ORIGINAL PAPER

A MODELLING-ORIENTED FRAMEWORK FOR TEACHING THE LAPLACE TRANSFORM WITH APPLICATIONS TO ELECTRICAL CIRCUITS

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Abstract. *The Laplace Transform is a fundamental analytical tool in electrical engineering for modelling and analysing the transient behaviour of linear systems such as RL, RC, and RLC circuits. However, traditional instruction often emphasizes procedural computation with limited support for modelling interpretation and validation. This study proposes a modelling-oriented instructional framework that treats the Laplace Transform as a mathematical modelling tool. The instructional process consists of six stages: (1) presentation of an authentic engineering problem; (2) understanding and simplifying the real situation; (3) mathematical modelling of the system; (4) application of the Laplace Transform; (5) interpretation and validation of results; and (6) refinement and extension of the model. The framework is implemented through electrical circuit problems and is supported by numerical simulations in MATLAB. A one-group pretest–posttest quasi-experimental study was conducted with 80 undergraduate electrical engineering students. Quantitative results show statistically significant improvements in students' conceptual understanding of the Laplace Transform, while qualitative findings indicate enhanced engagement in mathematization, interpretation, and model validation. From an applied mathematics perspective, the results confirm the effectiveness of a modelling-oriented approach in Laplace Transform instruction.*

Keywords: *Laplace Transform; mathematical modelling; engineering mathematics; electrical engineering.*

1. INTRODUCTION

Mathematics occupies a central role in electrical engineering because it provides the conceptual and analytical tools needed to describe and investigate engineering systems. Mathematical topics such as differential equations, integral transforms, and modelling techniques are widely used to analyse electrical circuits, control processes, signal transmission, and power electronic systems [1]. The importance of this application-oriented perspective has been recognized in numerous studies published in the Journal of Science and Arts, in which mathematical modelling and simulation have been employed to explore and explain complex systems in engineering, economics, and related applied fields [2-4].

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The Laplace Transform is widely used in electrical engineering, especially when analysing the response of linear systems such as RL, RC, and RLC circuits. Replacing differential equations with equivalent algebraic forms in the complex domain allows engineers to examine system behaviour and transient responses more systematically. Its usefulness has been demonstrated in several studies published in the Journal of Science and Arts, in which Laplace-based approaches and other integral-transform techniques have been employed to solve differential equations and investigate practical phenomena [5-7].

Despite its importance, research in engineering education indicates that students often approach the Laplace Transform in a predominantly procedural manner, focusing on symbolic manipulation and transformation tables while lacking conceptual understanding of its mathematical meaning and physical interpretation [8,9]. This procedural emphasis limits students' ability to relate Laplace-domain expressions to the actual behaviour of electrical circuits, particularly in interpreting initial conditions, system poles, time constants, and transient responses. Such difficulties are closely associated with motivational issues, including reduced confidence, increased mathematics anxiety, and lower engagement in learning advanced engineering mathematics [10,11].

From a motivational perspective, Expectancy–Value Theory suggests that students' engagement depends on their beliefs about the likelihood of success and the perceived value of learning tasks [12,13]. When mathematical content is presented in an abstract and decontextualized manner, students often struggle to recognize its relevance to engineering practice, resulting in lower task value and weakened motivation. Conversely, instructional approaches grounded in authentic contexts and applications can enhance perceived usefulness and engagement.

Mathematical modelling offers a coherent theoretical framework to address these challenges. Modelling-oriented approaches describe mathematical analysis as an iterative cycle that involves understanding real situations, constructing mathematical models, performing analysis, interpreting results, and validating or refining the model [14,15]. Empirical studies show that modelling tasks enable engineering students to connect symbolic mathematics to physical system behaviour and to develop a deeper conceptual understanding [16,17]. Consistently, JOSA publications on modelling and simulation emphasize interpretation, validation, and system-level reasoning as essential components of applied mathematical inquiry [2,3].

Mathematical modelling may also contribute to the development of students' motivation. Learning activities based on meaningful engineering situations help students recognize the relevance of mathematics to professional practice. Through constructing, testing, and refining models, students gain experience in applying mathematical ideas to practical problems, which can enhance their confidence and engagement in learning [18, 19]. In addition, simulation and visualization technologies commonly reported in JOSA studies provide opportunities to explore system behaviour and compare analytical findings with numerical results [2,20].

Although modelling and simulation have been widely investigated, relatively little attention has been paid to instructional approaches that integrate the full modelling cycle into Laplace Transform teaching and examine students' motivation. Most publications in the Journal of Science and Arts focus primarily on mathematical methods and applications, whereas instructional design and student learning processes receive less attention. This indicates a clear research gap at the intersection of Laplace Transform instruction, mathematical modelling, and learning motivation in electrical engineering education.

To address this gap, the present study proposes and examines a modelling-oriented instructional framework for teaching the Laplace Transform through authentic electrical engineering contexts, particularly RL, RC, and RLC circuits. The instructional process consists of six stages: (1) presentation of an authentic engineering problem; (2) understanding and simplifying the real situation; (3) mathematical modelling of the system; (4) application of the Laplace Transform; (5) interpretation and validation of results; and (6) refinement and extension of the model. Using a one-group pretest–posttest quasi-experimental design, this study investigates the effects of the proposed framework on students' conceptual understanding of the Laplace Transform and key components of learning motivation. By aligning modelling-oriented instruction with the applied mathematics tradition of JOSA, the study aims to contribute both empirical evidence and a transferable instructional framework for engineering mathematics education.

This study contributes to the literature in three important ways. First, it proposes a six-stage modelling-oriented instructional framework specifically designed for teaching the Laplace Transform in electrical engineering contexts. Second, it demonstrates how the framework can be implemented through authentic RL, RC, and RLC circuit-modelling activities, supported by MATLAB-based simulation and validation. Third, it provides empirical evidence on the effects of modelling-oriented instruction on students' conceptual understanding and motivation to learn in engineering mathematics education.

