*ORIGINAL PAPER*

# NUMERICAL SOLUTIONS OF BURGERS' AND COUPLED BURGERS' EQUATIONS BY MODIFIED LAPLACE ADOMIAN DECOMPOSITION METHOD

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***Abstract.*** *This study demonstrates the effective use of the modified Laplace Adomian decomposition method (MLADM) to numerically solve the Burgers' and coupled Burgers' equations. The proposed method integrates the Laplace transform with the Adomian decomposition to effectively address nonlinear terms, avoiding linearisation, perturbation, or discretisation. The method yields rapidly convergent series solutions that achieve high accuracy with minimal terms. Comparisons between exact solutions and approximation solutions illustrate the enhanced accuracy, simplicity, and computational efficiency of MLADM in addressing both single and coupled nonlinear Burger's equations.*

***Keywords:*** *Burgers' and Coupled Burgers' Equations; Modified Laplace Adomian Decomposition Method; Nonlinear PDEs; Laplace Transform; Adomian Polynomials.*

***Mathematics Subject Classification:*** *65A05; 65M06; 35Q35.*

## 1. INTRODUCTION

Over the past decade, the significance of studying nonlinear partial differential equations modelling physical phenomena has increased. Nonlinear phenomena are important in many areas of engineering and science. Most of the problems we encounter in real life are modelled by nonlinear events. Travelling wave solutions are an important topic in nonlinear research. The nearness of Burgers' equation and its steady states was first recognised by Bateman [1]. Burgers gave it that review [2]. The inverse scattering transform [3], sine-cosine method [4], homotopy perturbation method [5], variational iteration method [6], homotopy analysis method [7,8], tanh-function method [9,10], tanh-coth method [11], Backlund transformation [12], (G′/G)-expansion method [13], and other potent techniques have been introduced.

George Adomian introduced a novel method for resolving nonlinear functional equations in 1980. This methodology, now called the Adomian decomposition method (ADM), has been the subject of numerous investigations [14-16]. To perform the ADM, the examined equation must be separated into linear and nonlinear components. This method generates a solution in the form of a series, with each term determined by a recursive relationship utilising Adomian polynomials. Several key studies on different aspects of variations of Adomian's decomposition method have been conducted by Andrianov [17] and Venkatarangan [18, 19]. A modified Laplace decomposition approach was initially introduced by Khuri [20, 21]. This technique was later applied by Agadjanov [22] to solve the Duffing

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equation. This numerical method demonstrates how the decomposition approach can be adapted with the Laplace transform to approximate solutions to nonlinear partial differential equations. Elgasery [23] applied the Laplace decomposition method to solve the Falkner-Skan problem. Hussain and Khan [24] used a modified version of the Laplace decomposition method to address specific PDEs. Burgers' formula is referenced in [25, 26].

$$\nu_\tau + \nu\nu_\xi = \nu_{\xi\xi} \tag{1}$$

is a second-order nonlinear partial differential equation that is used in many applied mathematics fields, including traffic flow, boundary layer behaviour, turbulence, fluid dynamics modelling, and shock wave production [27].

The coupled system was created by Esipov [28]. It acts as a basic model to explain sedimentation, or how the concentration of two kinds of particles changes over time in fluid suspensions or colloids, which is affected by gravity [29, 30].

Khuri [20] created the Laplace Adomian decomposition method and used it to solve nonlinear differential equations. This method combines two strong techniques: the Laplace transform and the Adomian decomposition method. Ahmad et al. [31] applied the Laplace decomposition method to solve the Volterra nonlinear integral equation. The Laplace Adomian decomposition approach is therefore applied in many different contexts [32]. Additionally, some authors have enhanced and changed the approach from many aspects [33] Kumar et al. [34].

In this paper, we describe the MLADM to solve the Burger's and coupled Burger's equations numerically. In Section 2, MLADM is described in detail. In Section 3, we apply the MLADM to solve Burgers' and coupled Burgers' equations. Lastly, Section 4 is the conclusion of this paper.

## 2. METHOD OF MODIFIED LAPLACE ADOMAIN DECOMPOSITION

This section explains the modified Laplace decomposition process for the integro-differential equation. We consider the general form of second-order nonlinear partial differential equations along with their starting conditions in the form

$$\mathrm{M}\nu(\xi,\tau) + \mathrm{P}\nu(\xi,\tau) + \mathrm{Q}\nu(\xi,\tau) = \Psi(\xi,\tau), \qquad \nu(\xi,0) = \mathrm{p}(\xi), \nu_\tau(\xi,0) = \mathrm{q}(\xi) \tag{2}$$

In this context, M denotes the second-order differential operator, defined as $M_{\xi\xi} = \frac{\partial^n}{\partial x^n}$, P represents the remaining linear operator, Q is a general nonlinear differential operator, and $\Psi(\xi,\tau)$ is the source term. By applying the Laplace transform (denoted by $\boldsymbol{\mathcal{L}}$) to both sides of Equation (2), the equation is modified as follows.

