ORIGINAL PAPER

ON G-J_S OPEN SETS IN GRILL TOPOLOGICAL SPACES

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Abstract. *This research paper aims to define and analyze a new class of sets, named* $G - J_S$ *Open sets with a grill set* G *of* X *in a grill topological space* (X, τ, G). *Also, we track down the characteristics and some of the properties work in the above-mentioned set.*
Keywords: *Grill;* J_S *open sets;* $G - J_S$ *open sets; generalized closed set.*

1. INTRODUCTION

Grill topological spaces have applications in various areas of mathematics, including *algebraic topology*, representation theory, and dynamical systems. They allow us to study the interplay between topological structures and group actions, shedding light on the symmetries and geometric properties of the underlying space. Grills provide a way to study the "nice" subsets of the space that are preserved by the group action. Levine [1], on the other hand, a more recent effort made with the same motivation by Hatir and Jafari [2] has led to the introduction and investigation of φ – open sets, for a suitable operator φ. The operator φ: $P(X) \rightarrow P(X)$ was first defined in terms of grill. Grill set theory was initially introduced by Choquet [3]. In this paper we defined and studied a different kind of generalized closed sets, the definition being formulated in terms of grills.

Throughout the entirety of the paper, the term "Top. Sps" refers exclusively to a Topological Space (X, τ) for which no separation properties are assumed. On the off chance that $M \subseteq X$, we will embrace the typical notations *int*(*M*) and *cl*(*M*) separately for the interior and closure of M in (X, τ). Again $\tau_G - cl(M)$ and $\tau_G - int(M)$ will separately mean the closure and interior of M in (X, τ_G).

2. PRELIMINARIES

Definition 2.1. [4] In a Top. Sps (X, τ), G be a $grill$ set on X.

(i) The mapping $\varphi: P(X) \rightarrow P(X)$ denoted by $\varphi(J_1)$ defined as $\varphi_G(J_1)= \varphi(J_1)= \{x \in X: J_1 \cap N \in G, \forall N \in \tau(x)\}$.

(ii) The map $\psi: P(X) \rightarrow P(X)$ as $\psi(J_1) = J_1 \cup \varphi(J_1)$, for all $J_1 \in P(X)$.

Definition 2.2. [5] In a Top. Sps (X, τ), a subset J_1 of X is called as

(1) a $semi\ closed$ set if $int\ cl(J_1) \subseteq J_1$

(2) a $generalized\ closed$ ($g\ closed$) set if $cl(J_1) \subseteq K$ whenever $J_1 \subseteq K$ and K is $open$ in X.

(3) a $generalized\ semi\ closed$ ($gs\ closed$) set if $scl\ J_1 \subseteq K$ whenever $J_1 \subseteq K$

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and K is $open$ in X.

The complements of aforementioned $closed$ sets are respective $open$ sets.

Proposition 2.3. [6] In a Top. Sps (X,τ), G be a $grill$ set on X. Then for the subsets J_1 and J_2 of X, the following hold:

i. $J_1 \subseteq J_2$, $\varphi(J_1) \subseteq \varphi(J_2)$

ii. $\varphi(J_1 U J_2) = \varphi(J_1)\ U\ \varphi(J_2)$

iii. $\varphi(\varphi(J_1)) \subseteq \varphi(J_1) \subseteq Cl(\varphi(J_1)) \subseteq Cl(J_1)$

Definition 2.4. [7] In a Top. Sps (X,τ), G be a $grill$ set on X. Then the subset J_1 of Top. Sps (X,τ), called as $G\ g-closed$ (i.e. $generalized\ closed\ set\ in\ grill\ topological\ space$) if $\varphi(J_1) \subseteq U$ whenever $J_1 \subseteq U$ and U is $open$ in X. The complement of $G\ g-closed$ set is called $G\ g-open$ set.