Research Questions – To guide the investigation, the study addresses the following research questions:

RQ1. How does the proposed modelling-oriented instructional framework affect students' conceptual understanding of the Laplace Transform in electrical engineering contexts?

RQ2. How does the framework influence students' learning motivation in terms of expectancy for success, task value, and intrinsic motivation?

RQ3. How do students engage in the different phases of the mathematical modelling cycle when learning the Laplace Transform through electrical circuit applications?

2. MATERIALS AND METHODS

2.1. RESEARCH DESIGN

This study employed a mixed-methods research design, integrating quantitative and qualitative approaches to examine the effects of a modelling-oriented instructional framework on students' conceptual understanding and learning motivation in Laplace Transform instruction. Mixed-methods designs are particularly suitable for applied mathematics education research, as they allow researchers to capture both measurable learning outcomes and the modelling processes that underlie them [21].

The quantitative component focused on measuring changes in students' conceptual and modelling-related understanding of the Laplace Transform as well as motivational variables. The qualitative component aimed to capture students' modelling behaviours, classroom engagement, and interaction with modelling tasks. Such integration is recommended in modelling-oriented mathematics education research, where learning outcomes cannot be fully interpreted without considering students' modelling processes [14,22].

A one-group pretest–posttest quasi-experimental design was adopted. This design is commonly used in authentic higher-education contexts, where random assignment and the establishment of control groups are often constrained by institutional and curricular factors [23].

This design was selected to preserve the integrity of the applied modelling process within an authentic engineering mathematics course, where isolating control conditions may disrupt coherence and continuity in modelling activities.

The pretest was administered before the instructional intervention, and the posttest was administered after the modelling-oriented teaching sequence was completed. The instructional framework and research instruments were developed based on theories of mathematical modelling and learning motivation, and were reviewed by three experts in mathematics education and electrical engineering education to ensure content validity and pedagogical coherence [13,15].

2.2. PARTICIPANTS

The participants were 80 undergraduate students majoring in Electrical Engineering at Ho Chi Minh City University of Industry. All participants were enrolled in a compulsory Engineering Mathematics course that covered the Laplace Transform.

Prior to the study, all students had successfully completed prerequisite courses in Calculus and Basic Circuit Theory, ensuring adequate foundational knowledge for engaging with Laplace Transform applications in electrical circuits. Such prerequisite alignment is considered essential for research examining conceptual change in advanced engineering mathematics [1,8].

The participants were relatively homogeneous in terms of academic background and year of study, which contributed to internal consistency within the sample. All students participated voluntarily and completed both the pretest and posttest; no participant data were excluded from the analysis.

2.3. INSTRUCTIONAL INTERVENTION

The instructional intervention was implemented over four consecutive weeks and structured around a six-stage, modelling-oriented instructional process designed to teach the Laplace Transform in electrical engineering contexts. This design was motivated by prior research indicating that traditional, procedure-driven instruction often fails to help students connect Laplace-domain techniques to the physical behaviour of electrical systems, leading to fragmented understanding and reduced motivation to learn [9,8].

Research on mathematical modelling emphasizes that embedding abstract mathematical concepts within authentic engineering contexts supports conceptual understanding, analytical reasoning, and perceived task value [14,24]. Accordingly, the proposed instructional process integrates the Laplace Transform into a coherent modelling cycle that reflects professional engineering practice.

The instructional process consisted of six stages: (1) presentation of an authentic engineering problem; (2) understanding and simplifying the real situation; (3) mathematical modelling of the system; (4) application of the Laplace Transform; (5) interpretation and validation of results; (6) refinement and extension of the model (Figure 1).

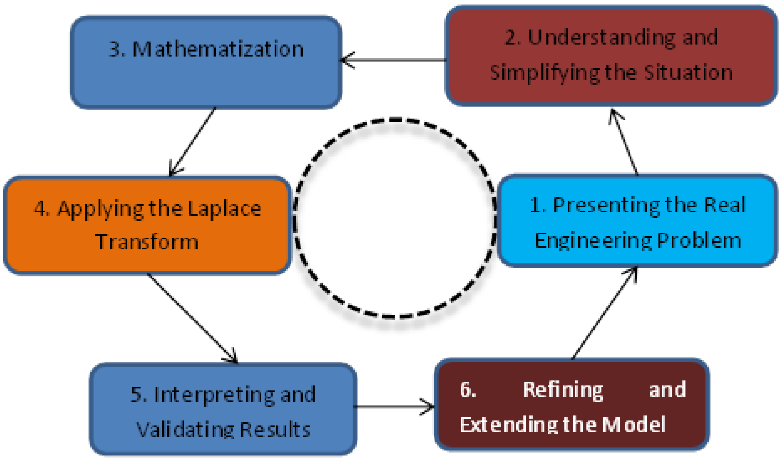

Figure 1. 6-step process of teaching the Laplace transform (feedback loop for model verification and adjustment).

To facilitate the implementation of the proposed instructional framework and enhance its transferability to other engineering mathematics courses, the intervention was organized into a four-week sequence of modelling activities. Each week focused on a different aspect of circuit modelling and Laplace Transform applications while maintaining alignment with the six-stage modelling cycle.

Table 1. Four-week implementation schedule of the modelling-oriented instructional intervention

Week	Circuit Topic	Modelling Focus	MATLAB Activity	Expected Student Product
1	RL circuits	Understanding authentic engineering problems and mathematization of first-order circuit models	Simulation of RL transient response and parameter exploration	RL modelling report
2	RC circuits	Application of the Laplace Transform to derive and analyse circuit responses	RC charging and discharging simulation	RC modelling worksheet
3	RLC circuits	Interpretation and validation of analytical solutions through modelling activities	Comparison of analytical and numerical responses	Group presentation and validation report
4	Extended modelling tasks	Parameter variation, model refinement, and engineering interpretation	Sensitivity analysis and scenario testing	Final modelling project

The sequence was designed to progressively develop students' modelling competencies, beginning with model construction and mathematization, followed by analytical solution, interpretation, validation, and model refinement. MATLAB activities were integrated throughout the intervention to support visualization, verification, and engineering interpretation of mathematical results.