$$\boldsymbol{\mathcal{L}}\,[\mathrm{M}\nu(\xi,\tau)] + \boldsymbol{\mathcal{L}}\,[\mathrm{P}\nu(\xi,\tau)] + \boldsymbol{\mathcal{L}}\,[\mathrm{Q}\nu(\xi,\tau)] = \boldsymbol{\mathcal{L}}\,[\Psi(\xi,\tau)]$$

and by applying the Laplace transform differentiation property, we get

$$s^2\boldsymbol{\mathcal{L}}\,[\nu(\xi,\tau)] + \mathrm{sp}(\xi) - \mathrm{q}(\xi) + \boldsymbol{\mathcal{L}}\,[\mathrm{P}\nu(\xi,\tau)] + \boldsymbol{\mathcal{L}}\,[\mathrm{Q}\nu(\xi,\tau)] = \boldsymbol{\mathcal{L}}\,[\Psi(\xi,\tau)]$$

and so

$$\boldsymbol{\mathcal{L}}\,[\nu(\xi,\tau)] = \frac{\mathrm{p}(\xi)}{s} + \frac{\mathrm{q}(\xi)}{s^2} - \frac{1}{s^2}\boldsymbol{\mathcal{L}}\,[\mathrm{P}\,\nu(\xi,\tau)] - \frac{1}{s^2}\boldsymbol{\mathcal{L}}\,[\mathrm{Q}\nu(\xi,\tau)] + \frac{1}{s^2}\boldsymbol{\mathcal{L}}\,[\Psi(\xi,\tau)] \tag{3}$$

Presenting the solution as an infinite series is the next step in the Laplace decomposition method, as seen below.

$$\mathrm{v}(\xi,\tau) = \sum_{n=0}^{\infty} \nu_n(\xi,\tau) \tag{4}$$

Decomposed nonlinear operator

$$\mathrm{Qv}(\xi,\tau) = \sum_{n=0}^{\infty} B_n(\xi,\tau) \tag{5}$$

ehere $B_n$ is the Adomian polynomial given below

$$B_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\,\mathrm{Q}\left(\sum_{i=0}^{\infty} \lambda^i\,\nu_i\right)\Big|_{\lambda=0}, \qquad \text{for every n} \in N$$

Using Equations (3), (4) and (5), we get

$$\sum_{n=0}^{\infty} \mathcal{L}\,[\nu_n(\xi,\tau)] = \frac{\mathrm{p}(\xi)}{s} + \frac{\mathrm{q}(\xi)}{s^2} - \frac{1}{s^2}\mathcal{L}\,[\mathrm{Pv}(\xi,\tau)] - \frac{1}{s^2}\mathcal{L}\,[\sum_{n=0}^{\infty} B_n(\xi,\tau)] + \frac{1}{s^2}\mathcal{L}\,[\Psi(\xi,\tau)] \tag{6}$$

Comparing both sides in Equation (6), we have

$$\mathcal{L}\,[\nu_0(\xi,\tau)] = \phi_1(\xi,s) \tag{7}$$

$$\mathcal{L}\,[\nu_1(\xi,\tau)] = \phi_2(\xi,s) - \frac{1}{s^2}\mathcal{L}\,[Q_0\mathrm{v}(\xi,\tau)] - \frac{1}{s^2}\mathcal{L}\,[B_0(\xi,\tau)] \tag{8}$$

$$\mathcal{L}\,[\nu_{n+1}(\xi,\tau)] = -\frac{1}{s^2}\mathcal{L}\,[Q_n\mathrm{v}(\xi,\tau)] - \frac{1}{s^2}\mathcal{L}\,[B_n(\xi,\tau)] \qquad \mathrm{n} \geq 1, \tag{9}$$

where $\phi_1(\xi,s)$ and $\phi_2(\xi,s)$ represent the Laplace transform of $\phi_1(\xi,\tau)$ and $\phi_2(\xi,\tau)$ respectively. By applying the inverse Laplace transform to Equations (7) to (9), we obtain the desired recursive relation.

$$\nu_1(\xi,\tau) = \phi_2(\xi,s) - \mathcal{L}^{-1}\left[\frac{1}{s^2}\mathcal{L}\,[Q_0\mathrm{v}(\xi,\tau) - \frac{1}{s^2}\mathcal{L}\,[B_0(\xi,\tau)]\right] \tag{10}$$

$$\nu_{n+1}(\xi,\tau) = -\mathcal{L}^{-1}\left[\frac{1}{s^2}\mathcal{L}\,[Q_n\mathrm{v}(\xi,\tau)] - \frac{1}{s^2}\mathcal{L}\,[B_n(\xi,\tau)]\right], \mathrm{n} \geq 1. \tag{11}$$

$$\nu_0(\xi,\tau) = \phi_1(\xi,\tau) \tag{12}$$

The solution using the modified Adomian decomposition method (MADM) is heavily influenced by the choice of $\phi_0(\xi,\tau)$ and $\phi_1(\xi,\tau)$. These terms are derived from the source function in the equation and the provided initial conditions.