Definition 2.5. [8] In a Top. Sps (X,τ), G be a $grill$ set on X. Then the subset J_1 of Top. Sps (X,τ), called as $(gs)*\ closed\ with\ respect\ to\ grill\ G$ (i.e. $G(gs)*\ Closed\ Set$) if $\varphi(J_1) \subseteq U$ whenever $J_1 \subseteq U$ and U is $gs-open$ in X. The complement of $G(gs)*\ Closed\ Set$ is called $G(gs)*-open$ set.

3. TOPOLOGY INDUCED BY A *G - GENERALIZED OPEN* SET

In a Top. Sps (X,τ), G be a $grill$ set on X. Then the mapping $\varphi^*: P(X) \to P(X)$ denoted by $\varphi^*(Q_1)$ is defined as $\varphi^*_G(Q_1)$= $\varphi^*(Q_1)$= $\{x \in X: Q_1 \cap M \in G,\ \forall\ M \in G-go\ (x)\}$.

Remark 3.1.

(i) If Q_1 and Q_2 are two subsets of X such that $Q_1 \subseteq Q_2$ then $\varphi^*(Q_1) \subseteq \varphi^*(Q_2)$

(ii) If G_1 and G_2 are two $grills$ on X with $G_1 \subseteq G_2$ then
$\varphi^*_{G_1}(Q_1)$= $\{x \in X: Q_1 \cap M \in G_1, \forall M \in G-go\ (x)\}$
$\subseteq \{x \in X: Q_1 \cap M \in G_2, \forall M \in G-go\ (x)\}$ = $\varphi^*_{G_2}(Q_1)$

(iii) For any $Grill\ G$ on X and $Q_1 \subseteq X$ if $Q_1 \notin G$ then $\varphi^*_G(Q_1) = \emptyset$

Proposition 3.2. In a Top. Sps (X,τ), G be a $grill$ set on X. Then for all $Q_1, Q_2 \subseteq X$

(i) $\varphi^*(Q_1 \cup Q_2) = \varphi^*(Q_1) \cup \varphi^*(Q_2)$

(ii) $\varphi^*(\varphi^*(Q_1)) \subseteq \varphi^*(Q_1) = G-go-cl(\varphi^*(Q_1)) \subseteq G-go-cl(Q_1)$

Proof:

(i). by (i) of Remark 3.1, $Q_1 \subseteq Q_1 \cup Q_2$ and $Q_2 \subseteq Q_1 \cup Q_2$ implies $\varphi^*(Q_1) \subseteq \varphi^*(Q_1 \cup Q_2)$ and $\varphi^*(Q_2) \subseteq \varphi^*(Q_1 \cup Q_2)$. Thus $\varphi^*(Q_1) \cup \varphi^*(Q_2) \subseteq \varphi^*(Q_1 \cup Q_2)$. Enough to prove, $\varphi^*(Q_1 \cup Q_2) \subseteq \varphi^*(Q_1) \cup \varphi^*(Q_2)$. Let $x \notin \varphi^*(Q_1) \cup \varphi^*(Q_2)$. Then there exists $U, V \in G-go\ (x)$ such that $Q_1 \cap U \notin G$ and $Q_2 \cap V \notin G$. This implies $(Q_1 \cap U) \cup (Q_2 \cap V) \notin G$. Further $U \cap V \in G-go\ (x)$. And $(Q_1 \cup Q_2) \cap (U \cap V) \subseteq (Q_1 \cap U) \cup (Q_2 \cap V) \notin G$. This proves $x \notin \varphi^*(Q_1 \cup Q_2)$. Therefore $\varphi^*(Q_1 \cup Q_2) \subseteq \varphi^*(Q_1) \cup \varphi^*(Q_2)$. Hence, $\varphi^*(Q_1 \cup Q_2) = \varphi^*(Q_1) \cup \varphi^*(Q_2)$.