Learning activities were centred on RL, RC, and RLC circuits, which are canonical systems in electrical engineering and engineering mathematics. Digital tools, particularly MATLAB, were integrated to support simulation, visualization, and model validation. Such integration aligns with applied-modelling traditions commonly emphasized in the *Journal of Science and Arts*, where modelling, interpretation, and validation are regarded as essential components of mathematical inquiry [2,6].

2.4. DATA COLLECTION INSTRUMENTS

2.4.1. Conceptual–Modelling Test

A Conceptual–Modelling Test was developed to assess students' understanding of the Laplace Transform and its applications in electrical circuits from a modelling perspective. The test consisted of 12 items designed to evaluate three integrated dimensions:
(1) conceptual understanding of the Laplace Transform,
(2) analytical and procedural modelling skills, and
(3) interpretation, validation, and parameter reasoning in circuit models.

Items were scored using a 0.25-point increment scale, and total scores were normalized to a 10-point scale. Content validity was established through expert review by three specialists in mathematics education and electrical engineering, with particular attention to modelling phases such as mathematization, interpretation, and validation [14,15].

2.4.2. Learning Motivation Questionnaire

Learning motivation was measured using a questionnaire grounded in Expectancy–Value Theory, with specific adaptation to modelling-oriented learning contexts. Rather than treating motivation as a general educational construct, the instrument focused on students' engagement with modelling activities involving the Laplace Transform and electrical circuit analysis.

The questionnaire comprised 15 Likert-scale items (1 = strongly disagree to 5 = strongly agree), organized into three dimensions:
(1) Expectancy for success in performing modelling-related tasks (e.g., formulating circuit models, applying the Laplace Transform, and validating results);
(2) Task value, reflecting students' perceptions of the usefulness and relevance of modelling activities for understanding electrical systems and engineering practice;
(3) Intrinsic motivation, associated with students' interest and satisfaction when engaging in modelling, interpretation, and simulation-based validation.

The items were explicitly contextualized within modelling activities, such as interpreting Laplace-domain solutions, analysing system parameters, and comparing analytical results with numerical simulations. This contextualization ensured alignment between motivational measures and the applied mathematical nature of the instructional framework.

The questionnaire was adapted from established Expectancy–Value instruments [13] and reviewed by experts in mathematics education and electrical engineering to ensure content relevance and consistency with modelling-oriented instruction.

To evaluate internal consistency, Cronbach's alpha coefficients were calculated for each motivational dimension. The results indicated satisfactory reliability for all scales, exceeding the commonly accepted threshold of 0.70 for educational research. Cronbach's alpha coefficients were calculated using responses from the pre-intervention administration of the questionnaire (n = 80).

Table 2. Reliability coefficients of the learning motivation questionnaire

Scale	Number of Items	Cronbach's α
Expectancy for Success	5	0.828
Task Value	5	0.804
Intrinsic Motivation	5	0.809

These results indicate good internal consistency and support the questionnaire's reliability in measuring students' modelling-oriented learning motivation in the context of Laplace Transform instruction.

Examples of questionnaire items include: "I am confident that I can formulate a mathematical model for an electrical circuit problem" (Expectancy for Success), "Modelling activities help me understand the practical relevance of the Laplace Transform" (Task Value), and "I enjoy comparing analytical solutions with MATLAB simulations" (Intrinsic Motivation).

2.4.3. Classroom Observation and Student Artefacts

Qualitative data were collected through structured classroom observations, students' written modelling products, group discussion records, and simulation outputs. Observations were guided by a modelling-competency checklist aligned with established modelling frameworks, focusing on mathematization, model solution, interpretation, and validation [22].

2.5. DATA ANALYSIS

Quantitative data were analysed using SPSS 26 to examine changes in students' conceptual and modelling-related understanding of the Laplace Transform before and after the instructional intervention. Paired-sample *t*-tests were employed to compare pretest and posttest scores, as this method is appropriate for within-group comparisons in quasi-experimental designs [25].

Beyond statistical significance, effect sizes were calculated using Cohen's *d* to evaluate the magnitude of observed changes. In line with an applied mathematics perspective, effect sizes were interpreted not only as indicators of educational impact, but also as measures of meaningful improvement in students' ability to model, analyse, interpret, and validate electrical circuit behaviour using the Laplace Transform. Large effect sizes were therefore considered evidence of substantive gains in modelling competence rather than solely test performance.

Qualitative data from classroom observations, student modelling artefacts, and simulation outputs were analysed using thematic analysis. Coding followed a systematic procedure focusing on key modelling phases, including mathematization, model formulation, analytical solution, interpretation, validation, and model refinement [26]. This analytical focus ensured that qualitative findings captured students' engagement with the full modelling cycle, in a way consistent with the instructional framework.

Finally, data triangulation across quantitative results, qualitative themes, and modelling artefacts was employed to enhance the trustworthiness and coherence of the findings, particularly regarding modelling engagement and applied mathematical reasoning.

Two researchers independently coded the qualitative data. Coding disagreements were discussed until consensus was reached. The coding framework was subsequently reviewed by one expert in mathematics education to enhance credibility.

3. RESULTS AND DISCUSSION

3.1. RESULTS

3.1.1. Implementation of the Modelling-Oriented Instructional Process

The six-stage modelling-oriented instructional process described in Section 2.3 was implemented consistently throughout the intervention period. During classroom activities, students engaged in modelling tasks focused on RL, RC, and RLC circuits, working individually and in small collaborative groups.

Classroom observations and student artefacts indicate that, in the early stages, most student groups were able to identify relevant physical quantities, formulate modelling assumptions, and translate circuit diagrams into appropriate mathematical variables. This behaviour is consistent with findings from modelling research, which emphasize the importance of mathematization as a critical step in connecting real systems to mathematical representations [14].