## 3. APPLICATIONS

### *3.1. ONE-DIMENSIONAL BURGER'S EQUATION*

Consider that the one-dimensional Burger's equation has the form [35]

$$\nu_\tau + \nu\nu_\xi = \nu_{\xi\xi}\,, \qquad \nu(\xi,0) = \frac{1}{2} - \frac{1}{2}\tanh\left(\frac{\xi}{4}\right). \tag{13}$$

Applying the Laplace Transform ($\mathcal{L}$), we have

$$s\nu(\xi,s) - \nu(\xi,0) = -\mathcal{L}\,(\nu\nu_\xi) + \mathcal{L}(\nu\nu_{\xi\xi}),$$

or

$$\nu(\xi,s) = \frac{1}{s}\nu(\xi,0) - \frac{1}{s}\mathcal{L}\,(\nu\nu_\xi) + \frac{1}{s}\mathcal{L}(\nu\nu_{\xi\xi}). \tag{14}$$

Using the initial condition in Equation (14), we get

$$\nu(\xi,s) = \frac{1}{2s} - \frac{1}{2s}\tanh\left(\frac{\xi}{4}\right) - \frac{1}{s}\mathcal{L}\,(\nu\nu_\xi) + \frac{1}{s}\mathcal{L}(\nu\nu_{\xi\xi}).$$

Using the inverse Laplace transform, we obtain the results needed for our analysis.

$$\nu(\xi,s) = \frac{1}{2s} - \frac{1}{2s}\tanh\left(\frac{\xi}{4}\right) - \mathcal{L}^{-1}\left[-\frac{1}{s}\mathcal{L}\,(\nu\nu_\xi) + \frac{1}{s}\mathcal{L}(\nu\nu_{\xi\xi})\right]. \tag{15}$$

We break down the solution into the following infinite sum

$$\nu(\xi,\tau) = \sum_{n=0}^{\infty}\nu_n(\xi,\tau). \tag{16}$$

Using Equation (16) on Equation (15), we get

$$\sum_{n=0}^{\infty}\nu_n(\xi,\tau) = \frac{1}{2s} - \frac{1}{2s}\tanh\left(\frac{\xi}{4}\right) + \mathcal{L}^{-1}\left[-\frac{1}{s}\mathcal{L}\left(\sum_{n=0}^{\infty}B_n(\tau)\right) + \frac{1}{s}\mathcal{L}\left(\sum_{n=0}^{\infty}\nu_{n,\xi\xi}(\xi,\tau)\right]\right.$$

in which $B_n = \sum_{n=0}^{\infty}\nu_j\nu_{n-j,x}$. The recursive relationship is as follows.

(I). $\nu_0(\xi,\tau) = \frac{1}{2} - \frac{1}{2}\theta$,

(II). $\nu_1(\xi,\tau) = \frac{1}{16}\tau(1-\theta^2)$,

(III). $\nu_2(\xi,\tau) = \frac{1}{128}\tau^2(\theta-\theta^3)$,

(IV). $\nu_3(\xi,\tau) = \frac{1}{384}\tau^3(-1+5\theta^2-4\theta^4)$,

(V). $\nu_4(\xi,\tau) = \frac{1}{10240}\tau^4(-23\theta+58\theta^3-35\theta^5)$,

(VI). $\nu_5(\xi,\tau) = \frac{1}{61440}\tau^5(57-385\theta^2+518\theta^4-190\theta^6)$,

(VII). $\nu_6(\xi,\tau) = \frac{1}{9175040}\tau^6(32215\theta-11319\theta^3+127743\theta^5-3639\theta^7)$,

(VIII). $\nu_7(\xi,\tau) = \frac{1}{11468800}\tau^7(-19757+170765\theta^2-315315\theta^4+213445\theta^6-44055\theta^8)$,

$$\vdots$$

and so on. Where $\theta = \tanh(\frac{\xi}{4})$ and we use an approximation with 8 terms and set

$$Approx^{7}_{(MLADM)} = \nu_0 + \nu_1 + \nu_2 + \nu_3 + \cdots + \nu_7.$$

When we use Taylor's Expansion to expand $Approx^{7}_{(MLADM)}$ about (0,0) [36], we would have

$$\mathrm{App}rox(\xi,\tau) \cong \frac{1}{2} - \frac{1}{8}\xi + \frac{1}{16}\tau + \frac{1}{384}\xi^3 - \frac{1}{256}\tau\xi^2 + \frac{1}{512}\tau^2\xi - \frac{1}{3072}\tau^3 + \frac{1}{491520}\tau^5 - \frac{1}{49152}\tau^4\xi + \frac{1}{12288}\tau^3\xi^2 - \frac{1}{12288}\tau^2\xi^3 + \cdots,$$

The Exact solution comes from the Taylor series

$$\mathrm{v}(\xi,\tau) = \frac{1}{2} - \frac{1}{2}\tanh\left[\frac{1}{4}\left(\xi - \frac{1}{2}\tau\right)\right]. \tag{17}$$

The behaviour of the exact and approximate solutions of 8 terms obtained by MLADM in MATLAB is shown in Table 1 and Figures 1-3. We examined the ξ-values ranging from −2 to 2, with a step size of Δξ = 0.5 and a τ value of 0.1.

