ORIGINAL PAPER

ANALYSIS OF SOLITON DYNAMICS IN A FRACTIONAL-ORDER (2+1)-DIMENSIONAL CHAFFEE–INFANTE MODEL FOR COMBUSTION IN HOMOGENEOUS MEDIA

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Abstract. *This paper explores the fractional (2+1)-dimensional Chaffee-Infante (CI) equation using the Exp-function method. The classical Chaffee-Infante equation, widely employed to model reaction-diffusion, combustion, and gas diffusion processes in homogeneous media, is extended here to incorporate a fractional modified Riemann-Liouville derivative. This extension allows for a more accurate representation of complex diffusion behaviors observed in real-world systems. By applying the Exp-function method, we derive exact solutions that illuminate the system's intricate dynamics. The introduction of fractional order provides additional flexibility in modelling phenomena with memory effects and anomalous diffusion. Our results demonstrate the method's effectiveness in solving fractional nonlinear partial differential equations (PDEs), with potential applications across various scientific and engineering fields. The solutions are validated through comparison with known results, confirming the accuracy and utility of the approach. These findings have important implications for the modelling of nonlinear phenomena in mathematical physics and engineering disciplines.*

Keywords: *Fractional (2+1)-Dimensional Chaffee–Infante equation; Exp-function method; fractional wave transform; soliton solution.*

Mathematics Subject Classification: *35C08; 35R11; 35K57.*

1. INTRODUCTION

Fractional PDEs are multidisciplinary and indicate physical aspects of complex phenomena, and they form the basis of most mathematical and physical simulations in real-world applications. Numerous real-life phenomena have been explained using fractional derivatives, such as signal processing, ocean waves, optical fibres, photonics, nuclear physics, and other sciences. Fractional PDEs have broad research potential for numerous mathematicians. They have also been formulated as complicated nonlinear equations that describe real-life difficulties in mathematics, physics, engineering, and medical imaging, and many examples of fractional PDEs include the fractional diffusion equation, the fractional Schrödinger equation, and the fractional wave equation [1-9].

Solitons are solutions to certain nonlinear fractional PDEs and have been observed in several physical contexts such as plasma physics and fluid dynamics. Solitary wave solutions are renowned solutions to certain nonlinear PDEs that explain a wave packet that maintains its shape while travelling at constant velocity. Solitary waves, when stable and able to interact

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with other solitary waves without changing form, are termed solitons. These solutions are particularly significant in the study of nonlinear systems. Some characteristics of solitary waves are shape, preservation, stability, and nonlinear interaction. Solitary waves play a critical role in various scientific and computational engineering fields, providing insight into the dynamics of nonlinear systems. Travelling-wave solutions are fundamental to understanding and describing wave phenomena in many physical systems. They provide a simplified, powerful way to predict the behaviour of waves in many contexts. These solutions are essential in various fields of physics and engineering, including fluid dynamics, acoustics, and electromagnetism. Travelling-wave solutions have many applications, such as in fluid dynamics and electromagnetism. The exponential function method is a powerful mathematical tool for solving various problems, particularly growth and decay processes, linear differential equations, and data modelling [10-17].

This work is about utilising the exponential function method for obtaining solitary wave solutions. This method has many applications such as radioactive decay, population growth, compound interest, and heat transfer. The exponential function method is versatile for solving differential equations, modelling real-world phenomena, and fitting data. This method has been used by many researchers to obtain travelling and non-travelling wave solutions. The proposed method has been used in various contexts, such as solving differential equations, optimization problems, and fitting data to an exponential model [18-29].

The CI equation is a well-known PDE in the study of reaction-diffusion systems. It is used to explain numerous physical phenomena such as heat conduction, population dynamics, and chemical reactions. This equation has extensive applications in physical science and mathematical engineering, including fluid dynamics and signal processing. PDEs can give many different types of wave solutions. Soliton solutions are often associated with nonlinear PDES. They are typically found in integrable systems, a special class of nonlinear systems with many conserved quantities. Common examples of equations that admit lump and soliton solutions include the KP equation. The present study focuses on the (2+1)-dimensional fractional CI equation because multidimensional nonlinear interactions and fractional dynamics provide a more accurate representation of real combustion processes than classical integer-order models. The derived soliton solutions help explain the propagation of localized nonlinear waves and provide useful insight into the influence of fractional effects on combustion behavior. Solitary wave solutions can model phenomena such as solitary water waves and optical pulses in nonlinear media. The given technique is used especially to find exact solitary-wave solutions and to understand the structure of the equations. Soliton solutions are a special type of localized periodic solutions to certain nonlinear fractional PDEs. They exhibit localized oscillations in both space and time and are particularly significant in the study of nonlinear wave equations and integrable systems. Several approaches have been used in the literature to solve the CI equation, yielding different types of solutions [30-37]. In this study, we work on the fractional-order CI equation to find analytic solutions, which provide a brief analysis of the dynamical behaviour of the problem, and, at the end, we obtain new and more general results. Although several studies have investigated the classical and variable-coefficient CI equation using different analytical techniques, limited work has been reported on the fractional (2+1)-dimensional form of the equation. In particular, the application of the Exp-function method to obtain exact soliton solutions incorporating fractional dynamics and memory effects remains largely unexplored. This study aims to fill this gap by deriving new analytical solutions and analyzing their dynamical behavior.

The modified Riemann-Liouville derivative of order α is used in this research, which is defined as [38]:

$$D_t^{\alpha} f(t) = \begin{cases} \dfrac{1}{\Gamma(1-\beta)} \dfrac{d}{dt} \displaystyle\int_0^t (t-\xi)^{-\alpha} \left(f(\xi) - f(0) \right) d\xi, \ 0 < \alpha < 1, \\ \left(f^n(t) \right)^{(\alpha - n)}, \ \ n \le \alpha < n+1, \ n \ge 1. \end{cases} \tag{1}$$

There are 5 sections in this work, which are ordered as follows: Methodology is given in Section 2, application of the proposed method is given in Section 3, Section 4 is about the results and discussion, and conclusion is given in Section 5.

2. MATERIALS AND METHODS

2.1. METHOD DESCRIPTION

We will examine general nonlinear partial differential Equations.

$$E\left(u, u_x, u_t, D_t^{\alpha}, ...\right) = 0, \ 0 < \alpha \le 1. \tag{2}$$

The fractional wave transformation is given by

$$u(x,t) = V(\eta), \ \ \eta = \frac{k_1 x^{\alpha}}{\alpha} + \frac{k_2 y^{\alpha}}{\alpha} - \frac{\omega t^{\alpha}}{\alpha}, \tag{3}$$

where k_1 *and* k_2 are arbitrary constants that represent the wave number, and ω represents the wave velocity. Adding Equation (3) into Equation (2) yields

$$Q\left(V, V', V'', V''', ...\right) = 0. \tag{4}$$

Suppose the solitary wave solution as

$$V(\eta) = \frac{\sum\limits_{m=-q}^{s} a_m \exp[m\eta]}{\sum\limits_{l=-g}^{h} b_l \exp[l\eta]}, \tag{5}$$

where: q, s, g *and* h are constants. With the help of computational software Maple, we resolve Equation (4) by putting the value of Equation (5) in Equation (4), and we get a system of equations which are further resolved by comparing their coefficients to zero and further elucidated as required.

