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ORIGINAL PAPER

STATISTICAL ROUGH ORDER CONVERGENCE ON RIESZ SPACES

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Abstract. *In this paper, we introduce the concept of statistical rough order convergence on Riesz spaces by combining the notions of order convergence and rough convergence. We explore fundamental results and develop the theory of statistical rough order convergence in detail. Our findings show that statistical rough order convergence offers a flexible approach to convergence, accommodating both statistical and rough approximations. Additionally, we derive results on statistical rough order limit points.*

Keywords: *Statistical rough order convergence; rough convergence; statistical convergence; order convergence; Riesz space.*

Mathematics Subject Classification: *46A40; 40A35; 40A05.*

1. INTRODUCTION

The notion of Riesz space, also known as a vector lattice, was initially introduced by Riesz [1]. Since its early stages, numerous scholars have contributed to its development. Among the early contributors to the theory were Freudenthal [2] and Kantorovich [3]. Riesz spaces are ubiquitous in analysis and play significant roles in Banach space theory, measure theory, and operator theory [4-7], and they also find applications in economics [8]. Central to the examination of Riesz spaces is the concept of order convergence, which gives rise to the notion of order continuity. However, it is crucial to note that order convergence in a Riesz space does not necessarily align with a topology [9]. This means that order and topological convergences are not necessarily equivalent. A thorough treatment of these notions can be found in Zaanen's book [10], which provides a comprehensive investigation of the topic.

Convergence is a cornerstone of mathematical analysis, a fundamental tool for understanding the diverse properties of sequences and nets. Although topological convergence within a topological space represents the prevalent form of convergence, there exist notable non-topological convergences of equal significance. Ordman showed that almost everywhere convergence cannot be characterized by a topology [11]. Hyland conducted investigations into convergences beyond the realm of topology [12]. Additionally, Beattie and Butzmann contributed valuable insights to the theory of convergence structures [13]. Aydın et al. introduced full lattice convergence in Riesz spaces [14], while O'Brien et al. introduced the theory of net convergence structures, which is related to filter convergence theory [15].

The theory of statistical convergence remains an active field of exploration. Initially proposed by Fast [16] and Steinhaus [17], it extends the concept of conventional convergence to real sequences. Numerous scholars have since introduced its intricacies, with notable

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applications highlighted in various studies. Furthermore, its scope extends beyond real sequences to encompass broader abstract spaces, including locally convex spaces, Banach spaces, and fuzzy number spaces [18-21]. Ercan introduced the concept of statistical convergence in Riesz spaces, presenting the idea of statistically u-uniformly convergent sequences [22]. Şençimen and Pehlivan expanded upon this by extending statistical convergence to Riesz spaces in relation to order convergence [23]. Aydın et al. conducted studies on statistical convergence in both Riesz spaces and locally solid Riesz spaces [24-26].

Phu, building upon previous advancements, introduced a new concept of convergence within normed spaces, which he termed rough convergence [27]. This concept was specifically applied to sequences within finite-dimensional normed linear spaces. Moreover, Phu explored the rough continuity of linear operators [28]. The application of rough convergence extends to weighted statistical convergence in locally solid Riesz spaces [29], and it has been studied alongside various types of convergence across different spaces [30]. The aim of this study is to establish a comprehensive theory of statistical rough order convergence on Riesz spaces and to provide fundamental results regarding statistical rough limit points. Thus, this investigation lays the groundwork for future research into statistical rough order convergence on Riesz spaces.

The paper is organized as follows: Section 2 provides a foundation for our work by presenting the key definitions and essential theorems concerning the theory of Riesz spaces and the concept of statistical convergence. In Section 3, we introduce the central notion of this study: statistical rough order convergence. We then explore its fundamental properties, including an examination of its relationship with other known convergence types in Remark 3. Theorem 3.3 demonstrates how key vector lattice concepts are realized within the framework of statistical rough order convergence, while Theorem 3.4 investigates its linearity. Furthermore, Theorem 3.6 establishes that disjointness is preserved under this mode of convergence. The conditions for the uniqueness of the statistical rough order limit are provided in Proposition 3.7 and Theorem 3.8. Finally, the last section is dedicated to an analysis of the set of all statistical rough order limit points for a sequence in a Riesz space. We investigate properties of this set, such as order closure, convexity, and boundedness, and establish results on inclusion, supremum bounds, and equivalence under specific conditions.

2. PRELIMINARIES

We initially enumerate several fundamental concepts associated with the theory of rough convergence and Riesz spaces, with further elaboration available in references [4-6,8,10,18,27].

Definition 2.1. *A sequence (x_n) in a normed space $(X, \|\cdot\|)$ is defined as rough convergent (or r-convergent) to $x \in X$, where r is a fixed nonnegative real number, if for every $\varepsilon > 0$ there exists $n_\varepsilon \in \mathbb{N}$ such that*

$$\|x_n - x\| \leq r + \varepsilon$$

holds for all natural numbers $n > n_\varepsilon$.

Equivalently, rough convergence can be formulated as $\limsup\|x_n - x\| \leq r$, where r denotes the convergence degree of the sequence (x_n). It is evident that every sequence converging in norm is also rough convergent to the norm limit point for all $r \geq 0$. However, the reverse implication may not universally apply. Rough convergence can be interpreted as a more relaxed criterion for convergence when contrasted with norm convergence.

Definition 2.2. *A real vector space E equipped with a partial order "$\leq$" is defined as an ordered vector space if E is partially ordered such that the vector space operations satisfy*:
1) $x \leq y$ implies $x + z \leq y + z$ for all $x, y, z \in E$,
2) $x \geq \theta$ implies $\alpha x \geq \theta$ for every $\alpha \geq 0$ in $\mathbb{R}$, where θ is the zero vector of E.