(ii). Let $x \notin G-go-cl(Q_1)$ implies there exists a $open$ set $U \in G-go\ (x)$ such that $U \cap Q_1 = \emptyset \notin G \Rightarrow x \notin \varphi^*(Q_1)$. Thus $\varphi^*(Q_1) \subseteq G-go-cl(Q_1)$. And $\varphi^*(Q_1) \subseteq G-go-cl(\varphi^*(Q_1))$. To show that $G-go-cl(\varphi^*(Q_1)) \subseteq \varphi^*(Q_1)$. Let $x \in G-go-cl(\varphi^*(Q_1))$ and $U \in G-go\ (x)$ implies $U \cap \varphi^*(Q_1) \neq \emptyset$. Let $y \in U \cap \varphi^*(Q_1)$. Then $y \in U$ and $y \in \varphi^*(Q_1) \Rightarrow U \cap Q_1 \in G$. Therefore, $x \in \varphi^*(Q_1)$. Hence, $G-go-$

$cl(\varphi^*(Q_1)) = \varphi^*(Q_1)$. Further, $\varphi^*(\varphi^*(Q_1)) \subseteq G - go - cl(\varphi^*(Q_1))$ and $G - go - cl(\varphi^*(Q_1)) = \varphi^*(Q_1)$ and $\varphi^*(Q_1) \subseteq G - go - cl(Q_1)$ implies $\varphi^*(\varphi^*(Q_1)) \subseteq \varphi^*(Q_1) = G - go - cl(\varphi^*(Q_1)) \subseteq G - go - cl(Q_1)$.

Definition 3.3. Let G be a $grill$ in a Top. Sps (X,τ). We define a map $\psi^*: P(X) \to P(X)$ as $\psi^*(D_1) = D_1 \cup \varphi^*(D_1)$, for all $D_1 \in P(X)$.

Theorem 3.4. The function $\psi^*(D_1)$ satisfies $Kuratowski's\ closure$ axioms.

Proof: By (iii) of Remark 3.1, we have $\varphi^*(\emptyset)=\emptyset$ and $\psi^*(\emptyset) = \emptyset \cup \varphi^*(\emptyset)= \emptyset$. $D_1 \subseteq \psi^*(D_1)$ for all $D_1 \subseteq X$. $\psi^*(D_1 \cup D) = (D_1 \cup D) \cup \varphi^*(D_1 \cup D)= D_1 \cup D \cup \varphi^*(D_1) \cup \varphi^*(D) = (D_1 \cup \varphi^*(D)) \cup (D \cup \varphi^*(D_1)) = \psi^*(D_1) \cup \psi^*(D)$. For any $D_1 \subseteq X$, $\psi^*(\psi^*(D_1))=\psi^*(D_1 \cup \varphi^*(D_1)) = (D_1 \cup \varphi^*(D_1)) \cup \varphi^*(D_1 \cup \varphi^*(D_1)) = D_1 \cup \varphi^*(D_1) \cup \varphi^*(D_1) \cup \varphi^*(\varphi^*(D_1))$. Since $\varphi^*(\varphi^*(D_1)) \subseteq \varphi^*(D_1)$, $\psi^*(\psi^*(D_1)) = D_1 \cup \varphi^*(D_1) = \psi^*(D_1)$.

Definition 3.5. Corresponding to a $grill\ G$ in the Top. Sps (X,τ) we define a topology τ_G^* on X as $\tau_G^* = \{K \subseteq X: \psi^*(X-K) = X-K\}$ where $K \subseteq X, \psi^*(K) = K \cup \varphi^*(K) = \tau_G^* - cl(K)$.

(i) $\emptyset \subseteq X, \psi^*(X) = X \Rightarrow \psi^*(X-\emptyset) = X - \emptyset \Rightarrow \emptyset \in \tau_G^*$ and

(ii) $X \subseteq X, \psi^*(\emptyset) = \emptyset \Rightarrow \psi^*(X-X) = X-X \Rightarrow X \in \tau_G^*$

