

ZhM* TRANSFORM TO SOLVE LINEAR ORDINARY DIFFERENTIAL OPERATOR EQUATIONS OF THE MOMENT PARETO DISTRIBUTION

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Abstract. Because of this, many researchers have proposed methods, procedures, and mathematical algorithms to facilitate finding accurate solutions. In this article, we introduce the ZhM* integral transform, a novel integral transform for evaluating the exact solution of linear ordinary differential operator equations with trigonometric functions as a variable coefficient, as well as linear ordinary differential operator equations of the moment Pareto distribution (MPD); the exact solutions of the linear ordinary differential operator equations of its hazard and survival functions will be found. In this paper, we are going to use a general integral transform to find the solution of the linear ordinary differential operator equation with a sine coefficient, which has the general formula:

$$C_0(\sin N)^n \frac{d^n y(\sin N)}{dN^n} + C_1(\sin N)^{n-1} \frac{d^{n-1} y(\sin N)}{dN^{n-1}} + C_2(\sin N)^{n-2} \frac{d^{n-2} y(\sin N)}{dN^{n-2}} + \dots + C_n y(\sin N) = \eta(\sin N),$$

The above equation is known as the Ali and Zafar equation. This article is also important because the parameter functions can be chosen based on the application or real-life problem, as many mathematical and engineering applications relate to these parameters.

Keywords: Generalization of AMK Transform; hazard function; inverse of ZhM*-transform; Moment Pareto Distribution; survival function; ZhM*-transform

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1. INTRODUCTION

An integral transform is a mathematical operation that converts a function into another function by integration. This approach involves multiplying the original function by a specific "kernel of the transform" and integrating the result over a given range to produce a new function, which often resides in a different mathematical domain (such as the frequency domain instead of the time t domain). The aim is to simplify difficult problems or applications, such as differential operator equations, by transforming them into more manageable algebraic models in a new domain [1-30]. In this context, a general integral transform was proposed by M. F. Salman and Qin Zhao [11]. We note that this integral transform differs from previous transforms in the transformation interval and kernel, as the interval is not reciprocal. It is between 0 and $\pi/2$, and the kernel is not expressed in

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terms of the exponential function. This means that this transform will be used in important applications, as will be mentioned in the applications section.

2. MATERIALS AND METHODS

2.1. DEFINITION [12]

If the function $F(s)$ represents the convergent integral transform of η (where η is a function of N) that lies in the open interval (a, β) , then $F(s)$ is defined as:

$$IT\{\eta(n)\} = F(s) = \int_a^\beta ke(s, N) \cdot \eta(N) dN$$

where α and β belong to $\mathbb{R}$ and ke is the kernel of the transform.

2.2. DEFINITION OF THE AMK INTEGRAL TRANSFORM [31]

For the function $\eta(\sin(N))$, where $N \in [0, \pi/2]$, the convergent AMK integral transform is defined as:

$$AMK\{\eta(\sin N)\} = \int_0^{\frac{\pi}{2}} \cos N (\sin N)^\rho \eta(\sin N) dN = F(\rho),$$

where ρ is a constant.

2.3. DEFINITION [11]

ZhM* transform transform for the function $\eta(\sin N)$ where N belongs to closed interval $[0, \pi/2]$ is known as:

$$ZhM^*\{\eta(\sin N)\} = f(\rho) \int_{N=0}^{\frac{\pi}{2}} \cos N (\sin N)^{g(\rho)} \eta(\sin N) dN = F(\rho).$$

where $f(\rho)$ and $g(\rho)$ are functions of the parameter ρ , $f(\rho), g(\rho) \neq 0$, and the above definite integral is convergent in $[0, \pi/2]$.

2.4. DEFINITION [11]

Assume that $\eta(\sin N)$ is a function and $ZhM^*\{\eta(\sin N)\} = F(\rho)$, so $\eta(\sin N)$ is said to be the inverse of the function $F(\rho)$ and is known as:

$$(ZhM^*)^{-1}\{F(\rho)\} = \eta(\sin N).$$

2.5. THEOREM [11]

Suppose that $F(\rho) = \text{ZhM}^* \{ \eta(\sin N) \}$ and α is constant, so

$$(\text{ZhM}^*)^{-1} F(\rho), g(\rho) + \alpha = (\sin N)^\alpha (\text{ZhM}^*)^{-1} F(\rho), g(\rho)$$

2.6. ZhM* INTEGRAL TRANSFORM TRANSFORMS FOR SOME IMPORTANT FUNCTIONS [11]

Table 1. Some important functions of the ZhM* integral transform.