Over the course of the intervention, students showed noticeable improvement in their ability to formulate differential-equation models and solve them using the Laplace Transform. As they became more familiar with the modelling process, greater attention was given to interpreting the meaning of Laplace-domain solutions and relating them to the transient behaviour of electrical circuits. MATLAB simulations were frequently used to compare numerical responses with analytical predictions. Rather than serving solely as a verification tool, these comparisons encouraged students to reflect on the adequacy of their models and the plausibility of their conclusions. Similar observations have been reported in previous JOSA studies addressing modelling and simulation in applied contexts [6,7].

Furthermore, many student groups engaged in exploratory activities involving parameter variation and model extension, reflecting higher-level modelling behaviours. Such activities align with research indicating that modelling-oriented instruction supports the development of advanced engineering thinking rather than mere procedural competence [2,24].

3.1.2. Illustrative Example: Transient Analysis of an RC Circuit in an Electro-Mechanical Actuator

Problem Statement: A 24 V DC power supply energizes an electromechanical actuator in an industrial production line. The actuator can be approximated by a series RC circuit, where the capacitor represents the actuator's energy-storage characteristic and the resistor represents energy dissipation. When the switch closes at $t = 0$, the capacitor voltage $v_C(t)$ evolves.

Students are required to:

1. Formulate the mathematical model describing $v_C(t)$.
2. Apply the Laplace Transform to find the analytical expression of $v_C(t)$.
3. Simulate the response in MATLAB and compare it with the analytical result.
4. Predict how changes in R and C affect the system response.
5. Discuss the engineering significance: “How does the RC time constant affect the actuator delay?”

Phase 1. Presenting the Real Engineering Problem

Purpose: To introduce an authentic engineering problem that enables students to identify the task to be solved, understand the operational context, and recognize the practical relevance of the topic.

Problem context: Students are presented with a familiar engineering scenario: an electrical actuator in a control system exhibits a delay when powered by a DC source. The task is to analyse the system's transient response and determine the capacitor voltage as a function of time, using realistic RC circuit parameters.

Through brief group discussion, students identify the target variable, predict the qualitative form of the response, and represent the system using an appropriate circuit diagram.

Pedagogical Significance: This phase corresponds to the "understanding and simplifying the real situation" stage of the modelling cycle [14] and initiates the transition from an industrial context to an analyzable mathematical problem. Authentic tasks enhance perceived task value [13], while collaborative work supports relatedness and intrinsic motivation [27].

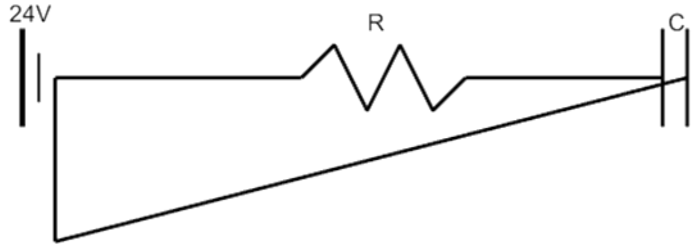

Figure 2. Standard RC Circuit Diagram

Phase 2. Understanding and Simplifying the Situation

Purpose: To transform the engineering context into a quantitative form, preparing for mathematization.

Classroom Procedure: Students identify: Relevant quantities: $V_S, v_C(t), i(t), R, C,$ assumptions: ideal DC source, negligible wire resistance, $v_C(0) = 0$, the circuit becomes closed at $t = 0$, redraw the circuit using IEEE conventions with clear labels and current direction.

Pedagogical Significance: Idealization: A key phase in modelling [15]; Expectancy for success: Students perceive the task as manageable; Autonomy: Students make modelling decisions themselves.

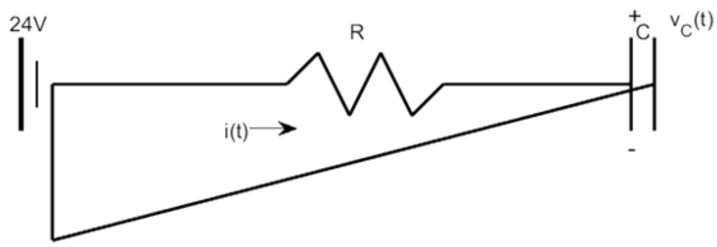

Figure 3. Standard RC circuit with current direction and capacitor polarity

Phase 3. Mathematization
Purpose: To translate the technical description of the RC circuit into a mathematical model, including voltage–current relations, a governing differential equation, and the initial condition.
Implementation: Based on fundamental circuit relations and Kirchhoff's laws, students formulate the differential equation describing the capacitor voltage:

$$\frac{dv_C}{dt} + \frac{1}{RC} v_C(t) = \frac{V_S}{RC}, \tag{1}$$

with the initial condition $v_0(0) = 0$.

Pedagogical significance: This phase represents the core of modelling competency, as students transition from technical reasoning to mathematical reasoning [15]. Constructing the model fosters students' sense of competence and task value, thereby supporting learning motivation [13].

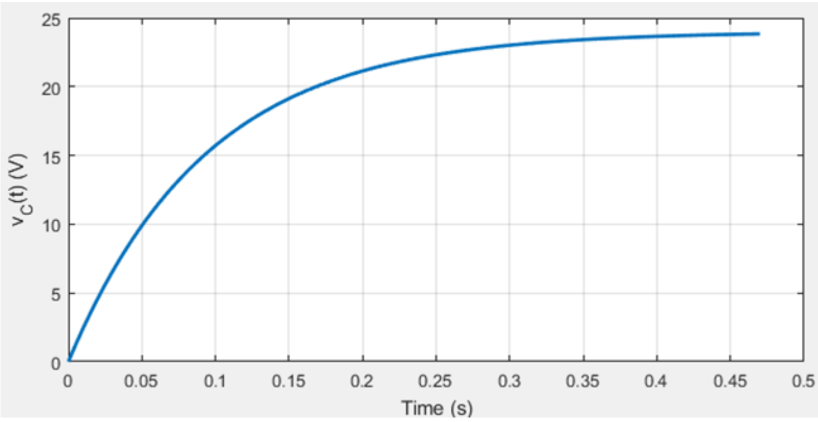

Figure 4. RC charging curve derived from the differential equation.