2.2. IMPLEMENTATION OF THE METHOD

This equation under the assumption is the fractional (2+1) dimensional CI equation.

$$D_{xt}^{2\alpha}u + (\beta u^3 - D_{xx}^{2\alpha}u - \beta u)_x + \sigma u_{yy} = 0, \qquad 0 < \alpha \leq 1. \tag{6}$$

To explain this equation, let's break down each term and the significance of the coefficients: β is the coefficient of diffusion, and σ degradation coefficient. u is the dependent variable, which could represent a physical quantity t represents time x and y are spatial coordinates. This PDE appears to model a system where nonlinear, dispersive, and linear effects are present and interact with each other. The specific nature of the system depends on the context, which could be in fluid dynamics. The CI equation is a distinguished PDE in the study of reaction-diffusion systems. It is used to describe various physical phenomena such as heat conduction, population dynamics, and chemical reactions.

Substituting Equation (3) in Equation (6), and integrating, we get

$$-k_1^3 V'' + \left(k_2^2\sigma - k_1\omega\right)V' + \beta k_1 V^3 - \beta k_1 V = 0. \tag{7}$$

We get $r = s = g = h = 1$ with the help of homogeneous balance principal and then Equation (5) reduces to

$$V(\eta) = \frac{a_{-1}\exp[-\eta] + a_0 + a_1\exp[\eta]}{b_{-1}\exp[-\eta] + b_0 + b_1\exp[\eta]}. \tag{8}$$

By substituting Equation (8) into Equation (7) and comparing the coefficients of $\exp(\eta)$ to zero, we find a system of equations.

$$\begin{aligned}
&A_0 = k_1\beta a_{-1} \;\; a_{-1} - b_{-1} \;\; a_{-1} + b_{-1} \;\; = 0,\\
&A_1 = \;\; a_{-1}b_{-1}b_0 - a_0 b_{-1}^2 \;\; k_1^3 + \begin{pmatrix} -2b_{-1}a_{-1}\left(\beta - \dfrac{\omega}{2}\right)b_0 + \\ 3a_0\left(\left(-\dfrac{\beta}{3} - \dfrac{\omega}{3}\right)b_{-1}^2 + \beta a_{-1}^2\right) \end{pmatrix} k_1\\
&+k_2^2\sigma b_{-1} \;\; -a_{-1}b_0 + a_0 b_{-1} \;\; = 0,\\
&A_2 = \;\; 4a_{-1}b_{-1}b_1 - a_{-1}b_0^2 + a_0 b_0 b_{-1} - 4a_1 b_{-1}^2 \;\; k_1^3 +\\
&\begin{pmatrix} -a_{-1} \;\; \beta - \omega \;\; b_0^2 - 2b_{-1}a_0 b_0\left(\beta + \dfrac{\omega}{2}\right) - a_1 (\beta - 2\omega)\, b_1^2 \\ -2b_1 a_{-1} (\beta - \omega)\, b_1 + 3\beta a_{-1} (a_1 a_{-1} + a_0^2) \end{pmatrix} k_1\\
&-2\begin{pmatrix} a_{-1}b_{-1}b_1 - \dfrac{1}{2}a_0 b_0 b_{-1} \\ +\dfrac{1}{2}a_{-1}b_0^2 - a_1 b_{-1}^2 \end{pmatrix} k_2^2\sigma = 0,\\
&A_3 = \;\; -3a_{-1}b_1 - 3a_1 b_{-1} \;\; b_0 + 6a_0 b_1 b_{-1} \;\; k_1^3 +\\
&\begin{pmatrix} -a_1 \;\; \beta + \omega \;\; b_0^2 - 2b_1 a_0 b_0\left(\beta - \dfrac{\omega}{2}\right) - a_{-1} \;\; \beta - 2\omega \;\; b_1^2 - 2b_1 a_1 \;\; \beta + \omega \;\; b_{-1} \\ +3\beta a_1 \;\; a_1 a_{-1} + a_0^2 \end{pmatrix} k_1
\end{aligned} \tag{9}$$

$$-2\begin{pmatrix}\frac{1}{2}a_0b_0b_1-\frac{1}{2}a_1b_0^2 \\ -a_1b_{-1}^2+b_1\left(a_{-1}b_1-a_1b_{-1}\right)\end{pmatrix}k_2^2\sigma=0,$$

$$A_5=\left(-a_0b_1^2+a_1b_0b_1\right)k_1^3+\left(-2b_1a_1b_0\left(\beta+\frac{\omega}{2}\right)-a_0\left(\beta-\omega\right)b_1^2-3\beta a_1^2\right)k_1$$

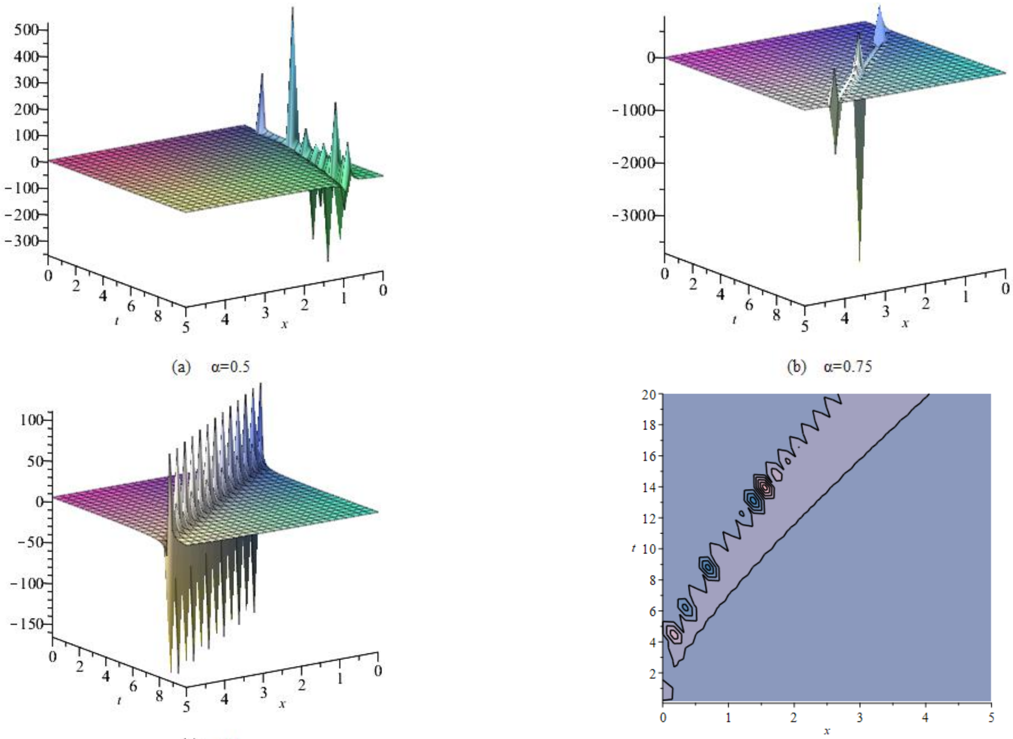
$$-b_1k_2^2\sigma\left(a_0b_1-a_1b_0\right)=0,$$

$$A_6=-k_1\beta a_1\left(b_1-a_1\right)\left(b_1+a_1\right)=0.$$

Case 1.