Function $\eta(\sin N)$	$\text{ZhM}^* \{ \eta(\sin N) \} = f(\rho) \int_{N=0}^{\frac{\pi}{2}} (\sin N)^{g(\rho)} \cos N \eta(\sin N) dN$
k where k is a constant	$\frac{k f(\rho)}{g(\rho)+1}, g(\rho) > -1.$
(sin(N))ⁿ	$\frac{f(\rho)}{g(\rho)+(n+1)}, g(\rho) > -(n+1).$
cos(a ln(sinN))	$\frac{f(\rho) \cdot (g(\rho)+1)}{(g(\rho)+1)^2 + a^2}, a \text{ is constant.}$
sin(a ln(sinN))	$\frac{a f(\rho)}{(g(\rho)+1)^2 + a^2}, a \text{ is constant.}$
cosh(a ln(sinN))	$\frac{f(\rho) \cdot (g(\rho)+1)}{(g(\rho)+1)^2 - a^2}, a \text{ is constant.}$
sinh(a ln(sinN))	$\frac{-f(\rho) a}{(1+g(\rho))^2 - a^2}, a \text{ is constant.}$
[ln(sinN)]ⁿ where $n \in \mathbb{N}$	$\frac{f(\rho) n! (-1)^n}{(1+g(\rho))^{n+1}}, g(\rho) > -1.$

2.7. THEOREM [11]

If the $\eta(\sin(N))$ is defined as $N > 0$ and its derivatives $\frac{d\eta(\sin(N))}{dN}, \frac{d^2\eta(\sin(N))}{dN^2}, \dots, \frac{d^n\eta(\sin(N))}{dN^n}$ exist, then:

$$\begin{aligned} \text{ZhM}^* \{ (\sin(N))^n \eta^{(n)}(\sin(N)) \} \\ = f(\rho) [\eta^{(n-1)}(1) \\ - (n + g(\rho)) \eta^{(n-2)}(1) \\ + (-1)^{n-1} (n + g(\rho)) ((n-1) \\ + g(\rho)) \eta^{(n-3)}(1) + \dots \\ + (g(\rho) + n) ((n-1) + g(\rho)) \dots \\ + (g(\rho) + 2) \eta(1) \\ + (-1)^n [(g(\rho) + n)!] F[f(\rho), g(\rho)]. \end{aligned}$$

where $n \in \mathbb{N}$.

3. RESULTS AND DISCUSSION

3.1. RESULTS

3.1.1. Survival function of Moment Pareto Distribution

$$\eta(N) = \alpha^{(\beta-1)} N^{-(\beta+1)}, \alpha > 0, \alpha < N < \infty, \beta > 0$$

ZhM*-transform of the first-order differential equation of the survival function of the Moment Pareto Distribution. The first-order differential equation (O.D.E.) of the survival function of the Moment Pareto Distribution is given as:

$$N\eta'(N) + (\beta + 1)\eta(N) = 0$$

where $\eta(1) = \alpha^{(\beta-1)}$, $\eta'(1) = -(\beta + 1)\alpha^{(\beta-1)}$.

Solution:

Let $N = \sin(x)$, so the equation will become:

$$\sin(x)\eta'(\sin(x)) + (\beta + 1)\eta(\sin(x)) = 0,$$

By using the ZhM*-integral transform and applying Theorem (2.6), we get:

$$\begin{aligned} ZhM^*\{\sin(x)\eta'(\sin(x))\} + (\beta + 1)ZhM^*\{\eta(\sin(x))\} &= 0, \\ f(\rho)\eta(1) - (g(\rho) + 1)F + (\beta + 1)F &= 0, \\ [(\beta + 1) - (g(\rho) + 1)]F &= -f(\rho)\eta(1), \\ (\beta + 1 - g(\rho) - 1)F &= -f(\rho)\eta(1), \\ (-g(\rho) + \beta)F &= -f(\rho)\eta(1), \\ (g(\rho) - \beta)F &= f(\rho)\alpha^{(\beta-1)}, \\ F &= \frac{f(\rho)\alpha^{(\beta-1)}}{g(\rho) - \beta} = \frac{f(\rho)\alpha^{(\beta-1)}}{g(\rho) - \beta - 1 + 1} = \frac{f(\rho)\alpha^{(\beta-1)}}{g(\rho) - (\beta + 1) + 1}, \end{aligned}$$

