}{n}^\alpha \\ & = M \left(\frac{rs}{n} (v_{n,r,s}(y) - v_{n,r,s}(x)) + \frac{n-rs}{n} (v_{n,r,s}(y) - v_{n,r,s}(x)) \right)^\alpha \\ & = M (v_{n,r,s}(y) - v_{n,r,s}(x))^\alpha \\ & \leq M \left(\frac{2n^2}{n(2n-1)-rs(r-1)}\right)^\alpha (y-x)^\alpha. \end{aligned}$$

So

$$V_{n,r,s}f \in Lip_{M\left(\frac{2n^2}{n(2n-1)-rs(r-1)}\right)} \alpha.$$

3. CONCLUSION

The operators introduced in this paper represent a Stancu-type generalization of the King operators. These operators interpolate the function at the endpoints 0 and 1, preserve the test functions e_0 and e_2 , and have certain shape-preserving properties.

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