$$\beta=0,\omega=\omega,\sigma=\sigma,,a_{-1}=\frac{a_1b_{-1}}{b_1},a_0=\frac{a_1b_0}{b_1},a_1=a_1,b_1=b_1,b_{-1}=b_{-1},b_0=b_0. \tag{10}$$

$$u_1(x,t)=\frac{\frac{a_1b_{-1}}{b_1}\exp\left[-\frac{k_1x^\alpha}{\alpha}-\frac{k_2y^\alpha}{\alpha}+\frac{\omega t^\alpha}{\alpha}\right]+\frac{a_1b_0}{b_1}+a_1\exp\left[\frac{k_1x^\alpha}{\alpha}+\frac{k_2y^\alpha}{\alpha}-\frac{\omega t^\alpha}{\alpha}\right]}{b_{-1}\exp\left[-\frac{k_1x^\alpha}{\alpha}-\frac{k_2y^\alpha}{\alpha}+\frac{\omega t^\alpha}{\alpha}\right]+b_0+b_1\exp\left[\frac{k_1x^\alpha}{\alpha}+\frac{k_2y^\alpha}{\alpha}-\frac{\omega t^\alpha}{\alpha}\right]}. \tag{11}$$



(a) α=0.5 (b) α=0.75 (c) α=1 (d) α=0.5

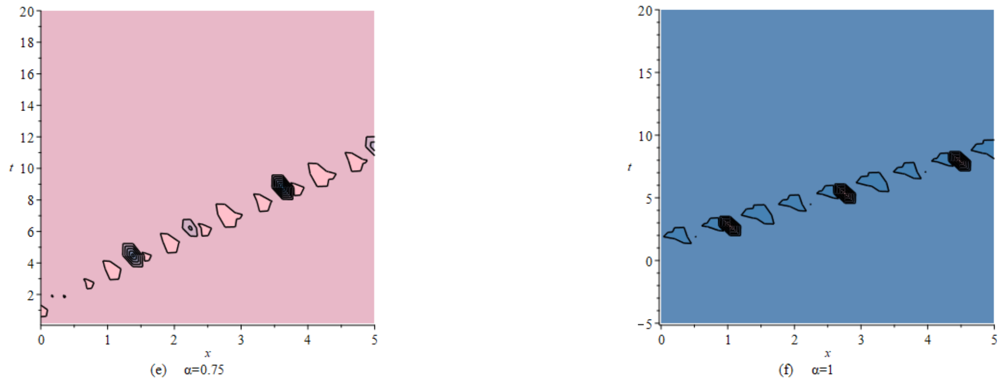
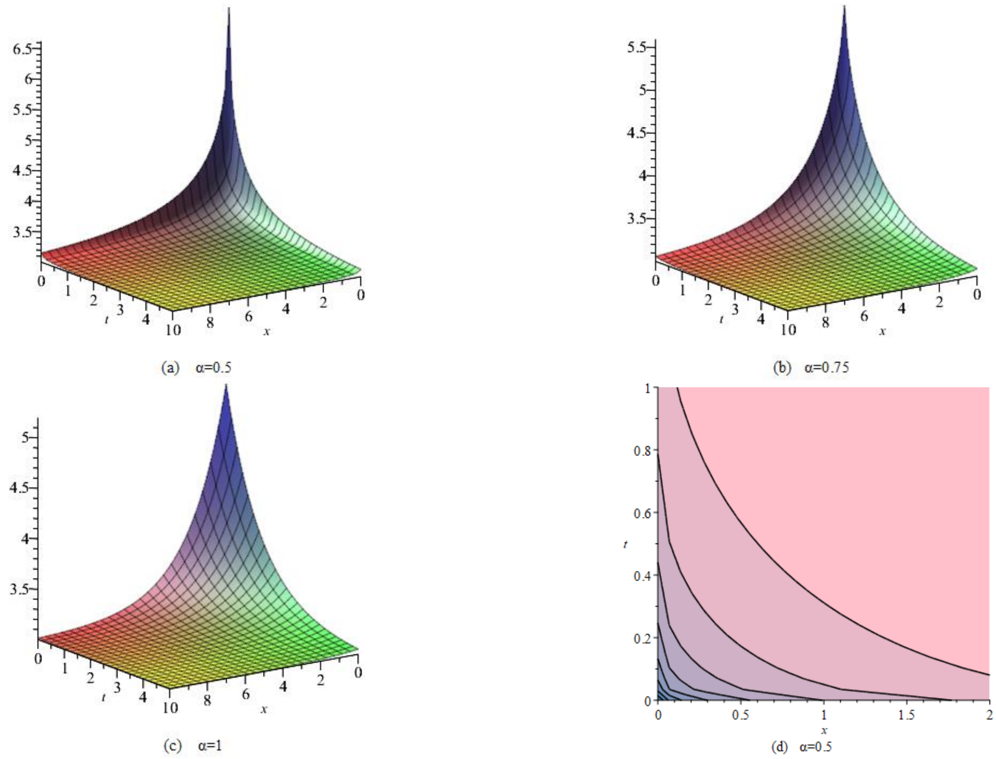


Figure 1. 3D and contour plots of $u_1(x,t)$ for different values of α .

Case 2.

$$\beta = 0, \omega = \frac{k_2^2\sigma - k_1^3}{k_1}, \sigma = \sigma,, a_{-1} = a_{-1}, a_0 = a_0, a_1 = 0, \\ b_1 = 0, b_{-1} = b_{-1}, b_0 = 0. \tag{12}$$

$$u_2(x,t) = \frac{a_{-1}\exp\left[-\frac{k_1 x^\alpha}{\alpha} - \frac{k_2 y^\alpha}{\alpha} + \left(\frac{k_2^2\sigma - k_1^3}{k_1}\right)\frac{t^\alpha}{\alpha}\right] + a_0}{b_{-1}\exp\left[-\frac{k_1 x^\alpha}{\alpha} - \frac{k_2 y^\alpha}{\alpha} + \left(\frac{k_2^2\sigma - k_1^3}{k_1}\right)\frac{t^\alpha}{\alpha}\right]}. \tag{13}$$