By applying inverse of ZhM*-transform, we obtain:

$$\eta(\sin(N)) = \alpha^{(\beta-1)}(\sin(x))^{-(\beta+1)}, \text{ as } N = \sin(x).$$

so

$$\eta(N) = \alpha^{(\beta-1)} N^{-(\beta+1)}.$$

which is the required PDF (Pareto Distribution Function).

3.1.2. ZhM*-transforms on second order O.D.E. of survival function of moment Pareto distribution

The second-order differential equation of the survival function of the Moment Pareto Distribution is given by:

$$N\eta''(N) + (\beta + 2)\eta'(N) = 0$$

with

$$\eta(1) = \alpha^{(\beta-1)}, \eta'(1) = -(\beta + 1)\alpha^{(\beta-1)},$$

and

$$\eta''(1) = (\beta + 1)(\beta + 2)\alpha^{(\beta-1)}.$$

Solution:

Let $N=\sin(x)$, so the equation becomes:

$$\sin(x)\eta''(\sin(x)) + (\beta + 2)\eta'(\sin(x)) = 0.$$

Multiply by $\sin(x)$ on both sides; we get:

$$\sin^2(x)\eta''(\sin(x)) + (\beta + 2)\sin(x)\eta'(\sin(x)) = 0.$$

By applying the ZhM*-transform and Theorem (2.6),

$$\begin{aligned} f(\rho)\eta'(1) - f(\rho)(g(\rho) + 2)\eta(1) + (g(\rho) + 2)(g(\rho) + 1)F + (\beta + 2)[f(\rho)\eta(1) - \\ (g(\rho) + 1)F] &= 0. \\ f(\rho)\eta'(1) - f(\rho)(g(\rho) + 2)\eta(1) + (g(\rho) + 2)(g(\rho) + 1)F + (\beta + 2)f(\rho)\eta(1) \\ - (\beta + 2)(g(\rho) + 1)F &= 0 \\ (g(\rho) + 1)[(g(\rho) + 2) - (\beta + 2)]F \\ &= f(\rho)[- \eta'(1) + (g(\rho) + 2)\eta(1) - (\beta + 2)\eta(1)]. \\ (g(\rho) + 1)(g(\rho) + 2 - \beta - 2)F &= f(\rho)[- \eta'(1) + (g(\rho) + 2)\eta(1) - (\beta + 2)\eta(1)]. \\ F &= \frac{f(\rho)[(\beta + 1)\alpha^{(\beta-1)} + (g(\rho) + 2)\alpha^{(\beta-1)} - (\beta + 2)\alpha^{(\beta-1)}]}{(g(\rho) + 1)(g(\rho) - \beta)}, \\ F &= \frac{f(\rho)\alpha^{(\beta-1)}[(\beta + 1) + (g(\rho) + 2) - (\beta + 2)]}{(g(\rho) + 1)(g(\rho) - \beta)}, \\ &= \frac{f(\rho)\alpha^{(\beta-1)}[\beta + 1 + g(\rho) + 2 - \beta - 2]}{(g(\rho) + 1)(g(\rho) - \beta)}, \\ &= \frac{f(\rho)\alpha^{(\beta-1)}(g(\rho) + 1)}{(g(\rho) + 1)(g(\rho) - \beta - 1 + 1)} = \frac{f(\rho)\alpha^{(\beta-1)}}{g(\rho) - (\beta + 1) + 1} \end{aligned}$$

By applying inverse of ZhM*-transform, we get:

$$\eta(\sin(x)) = \alpha^{(\beta-1)}(\sin(x))^{-(\beta+1)}.$$

As

$$N=\sin(x),$$

so, we have

$$\eta(N) = \alpha^{(\beta-1)}(N)^{-(\beta+1)}.$$

which is the required survival function.