Phase 4. Applying the Laplace Transform
Purpose: To transform the differential equation of the RC circuit into an algebraic equation in the Laplace domain, enabling efficient solutions and clearer insight into system behaviour.
Implementation: Starting from the governing differential equation:

$$\frac{dv_C(t)}{dt} + \frac{1}{RC} v_C(t) = \frac{V_S}{RC}, v_C(0) = 0, \tag{2}$$

The Laplace Transform is applied to both sides. Using the initial condition, the Laplace-domain expression is obtained:

$$V_C(s) = \frac{V_S}{s} \cdot \frac{1}{RCs + 1}. \tag{3}$$

This form explicitly reveals the first-order system structure with a pole at $s = -\frac{1}{RC}$. Applying the inverse Laplace Transform yields the time-domain response:

$$v_C(t) = V_S\left(1 - e^{-\frac{t}{RC}}\right). \tag{4}$$

Pedagogical significance: At this phase, students recognize the Laplace Transform not merely as a computational technique, but as a modelling tool that directly connects algebraic expressions in the Laplace-domain with the transient physical behaviour of the electrical circuit [28].

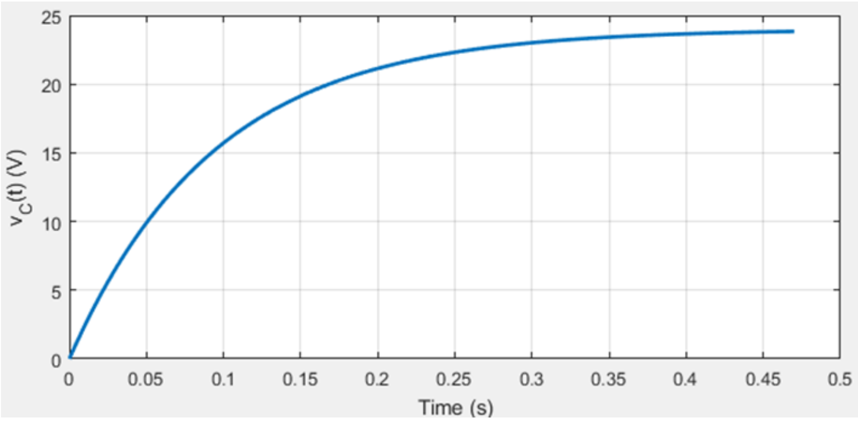

Figure 5. Analytical RC Charging Response.

Phase 5. Interpreting and Validating Results
Purpose: To obtain the time-domain solution, validate the analytical expression through numerical simulation, and interpret the physical meaning of the system response in relation to real engineering applications.
Results and interpretation: From the analytical solution,

$$v_C(t) = V_S\left(1 - e^{-\frac{t}{RC}}\right).$$

the capacitor voltage exhibits an exponential rise, characteristic of a first-order system. The RC time constant governs the response speed: the voltage reaches approximately 63% of its final value at $t = RC$ and approaches steady state after several time constants.

The analytical result is validated using MATLAB simulation, revealing close agreement between the mathematical solution and the numerical response (Figure 6). This agreement reinforces the interpretation of the Laplace Transform as a modelling tool that accurately captures the physical behaviour of electrical circuits.

Pedagogical significance: This phase strengthens the interpretation and validation stages of the modelling cycle, enhances students' trust in mathematical models, and increases intrinsic motivation through visualization supported by MATLAB.

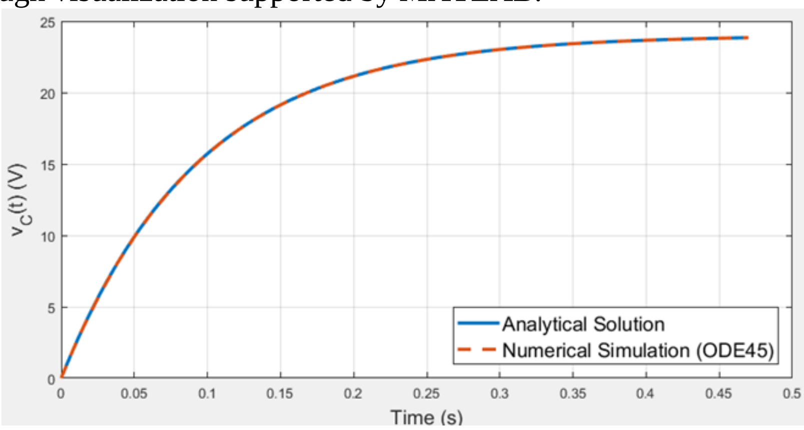

Figure 6. Comparison between analytical and MATLAB-simulated responses.

Phase 6. Refining and Extending the Model

Purpose: To refine and extend the model in order to represent more realistic operating conditions and to foster engineering design thinking through parameter analysis and structural extensions.

Parameter refinement: Starting from the standard RC response,

$$v_C(t) = V_S\left(1 - e^{-\frac{t}{RC}}\right)$$

students examine the influence of key parameters: increasing R leads to slower charging; increasing C increases energy storage and slows the response; increasing V_S raises the final voltage level.

This analysis highlights how system parameters govern the dynamics of first-order circuits.

Model Extensions: The model is further extended to common engineering scenarios. Introducing a parallel load R_L modifies the effective time constant τ_{eq}, resulting in a different charging behaviour. The model is also extended to charging–discharging operation, where the capacitor voltage decays exponentially after the source is disconnected. These extensions reflect practical applications in control circuits, delay networks, and pulse-generation systems.

Pedagogical Significance: This phase represents the highest level of modelling competency, as students evaluate, refine, and extend models. Such activities enhance autonomy and intrinsic motivation [27], while increasing perceived task value through authentic engineering relevance.