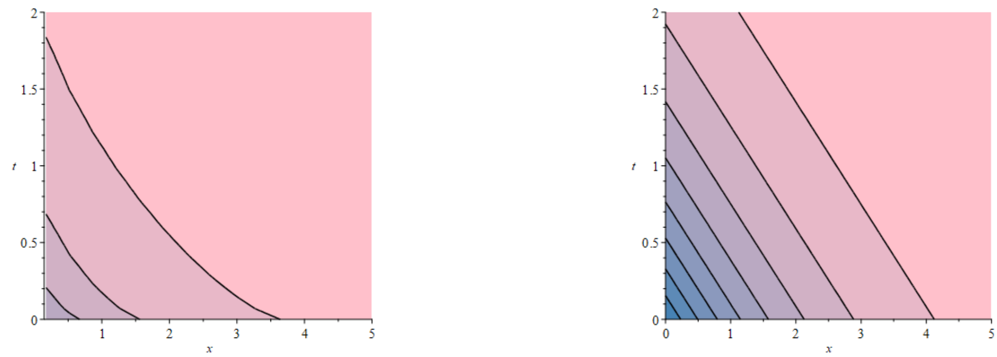
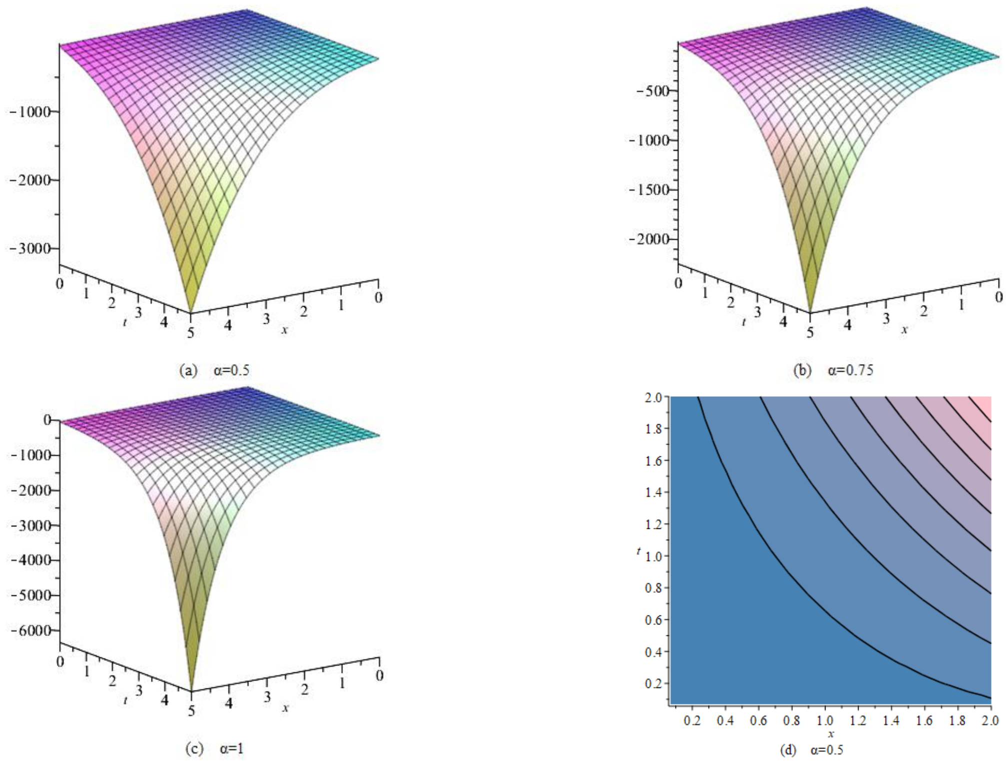


Figure 2. 3D and contour plots of $u_2(x,t)$ for different values of α .

Case 3.

$$\beta=0,\omega=\frac{k_2^2\sigma-k_1^3}{k_1},\sigma=\sigma,,a_{-1}=\frac{b_{-1}(a_0b_0-a_1b_{-1})}{b_0^2},a_0=a_0,a_1=a_1, \tag{14}$$
$$b_1=0,b_{-1}=b_{-1},b_0=b_0.$$

$$u_3(x,t)=\frac{\dfrac{b_{-1}(a_0b_0-a_1b_{-1})}{b_0^2}\exp\left[-\dfrac{k_1x^\alpha}{\alpha}-\dfrac{k_2y^\alpha}{\alpha}+\left(\dfrac{k_2^2\sigma-k_1^3}{k_1}\right)\dfrac{t^\alpha}{\alpha}\right]+a_0+a_1\exp\left[\dfrac{k_1x^\alpha}{\alpha}+\dfrac{k_2y^\alpha}{\alpha}-\left(\dfrac{k_2^2\sigma-k_1^3}{k_1}\right)\dfrac{t^\alpha}{\alpha}\right]}{b_{-1}\exp\left[-\dfrac{k_1x^\alpha}{\alpha}-\dfrac{k_2y^\alpha}{\alpha}+\left(\dfrac{k_2^2\sigma-k_1^3}{k_1}\right)\dfrac{t^\alpha}{\alpha}\right]+b_0}. \tag{15}$$



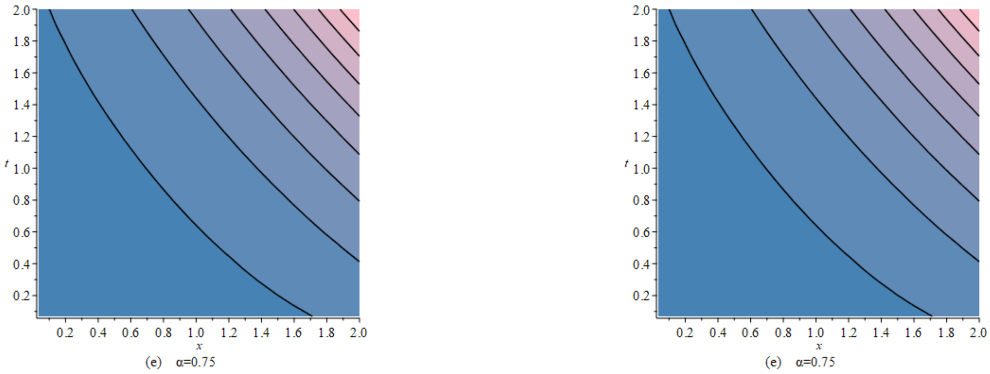
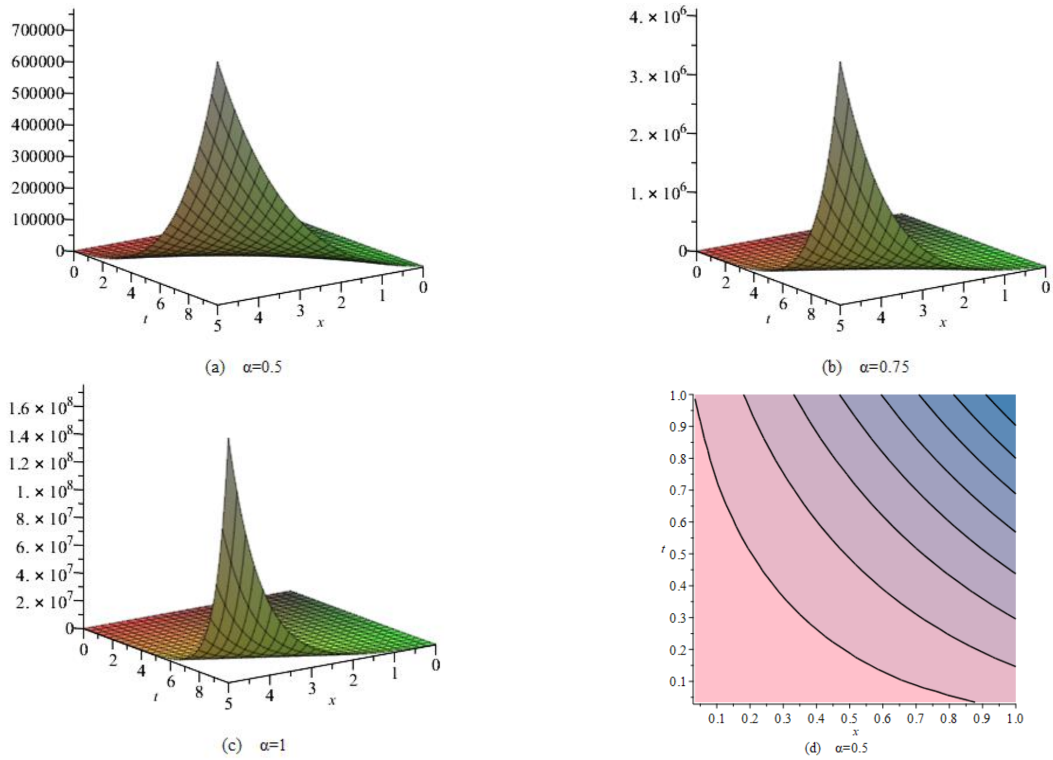