### *3.2. COUPLED BURGER'S EQUATIONS*

$$\begin{aligned} &\mathrm{u}_\tau - \mathrm{u}_{\xi\xi} - 2\mathrm{u}\mathrm{u}_\xi + (\mathrm{u}\mathrm{v})_\xi = 0,\\ &\nu_\tau - \nu_{\xi\xi} - 2\nu\nu_\xi + (u\nu)_\xi = 0. \end{aligned} \tag{18}$$

Initial conditions

$$\mathrm{u}(\xi,0) = \sin\xi, \qquad \mathrm{v}(\xi,0) = \sin\xi \tag{19}$$

Rewriting Equation (18), we get

$$\begin{aligned} u_\tau &= u_{\xi\xi} + 2uu_\xi - (u\nu)_\xi,\\ \nu_\tau &= \nu_{\xi\xi} + 2\nu\nu_\xi - (u\nu)_\xi. \end{aligned} \tag{20}$$

Applying the ($\boldsymbol{\mathcal{L}}$) Laplace transform in time τ with respect to τ

$$\begin{aligned} s\overline{u}(\xi,s) - \mathrm{u}(\xi,0) &= \overline{u}_{\xi\xi}(\xi,s) + 2\boldsymbol{\mathcal{L}}[uu_\xi] - \boldsymbol{\mathcal{L}}[(u\nu)_\xi],\\ s\overline{\nu}(\xi,s) - \mathrm{v}(\xi,0) &= \overline{\nu}_{\xi\xi}(\xi,s) + 2\boldsymbol{\mathcal{L}}[\nu\nu_\xi] - \boldsymbol{\mathcal{L}}[(u\nu)_\xi]. \end{aligned}$$

or

$$\begin{aligned} \overline{u}(\xi,s) &= \frac{1}{s}\mathrm{u}(\xi,0) + \frac{1}{s}\overline{u}_{\xi\xi}(\xi,s) + \frac{2}{s}\boldsymbol{\mathcal{L}}[uu_\xi] - \frac{1}{s}\boldsymbol{\mathcal{L}}[(u\nu)_\xi],\\ \overline{\nu}(\xi,s) &= \frac{1}{s}\mathrm{v}(\xi,0) + \frac{1}{s}\overline{\nu}_{\xi\xi}(\xi,s) + \frac{2}{s}\boldsymbol{\mathcal{L}}[\nu\nu_\xi] - \frac{1}{s}\boldsymbol{\mathcal{L}}[(u\nu)_\xi]. \end{aligned} \tag{21}$$

Using Equation (19) in Equation (21), we get

$$\overline{u}(\xi,s) = \frac{1}{s}\sin\xi + \frac{1}{s}\overline{u}_{\xi\xi}(\xi,s) + \frac{2}{s}\boldsymbol{\mathcal{L}}[uu_\xi] - \frac{1}{s}\boldsymbol{\mathcal{L}}[(u\nu)_\xi],$$

$$\overline{v}(\xi, s) = \frac{1}{s}\sin\xi + \frac{1}{s}\overline{v}_{\xi\xi}(\xi, s) + \frac{2}{s}\mathcal{L}\left[vv_{\xi}\right] - \frac{1}{s}\mathcal{L}\left[(uv)_{\xi}\right].$$

Applying the inverse Laplace transform ($\mathcal{L}^{-1}$), we get

$$u(\xi,\tau) = \sin\xi + \mathcal{L}^{-1}\left[\frac{1}{s}\overline{u}_{\xi\xi}(\xi,s) + \frac{2}{s}\mathcal{L}\left[uu_{\xi}\right] - \frac{1}{s}\mathcal{L}\left[(uv)_{\xi}\right]\right],$$
$$v(\xi,\tau) = \sin\xi + \mathcal{L}^{-1}\left[\frac{1}{s}\overline{v}_{\xi\xi}(\xi,s) + \frac{2}{s}\mathcal{L}\left[vv_{\xi}\right] - \frac{1}{s}\mathcal{L}\left[(uv)_{\xi}\right]\right].$$

The decomposition series is given below.

$$u(\xi,\tau) = \sum_{n=0}^{\infty} u_n(\xi,\tau), \qquad v(\xi,\tau) = \sum_{n=0}^{\infty} v_n(\xi,\tau)$$

and nonlinear terms decomposed using Adomian polynomials.

$$uu_{\xi} = \sum_{n=0}^{\infty} \alpha_n(\xi,\tau), \quad vv_{\xi} = \sum_{n=0}^{\infty} \beta_n(\xi,\tau), \quad (uv)_{\xi} = \sum_{n=0}^{\infty} \gamma_n(\xi,\tau).$$

From the inverse Laplace transform, the recursive scheme becomes

$$u_0(\xi,\tau) = \sin\xi,$$
$$v_0(\xi,\tau) = \sin\xi.$$

$$u_{n+1}(\xi,\tau) = \mathcal{L}^{-1}\left[\frac{1}{s}\left(\mathcal{L}\left[u_{n,\xi\xi} + 2\alpha_n - \gamma_n\right]\right)\right],$$
$$v_{n+1}(\xi,\tau) = \mathcal{L}^{-1}\left[\frac{1}{s}\left(\mathcal{L}\left[v_{n,\xi\xi} + 2\beta_n - \gamma_n\right]\right)\right]. \tag{22}$$