3.1.3. ZhM*-transforms on 3rd O.D.E. of survival function of Moment Pareto Distribution

The differential equation of the 3rd order of the survival function of the Moment Pareto Distribution is given by:

$$N\eta'''(N) + (\beta + 3)\eta''(N) = 0$$

and

$$\begin{aligned}\eta(1) &= \alpha^{(\beta-1)}, \eta'(1) = -(\beta + 1)\alpha^{(\beta-1)} \\ \eta''(1) &= (\beta + 1)(\beta + 2)\alpha^{(\beta-1)}.\end{aligned}$$

Solution:

Let $N = \sin(x)$, so the equation will become:

$$\sin(x)\eta'''(\sin(x)) + (\beta + 3)\eta''(\sin(x)) = 0.$$

Multiply by $\sin^2(x)$ on both sides of the equation, we get:

$$\sin^3(x)\eta'''(\sin(x)) + (\beta + 3)\sin^2(x)\eta''(\sin(x)) = 0.$$

By taking the ZhM*-transform of the above equation and using Theorem (2.6)

$$\begin{aligned}\text{ZhM}^*\{\sin^3(x)\eta'''(\sin(x))\} + (\beta + 3)\text{ZhM}^*\{\sin^2(x)\eta''(\sin(x))\} &= 0. \\ f(\rho)[\eta''(1) - (3 + g(\rho))\eta'(1) + (3 + g(\rho))(2 + g(\rho))\eta(1)] - (g(\rho) + 1)(g(\rho) + 2)(g(\rho) + 3)F &+ (\beta + 3)f(\rho)[\eta'(1) - (2 + g(\rho))\eta(1)] + (\beta + 3)(g(\rho) + 1)(g(\rho) + 2)F = 0. \\ -(g(\rho) + 1)(g(\rho) + 2)[-(\beta + 3) + (g(\rho) + 3)]F &= -f(\rho)[\eta''(1) - (g(\rho) + 3)\eta'(1) + (g(\rho) + 3) \\ (2 + g(\rho))\eta(1) + (\beta + 3)\eta'(1) - (\beta + 3)(2 + g(\rho))\eta(1)] &= 0. \\ (g(\rho) + 1)(g(\rho) + 2)[- \beta - 3 + g(\rho) + 3]F & \\ = f(\rho)\alpha^{(\beta-1)}[(\beta + 1)(\beta + 2) + (g(\rho) + 3)(\beta + 1) & \\ + (3 + g(\rho))(2 + g(\rho)) - (\beta + 3)(\beta + 1) - (\beta + 3)(2 + g(\rho))] & \\ F = \frac{f(\rho)\alpha^{(\beta-1)}[\beta^2 + 3\beta + 2 + \beta g(\rho) + g(\rho) + 3\beta + 3 + 6 + 5g(\rho) + g^2(\rho) - \beta^2 - 4\beta - 3 - 2\beta - \beta g(\rho) - 6 - 3g(\rho)]}{(g(\rho) + 1)g(\rho) + 2)g(\rho) - \beta)} & \\ F = \frac{f(\rho)\alpha^{(\beta-1)}[g^2(\rho) + 3g(\rho) + 2]}{(g(\rho) + 1)(g(\rho) + 2)(g(\rho) - \beta - 1 + 1)} & \\ F = \frac{f(\rho)\alpha^{(\beta-1)}(g(\rho) + 1)(g(\rho) + 2)}{(g(\rho) + 1)(g(\rho) + 2)(g(\rho) - \beta - 1 + 1)} & \\ F = \frac{f(\rho)\alpha^{(\beta-1)}}{(g(\rho) - \beta - 1 + 1)} & \\ F = \frac{f(\rho)\alpha^{(\beta-1)}}{(g(\rho) - (\beta + 1) + 1)} &\end{aligned}$$

Taking the inverse of the ZhM*-transform, we have

$$\eta(\sin(x)) = \alpha^{(\beta-1)} (\sin(x))^{-(\beta+1)}.$$

Since

$$N = \sin(x) \Rightarrow \eta(N) = \alpha^{(\beta-1)} N^{-(\beta+1)}$$

which is the required survival function.