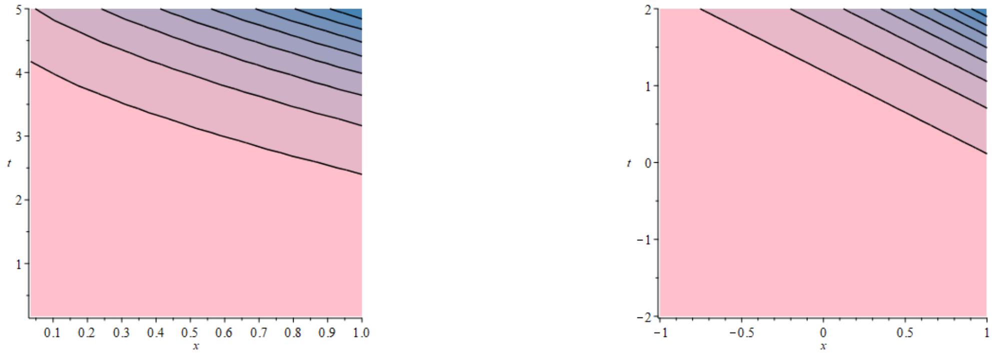
Figure 3. 3D and Contour plots of $u_3(x,t)$ for different values of α .

Case 4.

$$\beta=0,\omega=\frac{k_2^2\sigma+k_1^3}{k_1},\sigma=\sigma,\ a_0=a_0,a_{-1}=a_{-1},a_1=-\frac{b_1\left(a_{-1}b_1-a_0b_0\right)}{b_0^2}, \tag{16}$$
$$b_{-1}=0,b_0=b_0,b_1=b_1.$$

$$u_4(x,t)=\frac{a_{-1}\exp\left[-\frac{k_1x^\alpha}{\alpha}-\frac{k_2y^\alpha}{\alpha}+\left(\frac{k_2^2\sigma+k_1^3}{k_1}\right)\frac{t^\alpha}{\alpha}\right]+a_0-\frac{b_1\left(a_{-1}b_1-a_0b_0\right)}{b_0^2}\exp\left[\frac{k_1x^\alpha}{\alpha}+\frac{k_2y^\alpha}{\alpha}-\left(\frac{k_2^2\sigma+k_1^3}{k_1}\right)\frac{t^\alpha}{\alpha}\right]}{b_1\exp\left[\frac{k_1x^\alpha}{\alpha}+\frac{k_2y^\alpha}{\alpha}-\left(\frac{k_2^2\sigma+k_1^3}{k_1}\right)\frac{t^\alpha}{\alpha}\right]+b_0}. \tag{17}$$





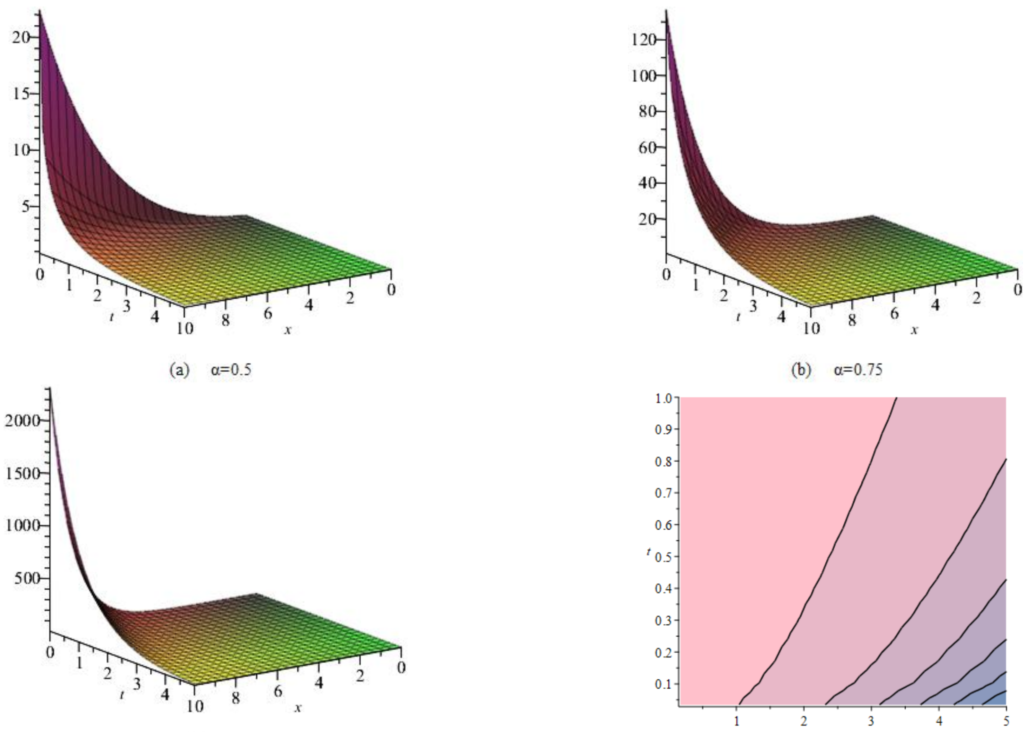
(e) α=0.75 (f) α=1

Figure 4. 3D and contour of $u_4(x,t)$ for different values of α.

Case 5.