Solve Equation (22) by using Adomian polynomials; we can obtain the following results:

$$u_0 = \sin\xi \qquad v_0 = \sin\xi$$
$$u_1 = -\tau\sin\xi \qquad v_1 = -\tau\sin\xi$$
$$u_2 = \frac{\tau^2}{2!}\sin\xi \qquad v_2 = \frac{\tau^2}{2!}\sin\xi$$
$$u_3 = -\frac{\tau^3}{3!}\sin\xi \qquad v_3 = -\frac{\tau^3}{3!}\sin\xi$$
$$\vdots \qquad \vdots$$
$$u_7 = \frac{(-1)^7\tau^7}{7!}\sin\xi \qquad v_7 = \frac{(-1)^7\tau^7}{7!}\sin\xi$$

and so on. We use an 8-term approximation and set

$$Approx_7^u = u_0 + u_1 + u_3 + \cdots + u_7$$
$$Approx_7^v = v_0 + v_1 + v_3 + \cdots + v_7$$

The two approximations $u(\xi,\tau)$ and $v(\xi,\tau)$ in closed form can be easily determined as follows.

$$\text{app}roxu(\xi,\tau) \cong \sin\xi\left[1 - \tau + \frac{\tau^2}{2!} - \frac{\tau^3}{3!} + \cdots + \frac{(-1)^7\tau^7}{7!}\right],$$
$$\text{app}roxv(\xi,\tau) \cong \sin\xi\left[1 - \tau + \frac{\tau^2}{2!} - \frac{\tau^3}{3!} + \cdots + \frac{(-1)^7\tau^7}{7!}\right]. \tag{23}$$

This expansion is accurate up to the final term, which can be verified by expanding the right solution of the coupled Burgers' Equations (18), which is

$$u(\xi,\tau) = \sin(\xi)\exp(-\tau),$$
$$v(\xi,\tau) = \sin(\xi)\exp(-\tau). \quad (24)$$

which is identical to the results obtained by the Adomian decomposition method [37].

The numerical results of coupled Burgers' equations are calculated as approximations and exact solutions of Equations (23) and (24), and $u(\xi,\tau)$ and $v(\xi,\tau)$ are both equal. The results are visualised using MATLAB, as shown in Table 2 and Figures 4-6, which compare the approximation and exact solutions.

**Table 1. Abbreviated comparison of exact and approximation solutions for Burger's equation using MLADM at $\tau = 0.1$. We considered the ξ-Values from $-2$ to 2, with a step size of $\Delta\xi = 0.5$.**

| ξ | Exact | Approximation | Error |
|---|---|---|---|
| -2.00 | 0.7338216187 | 0.7359453939 | 0.0021237753 |
| -1.50 | 0.6840553416 | 0.6846015003 | 0.0005461587 |
| -1.00 | 0.6282362488 | 0.6283161883 | 0.0000799395 |
| -0.50 | 0.5683168549 | 0.5683199835 | 0.0000031286 |
| 0.00 | 0.5062496745 | 0.5062496745 | 0.0000000000 |
| 0.50 | 0.4439872223 | 0.4439861095 | 0.0000011129 |
| 1.00 | 0.3834820130 | 0.3834334955 | 0.0000485175 |
| 1.50 | 0.3266865611 | 0.3262929005 | 0.0003936606 |
| 2.00 | 0.2755533814 | 0.2738850187 | 0.0016683626 |

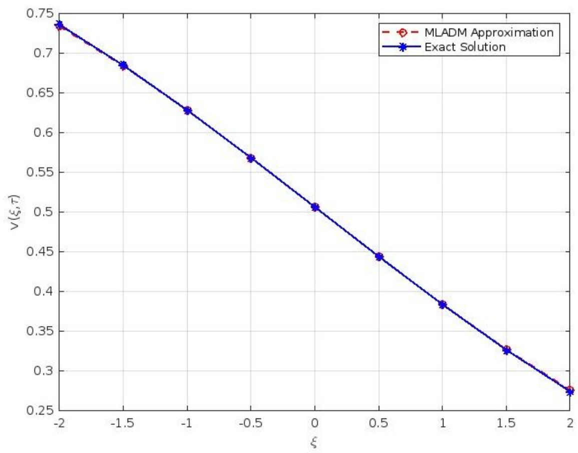
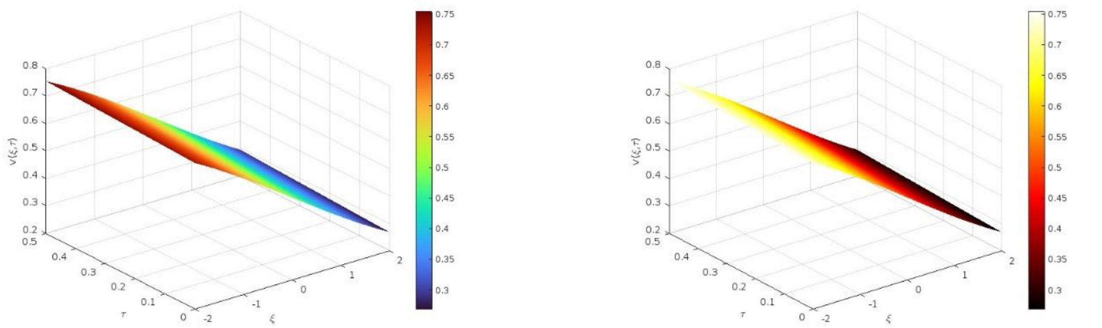



**Figure 1. Comparison of Exact and MLADM Approximation solutions at $\tau = 0.1$.**



(a) (b)

