ORIGINAL PAPER

A COMPARATIVE ANALYSIS FOR HOMOGENEITY OF VARIANCE TESTS

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Abstract. *The testing of homogeneity of variances has an important place in data analysis, especially in determining the test statistics in the analysis of variance. If the F test, which is a parametric technique, is desired to be used as test statistics in variance analysis, group variances should be homogeneous. To test for homogeneity of variance, several tests can be used. The non-parametric tests among these do not require the assumption that the groups are normally distributed. In this study, the performance of five different non-parametric tests for homogeneity of variances which are mean-based Levene, median-based Levene also known as Brown-Forsyte, trimmed-mean-based Levene, Fligner-Killeen, and nonparametric Levene tests, are investigated in terms of empirical type 1 error rates and powers for different sample sizes and various symmetric and asymmetric distributions by Monte Carlo simulation studies with free and open-source software R. The simulation results show that the Brown Forsyte test generally has the lowest type-1 error rates while the Median-based Levene test has the highest power for both symmetric and asymmetric distributions in most cases.*

Keywords: *Brown-Forsyte; homogeneity of variance; Levene's Test; nonparametric Levene; power of test; type-1 error rate.*

1. INTRODUCTION

Different statistical analyzes are applied to data obtained from many fields such as education, health, agriculture, veterinary medicine, psychology, and so on. The theory of some tests regarding these analyses, such as the F test in analysis of variance (ANOVA) used to compare two or more groups, is based on the assumption that the groups have homogeneous variances. Therefore, the homogeneity test of variances has an important place in statistical inference. Also, it should be noted that the test of homogeneity of variances is also referred to as the test of equality of variances in the literature.

It should be checked whether the assumptions of normality, independence of errors, and homogeneity of variances are satisfied when conducting ANOVA. Under these assumptions, the ANOVA F test is known to be the optimal test Lehmann and Romano [1]. However, violation of any assumption would impair the utility of the test and lead to wrong and invalid conclusions in case of unequal group sizes [2-5]. For example, the heterogeneity

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of variances causes each group to have measurably different contributions in the estimation of within-group variances, and thus, the sum of the squares within the group will be a biased estimate for the population variance parameter. This causes error rates of Type-1 increase and the power of the test decreases, especially when the group sizes are unequal [6-9].

It is commonly known that the classical F test for testing homogeneity variances is overly sensitive to the assumption that the population distributions are normal [10-12]. Therefore, several tests of homogeneity of variance have been developed such as Neyman and Pearson [13], Bartlett [14], Cochran [3], Hartley [15], Levene [16], Capon [17], median-based Levene known also as Brown-Forsythe and trimmed mean-based Levene [18], Layard [19], Fligner and Killeen [20], nonparametric Levene [21]. Levene tests are the most commonly used tests among these tests in the literature. The original Levene test involving means was found to be quite robust to departures from normality [16]. However, later studies [11, 22] have shown that in case of violation of the assumption of normality, the Type-1 error rate of the Levene test increases. Carroll and Schneider [23] showed mathematically the original Levene test maintains its nominal Type-1 error rate only for symmetric distributions while the median-based Levene test-Brown Forsythe (BF) test maintains its Type-1 error rate for asymmetric distributions. Conover, Johnson [24] demonstrated that the Brown Forsythe test and Fligner Killeen (FK) test are the most performing test among total of 56 tests for homogeneity of variances. Vorapongsathorn, Taejaroenkul [2] found that Levene's test was quite good for both equal and small sample sizes when compared to Bartlett's and Cochran's tests. Non-parametric Levene (NPL) test was developed by Nordstokke and Zumbo [21] as an extension of the mean-based Levene where a rank transformation is applied to the data before conducting the ANOVA. Nordstokke and Colp [12] compared BF and NPL tests for the Chi-square distribution with different degrees of freedom and they showed that the NPL has good statistical properties when population distributions are heavily skewed, the sample size is small and groups are unbalanced.

As mentioned above, various simulation studies have been carried out to compare the homogeneity tests of variances widely used in the literature. However, in these studies, it was observed that the tests for various distributions did not have consistent results in terms of Type-1 error rate and power performances. Therefore, a comprehensive simulation study involving various experimental conditions is essential to investigate the performance of these tests more thoroughly.