(iii) Let $\{U_i\}_{i\in I} \in \tau_G^*$ then $\psi^*(X-U_i) = X - U_i\ \forall\ i$ which implies $(X-U_i) \cup \varphi^*(X-U_i) = X - U_i\ \forall\ i$. Thus $\varphi^*(X-U_i) \subseteq X - U_i\ \forall\ i$. Now, $\varphi^*(\cap (X-U_i)) \subseteq \cap (X-U_i)\ \forall\ i$. $\psi^*(\cap (X-U_i)) = (\cap (X-U_i)) \cup \varphi^*(\cap (X-U_i)) \Rightarrow \psi^*(\cap (X-U_i)) = (\cap (X-U_i)) \Rightarrow \psi^*(X - \cup U_i) = (X - \cup U_i) \Rightarrow \cup U_i \in \tau_G^*$

(iv) Let $U_1, U_2 \in \tau_G^*$ then $\psi^*(X-U_1) = (X-U_1)$ and $\psi^*(X-U_2) = (X-U_2)$. Now $\psi^*(X-(U_1 \cap U_2)) = \psi^*(X-U_1) \cup \psi^*(X-U_2) = (X-U_1) \cup (X-U_2) = X-(U_1 \cap U_2)$. Hence, $U_1 \cap U_2 \in \tau_G^*$.

Therefore, τ_G^* forms a topology.

Theorem 3.6.

(i) If P and Q are two $grills$ in a Top. Sps (X,τ) with $P \subseteq Q$ then $\tau_P^* \subseteq \tau_Q^*$

(ii) If G is a $grill$ in a Top. Sps (X,τ) and $E \notin G$, then E is a $\tau_G^* - closed$ in (X,τ_G^*).

(iii) In a Top. Sps (X,τ) and a $grill\ G$ on X,
$\varphi^*(S)\ is\ \tau_G^* - closed\ for\ any\ subset\ S\ of\ X.$

Proof:

(i) Let $S_1 \in \tau_Q^*$ then $\psi_Q^*(X-S_1) = (X-S_1) \cup \varphi_Q^*(X-S_1) \Rightarrow X - S_1 = (X-S_1) \cup \varphi_Q^*(X-S_1)$. Thus, $\varphi_Q^*(X-S_1) \subseteq X - S_1 \Rightarrow \varphi_P^*(X-S_1) \subseteq X - S_1 \Rightarrow X - S_1 = (X-S_1) \cup \varphi_P^*(X-S_1) \Rightarrow \psi_P^*(X-S_1) = (X-S_1) \cup \varphi_P^*(X-S_1)$. Therefore, $S_1 \in \tau_P^*$. Thus, $\tau_Q^* \subseteq \tau_P^*$.

(ii) If $S_2 \notin G \Rightarrow \varphi^*(S_2) = \emptyset$ then $\tau_G^* - cl(S_2) = \psi^*(S_2) = S_2 \cup \varphi^*(S_2) = S_2$. Therefore, S_2 is $\tau_G^* - closed$.

(iii) $\psi^*(\varphi^*(S_3)) = \varphi^*(S_3) \cup \varphi^*(\varphi^*(S_3)) = \varphi^*(S_3) \Rightarrow \varphi^*(S_3)$ is $\tau_G^* - closed$.

Theorem 3.7.
Let G be a $grill$ in a Top. Sps (X,τ). If $E_P \in G - go\ (X)$ then $E_P \cap \varphi^*(P) = E_P \cap \varphi^*(E_P \cap P)$, for any $P \subseteq X$.

Proof: We have $E_P \cap \varphi^*(P) \supseteq E_P \cap \varphi^*(E_P \cap P)$. Let $x \in E_P \cap \varphi^*(P)$ and $V \in G - go(x)$. Then $E_P \cap V \in G - go(x)$ and $x \in \varphi^*(P) \Rightarrow (E_P \cap V) \cap P \in G$. Thus $(E_P \cap P) \cap V \in$ G. $\Rightarrow x \in \varphi^*(E_P \cap P) \Rightarrow x \in E_P \cap \varphi^*(E_P \cap P)$. Therefore, $E_P \cap \varphi^*(P) = E_P \cap \varphi^*(E_P \cap P)$.

Theorem 3.8.
If G is a $grill$ in a Top. Sps (X, τ) with $G - go\ (X) - \emptyset \subseteq G$, then for all $N_1 \in G - go\ (X)$, $N_1 \subseteq \varphi^*(N_1)$.