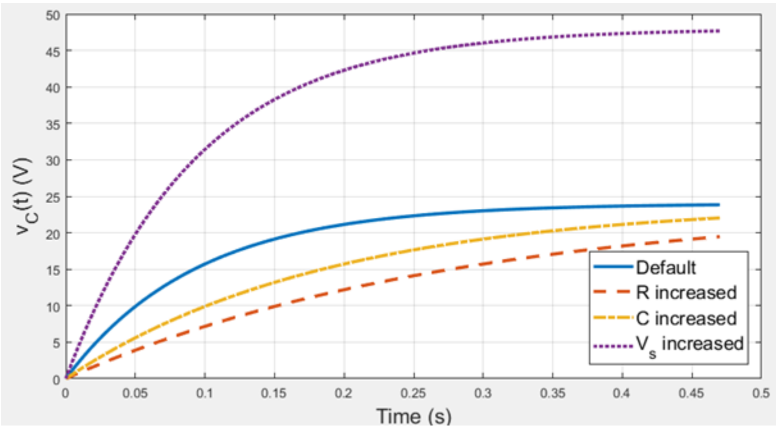

Figure 7. Effect of R, C, V_S on Charging Curve

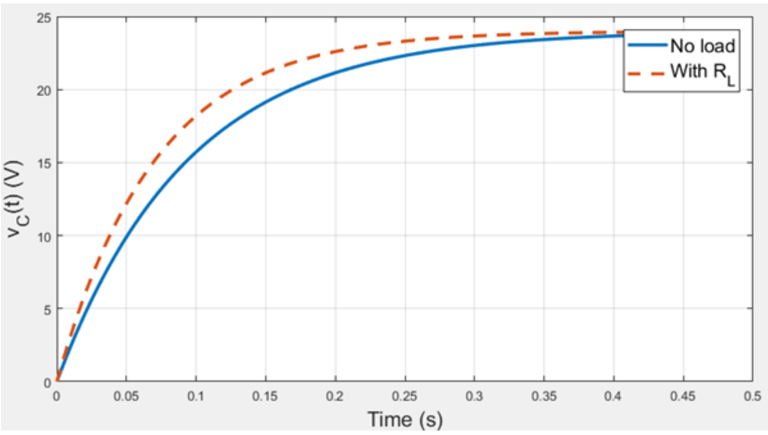

Figure 8. RC Charging with Parallel Load R_L

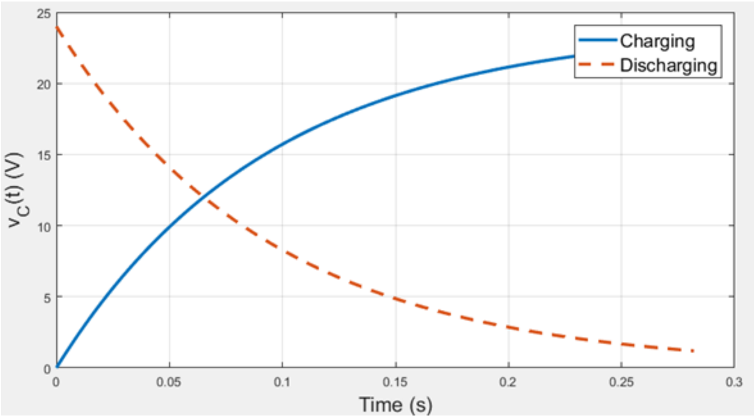

Figure 9. Charging and Discharging of RC Circuit

Although the RC circuit is used as the primary illustrative example in this paper, the proposed modelling-oriented framework was implemented across RL, RC, and RLC circuit contexts during the instructional intervention. The applicability of the framework to different circuit types is summarized in Table 3.

Table 3. Application of the modelling-oriented framework to different electrical circuit types.

Circuit Type	Mathematical Model	Typical Laplace Application	Engineering Interpretation
RL Circuit	First-order differential equation	Analysis of current response	Inductor energy storage and transient current behaviour
RC Circuit	First-order differential equation	Analysis of voltage response	Capacitor charging, discharging, and system delay
RLC Circuit	Second-order differential equation	Oscillation and transient-response analysis	Damping, resonance, and dynamic system behaviour

The RC circuit was selected as the main illustrative example because it provides a clear and accessible context for demonstrating all six stages of the modelling-oriented instructional framework. Nevertheless, the same modelling cycle was applied to RL and RLC circuits during classroom activities, enabling students to experience a broader range of engineering applications of the Laplace Transform.

3.1.3. Results of Pedagogical Experiments

a) Effects of the Modelling-Oriented Instruction on Conceptual Understanding

To examine the impact of the modelling-oriented instructional framework on students' conceptual and modelling-related understanding of the Laplace Transform, pretest and posttest scores were compared using a paired-sample *t*-test.

Table 4. Descriptive and inferential statistics for conceptual understanding scores (n = 80)

Statistic	Pretest	Posttest
Mean	5.25	6.97
Median	5.20	7.00
Standard Deviation	1.45	1.61
Minimum	2.00	3.50
Maximum	8.00	10.00
Mean Difference	1.72	-
t(79)	10.36	-
p-value	<0.001	-
Cohen's d	1.12	-
95% CI of Mean Difference	[1.39, 2.05]	-

As shown in Table 4, the mean conceptual-understanding score increased from 5.25 (SD = 1.45) in the pretest to 6.97 (SD = 1.61) in the posttest. The mean gain was 1.72 points, with a statistically significant difference, $t(79) = 10.36$, $p < 0.001$. The 95% confidence interval for the mean difference ranged from 1.39 to 2.05. The effect size was large (Cohen's d = 1.12), indicating substantial improvement in students' conceptual and modelling-related understanding of the Laplace Transform.

From an applied mathematics perspective, this magnitude of effect suggests that students' improvement was not limited to procedural fluency in computing Laplace transforms. Rather, students demonstrated enhanced conceptual and modelling-related understanding of the Laplace Transform as a tool for describing transient behaviour in electrical circuits.