3.1.4. Transforms on O.D.E. of hazard function of Moment Pareto Distribution

The hazard function of the Moment Pareto Distribution is defined as:

$$\eta(N) = \frac{\beta-1}{N}, N > 0, \beta > 0.$$

The first-order differential equation of the hazard function of the Moment Pareto Distribution is given as:

$$N\eta'(N) + \eta(N) = 0 \text{ and } \eta(1) = \beta - 1, \eta'(1) = -(\beta - 1), \eta''(1) = 2(\beta - 1).$$

Solution:

Let $N = \sin(x)$ So, the equation will become:

$$\sin(x)\eta'(\sin(x)) + \eta(\sin(x)) = 0.$$

By using the ZhM*-transform and applying Theorem (2.6):

$$\begin{aligned} \text{ZhM}^*\{\sin(x)\eta'(\sin(x))\} + \text{ZhM}^*\{\eta(\sin(x))\} &= 0, \\ f(\rho)\eta(1) - (g(\rho) + 1)F + F &= 0, \\ -[(g(\rho) + 1) - 1]F &= -f(\rho)\eta(1), \\ F &= \frac{f(\rho)(\beta-1)}{(g(\rho)+1)-1}, \end{aligned}$$

By applying inverse of ZhM*-transform, we get:

$$\eta(\sin(x)) = (\beta - 1)(\sin(x))^{-1} = \frac{(\beta-1)}{\sin(x)}.$$

As

$$N = \sin(x),$$

so

$$\eta(N) = \frac{(\beta-1)}{N}.$$

which is the required hazard function.

3.1.5. ZhM*-transforms on 2nd O.D.E. of hazard function of Moment Pareto Distribution

The differential equation of the 2nd order of the hazard function of the Moment Pareto distribution is given as:

$$N\eta''(N) + 2\eta'(N) = 0,$$

and

$$\eta(1) = (\beta - 1), \eta'(1) = -(\beta - 1), \eta''(1) = 2(\beta - 1),$$

Solution:

Let $N = \sin(x)$. So, the equation will become

$$\sin(x)\eta''(\sin(x)) + 2\eta'(\sin(x)) = 0,$$

multiply by $\sin(x)$ on both sides of the above equation, we get

$$\sin^2(x)\eta''(\sin(x)) + 2(\sin(x))\eta'(\sin(x)) = 0.$$

By using the ZhM*-transform and applying Theorem (2.6):

$$\begin{aligned} \text{ZhM}^*\{\sin^2(x)\eta''(\sin(x))\} + 2\text{ZhM}^*\{(\sin(x))\eta'(\sin(x))\} &= 0. \\ f(\rho)\eta'(1) - f(\rho)(2 + g(\rho))\eta(1) + (g(\rho) + 1)(g(\rho) + 2)F + 2f(\rho)\eta(1) - 2(g(\rho) + 1)F &= 0, \\ (g(\rho) + 1)[(g(\rho) + 2) - 2]F &= f(\rho)\eta'(1) + f(\rho)(2 + g(\rho))\eta(1) - 2f(\rho)\eta(1), \\ F &= \frac{f(\rho)[(\beta-1)+(2+g(\rho))(\beta-1)-2(\beta-1)]}{(g(\rho)+1)[g(\rho)+2-2]}, \\ F &= \frac{f(\rho)[(\beta-1)[1+(2+g(\rho))-2]}{(g(\rho)+1)[g(\rho)+2-2]} = \frac{f(\rho)(\beta-1)[g(\rho)+1]}{[g(\rho)+1](g(\rho)+1+1-1-1)} = \frac{f(\rho)[(\beta-1)]}{g(\rho)+1-1}. \end{aligned}$$

By applying inverse of ZhM*-transform, we get:

$$\eta(\sin(x)) = \frac{(\beta-1)}{\sin(x)}.$$

As

$$N = \sin(x), \text{ so } \eta(N) = \frac{(\beta-1)}{N}$$

which is the required hazard function.