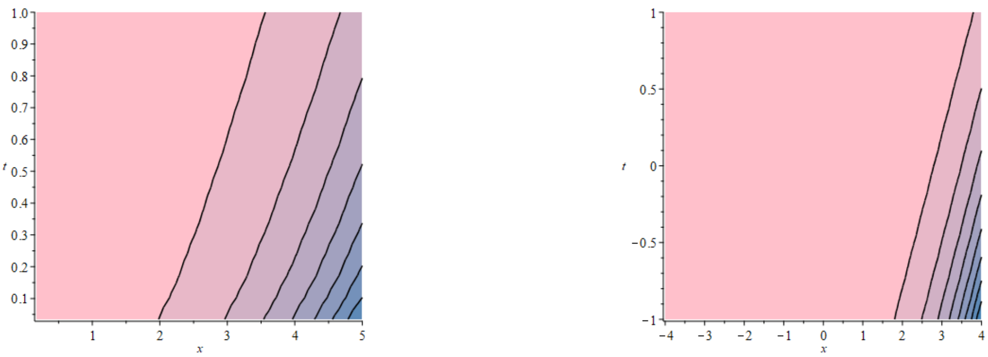
$$\beta=0,\omega=\frac{k_2^2\sigma+2k_1^3}{k_1},\sigma=\sigma,\ a_0=0,a_{-1}=a_{-1},\ a_1=a_1,\\ b_0=0,b_{-1}=0,b_1=b_1. \tag{18}$$

$$u_5(x,t)=\frac{a_{-1}\exp\left[-\frac{k_1x^\alpha}{\alpha}-\frac{k_2y^\alpha}{\alpha}+\left(\frac{k_2^2\sigma+2k_1^3}{k_1}\right)\frac{t^\alpha}{\alpha}\right]+a_1\exp\left[\frac{k_1x^\alpha}{\alpha}+\frac{k_2y^\alpha}{\alpha}-\left(\frac{k_2^2\sigma+2k_1^3}{k_1}\right)\frac{t^\alpha}{\alpha}\right]}{b_1\exp\left[\frac{k_1x^\alpha}{\alpha}+\frac{k_2y^\alpha}{\alpha}-\left(\frac{k_2^2\sigma+2k_1^3}{k_1}\right)\frac{t^\alpha}{\alpha}\right]}. \tag{19}$$



(a) α=0.5 (b) α=0.75

(c) α=1 (d) α=0.5



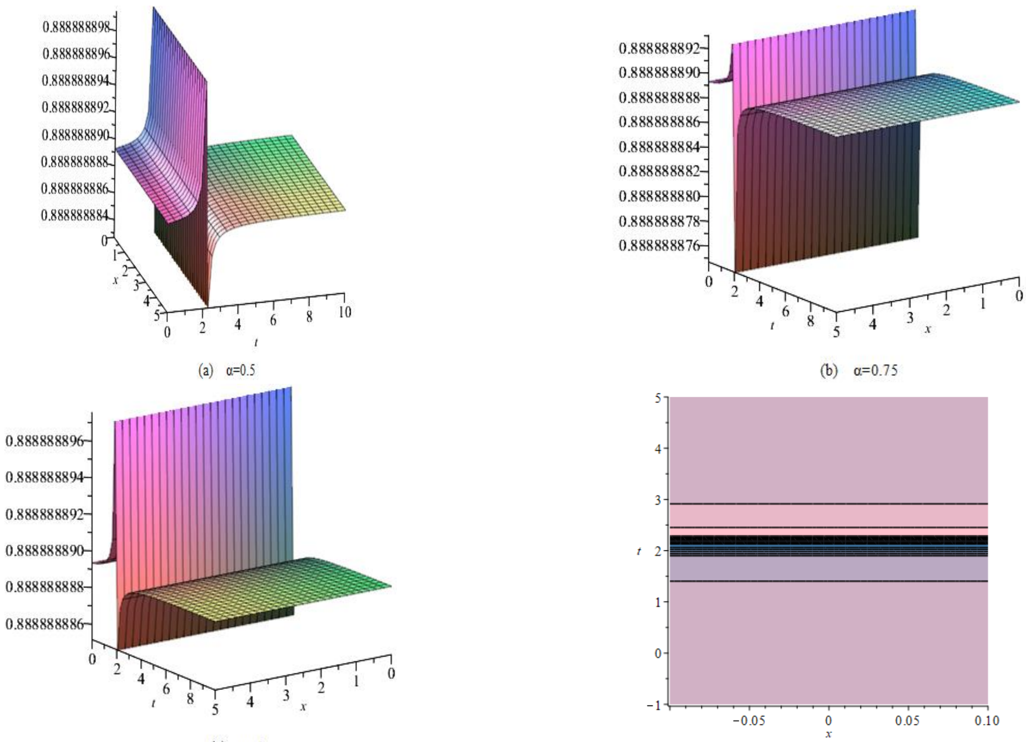
(e) α=0.75 (f) α=1

Figure 5. 3D and contour plots of $u_5(x,t)$ for different values of α.

Case 6.

$$\beta=\beta,\omega=\omega,\sigma=\sigma,\ a_0=a_0,a_{-1}=0,\ a_1=\frac{a_0b_1}{b_0},b_0=b_0,b_{-1}=0,b_1=b_1,k_1=0. \tag{20}$$

$$u_6(x,t)=\frac{a_0+\dfrac{a_0b_1}{b_0}\exp\left[\dfrac{k_2y^\alpha}{\alpha}-\dfrac{\omega t^\alpha}{\alpha}\right]}{b_0+b_1\exp\left[\dfrac{k_2y^\alpha}{\alpha}-\dfrac{\omega t^\alpha}{\alpha}\right]}. \tag{21}$$