Moreover, an ordered vector space E is denoted as a *Riesz space* or a *vector lattice* if, for any two vectors x and y in E, both the infimum $x \wedge y = \inf\{x, y\}$ and the supremum $x \vee y = \sup\{x, y\}$ exist within E. For an element x in a Riesz space E, the *positive part* of x is defined as $x^+ := x \vee \theta$, the *negative part* of x is defined as $x^- := (-x) \vee \theta$, and the *modulus* of x is $|x| := x \vee (-x)$. The set $E_+ := \{x \in E : \theta \leq x\}$ is called the *positive cone* of Riesz space E, and so the elements of E_+ are called positive elements of E. Furthermore, a Riesz space is labelled as *Dedekind complete* whenever every nonempty subset bounded above possesses a supremum (or, equivalently, whenever every nonempty subset bounded below possesses an infimum). If x and y are vectors in a Riesz space E with $x \leq y$, then the set $[x, y] := \{z \in E : x \leq z \leq y\}$ is called an *order interval*. Moreover, a subset A of a Riesz space is said to be $order\ bounded$ if there is an element $x \in E_+$ such that $A \subseteq [-x, x]$. A Riesz space E is said to be *Archimedean* if $\frac{1}{n}x \downarrow \theta$ remains true in E for every $x \in E_+$. In this paper, all vector lattices are assumed to be real and Archimedean unless stated otherwise.

It is important to note that order convergence is a crucial concept in Riesz spaces because it allows many topological properties to be defined without relying on topology, forming the basis for numerous studies. However, there are multiple definitions of order convergence, so we provide one of the most general definitions below. For a thorough examination of the various types of order convergence in Riesz spaces, readers should particularly consult [31], and it is often abbreviated as *o-convergence*.

Definition 2.3. *Let E be a Riesz space and (w_n) be a sequence in E.*
a) The sequence (w_n) is called *decreasing* whenever $w_{n+1} \leq w_n$ is correct for all natural numbers $n \in \mathbb{N}$. This is denoted by $w_n \downarrow$. Additionally, $w_n \downarrow w$ indicates that (w_n) is decreasing and $\inf w_n = w$, meaning (w_n) monotonically decreases to w.
b) The sequence (w_n) is said to be *order convergent* to $w \in E$ (denoted as $w_n \xrightarrow{o} w$) if there exists another sequence $q_n \downarrow \theta$ in E such that

$$|w_n - w| \leq q_n$$

stands for all $n \in \mathbb{N}$.
c) Let c be an arbitrary convergence in a Riesz space E. A net $(w_\alpha)_{\alpha \in A}$ is said to be *rough c-convergent* (or *rc-convergent*) to $w \in E$ for a fixed $u \in E_+$ if there exists a net $q_\alpha \xrightarrow{c} \theta$ in E such that

$$|w_\alpha - w| \leq q_\alpha + u$$

is fulfilled for each $\alpha \in A$. This is indicated as $w_\alpha \xrightarrow{rc} w$.

Now let us recall the main concepts related to statistical convergence. The following limit

$$\delta(K) := \lim_{n\to\infty} \frac{1}{n}\mathcal{K}(\{k \in K: k \leq n\})$$

represents the natural density of a subset $K \subseteq \mathbb{N}$ when it exists, where $\mathcal{K}(\{k \in K: k \leq n\})$ denotes the number of elements in K that are less than or equal to n [19,21]. A real sequence $x = (x_k)$ is termed as *statistically convergent* to L if the limit

$$\lim_{n\to\infty} \frac{1}{n}|\{k \leq n: |x_k - L| \geq \varepsilon\}| = 0$$

exists for every $\varepsilon > 0$.

Definition 2.4. *Let* (w_n) *be a sequence in a Riesz space* E. Then (w_n) is said to be
a) *statistically order decreasing* to θ if there exists a set $K = \{k_1 < k_2 < \cdots\} \subseteq \mathbb{N}$ with $\delta(K) = 1$ such that (w_{k_n}) is decreasing and $\inf_{k_n\in K}(w_{k_n}) = \theta$, abbreviated as $w_n \downarrow^{st_o} \theta$,
b) *statistically order convergent* (or st_o*-convergent*) to $w \in E$ if there exists a sequence $q_n \downarrow^{st_o} \theta$ with a set $K = \{k_1 < k_2 < \cdots\} \subseteq \mathbb{N}$ such that $\delta(K) = 1$ and $|w_{k_n} - w| \leq q_{k_n}$ for every $k_n \in K$, written as $w_n \xrightarrow{st_o} w$.

Note that if a sequence (w_n) satisfies a property $\mathcal{P}$ for all n except for a set with natural density zero, then we say that (w_n) satisfies property $\mathcal{P}$ for *almost all* n, and also, it is abbreviated as *a.a.n.* [19].

3. STATISTICAL ROUGH ORDER CONVERGENCE

We begin the section by giving the following crucial notion of the current paper.

Definition 3.1. *Let* (w_n) *be a sequence in a Riesz space* E *and u be a fixed positive element of* E. Then (w_n) is said to be *statistically rough order convergent* (or st_{ro}*-convergent*) to $w \in E$ if there exists a sequence $q_n \downarrow^{st_o} \theta$ in E with index set $\delta(K) = 1$ such that

$$|w_{k_n} - w| \leq q_{k_n} + u$$

applies to every $k_n \epsilon K$. This is abbreviated as $w_n \xrightarrow{st_{ro}} w$.