Proof: If $N_1 = \emptyset$ then $\varphi^*(N_1) = \emptyset = N_1$. If $G - go\ (X) - \emptyset \subseteq G$, then $\varphi^*(X)$=X. If $x \in \varphi^*(X) \Rightarrow \exists\ V \in G - go\ (X)$ such that $\cap X \notin G \Rightarrow V \notin G$, a contradiction. By using Theorem 3.7 we have for any $N_1 \in G - go\ (X) - \emptyset$, $N_1 \cap \varphi^*(X) = N_1 \cap \varphi^*(N_1 \cap X)$ and hence N_1= $N_1 \cap X$ = $N_1 \cap \varphi^*(X)$. Thus $N_1 \subseteq \varphi^*(N_1)$.

4. *G-J_S OPEN SETS* WITH RESPECT TO A GRILL

Definition 4.1. Suppose (X, τ) be a Top. Sps. Then Q a subset of (X, τ) is defined as J_s $closed$ set if $cl(Q) \subseteq U$ whenever $Q \subseteq U$ and U is $(gs) *\ open$ in X.

Definition 4.2. Suppose in a Top. Sps (X, τ), G be a $grill$ set on X. Then M a subset of (X, τ) is defined as J_s $closed$ set $with\ respect\ to\ a\ grill$ induced by a $generalized\ open$ set (G-$J_s{}^*$ $closed$) if $\varphi^*(M) \subseteq U$ whenever $M \subseteq U$ and U *is (gs)* open* in X.
The complement of G-$J_s{}^*$ $closed$ set *in X is G-$J_s{}^*$ open* set.

Definition 4.3. Suppose in a Top. Sps (X, τ), G be a $grill$ set on X. Then L a subset of (X, τ) is defined as J_s $closed$ set $with\ respect\ to$ a $grill$ (G-J_s $closed$) if $\varphi(L) \subseteq U$ whenever $L \subseteq U$ *and U is (gs)* open* in X.
The complement of G-J_s $closed$ set in X is G-J_s $open$ set.

Theorem 4.4. Every G-J_s $closed$ set is G-$J_s{}^*$ $closed\ set$.

Proof: Let D be a G-J_s $closed$ set and $D \subseteq U$ where U is (gs)* $open$ set.
Then $\varphi(D) \subseteq U$.
Let $x \notin \varphi(D)$ then there exists a $open$ set V of x such that $D \cap V \notin G$. But every $open$ set is a $generalized\ open$ set. This implies there exists a $generalized\ open$ set V of x such that $D \cap V \notin G \Rightarrow x \notin \varphi^*(D)$. Thus $\varphi^*(D) \subseteq \varphi(D)$. Therefore, $\varphi^*(D) \subseteq \varphi(D) \subseteq U \Rightarrow \varphi^*(D) \subseteq U$. Hence D is a G-$J_s{}^*$ $closed$ set.

Theorem 4.5. Suppose in a Top. Sps (X, τ), G be a $grill$ set on X. Then,
- (i) Every $closed$ set in X is G-J_s $closed$ set.
- (ii) Q is G-J_s $closed$ set then $\varphi(Q)$ is G-J_s $closed$ set.
- (iii) Every τ_G $closed$ set is a G-J_s $closed$ set.
- (iv) Any non-member of G is G-J_s $closed$ set.
- (v) Any J_s $closed$ set is G-J_s $closed$ set.

Proof:
- (i) Let Q be a $closed$ set then $\varphi(Q) = Q$. Let $Q \subseteq U$ where U is (gs)* $open$ set $\Rightarrow \varphi(Q) \subseteq U$ where U is (gs)* $open$ set. Thus Q is a G-J_s $closed$ set.
- (ii) Let Q be a G-J_s $closed$ set. Then $\varphi(Q) \subseteq U$ where U is (gs)* $open$ set. $\varphi\big(\varphi(Q)\big) \subseteq \varphi(Q) \subseteq U$ where U is (gs)* $open$ set. Thus $\varphi\big(\varphi(Q)\big) \subseteq U$ where U is (gs)* $open$ set. Therefore, $\varphi(Q)$ is a G-J_s $closed$ set.