(a) α=0.5 (b) α=0.75

(c) α=1 (d) α=0.5

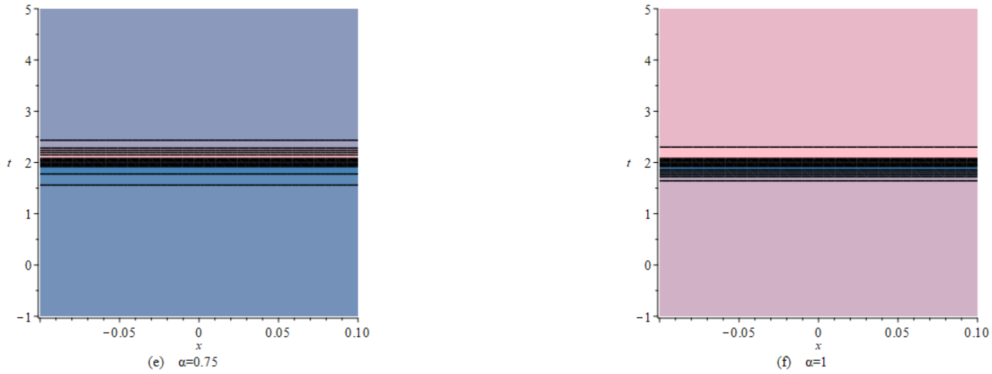
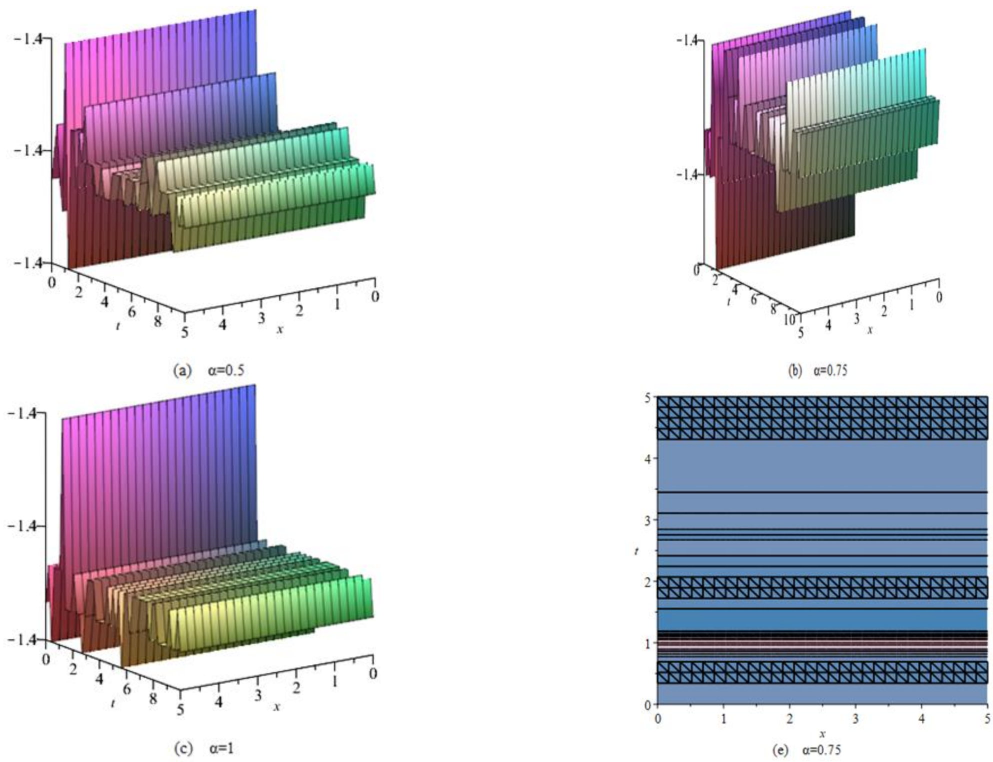


Figure 6. 3D and contour plots of $u_6(x,t)$ for different values of α .

Case 7.

$$\beta=\beta,\omega=\omega,\sigma=\sigma,\ a_0=0,a_{-1}=a_{-1},a_1=\frac{a_{-1}b_1}{b_{-1}},b_0=0,b_{-1}=b_{-1},\\ b_1=b_1,k_1=0. \tag{22}$$

$$u_7(x,t)=\frac{a_{-1}\exp\left[-\frac{k_2y^\alpha}{\alpha}+\frac{\omega t^\alpha}{\alpha}\right]+\frac{a_{-1}b_1}{b_{-1}}\exp\left[\frac{k_2y^\alpha}{\alpha}-\frac{\omega t^\alpha}{\alpha}\right]}{b_{-1}\exp\left[-\frac{k_2y^\alpha}{\alpha}+\frac{\omega t^\alpha}{\alpha}\right]+b_1\exp\left[\frac{k_2y^\alpha}{\alpha}-\frac{\omega t^\alpha}{\alpha}\right]}. \tag{23}$$



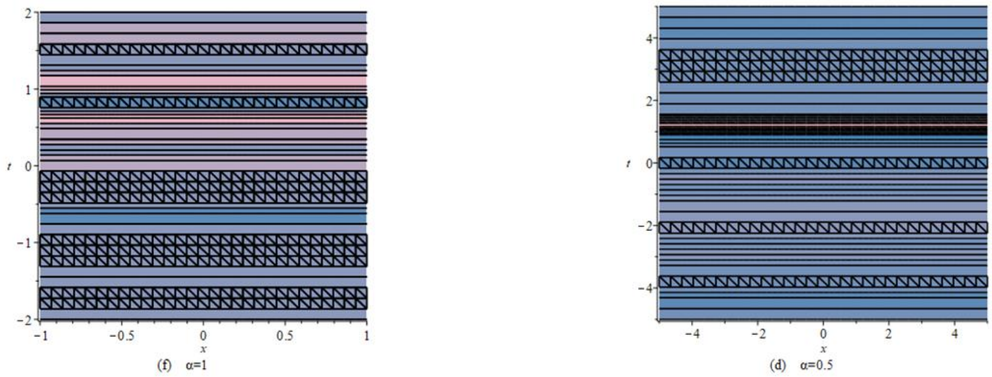
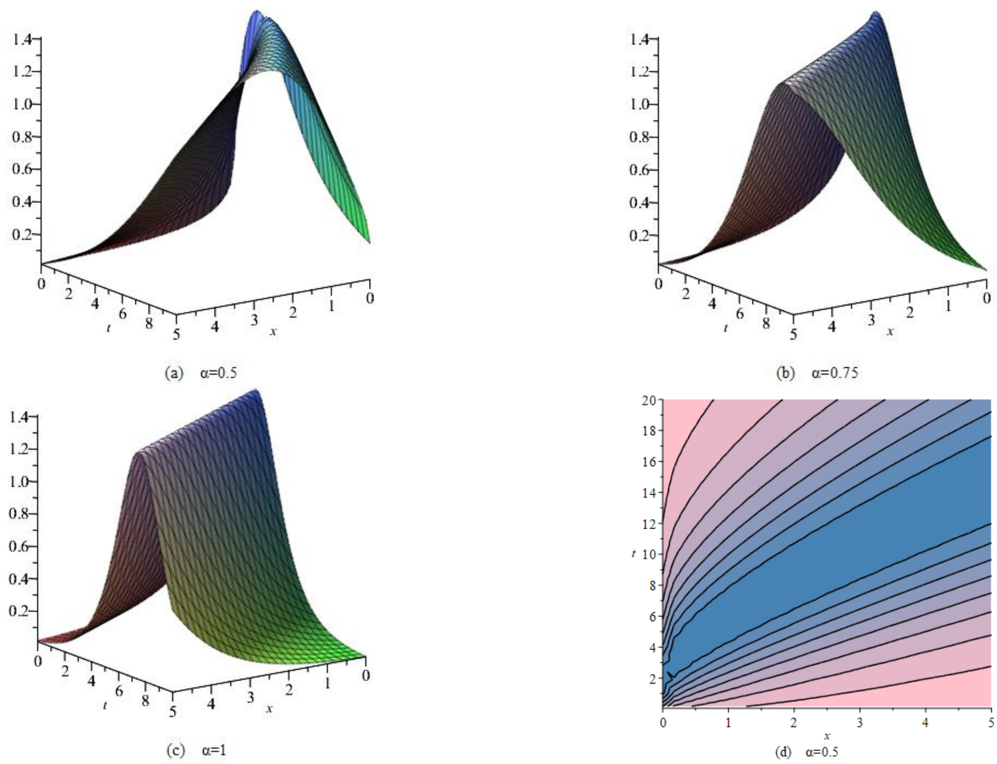


Figure 7. 3D and contour plots of $u_7(x,t)$ for different values of α .