In this context, the notion of st_{ro} −convergence is defined by introducing a rough degree denoted as u. It is evident that st_{ro}- and st_o- convergences coincide when u is θ. However, our primary interest lies in cases where u is a strictly positive element in any Riesz space because it is important to emphasize that the st_{ro}-limit point of a sequence (w_n) is typically non-unique, particularly when u contains strictly positive values. This focus arises from various considerations. For instance, consider two sequences (w_n) and (v_n) in a Riesz space such that $w_n \xrightarrow{st_{ro}} w$ and $|w_n - v_n| \leq u$ hold for all $n \in \mathbb{N}$. Then there exists a sequence $q_n \downarrow^{st_o} \theta$ with $\delta(K) = 1$ such that $|v_{k_n} - v| \leq q_{k_n}$ is satisfied for every $k_n \in K$. Consequently, from the inequality

$$|w_{k_n} - v| \leq |w_{k_n} - v_{k_n}| + |v_{k_n} - v| \leq u + |v_{k_n} - v| \leq u + q_{k_n}$$

is satisfied for every $k_n \in K$. It follows that (w_n) statistically rough order converges to v. In Dedekind complete Riesz spaces, statistical rough order convergence can equivalently be expressed as

$$\limsup |w_{k_n} - w| \leq u$$

for an index set $\delta(K) = 1$ with rough degree u of the sequence (w_n).

Remark 3.2. The following statements are correct for st_{ro}-convergence in Riesz spaces.
1. Every order convergent sequence is st_{ro}-convergent for all positive vectors u; see, for example, Remark 2.2 [32].
2. Convergence in the st_o-sense implies st_{ro}-convergence for any roughness degree u.
3. Each statistically order decreasing to zero sequence is st_{ro}-convergent zero for all u.
4. $w_n \xrightarrow{st_{ro}} w$ if and only if $(w_n - w) \xrightarrow{st_{ro}} \theta$ if and only if $|w_n - w| \xrightarrow{st_{ro}} \theta$.
5. If $w_n \xrightarrow{st_{ro}} w$ holds with rough degree u, then $w_n \xrightarrow{st_o} w + u$ is correct whenever $w + u \leq w_n$ is valid for all $n \in \mathbb{N}$.

Question 3.3. When does st_{ro}-convergence ensure st_o-convergence?

It is not hard to see that the converse statements in Remark 3.2 may not hold universally. To exemplify this, we provide the following instance.

Example 3.4. Let us consider the Euclidean Riesz space $\mathbb{R}^2$ equipped with coordinate-wise ordering. Now we establish a sequence (w_n) within this space as follows:

$$w_n := \begin{cases} (2n, 2n), & n = k^2 \\ (1 + \frac{1}{n}, 2 + \frac{1}{n}), & n \neq k^2 \end{cases}$$

for all $k, n \in \mathbb{N}$. Therefore, we obtain $w_n \xrightarrow{st_{ro}} (0,0)$ for the positive vector $u := (1,2) \in \mathbb{R}^2$ and the sequence $q_n = (\frac{1}{n}, \frac{1}{n})$. However, it is clear that (w_n) is not order convergent. Moreover, (w_n) is st_o-convergent to $(1,2)$, but it is not st_o-convergent to $(0,0)$.

It is important to note that a statistically rough order convergent sequence does not necessarily have to be order bounded, and a subsequence of such a sequence does not have to maintain statistical rough order convergence. These points are illustrated in the following example.

Example 3.5. Consider a real sequence (w_n) defined as follows:

$$w_n := \begin{cases} n + 2, & n = k^3 \\ n - 2, & n \text{ is prime} \\ \frac{1}{2n+1}, & \text{otherwise} \end{cases}$$

for all $k, n \in \mathbb{N}$. On the other hand, if we choose

$$q_n := \begin{cases} 2n, n = k^3 \\ \dfrac{1}{2n}, otherwise \end{cases}$$

then we have $q_n \downarrow^{st_o} \theta$. Hence, for the set $K = \{n \in \mathbb{N} : \text{ n is not a prime}\}$ and $u := 0$, we get $|w_n| \leq q_n + u$ for each $n \in K$. Therefore, we get $w_n \xrightarrow{st_{ro}} \theta$. It is clear that (w_n) is not order bounded, and also the subsequence containing elements of the set $\{w_n : n = k^3 \text{ for some } k \in \mathbb{N}\}$ is not statistically rough order convergent.

The linearity of convergence is a fundamental tool for the theory studies in general. However, as illustrated by the following example, the linearity property is not generally valid for st_{ro}-convergence.

Example 3.6. Let us consider the Riesz space $E := \mathbb{R}$ and sequences (w_n) and (v_n) in E defined by;

$$w_n := \begin{cases} u, & n \text{ is odd} \\ -u, & n \text{ is even} \end{cases} \text{ and } v_n := \begin{cases} -u, & n \text{ is odd} \\ u, & n \text{ is even} \end{cases}$$

for any fixed positive element u. Thus, both $w_n \xrightarrow{st_{ro}} 0$ and $v_n \xrightarrow{st_{ro}} 0$ are satisfied with rough degree u. However, $w_n - v_n \xrightarrow{st_{ro}} 0$ is true with rough degree $2u$.