**Figure 2. Modified Laplace Adomian Decomposition Method (MLADM) at time $\tau = 0.1$ and spatial domain $\xi \in [-2, 2]$, with a step size of $\Delta\xi = 0.5$ (a) MLADM Approximation (b) Exact solution.**

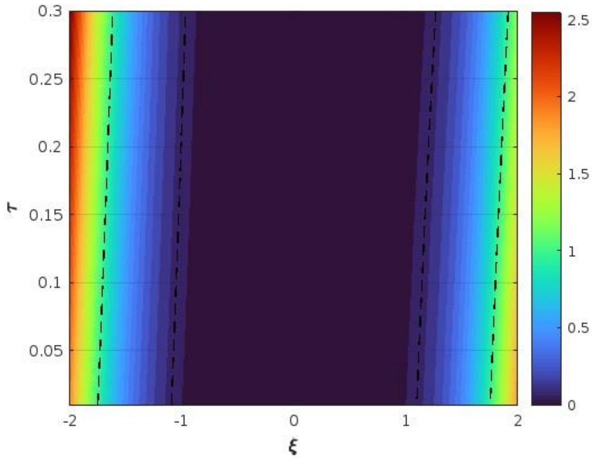



**Figure 3. An absolute error contour map comparing the exact solution of Burger's equation with the 8-term MLADM approximation.**

**Table 2. Comparison between exact and Approx[7] solutions for coupled Burger's equation using MLADM with a step size of $\Delta\xi = 0.5$ and $\tau = 0.3$.**

| ξ | **Approximation** | **Exact** | **Error** |
|---|---|---|---|
| 1.00 | 0.6233770364 | 0.6233770377 | 0.0000000013 |
| 1.50 | 0.7389624595 | 0.7389624611 | 0.0000000016 |
| 2.00 | 0.6736241004 | 0.6736241018 | 0.0000000014 |
| 2.50 | 0.4433590680 | 0.4433590689 | 0.0000000009 |
| 3.00 | 0.1045442731 | 0.1045442733 | 0.0000000002 |
| 3.50 | −0.2598666060 | −0.2598666066 | 0.0000000006 |
| 4.00 | −0.5606530768 | −0.5606530780 | 0.0000000012 |
| 4.50 | −0.7241721209 | −0.7241721224 | 0.0000000015 |
| 5.00 | −0.7103885734 | −0.7103885749 | 0.0000000015 |

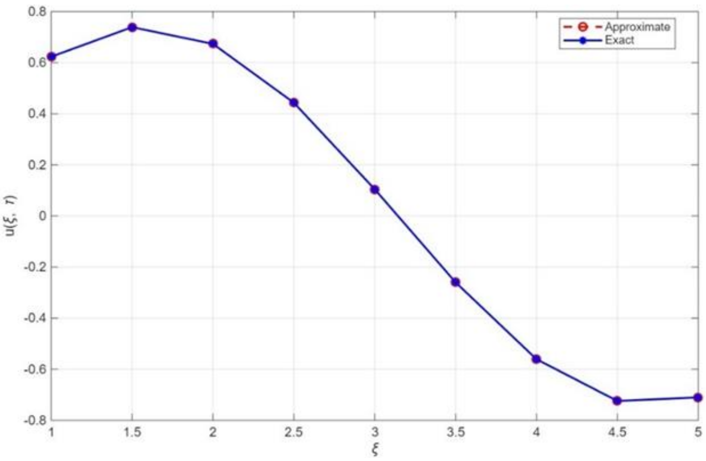
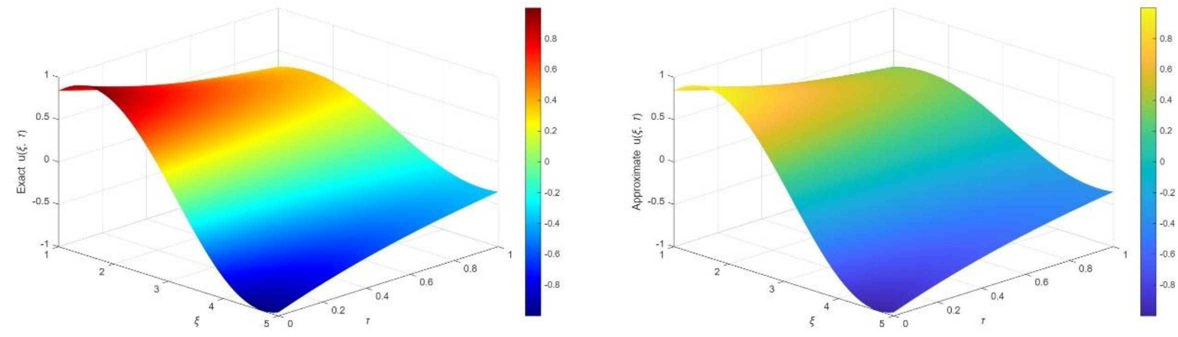



**Figure 4. Comparison of Approximation and Exact solutions at $\tau = 0.3$.**




(a) (b)