(iii) Let M be a τ_G $closed$ set. Then $\varphi(M) \subseteq M$. Let $M \subseteq U$ where U is (gs)* $open$ set $\Rightarrow$ $\varphi(M) \subseteq U$ where U is (gs)* $open$ set. Thus M is a G-J_s $closed$ set.

(iv) Let L be a non-member of G. Then $\varphi(L)$=$\emptyset$. Let $L \subseteq U$ where U is (gs)* $open$ set $\Rightarrow \emptyset = \varphi(L) \subseteq U$ where U is (gs)* $open$ set. Thus L is a G-J_s $closed$ set.

(v) Let Q be a J_s $closed$ set. Then $cl\ (Q) \subseteq U$ where U is (gs)* $open$ set. And $\varphi(Q) \subseteq$ cl $(Q) \Rightarrow \varphi(Q) \subseteq U$ where U is (gs)* $open$ set. Thus Q is a G-J_s $closed$ set.

Example 4.6. The converse of the Theorem 4.5 need not be true. It follows from the following observations

(i) Let $U = \{p, q, r\}$.

$\tau = \{\emptyset, \{p\}, \{p, q\}, X\}$

$G = \{\{p\}, \{p, q\}, \{p, r\}, X\}$

Here (U, τ) is the grill Top. Sps induced by the $grill\ G$.

Here $L = \{p, r\}$ is $G - J_s$ $closed$ but not $closed$.

$M = \{p, r\}$ is $G - J_s$ $closed$ but also a member of $Grill\ G$.

$N = \{q\}$ is $G - J_s$ $closed$ but not J_s $closed$.

(ii) Let $U = \{p, q, r\}$.

$\tau = \{\emptyset, \{q, r\}, X\}$

$G = \{\{p\}, \{q\}, \{r\}, \{p, q\}, \{p, r\}, \{q, r\}, X\}$

Here (U, τ) is the grill Top. Sps induced by the $grill\ G$.

For $H = \{r\}$

$\varphi(H) = X$ is $G - J_s$ $closed$ but H $= not$ $G - J_s$ $closed$.

Here $J = \{p, r\}$ is $G - J_s$ $closed$ but not τ_G $closed$.

Theorem 4.7. Suppose in a Top. Sps (X, τ), G be a $grill$ set on X. Then for $Q \subseteq X$, Q is $G - J_s$ $closed$ iff τ_G – $cl\ (Q) \subseteq U, Q \subseteq U$ and U is (gs)* $open$.

Proof: Suppose Q is $G - J_s$ $closed$. Then $\varphi(Q) \subseteq U$ where U is (gs)* $open$ set $\Rightarrow Q \cup \varphi(Q) \subseteq U$. Therefore, τ_G– $cl\ (Q) \subseteq U, Q \subseteq U$ and U is (gs)* $open$.

Conversely, τ_G– $cl\ (Q) \subseteq U, Q \subseteq U$ and U is (gs)* $open$.

Therefore $A \cup \varphi(Q) \subseteq$ U $\Rightarrow \varphi(Q) \subseteq U$. Hence Q is $G - J_s$ $closed$.

Theorem 4.8. Suppose in a Top. Sps (X, τ), G be a $grill$ set on X. If P is $\tau_G - dense$ in itself and $G - J_s$ $closed$ implies P is J_s $closed$.

Proof: Let P be $\tau_G -$ $dense$ in itself, then $\varphi\ (P) = cl(P)$.

Since P is $G - J_s$ $closed$, $\varphi\ (P) \subseteq U$ where U is (gs)* $open$ in X and $P \subseteq U$.

Therefore $cl(P) \subseteq U$ where U is (gs)* $open$ in X and $P \subseteq U$. Hence P is J_s $closed$.