Theorem 3.7. Let $w_n \xrightarrow{st_{ro}} w$ and $v_n \xrightarrow{st_{ro}} v$ in a Riesz space E and λ be an arbitrary scalar, then the following assertions hold true:
i. $\lambda w_n \xrightarrow{st_{ro}} \lambda w$ for any $|\lambda| \leq 1$, and $\lambda w_n \xrightarrow{st_{ro}} \lambda w$ with rough degree$|\lambda|u$ for each scalar $|\lambda| > 1$;
ii. The lattice operations $\vee$ and $\wedge$ on E are continuous with respect to the st_{ro}-convergence with the sum of the rough degrees of sequences;
iii. The mappings on E defined by $w \to w^+$, $w \to w^-$ and $w \to |w|$ are continuous with respect to the st_{ro}-convergence.

Proof: (i) Assume that (w_n) is a sequence in E such that $w_n \xrightarrow{st_{ro}} w$, and so there is a sequence $q_n \downarrow^{st_o} \theta$ with an index set $\delta(K) = 1$ such that $|w_{k_n} - w| \leq q_{k_n} + u$ holds for all $k_n \in K$. It is clear that $\lambda w_n \xrightarrow{st_{ro}} \lambda w$ for any $|\lambda| \leq 1$ because of $|\lambda w_n - \lambda w| = |\lambda||w_n - w| \leq |w_n - w|$. Take any scalar $|\lambda| > 1$. Then, by the inequality $|\lambda w_{k_n} - \lambda w| \leq |\lambda| q_{k_n} + |\lambda| u$ and $|\lambda| q_{k_n} \downarrow \theta$, we obtain the desired result.
(ii) It is obvious.
(iii) Since $w_n \xrightarrow{st_{ro}} w$ and $v_n \xrightarrow{st_{ro}} v$, there exist $q_n \downarrow^{st_o} \theta$ and $r_n \downarrow^{st_o} \theta$ with index sets $\delta(K) = \delta(J) = 1$ such that $|w_{k_n} - w| \leq q_{k_n} + u$ and $|v_{j_n} - v| \leq r_{j_n} + u$ for all $k_n \in K$ and $j_n \in J$. By applying Birkhoff's inequality see for example, Theorem 1.19 [4], we observe the following inequality:

$$\begin{aligned} \left|w_{i_n} \vee v_{i_n} - w \vee v\right| &= \left|w_{i_n} \vee v_{i_n} - v_{i_n} \vee w + v_{i_n} \vee w - w \vee v\right| \\ &\leq |w_{i_n} \vee v_{i_n} - v_{i_n} \vee w| + |v_{i_n} \vee w - w \vee v| \\ &\leq |w_{i_n} - w| + |v_{i_n} - v \\ &\leq q_{i_n} + r_{i_n} + 2u \end{aligned}$$

applies to every $i_n \in I := K \cap J$. It is clear that $\delta(I) = 1$, and hence we obtain $w_n \vee v_n \xrightarrow{st_{ro}} w \vee v$ with rough degree $2u$. The other case can be obtained by following similar steps.
(iv) It can be observed that the zero sequence $(\theta_n) = (\theta, \theta, \cdots)$ is st_{ro}-convergent to θ with rough degree $u = \theta$. Thus, by applying (iii), we have $w_n \vee \theta_n \xrightarrow{st_{ro}} w \vee \theta$, i.e., we get $w_n^+ \xrightarrow{st_{ro}} w^+$ with rough degree $u + \theta = u$. The other cases can be obtained similarly.

Theorem 3.8. The following statements hold for st_{ro}-convergence on Riesz spaces.
i. $w_n \xrightarrow{st_{ro}} w$ and $v_n \xrightarrow{st_{ro}} v$ imply $w_n + v_n \xrightarrow{st_{ro}} w + v$ with the sum of the rough degrees of the sequences.
ii. $w_n \xrightarrow{st_{ro}} w$ with rough degree u and $\lambda_n \to \lambda$ with the usual sense on $\mathbb{R}$ imply $\lambda_n w_n \xrightarrow{st_{ro}} \lambda w$ with rough degree ru for some $r \in \mathbb{R}^+$.

Proof: (i) Take sequences $w_n \xrightarrow{st_{ro}} w$ with rough degree u_1 and $v_n \xrightarrow{st_{ro}} v$ with rough degree u_2 respectively in a Riesz space E. Thus, there exist sequences $p_n \downarrow^{st_o} \theta$ and $q_n \downarrow^{st_o} \theta$ in E with a common index set $\delta(I) = 1$ such that

$$\left|w_{i_n} - w\right| \leq p_{i_n} + u_1 \text{ and } |v_{i_n} - v| \leq q_{i_n} + u_2$$

are satisfied for all $i_n \in I$. It follows that

$$|(w_{i_n} + v_{i_n}) - (w + v)| \leq |w_{i_n} - w| + |v_{i_n} - v| \leq p_{i_n} + q_{i_n} + u_1 + u_2$$

provides for all $i_n \in I$. By applying Theorem 4 (ii) [23], we have $(p_n + q_n) \downarrow^{st_o} \theta$, and so we obtain $w_n + v_n \xrightarrow{st_{ro}} w + v$ with rough degree $u_1 + u_2$.