This study aims to investigate the performance of mean-based Levene, median-based Levene also known as Brown-Forsyte, trimmed-mean-based Levene, Fligner-Killeen and nonparametric Levene tests for homogeneity of variance in terms of empirical type-1 error rates and powers for various sample sizes and different symmetric (Normal, t, Laplace, Cauchy and Logistic) distributions and asymmetric (Chi-square, Gamma, Lognormal and Laplace-square) distributions and variances through simulation studies. Thus, this study will help researchers choose the most robust test for homogeneity of variance in terms of Type-1 error rates and power among the tests commonly used in the literature. The comparisons of tests were conducted via free statistical software R.

The rest of the paper is organized as follows: In Section 2, tests for homogeneity of variances: Levene tests which are mean-based Levene, Brown-Forsyte and trimmed-mean-based Levene, Fligner-Killeen test, and nonparametric Levene tests are explained. In Section 3, simulation results are presented. Conclusions are finally given in Section 4.

2. HOMOGENEITY OF VARIANCE TESTS

This section describes methods which are most commonly used in the literature to check the homogeneity of variance tests.

2.1. MEAN-BASED LEVENE TEST

Mean-based Levene's test (ML) introduced by Draper and Hunter [25] is equivalent to a one-way analysis of variance F-test based on the absolute deviations of the X_{ij} from their sub-group mean $\bar{X}_{i.}$ shown as $Z_{ij} = \left| X_{ij} - \bar{X}_{i.} \right|$ for a given variable X [26]. The Mean-based Levene's test can be written as follows:

$$ANOVA\left(\left| X_{ij} - \bar{X}_{i.} \right| \right) \tag{1}$$

where, $\bar{X}_{i.}$ is the mean of the i th subgroup.

2.2. BROWN FORSYTE TEST

Brown-Forsyte Test introduced by Brown and Forsythe [18] is a median-based version of Levene's test for equal variances. This test also known as the median-based Levene test has been shown to perform well in situations wherein data are skewed. The Brown Forsyte test can be written as follows:

$$ANOVA\left(\left| X_{ij} - \tilde{X}_{i.} \right| \right) \tag{2}$$

where, $\tilde{X}_{i.}$ is the median of the i th subgroup.

2.3. TRIMMED MEAN LEVENE TEST

Trimmed Mean Levene test introduced by Brown and Forsythe [18] is a trimmed-mean-based version of Levene's test for equal variances. The Brown Forsyte test can be written as follows:

$$ANOVA\left(\left| X_{ij} - \bar{X}_{i.}^{l} \right| \right) \tag{3}$$

where, $\bar{X}_{i.}^{l}$ is the 10% trimmed mean of the i th subgroup. Here the 10% trimmed mean is the arithmetic mean calculated when the largest 10% and the smallest 10% of the cases have been eliminated. Eliminating extreme cases from the computation of the mean results in a better estimate of central tendency, especially when the data are non-normal [2, 27].

2.4. FLIGNER-KILLEEN TEST

The Fligner-Killeen test proposed by Fligner and Killeen [20] has been determined as one of the most robust against departures from normality among the many tests for homogeneity of variance via simulation study by Conover, Johnson [24]. Consider the pooled Z sample and compute the score $A_i = \Phi^{-1}\left(0.5 + 0.5C_i / \left(n+1\right)\right)$, where C_i is the rank of Z_i in Z. The FK statistic is

$$FK = \frac{\sum_{i=1}^{k} n_i \left(A_i - \bar{A}\right)}{S^2} \tag{4}$$

here, $\bar{A}_i = \sum_{j=1}^{n_i} A_j / n_i$, $\bar{A} = \sum_{j=1}^{n} A_j / n$ and $S^2 = \sum_{j=1}^{n} \left(A_j - \bar{A}\right)^2 / \left(n-1\right)$.

2.5. NON-PARAMETRIC LEVENE TEST

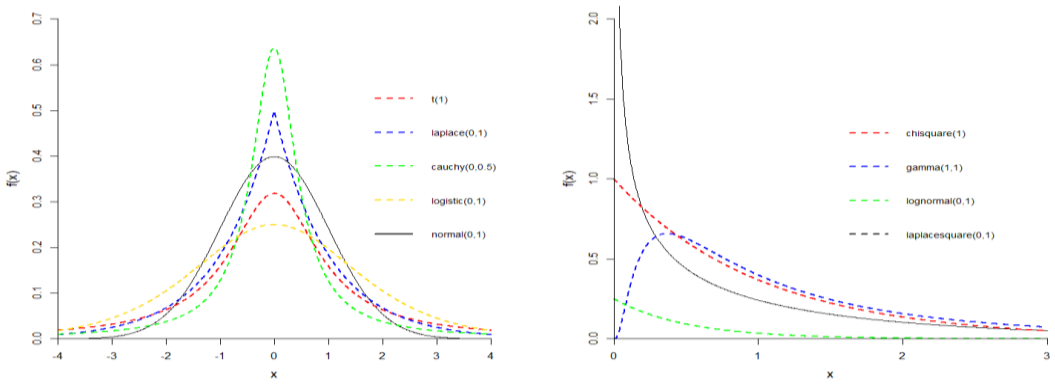
The non-parametric Levene (NPL) test was developed as an extension of the mean-based Levene by Nordstokke and Zumbo [21]. NPL applies a rank transformation to data before conducting ANOVA. This transformation strengthens the use of NPL for data from a non-normal population. The NPL test can be written as follows:

$$ANOVA\left(\left|R_{ij} - \bar{X}_i^R\right|\right) \tag{5}$$

which is a one-way analysis of variance that is conducted on the absolute value of the mean of the ranks for each group, denoted $\bar{X}_i^R$ subtracted from each individual's rank R_{ij} for j th case from i th group [12]. R statistical software program is used to compute the NPL for this study.

3. SIMULATION STUDIES

A simulation study was designed to compare the empirical type-1 errors and statistical powers of the five different homogeneity tests of variances. Firstly, random numbers are generated from total nine distributions whose probability density functions (pdf) are given in Figure 1, five from the symmetric distributions: Normal (0,1), t(1), Laplace(0,1), Cauchy(0,0.5) and Logistic (0,1) and three from the asymmetric distributions: Chi-square(1), Gamma(1,1), Lognormal(0,1) and Laplace-square (0,1).



(a)symmetric distributions (b) asymmetric distributions

Figure 1. Plots of probability density functions of the considered symmetric and asymmetric distributions.

Five different sample sizes ($N=(n_1+n_2+n_3)$): 30, 60, 90,120, and 150 and three levels of ratio of group sizes ($n_1/n_2/n_3=1/1/1$, $1/1/4$,$1/2/3$,and $3/2/1$) were investigated for each distribution. Thus, (5,5,20), (5,10,15), (10,10,10), (10,10,40), (10,20,30), (20,20,20), (15,15,60), (15,30,45), (30,30,30), (20,20,80), (20,40,60), (40,40,40), (25,25,100), (25,50,75), (50,50,50) group sizes for (n_1,n_2,n_3) were considered.

For the power comparisons of the test for homogeneity of variance, eight levels of variance ratios $\sigma_1^2/\sigma_2^2/\sigma_3^2$:$1/1/4$,$1/4/4$, $1/1/2$, $1/2/2$,$2/2/1$, $2/1/1$, $4/4/1$, and $4/1/1$ were investigated. Empirical type-1 error rates and power of tests were calculated with 10.000 repetitions for all cases. The results of the simulation for Type-1 error rates of tests are presented in Table 1-9 for considered distributions, respectively. The best results are in bold font.

Table 1. The empirical type-1 error rates of the homogeneity tests of variance for Normal distribution.

N	(n_1,n_2,n_3)	No. of Sample	ML	BF	TML	FK	NPL
30	**(5,5,20)**	**1**	0.0640	0.0220	0.0609	**0.0185**	0.1147
	(5,10,15)	**2**	0.0579	0.0223	0.0552	**0.0185**	0.0859
	(10,10,10)	**3**	0.0609	0.0303	0.0536	**0.0282**	0.0580
60	**(10,10,40)**	**4**	0.0528	**0.0361**	0.0497	0.0366	0.0716
	(10,20,30)	**5**	0.0544	**0.0361**	0.0524	0.0363	0.0655
	(20,20,20)	**6**	0.0585	0.0376	0.0569	**0.0361**	0.0526
90	**(15,15,60)**	**7**	0.0535	0.0403	0.0515	**0.0381**	0.0642
	(15,30,45)	**8**	0.0491	0.0361	0.0485	**0.0342**	0.0522
	(30,30,30)	**9**	0.0559	0.0444	0.0546	**0.0428**	0.0515
120	**(20,20,80)**	**10**	0.0520	0.0417	0.0503	**0.0414**	0.0573
	(20,40,60)	**11**	0.0542	0.0439	0.0524	**0.0402**	0.0545
	(40,40,40)	**12**	0.0530	0.0444	0.0519	**0.0430**	0.0484
150	**(25,25,100)**	**13**	0.0519	**0.0433**	0.0507	0.0438	0.0546
	(25,50,75)	**14**	0.0534	0.0434	0.0524	**0.0427**	0.0546
	(50,50,50)	**15**	0.0542	**0.0445**	0.0531	0.0451	0.0533

Table 2. The empirical type-1 error rates of the homogeneity tests of variance for t distribution.