**Figure 5. (a) Exact Solution (b) Approximate solution of the coupled Burgers' equation at τ = 0.3.**

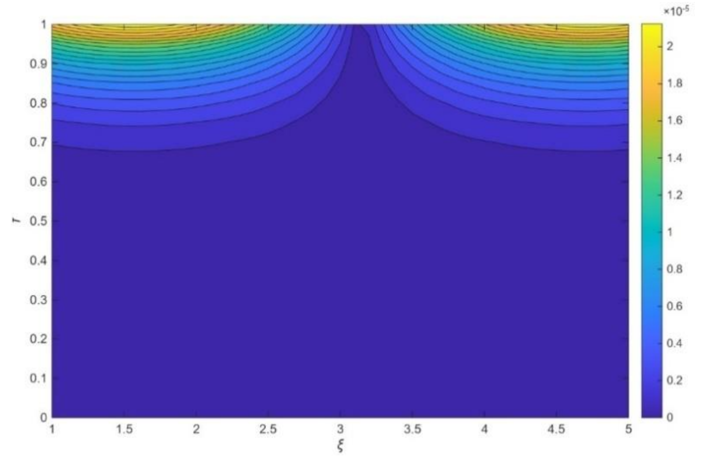



**Figure 6. This contour map shows the absolute error between the approximate and exact solutions of the coupled Burgers' equation obtained with MLADM at τ = 0.3.**

## 4. CONCLUSIONS

This paper successfully solves Burgers' and coupled Burgers' equations using the modified Laplace Adomian decomposition method (MLADM). For a suitable initial condition, the MLADM yields an infinite power series, which can then be expressed in a closed form, which is the precise solution. The findings demonstrate that the MLADM is a potent mathematical tool for resolving Burger's and coupled Burger's equations. It also shows promise as a technique for resolving other nonlinear equations. The resulting solutions are displayed graphically. In our work, we compute the series obtained via MLADM using MATLAB.

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## REFERENCES


[1] Bateman, H., Some recent research on the motion of fluids, *Monthly Weather Review*, **43**(4), 163, 1915. https://doi.org/10.1175/1520-0493(1915)43<163:SRROTM>2.0.CO;2

[2] Burgers, J.M., A mathematical model illustrating the theory of turbulence, *Advances in applied mechanics*, **1**, 171, 1948. https://doi.org/10.1016/S0065-2156(08)70100-5

[3] Ablowitz, M.J., Clarkson, P.A., *Solitons, nonlinear evolution equations and inverse scattering*, Cambridge University Press, Cambridge, 1991.

[4] Wazwaz, A.M., Travelling wave solutions for combined and double combined sine–cosine-Gordon equations by the variable separated ODE method, *Applied Mathematics and Computation*, **177**(2), 755, 2006. https://doi.org/10.1016/j.amc.2005.09.104

[5] Dehghan, M., Manafian, J., The solution of the variable coefficients fourth-order parabolic partial differential equations by the homotopy perturbation method, *Zeitschrift für Naturforschung A*, **64**(7-8), 420, 2009. https://doi.org/10.1515/zna-2009-7-803

[6] He, J.H., Variational iteration method–a kind of non-linear analytical technique: some examples, *International Journal of Non-Linear Mechanics*, **34**(4), 699, 1999. http://dx.doi.org/10.1016/S0020-7462(98)00048-1

[7] Dehghan, M., Manafian, J., Saadatmandi, A., Solving nonlinear fractional partial differential equations using the homotopy analysis method, *An International Journal*, **26**(2), 448, 2010. https://doi.org/10.1002/num.20460

[8] Dehghan, M., Manafian, J., Saadatmandi, A., The solution of the linear fractional partial differential equations using the homotopy analysis method, *Zeitschrift für Naturforschung-A*, **65**(11), 935-949, 2010.

[9] Bai, C.L., Zhao, H., Generalized extended tanh-function method and its application, *Chaos, Solitons and Fractals*, **27**(4), 1026, 2006. https://doi.org/10.1016/j.chaos.2005.04.069

[10] Fan, E., Extended tanh-function method and its applications to nonlinear equations, *Physics Letters A*, **277**(4-5), 212, 2000. https://doi.org/10.1016/S0375-9601(00)00725-8

[11] Heris, J.M., Zamanpour, I., Exact travelling wave solutions of the symmetric regularized long wave (SRLW) using analytical methods, *Optimization & Information Computing*, **2**(1), 47, 2014. https://doi.org/10.19139/soic.v2i1.69

[12] Meng, X.H., Liu, W.J., Zhu, H.W., Zhang, C.Y., Tian, B., Multi-soliton solutions and a Bäcklund transformation for a generalized variable-coefficient higher-order nonlinear Schrödinger equation with symbolic computation, *Physica A: Statistical Mechanics and its Applications*, **387**(1), 97, 2008. https://doi.org/10.1016/j.physa.2007.08.028

[13] Wang, M., Li, X., Zhang, J., The (G′ G)-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics, *Physics Letters A*, **372**(4), 417, 2007. https://doi.org/10.1016/j.physleta.2007.07.051

[14] Ansari, A., Ahmad, N., Numerical Accuracy of Fredholm linear Integro-differential equations by using Adomian Decomposition Method, Modified Adomian Decomposition Method and Variational Iteration Method, *Journal of Science and Arts*, **23**(3), 625, 2023. https://doi.org/10.46939/J.Sci.Arts-23.3-a05

[15] Ansari, A., Ahmad, N., Numerical Accuracy of Fredholm Integro-Differential Equations by Using Adomian Decomposition Method and Modified Adomian Decomposition Method, *Bulletin of Calcutta Mathematical Society*, **115**(5), 567, 2023.