(ii) We assume that $w_n \xrightarrow{st_{ro}} w$ with rough degree u in a Riesz space E and $\lambda_n \to \lambda$ with the usual sense on $\mathbb{R}$. Then there exists a sequence $q_n \downarrow^{st_o} \theta$ in E with an index set $\delta(K) = 1$ such that

$$|w_{k_n} - w| \leq q_{k_n} + u$$

is valid for all $k_n \in K$. Then, for some $n_0 \in \mathbb{N}$, we can find a scalar $\alpha \in \mathbb{R}$ by $\lambda_n \to \lambda$ such that $|\lambda_n| \leq |\lambda| + \alpha$ is true for all $n \geq n_0$. Hence, we observe the following inequality:

$$\begin{aligned} \left|\lambda_{k_n} w_{k_n} - \lambda w\right| &\leq |\lambda_{k_n}||w_{k_n} - w| + |\lambda_{k_n} - \lambda||w| \\ &\leq (|\lambda| + \alpha)(|w_{k_n} - w|) + |\lambda_{k_n} - \lambda||w| \\ &\leq (|\lambda| + \alpha)(q_{k_n} + u) + |\lambda_{k_n} - \lambda||w| \\ &= ((|\lambda| + \alpha)q_{k_n} + |\lambda_{k_n} - \lambda||w|) + (|\lambda| + \alpha)u \end{aligned}$$

for each $k_n \in K$ such that $k_n \geq n_0$. On the other hand, by using the linearity of order convergence and the Archimedean property of E, we get

$$\left((|\lambda| + \alpha)q_{k_n} + \left|\lambda_{k_n} - \lambda\right||w|\right) \xrightarrow{o} \theta$$

on$K$. As a result, we can see that $\lambda_{k_n} w_{k_n} \xrightarrow{st_{ro}} \lambda w$ with rough degree $(|\lambda| + \alpha)u$, and so $r := (|\lambda| + \alpha)u$.

In the following result, we get a relation between st_{ro}- and rc- convergences.

Theorem 3.9. *Let E be a Riesz space and (w_n) be a sequence in E.* Then (w_n) is st_{ro}-convergent to w if and only if there exists a sequence (v_n) such that $w_n = v_n$ for almost all n and (v_n) rc-converges to w.

Proof: We suppose that $w_n \xrightarrow{st_{ro}} w$ holds in E. Then there exists a sequence $q_n \downarrow^{st_o} \theta$ with set $\delta(K) = 1$ such that we have the inequality $|w_{k_n} - w| \leq q_{k_n} + u$ for all $k_n \in K$. Now, by choosing the sequence (v_n) as the subsequence $(w_{k_n})_{k_n \in K}$, we obtain the desired result.

Conversely, we assume that there exists a sequence (v_n) such that $v_n \xrightarrow{rc} w$ and $v_n = w_n$ for almost all n. Thus, there exists a sequence $s_n \downarrow \theta$ in E such that $|v_n - w| \leq s_n + u$ is correct for all n, and so we have

$$\delta(\{k \leq n : |v_k - w| \leq s_k + u\}) = 1.$$

On the other hand, by considering $v_n = w_n$ for almost all n, we have $\delta(\{k \leq n : v_k = w_k\}) = 1$. Hence, it follows from the inequality $|w_n - w| \leq |w_n - v_n| + |v_n - w|$ that we have the inclusion;

$$\{k \leq n : v_k = w_k\} \cap \{k \leq n : |v_k - w| \leq s_k + u\} \subseteq \{k \leq n : |w_k - w| \leq s_k + u\}.$$

Thus, we get $\delta(\{k \leq n : |w_k - w| \leq s_k + u\}) = 1$, i.e., we obtain the convergence of $w_n \xrightarrow{st_{ro}} w$.

Theorem 3.10. *Let (w_n), (v_n), and (t_n) be sequences in a Riesz space E such that $w_n \leq v_n \leq t_n$ is satisfied for all $n \in \mathbb{N}$.* Then $w_n \xrightarrow{st_{ro}} w$ and $t_n \xrightarrow{st_{ro}} w$ imply $v_n \xrightarrow{st_{ro}} w$.

Proof: Assume that the conditions in the statement are satisfied. Then there exist sequences $q_n \downarrow^{st_o} \theta$ and $p_n \downarrow^{st_o} \theta$ with a common index set $\delta(I)$ such that

$$|w_{i_n} - w| \leq q_{i_n} + u \text{ and } |t_{i_n} - w| \leq p_{i_n} + u$$

are satisfied for all $i_n \in I$. Now, for any $i_n \in I$, we have

$$w - q_{i_n} - u \leq w_n \leq v_n \leq t_n \leq w + p_{i_n} + u.$$

Therefore, for any $i_n \in I$, we observe

$$w - q_{i_n} - u \leq v_n \leq w + p_{i_n} + u.$$

Hence, we have $|v_n - w| \leq q_{i_n} + p_{i_n} + u$ for all $i_n \in I$, and so we get $v_n \xrightarrow{st_{ro}} w$ by applying $(p_n + q_n) \downarrow^{st_o} \theta$.

Theorem 3.11. *Let* $w_n \xrightarrow{st_{ro}} w$ *be a sequence and* v *be an element in a Riesz space* E. If the st_{ro}-limit of a sequence is unique and $|w_n| \wedge |v| = \theta$ is valid for each $n \in \mathbb{N}$, then $|w| \wedge |v| = \theta$.

Proof: Assume that $w_{\mathrm{n}} \xrightarrow{st_{ro}} w$ and $|w_n| \wedge |v| = \theta$ remains true for every $n \in \mathbb{N}$. It follows from Theorem 3.7. (iii)-(iv) that $|w_{\mathrm{n}}| \wedge |\mathrm{v}| \xrightarrow{st_{ro}} |w| \wedge |v|$ with rough degree u because a constant sequence st_{ro}-convergent with rough degree zero. Thus, by applying $|w_n| \wedge |v| = \theta_{\mathrm{n}} \xrightarrow{st_{ro}} \theta$ and the uniqueness of st_{ro}-limit, we obtain $|w| \wedge |v| = \theta$.

Proposition 3.12. If st_{ro}-convergence in a Riesz space possesses the additive property, then each constant sequence is uniquely st_{ro}-convergent if and only if st_{ro}-convergence is unique.