Case 8.

$$\beta=-k_1^2,\omega=\frac{k_2^2\sigma}{k_1},\sigma=\sigma,\ a_{-1}=0,a_0=a_0,a_1=0,$$
$$b_{-1}=\frac{a_0^2}{8b_1},b_0=0,b_1=b_1,k_1=k_1. \tag{24}$$

$$u_8(x,t)=\frac{a_0}{b_{-1}\exp\left[-\frac{k_1x^\alpha}{\alpha}-\frac{k_2y^\alpha}{\alpha}+\left(\frac{k_2^2\sigma}{k_1}\right)\frac{t^\alpha}{\alpha}\right]+b_0+b_1\exp\left[\frac{k_1x^\alpha}{\alpha}+\frac{k_2y^\alpha}{\alpha}-\left(\frac{k_2^2\sigma}{k_1}\right)\frac{\omega t^\alpha}{\alpha}\right]}. \tag{25}$$



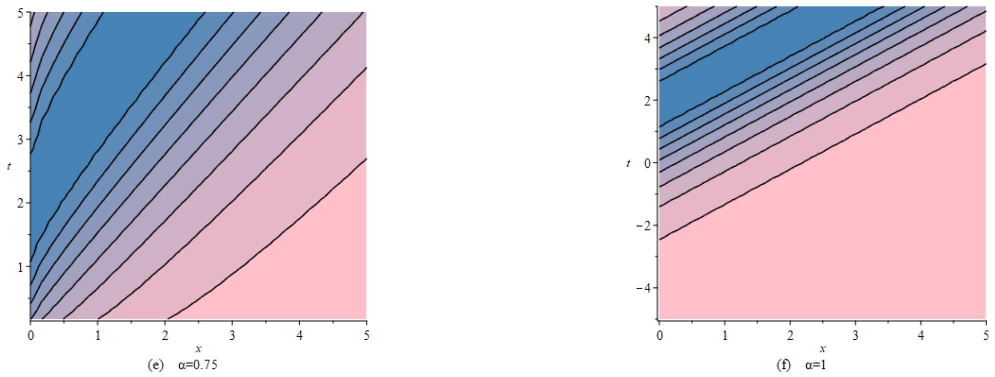


Figure 8. 3D and contour plots of $u_8(x,t)$ for different values of α .

3. RESULTS AND DISCUSSION

The fractional (2+1)-dimensional CI equation is investigated, yielding noteworthy solitary-wave solutions. The Exp-function approach is employed by using the fractional wave transformation to get more generalized solitary wave solutions. One of its major advantages is its ability to produce a wide variety of exact solutions, including soliton, periodic, and singular wave structures, directly and systematically. The method is algebraically straightforward and does not require linearization, perturbation assumptions, or discretization procedures. In the literature, various authors have explored different approaches to solving the (2+1)-dimensional CI equation and its variants. Specifically, the study by Cimpoiasu [36] introduced topological solutions, singular solitons, singular periodic waves, and solutions using the exponential function method. Sulaiman and colleagues focused on the variable-coefficient version of this equation and reported lump solutions [37]. Mao [38] derived exact solutions in terms of elliptic functions. Additionally, Sakthivel and Chun [39] employed the proposed technique to obtain travelling-wave solutions to the equation. Demiray and Bayrakci [40] used the extended tanh-coth approach to derive topological and solitary soliton solutions, while the sine-cosine function method was used to uncover singular periodic wave solutions [41]. This research extends the investigation of the problem in fractional order to include solitary-wave solutions and other new solutions by varying the parameters. Furthermore, substitution of the derived solutions into the original equation verifies that they satisfy the governing equation.

Different kinds of wave structures have been obtained by using different values of parameters, offered in the form of 3D-plots and contour plots as specified in Figures (1-8) at $\alpha = 0.5,\ \alpha = 0.7$ and $\alpha = 1$. Figure 1 indicates the consequence of $u_1(x,t)$ for $a_1 = 1.5, b_1 = \frac{1}{4}, b_0 = \frac{1}{3}, b_{-1} = -\frac{1}{2}, k_1 = 0.75, k_2 = 0.5,\ \sigma = 0.5.$ Figure 2 directs the solution of $u_2(x,t)$ at $a_0 = \frac{2}{3}, a_{-1} = 1.5, b_{-1} = \frac{1}{2},, k_1 = -0.5. k_2 = 0.5,\ \sigma = -2.$ Figure 3 specifies the result of $u_3(x,t)$ for $a_0 = \frac{2}{3}, a_1 = 0.1, b_0 = -1.75, b_{-1} = 0.1, k_1 = 1.2, k_2 = 0.5,\ \sigma = 2.$ Figure 4 specifies the result of $u_4(x,t)$for $a_0 = \frac{2}{3}, a_{-1} = \frac{1}{2}, b_0 = 0.1, b_1 = \frac{1}{2}, k_1 = -1.25, k_2 = 0.5,\ \sigma = 2.$ Figure 5 shows the outcome of $u_5(x,t)$ for $a_1 = 1.5, a_{-1} = \frac{1}{2}, b_1 = 1.75,, k_1 = -0.5, k_2 = 0.5,\ \sigma = 2.$ Figure 6 indicates the outcome of $u_6(x,t)$ for $a_0 = \frac{2}{3}, b_0 = 0.75, b_1 = -1.2,\ k_2 = 0.5,\ \sigma = 2.$ Figure 7 specifies the outcome of $u_7(x,t)$ for $a_{-1} = 1.75, b_1 = 1.5, b_{-1} = -1.25, k_2 = 0.5, \sigma = 2.$ Figure 8 directs the result of $u_8(x,t)$ for $a_0 = \frac{2}{3}, b_1 = \frac{1}{2},, k_1 = 0.75., k_2 = 0.5, \sigma = 2.$

4. CONCLUSIONS

The primary goal of this study was to apply the Exp-function method to the fractional (2+1)-dimensional Chaffee-Infante (CI) equation in order to derive analytical solutions and examine its dynamic behavior. By using transformations such as the fractional wave and fractional conformable transformations, we effectively converted the fractional partial differential equations into solvable nonlinear ordinary differential equations. This approach enabled us to obtain various analytical solutions, including solitary-wave solutions characterized by their localized and oscillatory nature in both space and time. These solutions provide valuable insights into complex wave phenomena within fractional systems. Our findings enhance the understanding of the fractional CI equation's behavior, revealing its solutions and underlying symmetries in detail. Overall, the application of the Exp-function method proves to be a powerful and effective tool for analyzing fractional differential equations, uncovering intricate dynamics and conservation properties that deepen our comprehension of nonlinear wave systems. The model may also be generalized to coupled nonlinear systems arising in combustion, plasma physics, and optical wave propagation.

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