[16] Ahmad, N., Singh, B., Ansari, A., Numerical Solution of Integral Equation by Using Picard Method, Homotopy and Modified Adomian Decomposition Method, *Ganita*, **74**(2), 349, 2024.

[17] Andrianov, I. V., Olevskii, V. I., Tokarzewski, S., A modified Adomian's decomposition method, *Journal of Applied Mathematics and Mechanics*, **62**(2), 309, 1998. https://doi.org/10.1016/S0021-8928(98)00040-9

[18] Venkatarangan, S.N., Rajalakshmi, K., A modification of Adomian's solution for nonlinear oscillatory systems, *Computers & Mathematics with Applications*, **29**(6), 67, 1995. https://doi.org/10.1016/0898-1221(95)00008-M

[19] Venkatarangan, S.N., Rajalakshmi, K., Modification of Adomian's decomposition method to solve equations containing radicals, *Computers & Mathematics with Applications*, **29**(6), 75, 1995. https://doi.org/10.1016/0898-1221(95)00009-N

[20] Khuri, S.A., A Laplace decomposition algorithm applied to a class of nonlinear differential equations, *Journal of Applied Mathematics*, **1**(4), 141-155, 2001. https://doi.org/10.1155/S1110757X01000183

[21] Khuri, S.A., A new approach to Bratu's problem, *Applied Mathematics and Computation*, **147**(1), 131, 2004. https://doi.org/10.1016/S0096-3003(02)00656-2

[22] Yusufoğlu, E., Numerical solution of Duffing equation by the Laplace decomposition algorithm, *Applied Mathematics and Computation*, **177**(2), 572, 2006.

[23] Elgazery, N.S., Numerical solution for the Falkner–Skan equation, *Chaos, Solitons & Fractals*, **35**(4), 738, 2008. https://doi.org/10.1016/j.chaos.2006.05.040

[24] Hussain, M., Khan, M., Modified Laplace decomposition method, *Applied Mathematical Sciences*, **4**(36), 1769, 2010.

[25] Burgers, J.M., *The nonlinear diffusion equation*, D. Reidel Publisher Company, Dordrecht, Holland, 1974.

[26] Veksler, A., Zarmi, Y., Wave interactions and the analysis of the perturbed Burgers equation, *Physica D: Nonlinear Phenomena*, **211**(1-2), 57, 2005. https://doi.org/10.1016/j.physd.2005.08.001

[27] Wazwaz, A.M., Analytic study on Burgers, Fisher, Huxley equations and combined forms of these equations, *Applied Mathematics and Computation*, **195**(2), 754, 2008.

[28] Esipov, S.E., Coupled Burgers equations: a model of polydispersive sedimentation, *Physical Review E*, **52**(4), 3711, 1995. https://doi.org/10.1103/PhysRevE.52.3711

[29] Mittal, R.C., Arora, G., Numerical solution of the coupled viscous Burgers' equation, *Communications in Nonlinear Science and Numerical Simulation*, **16**(3), 1304, 2011.

[30] Nee, J., Duan, J., Limit set of trajectories of the coupled viscous Burgers' equations, *Applied Mathematics Letters*, **11**(1), 57, 1998.

[31] Ahmad, N., Singh, B., Numerical solution of Volterra nonlinear integral equation by using Laplace Adomian decomposition method, *International Journal of Applied Mathematics*, **35**(1), 39, 2022. http://dx.doi.org/10.12732/ijam.v35i1.3

[32] Mishra, V., Rani, D., Modification of Laplace Adomian decomposition method for solving nonlinear Volterra integral and integro-differential equations based on Newton Raphson formula, *European Journal of Pure and Applied Mathematics*, **11**(1), 202, 2018. https://doi.org/10.29020/nybg.ejpam.v11i1.2645

[33] Heris, J.M., Solving the Integro-Differential Equations Using the Modified Laplace Adomian Decomposition Method, *Journal of Mathematical Extension*, **6**(1), 41, 2012.

[34] Kumar, S., Kumar, D., Abbasbandy, S., Rashidi, M. M., Analytical solution of fractional Navier–Stokes equation by using modified Laplace decomposition method, *Ain Shams Engineering Journal*, **5**(2), 569, 2014. https://doi.org/10.1016/j.asej.2013.11.004

[35] Heris, J. M., Zamanpour, I., Solving Burgers' equation by semi-analytical and implicit method, *Statistics, Optimization and Information Computing*, **2**(3), 222, 2014. https://doi.org/10.19139/soic.v2i3.70

[36] Naghipour, A., Manafian, J., Application of the Laplace Adomian decomposition and implicit methods for solving Burgers' equation, *TWMS Journal of Pure and Applied Mathematics*, **6**(1), 68, 2015.

[37] Kaya, D., An explicit solution of coupled viscous Burgers′ equation by the decomposition method, *International Journal of Mathematics and Mathematical Sciences*, **27**(11), 675, 2001. https://doi.org/10.1155/S0161171201010249