Proof: It is sufficient to demonstrate the adequate case. We assume that a sequence (w_n) st_{ro}-converges to w_1 and w_2 in a Riesz space E such that $w_1 \neq w_2$. Thus, by using the additive property of st_{ro}-convergence, we observe

$$(\theta_n) = (w_n - w_n) \xrightarrow{st_{ro}} (w_1 - w_2).$$

Hence, under this assumption, we obtain the desired conclusion $w_1 = w_2$.

The following result presents a st_{ro}-version of Proposition 2.2 [14] with a similar proof.

Theorem 3.13. Let st_{ro}-convergence be additive on a Riesz space. Then the following statements are equivalent:

i. st_{ro}-convergence is unique;

ii. If $w_{\mathrm{n}} \xrightarrow{st_{ro}} w$, $v_{\mathrm{n}} \xrightarrow{st_{ro}} v$, and $w_n \geq v_n$ for all n, then we get w ≥ v;

iii. E_+ is st_{ro}-closed.

Proof: $(i) \Longrightarrow (ii)$ Assume that st_{ro}-convergence is determined uniquely and $w_{\mathrm{n}} \xrightarrow{st_{ro}} w$, $v_{\mathrm{n}} \xrightarrow{st_{ro}} v$, and $w_n \geq v_n$ holds for every n. By considering the additive of st_{ro}-convergence, $(w_n - v_n) \xrightarrow{st_{ro}} w - v$. It follows from Remark 3.2(iv) that we have $|w_{\mathrm{n}} - \mathrm{v}_{\mathrm{n}}| \xrightarrow{st_{ro}} |w - v|$. From Theorem 1.7.(2) [8], we see the following equalities

$$\theta = (v_n - w_n)^+ = \frac{1}{2}(v_n - w_n + |v_n - w_n|) \text{ and } (v - w)^+ = \frac{1}{2}(v - w + |v - w|).$$

Hence, we have $\theta_n \xrightarrow{st_{ro}} (v - w)^+$. As a result, by considering Proposition 3.12, we conclude that $(v - w)^+ = \theta$, i.e., we get $w \geq v$.

$(ii) \Longrightarrow (iii)$ Take a sequence (w_n) in E_+ such that $w_{\mathrm{n}} \xrightarrow{st_{ro}} w$ and the constant sequence $v_n = \theta_{\mathrm{n}} \xrightarrow{st_{ro}} \theta$. Therefore, we have $\theta_{\mathrm{n}} = \mathrm{v}_{\mathrm{n}} \leq \mathrm{w}_{\mathrm{n}} \xrightarrow{st_{ro}} w$, i.e., by using (ii), we get $w \geq \theta$.

$(iii) \Longrightarrow (ii)$ Let $w_{\mathrm{n}} \xrightarrow{st_{ro}} w$, $v_{\mathrm{n}} \xrightarrow{st_{ro}} v$, and $v_n \leq w_n$ for all $n \in \mathbb{N}$. By using the additivity of st_{ro}-convergence, we conclude that $\theta \leq w_{\mathrm{n}} - \mathrm{v}_{\mathrm{n}} \xrightarrow{st_{ro}} w - v$. It follows that $w - v \in E_+$, leading to $v \leq w$ as desired because E_+ is closed under the st_{ro}-convergence.

$(ii) \Longrightarrow (i)$ We suppose, by contradiction, that the st_{ro}-convergence is not uniquely determined. Thus, we have a constant sequence $w_n := w$, by Proposition 3.12, in E such that $\mathrm{w_n} \xrightarrow{st_{ro}} v$, for some elements $v \neq w$. On the other hand, one can see that $(\theta_n) = |\mathrm{w_n} - \mathrm{v_n}| \xrightarrow{st_{ro}} |w - v|$ by Remark 3.2(iv). Now, for the constant sequences $(t_n) = (r_n) = (\theta, \theta, \cdots)$, we have $t_\mathrm{n} \xrightarrow{st_{ro}} \theta$ and $\mathrm{r_n} \xrightarrow{st_{ro}} |w - v| \neq \theta$. So, we obtain a contradiction with (ii), and hence st_{ro}-convergence is unique.

Proposition 3.14. If $|w_n - v_n| \leq u$ remains true for some $u \in E_+$ and all $n \in \mathbb{N}$, and $\mathrm{v_n} \xrightarrow{st_o} w$ satisfies in a Riesz space, then (w_n) is st_{ro}-convergence to w.

Proof: We assume that the conditions of the statement hold. Then there exists a sequence $q_n \downarrow^{st_o} \theta$ in E with indexes set $\delta(K) = 1$ such that $\left|v_{k_n} - w\right| \leq \ q_{k_n}$ applies to every $k_n \in K$. Therefore, we have the following inequality;

$$|w_{k_n} - w| \leq |w_{k_n} - v_{k_n}| + |v_{k_n} - w| \leq u + q_{k_n}$$

for each $k_n \in K$, and so we get $\mathrm{w_n} \xrightarrow{st_{ro}} w$ with rough degree u.