Analysis of test items revealed notable gains in students' ability to interpret initial conditions, identify system poles and time constants, and relate algebraic expressions in the Laplace-domain to physical circuit behaviour. These results indicate meaningful development in modelling competence, particularly in interpretation and validation stages of the modelling cycle. These findings are consistent with previous studies reporting that modelling-oriented instruction supports conceptual learning in advanced engineering mathematics [16, 29]. The present study extends this line of research by providing empirical evidence of the effectiveness of this approach in Laplace Transform instruction, a topic traditionally taught in a predominantly procedural manner.

b) Changes in Students' Learning Motivation

Students' learning motivation was analysed across three modelling-oriented components of Expectancy–Value Theory: expectancy for success in modelling tasks, task value of modelling activities, and intrinsic motivation during modelling engagement.

Table 5. Learning motivation scores before and after the intervention (n = 80)

Component	Pretest Mean ± SD	Posttest Mean ± SD	Mean Difference	t(79)	p	Cohen's d	95% CI
Expectancy for Success	3.01 ± 0.51	3.53 ± 0.55	0.52	10.09	<0.001	1.13	[0.42, 0.62]
Task Value	3.08 ± 0.54	3.63 ± 0.58	0.55	9.06	<0.001	1.01	[0.43, 0.67]
Intrinsic Motivation	2.98 ± 0.57	4.20 ± 0.61	1.22	9.39	<0.001	1.05	[0.96, 1.48]

All three components showed statistically significant increases with large effect sizes. Expectancy for success increased, indicating greater confidence in performing modelling tasks involving Laplace-domain analysis and validation. Task value gains suggest that students increasingly perceived modelling activities as relevant to electrical engineering practice. Intrinsic motivation showed the largest mean increase among the three motivational components, reflecting heightened interest and satisfaction during modelling and simulation-based validation.

These results indicate that the modelling-oriented instructional design effectively addressed motivational barriers by embedding mathematical analysis within authentic modelling activities emphasizing interpretation, validation, and system behaviour.

c) Qualitative Findings

Qualitative findings triangulated classroom observations, student modelling artefacts, and reflective statements. To provide a more systematic account of students' modelling experiences, qualitative data were organized into several recurring themes identified through classroom observations, student artefacts, and reflective statements.

Table 6. Summary of qualitative themes emerging from modelling activities

Theme	Evidence from observations and student artefacts	Illustrative quotation
Mathematization	Most groups successfully identified variables, parameters, assumptions, and circuit relationships before constructing mathematical models.	"The circuit diagram helped us formulate the equation more easily." (Student 12)
Interpretation	Students increasingly connected poles, time constants, and transient responses to physical circuit behaviour.	"I finally understood why the RC time constant determines the delay of the system." (Student 24)
Validation	Students frequently compared analytical solutions with MATLAB simulations and discussed discrepancies.	"The matching graphs increased my confidence in the mathematical solution." (Student 31)
Model Extension	Many groups explored parameter changes and discussed their implications for engineering design.	"Changing R and C showed how design decisions affect system performance." (Student 17)

The qualitative findings indicate that students did not merely apply computational procedures but actively engaged in key phases of the modelling cycle, including mathematization, interpretation, validation, and model refinement. These observations support the quantitative results and provide additional evidence of the development of modelling competence during the intervention.

Student engagement across modelling phases. Consistent with the themes summarized in Table 6, student engagement increased progressively across the six modelling stages. Based on structured observations of 20 student groups (4 students per group), 16 groups (80%) correctly identified variables, parameters, initial conditions, and modelling assumptions during the mathematization phase. All groups successfully applied Laplace Transform procedures and compared analytical solutions with MATLAB simulations. During the interpretation and validation phase, 12 groups (60%) were able to articulate the physical meaning of the obtained solutions and discuss possible sources of discrepancies between mathematical models and real-world system behaviour.

These observations suggest that students increasingly moved beyond procedural application of the Laplace Transform and became more engaged in interpretation, validation, and model-based reasoning throughout the intervention.

A representative student reflection illustrates this:

"Seeing the MATLAB plot match our derived equation made me feel confident for the first time that I understood what the Laplace Transform is actually *doing*."

Development of modelling competence. Evidence from classroom observations, student reports, and reflective comments suggests that students gradually became more capable of engaging in different phases of the modelling process. Early activities mainly focused on translating circuit situations into mathematical representations and applying Laplace Transform procedures. As the intervention progressed, many students began to discuss the meaning of their solutions in relation to circuit behaviour and to use simulation results when evaluating the adequacy of their models. Several groups also demonstrated an awareness that mathematically convenient assumptions do not always fully reflect the complexity of real engineering systems.

Perceived relevance and learning challenges. Students frequently referred to the practical value of the Laplace Transform when working with RL, RC, and RLC circuit problems. Reflections collected near the end of the intervention indicate that many participants viewed the topic as more closely connected to engineering practice than they had initially expected. Difficulties that were commonly observed during the first weeks, particularly those related to incorporating initial conditions and interpreting simulation outputs, became less evident as students gained experience with the modelling tasks. These observations are consistent with the overall pattern of improvement found in both the quantitative and qualitative data. Representative comments included:

"For the first time, the Laplace Transform seemed connected to an actual circuit rather than a set of symbolic procedures."

"The modelling activities helped me think about the system the way an engineer would, not only as a mathematics student."

3.2. DISCUSSION

3.2.1. Interpretation of the main findings from an applied mathematics perspective

The findings suggest that the proposed framework may help improve students' understanding of the Laplace Transform when it is presented as a modelling tool rather than solely as a symbolic procedure. The large effect size observed in the Conceptual–Modelling Test reflects meaningful progress in students' ability to use the Laplace Transform for modelling and analysing transient behaviours in electrical circuits [25].