Theorem 4.9. Suppose in a Top. Sps (X, τ), G be a $grill$ set on X then the following are equivalent.

(a) If $Q \subseteq X$ then Q is a $G - J_s$ $closed$ set.

(b) If Q is (gs)* $open$ subset of (X, τ) then Q is a τ_G – $closed$ set.

Proof: $(a) \Rightarrow (b)$

Let Q be (gs)* $open$ in (X, τ). Then by (a), Q is $G - J_s$ $closed$, so that $\varphi\ (Q) \subseteq Q$. Therefore, Q is $\tau_G - closed$.

$(b) \Rightarrow (a)$

Let $Q \subseteq X$ and U be (gs)* $open$ in (X, τ) such that $Q \subseteq U$. Then (b), $\varphi\ (U) \subseteq U$.

Also, $Q \subseteq U \Rightarrow \varphi\ (Q) \subseteq \varphi\ (U) \subseteq U$. Therefore, Q is $G - J_s$ $closed$.

Theorem 4.10. Suppose in a Top. Sps (X, τ), G be a $grill$ set on X and S, D be subsets of X such that $S \subseteq D \subseteq \tau_G - cl\ (S)$. If S is $G - J_s\ closed$ then D is $G - J_s\ closed$.

Proof: Suppose $D \subseteq U$ and U is (gs)* $open$ in X. Since S is $G - J_s\ closed$.
$\varphi\ (S) \subseteq U \Rightarrow \tau_G - cl\ (S) \subseteq U$ …. (1)

Now, $S \subseteq D \subseteq \tau_G - cl\ (S)$ which implies $\tau_G - cl(S) \subseteq \tau_G - cl\ (D) \subseteq \tau_G - cl(S)$.
Therefore $\tau_G - cl(S) = \tau_G - cl(D)$
Therefore by (1) $\tau_G - cl(D) \subseteq U$. Hence D is $G - J_s\ closed$.

Corollary 4.11. $\tau_G - closure$ of every $G - J_s\ closed$ set is $G - J_s\ closed$.

Theorem 4.12. Suppose in a Top. Sps (X, τ), G be a $grill$ set on X and L, M are two subsets of X satisfying $L \subseteq M \subseteq \varphi\ (L)$. If L is $G - J_s\ closed$ then L and M are (gs)* $closed$.

Proof: Let $L \subseteq M \subseteq \varphi\ (L)$, then $L \subseteq M \subseteq \tau_G - cl(L)$. By Theorem 4.10, M is $G - J_s\ closed$. Again $L \subseteq M \subseteq \varphi\ (L)$
$\Rightarrow \varphi\ (L) \subseteq \varphi\ (M) \subseteq \varphi\ (\varphi\ (L)) \subseteq \varphi\ (L)$.

This implies that $\varphi\ (L) = \varphi\ (M)$. Thus, L and M are (gs)* $closed$.

Theorem 4.13. Suppose in a Top. Sps (X, τ), G be a $grill$ set on X. Then a subset M of X is $G - J_s\ open$ iff $F \subseteq \tau_G - int\ (M)$ whenever $F \subseteq M$ and F is (gs)* $closed$.

Proof: Let M be $G - J_s\ open$ set and $F \subseteq M$ where F is $(gs)^*\ closed$. Then $X \setminus M \subseteq X \setminus F$. Thus we get $\varphi\ (X \setminus M) \subseteq \varphi\ (X \setminus F) = X \setminus F$. Therefore $\tau_G - cl(X \setminus M) \subseteq X \setminus F$. It follows that $F \subseteq \tau_G - int(M)$.

Conversely, $\subseteq \tau_G - (int\ (M))$, $\tau_G - cl\ (X \setminus M \subseteq X \setminus F$, $\varphi\ (X \setminus M) \subseteq X \setminus F$, M is $G - J_s\ open$.

5. CONCLUSION

In this paper we defined and analysed the $G - J_S\ Open\ sets$ with a grill set G of X in a grill topological space (X, τ, G). Also, we tracked down the characteristics and some of the properties of the above-mentioned set.

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