3.1.6. ZhM*-transforms on 3rd O.D.E. of hazard function of Moment Pareto Distribution

The differential equation of the 3rd order of the hazard function of the moment Pareto distribution is given as:

$$N\eta'''(N) + 2\eta''(N) = 0$$

with

$$\eta(1) = (\beta - 1), \eta'(1) = -(\beta - 1), \eta''(1) = 2(\beta - 1).$$

Solution:

Let $N = \sin(x)$. So, the equation will become:

$$\sin(x)\eta'''(\sin(x)) + 3\eta''(\sin(x)) = 0.$$

Multiply by $\sin^2(x)$ on both sides of the above equation, we get

$$\sin^3(x)\eta'''(\sin(x)) + 3(\sin^2(x))\eta''(\sin(x)) = 0,$$

By using the ZhM*-transform and applying Theorem (2.6), we get:

$$\begin{aligned} & \text{ZhM}^*\{\sin^3(x)\eta'''(\sin(x))\} + 3\text{ZhM}^*\{(\sin^2(x))\eta''(\sin(x))\} = 0, \\ & f(\rho)\eta''(1) - f(\rho)(3 + g(\rho))\eta'(1) + f(\rho)(3 + g(\rho))(2 + g(\rho))\eta(1) \\ & \quad - (g(\rho) + 3)(g(\rho) + 2)(g(\rho) + 1)F + 3f(\rho)\eta'(1) - 3f(\rho)(2 + g(\rho))\eta(1) \\ & \quad + 3(g(\rho) + 1)(g(\rho) + 2)F = 0. \\ & (g(\rho) + 1)(g(\rho) + 2)[-(g(\rho) + 3) + 3]F \\ & \quad = -f(\rho)\eta''(1) + f(\rho)(3 + g(\rho))\eta'(1) - f(\rho)(3 + g(\rho))(2 + g(\rho))\eta(1) \\ & \quad - 3f(\rho)\eta'(1) + 3f(\rho)(2 + g(\rho))\eta(1) = 0. \\ & -(g(\rho) + 1)(g(\rho) + 2)[(g(\rho) + 3) - 3]F = -f(\rho)(\beta - 1)[2 + (3 + g(\rho) + \\ & \quad (3 + g(\rho))(2 + g(\rho)) - 3 - 3(2 + g(\rho))] = 0. \\ & F = \frac{f(\rho)(\beta - 1)[2 + 3 + g(\rho) + 6 + 5g(\rho) + g^2(\rho) - 3 - 6 - 3g(\rho)]}{(g(\rho) + 1)(g(\rho) + 2)[g(\rho) + 3 - 3]}. \\ & F = \frac{f(\rho)(\beta - 1)[g^2(\rho) + 3g(\rho) + 2]}{(g(\rho) + 1)(g(\rho) + 2)[g(\rho) + 1 - 1]}. \\ & F = \frac{f(\rho)[(\beta - 1)]}{(g(\rho) + 1 - 1)}. \end{aligned}$$

By applying the inverse transform:

$$\eta(\sin(x)) = \frac{(\beta - 1)}{\sin(x)}.$$

As

$$N = \sin(x),$$

so

$$\eta(N) = \frac{(\beta - 1)}{N},$$

which is the required hazard function.

3.2. DISCUSSION

This paper presents solutions to linear ordinary differential equations with constant coefficients of the first, second, and third orders and applies them to important problems using an integral transform other than the Laplace or Fourier transforms, or modifications of them. This transform is considered new because it differs from the Laplace and Fourier transforms

in its interval and kernel. Furthermore, compared with classical integral transforms such as the Laplace and Elzaki transforms, the ZhM*-transform provides a simpler kernel and a restricted interval $[0, \pi/2]$, making it particularly efficient for solving oscillatory problems and systems with trigonometric coefficients. This makes the transform a potential tool for future extensions in mathematical physics and engineering modeling.

4. CONCLUSION

The ZhM*-transform is a simple procedure for evaluating exact solutions of ordinary differential operator equations. Here, we introduce the ZhM*-transform for various functions and reveal some important results. We also apply the generalization to the AMK transform (ZhM*-transform) to find exact solutions of the linear ordinary differential equation of the Moment Pareto Distribution, as well as the linear ordinary differential operator equations for its hazard and survival functions.

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