The relatively large effect sizes observed in this study may be explained by the combined influence of three instructional features embedded within the proposed framework. First, authentic electrical engineering contexts increased the perceived relevance and usefulness of mathematical analysis. Second, iterative modelling activities required students to repeatedly connect mathematical representations with physical system behaviour, thereby strengthening conceptual understanding. Third, simulation-based validation using MATLAB enabled students to compare analytical solutions with numerical results, reinforcing interpretation and confidence in the modelling process. Together, these elements supported both conceptual understanding and learning motivation, which may account for learning gains that are stronger than those typically reported in procedure-oriented instruction.

From an applied mathematics perspective, these gains indicate improved capacity to connect differential-equation models with Laplace-domain representations and to interpret system behaviour in physical terms. In particular, students demonstrated increased competence in handling initial conditions, identifying system poles and time constants, and relating algebraic expressions in the complex domain to observable circuit dynamics. Such competencies are widely recognized as core components of mathematical modelling in engineering contexts [14,15].

Qualitative findings further support this interpretation. Students' engagement in interpretation, validation, and parameter analysis reflects a shift toward system-oriented reasoning, which is a defining feature of applied mathematical practice. The systematic comparison between analytical solutions and numerical simulations using MATLAB served as an external validation mechanism, reinforcing trust in mathematical models and highlighting the modelling role of the Laplace Transform [2,6].

3.2.2. Modelling-oriented instruction and the role of the Laplace Transform

A key contribution of this study is to demonstrate that the Laplace Transform can be effectively taught as a modelling tool embedded within a coherent modelling cycle. Unlike traditional approaches that prioritize computational procedures, the proposed six-stage framework emphasizes interpretation, validation, and model refinement as integral components of Laplace-based analysis. This orientation aligns with modelling frameworks widely adopted in applied mathematics education [14,22].

The progression evident in students' modelling activities suggests that they became increasingly capable of moving beyond mathematical formulation toward interpretation, validation, and model refinement. As students examined parameter effects, compared analytical and simulated responses, and discussed the limitations of simplified assumptions, they engaged in reasoning characteristic of engineering modelling practice. Such experiences are important because engineers are required not only to construct mathematical models but also to evaluate their adequacy in representing real system behaviour [16, 29].

The model-extension activities illustrated in Figures 8 and 9 further demonstrate how the proposed framework encourages students to investigate the effects of parameters and more realistic circuit behaviours beyond standard textbook models. Through analysing RC circuits with parallel loads and charging–discharging processes, students were required to refine initial models, examine alternative system configurations, and interpret the engineering implications of parameter changes. These activities reflect advanced stages of the modelling cycle and support the development of higher-order modelling competence.

3.2.3. Motivation, modelling engagement, and applied mathematical reasoning

The significant gains in expectancy for success, task value, and intrinsic motivation may be interpreted as outcomes of students' increased engagement in modelling activities. When Laplace Transform was embedded in authentic circuit problems and validated using MATLAB, students perceived mathematical analysis as more meaningful and relevant to engineering practice [13, 18]. From an applied mathematics perspective, this enhanced relevance strengthened students' confidence in using mathematical tools to explain and predict system behaviour, thereby sustaining engagement throughout the modelling cycle [2, 20].

3.2.4. Comparison with previous studies

This study both builds upon and extends previous research on mathematical modelling and applied mathematics education in several important ways. Compared to modelling-based studies in differential equations education [29], the present research confirms that similar pedagogical and modelling-related benefits can be achieved in teaching the Laplace Transform, a topic that is mathematically more abstract and often underexplored from a modelling perspective.

Unlike traditional Laplace Transform instruction that primarily emphasizes symbolic manipulation and computational procedures, the present study adopts a modelling-oriented perspective integrating contextualization, mathematization, interpretation, validation, and model refinement. This approach is consistent with contemporary trends in applied mathematics education that emphasize the connection between mathematical analysis and real-world problem solving [6,7].

Furthermore, the findings suggest that modelling-oriented instruction can simultaneously enhance conceptual understanding and learning motivation. This integrated outcome extends previous research by highlighting the complementary roles of modelling activities and simulation-based validation in learning engineering mathematics.

4. CONCLUSIONS

The present study explored the use of a six-stage modelling framework for teaching the Laplace Transform in electrical engineering contexts. Rather than treating the Laplace Transform primarily as a symbolic procedure, the instructional approach encouraged students to use it as a tool for modelling, analysing, and interpreting the behaviour of electrical circuits.

The results suggest that embedding Laplace-based analysis within modelling activities may support students' conceptual understanding of the topic. Improvements were observed not only in procedural performance but also in students' ability to interpret initial conditions, analyse system poles and time constants, and relate Laplace-domain representations to the physical behaviour of electrical systems. Students also appeared to benefit from opportunities to interpret analytical results and compare them with simulation outputs, thereby connecting mathematical reasoning with engineering applications.

In addition, the study revealed significant gains in students' learning motivation, particularly in expectancy for success, task value, and intrinsic motivation. From an applied mathematics perspective, these motivational improvements can be understood as a consequence of increased engagement in authentic modelling activities supported by numerical simulation. The systematic comparison between analytical solutions and MATLAB simulations played a central role in reinforcing confidence in mathematical models and sustaining engagement throughout the modelling process.

From a broader perspective, this research contributes to applied mathematics education by demonstrating that modelling-oriented instruction is highly effective not only for advanced application topics but also for core engineering mathematics content, which is often perceived as abstract and disconnected from practice. By aligning instructional design with modelling principles emphasized in the *Journal of Science and Arts*, the study extends existing work on mathematical modelling and simulation to include empirical insights into instructional processes and student learning outcomes.

Several limitations should be acknowledged. The study employed a one-group pretest–posttest design within a single institutional context, which may limit the generalizability of the findings. Future research may adopt controlled or longitudinal designs to examine retention, transfer, and the long-term development of modelling competence. Further studies could also explore the application of the proposed framework to other transform-based methods and system-analysis topics in engineering mathematics.

Overall, the results provide strong evidence that a modelling-oriented instructional framework offers a coherent and effective approach to Laplace Transform instruction, reinforcing the role of mathematical modelling as a unifying concept in applied mathematics and engineering education.

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