4. STATISTICAL ROUGH ORDER LIMIT POINTS

It is important to observe that the st_{ro}-limit of a sequence is not necessarily, particularly for strictly positive elements of u, unique. Therefore, we define the set of all st_{ro}-limit points of a sequence $\widetilde{\mathrm{w}} :=$ (wn) as follows:

$$LIM_{\widetilde{\mathrm{w}}}^{u} \coloneqq \{w \in E : \mathrm{w_n} \xrightarrow{st_{ro}} w\}.$$

This indicates that for each $w \in LIM_{\widetilde{\mathrm{w}}}^{u}$ there is a sequence $q_n \downarrow^{st_o} \theta$ with an index set $\delta(K) = 1$ such that $\left|w_{k_n} - w\right| \leq q_{k_n} + u$ for all $\mathrm{k_n} \in K$. On the other hand, it is possible that $LIM_{\widetilde{\mathrm{w}}}^{u} \cap\ LIM_{\widetilde{\mathrm{v}}}^{u} = \emptyset$ hold for any st_{ro}-convergent sequence $\widetilde{w} := (\mathrm{w_n})$ and $\tilde{v} := (\mathrm{v_n})$ in Riesz spaces.

Theorem 4.1. Let $\widetilde{w}$ be a sequence, and u_1 and u_2 be positive elements in a Riesz space E. Then we have $LIM_{\widetilde{w}}^{u_1} \subseteq LIM_{\widetilde{w}}^{u_2}$ under the condition $u_1 \leq u_2$.

Theorem 4.2. If $\underset{w,w' \in LIM_{\widetilde{w}}^{u}}{sup} |w - w'|$ exists for an arbitrary sequence $\widetilde{w}$ in a Riesz space, then the supremum is less than $2u$.

Proof: Assume that $\sup\limits_{w,w' \in LIM_{\widetilde{\mathrm{w}}}^{u}} |w - w'| = \mathrm{e}$ in a Riesz space E, and so it is clear $e \in E_+$. We suppose $e \nleq 2u$, and hence we have some elements $w, w' \in LIM_{\widetilde{\mathrm{w}}}^{u}$ such that $|w - w'| \nleq 2u$. On the other hand, there exist sequences $q_n \downarrow^{st_o} \theta$ and $p_n \downarrow^{st_o} \theta$ with a common index set $\delta(I) = 1$ such that

$$|\mathrm{w}_{\mathrm{i_n}} - \mathrm{w}| \leq \mathrm{q}_{\mathrm{i_n}} + \mathrm{u} \text{ and } |\mathrm{w}_{i_n} - \mathrm{w}'| \leq \mathrm{p}_{\mathrm{i_n}} + \mathrm{u}$$

are true for all $i_n \in I$ because of $\mathrm{w_n} \xrightarrow{st_{ro}} w$ and $\mathrm{w_n} \xrightarrow{st_{ro}} w'$. It follows that

$$|w - \mathrm{w}'| \leq |\mathrm{w_n} - w| + |\mathrm{w_n} - \mathrm{w}'| \leq \mathrm{q_{i_n}} + \mathrm{p_{i_n}} + 2u.$$

Since order convergence is additive, we get $(\mathrm{q_{i_n}} + \mathrm{p_{i_n}}) \downarrow \theta$ on I, and so we obtain $|w - \mathrm{w}'| \leq 2u$. It means that $2u$ is indeed an upper bound for the set considered in the assumption.

Theorem 4.3. Let $\widetilde{w}' := (w_{j_n})$ be a subsequence of a $\widetilde{w} := (w_n)$ in a Riesz space E. Then we have the following inclusion $LIM^u_{\widetilde{w}'} \subseteq LIM^u_{\widetilde{w}}$.

Theorem 4.4. Let $\widetilde{w} := (w_n)$ be a sequence in a Dedekind complete Riesz space.
i. If (w_n) is order bounded, then $LIM^u_{\widetilde{w}}$ is a nonempty set for some positive elements u.
ii. If $LIM^u_{\widetilde{w}}$ is nonempty for some positive elements u, then (w_n) is statistically order bounded.

Proof: (i) We assume that (wn) is an order bounded sequence in a Dedekind complete Riesz space E. Thus, the supremum of the sequence exists, and we choose $\mathrm{u} := \lim_{\mathrm{n} \in \mathbb{N}} |\mathrm{w_n}|$. Therefore, the inequality $|w_n| = |w_n - \theta| \leq q_n + u$ satisfies for each sequence $q_n \downarrow \theta$ in E, and so we obtain $\theta \in LIM^u_{\widetilde{w}}$.
(ii) We suppose that $LIM^u_{\widetilde{w}} \neq \emptyset$ holds for some $u \in \mathrm{E}_+$. Then we have $\mathrm{w_n} \xrightarrow{st_{ro}} w$ for some $w \in E$, i.e., there exists a sequence $q_n \downarrow^{st_o} \theta$ with an index set $\delta(K) = 1$ such that $|w_{\mathrm{k_n}} - w| \leq \mathrm{q_{k_n}} + u \leq \mathrm{q_{k_1}} + u$ is true for every $\mathrm{k_n} \in K$. Hence, we have the following inequality;

$$|\mathrm{w_{k_n}}| \leq |\mathrm{w_{k_n}} - w| + |w| \leq \mathrm{q_{k_1}} + u + |w|$$

for each $\mathrm{k_n} \in K$. So, we obtain the statistical order boundedness of $(\mathrm{w_n})$, which is the desired result.

Proposition 4.5. $LIM^u_{\widetilde{w}}$ is an order-closed set for any sequence $\widetilde{w} = (w_n)$ and positive element u in a Riesz space.

Proof: Let $\widetilde{w}$ be a sequence in a Riesz space E, and (f_m) be a sequence in $LIM^u_{\widetilde{w}}$ for some $u \in E$ such that $f_m \xrightarrow{o} f$. It is enough to show that $f \in LIM^u_{\widetilde{w}}$. Under our assumption, for each $m \in \mathbb{N}$, we have $\mathrm{w_n} \xrightarrow{st_{ro}} f_m$. Fix any $m \in \mathbb{N}$. Thus, there exists a sequence $q^m_n \xrightarrow{st_{ro}} \theta$ with an index set $\delta(K) = 1$ such that $|\mathrm{w_{k_n}} - \mathrm{f_m}| \leq q^m_{k_n} + u$ is valid for all $k_n \in K$. Then we have

$$|w_{k_n} - f| \leq |w_{k_n} - f_m| + |f_m - f| \leq q^m_{k_n} + u + |f_m - f|.$$

for all $k_n \in K$. It follows from $(q^m_{k_n} + |f_m - f|) \xrightarrow{o} \theta$ that $w_n \xrightarrow{st_{ro}} f$, i.e., we obtain $f \in LIM^u_{\widetilde{w}}$.

Corollary 4.6. $LIM^u_{\widetilde{w}}$ is a statistical order-closed set in Riesz spaces.

Theorem 4.7. *The following statements are satisfied for a sequence* $\widetilde{w} := (\mathrm{w_n})$ *in a Riesz space* E*:*

i. $\gamma w_1 + (1-\gamma) w_2 \in LIM_{\tilde{w}}^{\gamma u_1 + (1-\gamma) u_2}$ satisfies for each scalar $0 \leq \gamma \leq 1$, and $w_1 \in LIM_{\tilde{w}}^{u_1}$ and $w_2 \in LIM_{\tilde{w}}^{u_2}$,
ii. $LIM_{\tilde{w}}^{u}$ is a convex set.

Proof: (i) Let u_1 and u_2 be positive elements in E, and $w_1 \in LIM_{\tilde{w}}^{u_1}$ and $w_2 \in LIM_{\tilde{w}}^{u_2}$. Then we have some sequences $q_n \downarrow^{st_o} \theta$ and $p_n \downarrow^{st_o} \theta$ with a common index set $\delta(I)$ such that

$$|w_{i_n} - w_1| \leq q_{i_n} + u_1 \text{ and } |w_{i_n} - w_2| \leq p_{i_n} + u_2$$

for each $i_n \in$ I. Then we have the following inequality;

$$\begin{aligned} \left|w_{i_n} - (\gamma w_1 + (1-\gamma) w_2)\right| &\leq \gamma\left|w_{i_n} - w_1\right| + (1-\gamma)\left|w_{i_n} - w_2\right| \\ &\leq \gamma q_{i_n} + (1-\gamma) p_{i_n} + \gamma u_1 + (1-\gamma) u_2 \end{aligned}$$

for all $i_n \in$ I and $\gamma \in [0,1]$. It follows from $(\gamma q_{i_n} + (1-\gamma) p_{i_n}) \downarrow \theta$ that we get the desired result $\gamma w_1 + (1-\gamma) w_2 \in LIM_{\tilde{w}}^{\gamma u_1 + (1-\gamma) u_2}$.

(ii) If we take $u_1 = u = u_2$, then from (i) we can conclude that $LIM_{\tilde{w}}^{u}$ is convex.

Recall that a subset A of a Riesz space E is called order bounded whenever there exists a positive vector $e \in E_+$ such that $|a| \leq e$ for all $a \in A$. Now consider an element $u \geq 2e$. Then, for a sequence (a_n) and an element a in A, we observe the following inequality;

$$|a_n - a| \leq |a_n| + |a| \leq e + e \leq u < u + q_n$$

for all n and any $q_n \downarrow^{st_o} \theta$. Hence, we get $\alpha_n \xrightarrow{st_{ro}} \alpha$. As a result, we see that every sequence in an order-bounded set is st_{ro}-convergent.

Proposition 4.8. If $w_n \xrightarrow{o} w$ is satisfied in a Riesz space, then $w \in LIM_{\tilde{w}}^{u}$ holds for all $u \in E_+$.

Theorem 4.9. Let $\tilde{w} := (w_n)$ and $\tilde{w}' := (w'_n)$ be two sequences in a Riesz space. If $|w_n - w'_n| \xrightarrow{o} \theta$ holds, then we have $LIM_{\tilde{w}}^{u} = LIM_{\tilde{w}'}^{u}$.

Proof: Take any $w \in LIM_{\tilde{w}}^{u}$. Then there exists a sequence $q_n \xrightarrow{st_{ro}} \theta$ with an index set $\delta(K) = 1$ such that $|w_{k_n} - w| \leq q_{k_n} + u$ is satisfied for all $k_n \in K$. Then we have

$$|w'_{k_n} - w| \leq |w'_{k_n} - w_{k_n}| + |w_{k_n} - w| \leq |w'_{k_n} - w_{k_n}| + q_{k_n} + u$$

holds for all $k_n \in K$. By applying the linearity of order convergence, we get $w \in LIM_{\tilde{w}'}^{u}$. The same steps can be applied to show the reverse inclusion and are therefore omitted.

Corollary 4.10. Let $\tilde{w} := (w_n)$ and $\tilde{w}' := (w'_n)$ be two sequences in a Riesz space. If $|w_n - w'_n| \xrightarrow{st_o} \theta$ holds, then we have $LIM_{\tilde{w}}^{u} = LIM_{\tilde{w}'}^{u}$.

4. CONCLUSIONS

This paper introduces and develops the theory of statistical rough order convergence in Riesz spaces, combining the concepts of statistical convergence, rough convergence, and order convergence. We establish fundamental properties of this new convergence, including its behavior under lattice operations and linear transformations, and demonstrate that it provides a more flexible framework than classical statistical order convergence. We show the characterization of statistical rough limit points and conditions for the uniqueness of limits. The results lay a solid foundation for further research into the applications and extensions of statistical rough order convergence in the related fields.

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