N	(n_1,n_2,n_3)	No. of Sample	ML	BF	TML	FK	NPL
30	**(5,5,20)**	**1**	0.2588	0.0433	0.2554	**0.0218**	0.1221
	(5,10,15)	**2**	0.2564	**0.0282**	0.154	0.0301	0.0895
	(10,10,10)	**3**	0.3132	**0.0165**	0.0553	0.0447	0.0470
60	**(10,10,40)**	**4**	0.2526	**0.0459**	0.0677	0.0503	0.0751
	(10,20,30)	**5**	0.2471	**0.0301**	0.0451	0.0526	0.0644

N	(n_1,n_2,n_3)	No. of Sample	ML	BF	TML	FK	NPL
	(20,20,20)	**6**	0.2924	**0.0161**	0.0266	0.0461	0.0493
	(15,15,60)	**7**	0.2560	0.0458	0.0676	**0.0447**	0.0668
90	**(15,30,45)**	**8**	0.2459	**0.0304**	0.0442	0.0439	0.0600
	(30,30,30)	**9**	0.2757	**0.0169**	0.0206	0.0464	0.0466
	(20,20,80)	**10**	0.2536	0.0521	0.0591	**0.0520**	0.0594
120	**(20,40,60)**	**11**	0.2308	**0.0298**	0.0333	0.0480	0.0514
	(40,40,40)	**12**	0.271	**0.0147**	0.0174	0.0464	0.0461
	(25,25,100)	**13**	0.2534	0.0515	0.0577	**0.0490**	0.0577
150	**(25,50,75)**	**14**	0.2292	**0.0293**	0.0333	0.0462	0.0529
	(50,50,50)	**15**	0.2666	**0.0169**	0.0179	0.0463	0.0486

Table 3. The empirical type-1 error rates of the homogeneity tests of variance for the Laplace distribution.

N	(n_1,n_2,n_3)	No. of Sample	ML	BF	TML	FK	NPL
30	**(5,5,20)**	**1**	0.0805	0.0239	0.0773	**0.0174**	0.1126
	(5,10,15)	**2**	0.0778	0.0307	0.0706	**0.0251**	0.0896
	(10,10,10)	**3**	0.0765	**0.0373**	0.0631	0.0379	0.0528
60	**(10,10,40)**	**4**	0.0660	**0.0415**	0.0559	0.0441	0.0741
	(10,20,30)	**5**	0.0624	**0.0395**	0.0526	0.0415	0.0651
	(20,20,20)	**6**	0.0649	0.0399	0.056	**0.0393**	0.0485
90	**(15,15,60)**	**7**	0.0609	0.0410	0.0542	**0.0403**	0.0648
	(15,30,45)	**8**	0.0578	0.0406	0.0514	**0.0397**	0.0523
	(30,30,30)	**9**	0.0610	**0.0436**	0.0529	0.0438	0.0462
120	**(20,20,80)**	**10**	0.0614	**0.0460**	0.0542	0.0475	0.0618
	(20,40,60)	**11**	0.0589	0.0465	0.0533	**0.0461**	0.0522
	(40,40,40)	**12**	0.0574	0.0444	0.0521	**0.0428**	0.0513
150	**(25,25,100)**	**13**	0.0588	**0.0457**	0.0534	0.0459	0.0583
	(25,50,75)	**14**	0.0564	0.0460	0.051	**0.0435**	0.0496
	(50,50,50)	**15**	0.0523	**0.0442**	0.0487	0.0459	0.0488

Table 4. The empirical type-1 error rates of the homogeneity tests of variance for the Cauchy distribution.

N	(n_1,n_2,n_3)	No. of Sample	ML	BF	TML	FK	NPL
	(5,5,20)	**1**	0.2539	0.0451	0.2506	**0.0232**	0.1135
30	**(5,10,15)**	**2**	0.2590	**0.0295**	0.1522	0.0300	0.0886
	(10,10,10)	**3**	0.3214	**0.0190**	0.0574	0.0469	0.0509
	(10,10,40)	**4**	0.2530	**0.0508**	0.0713	0.0529	0.0731
60	**(10,20,30)**	**5**	0.2412	**0.0305**	0.0462	0.0461	0.0615
	(20,20,20)	**6**	0.2868	**0.0164**	0.0276	0.0439	0.049
	(15,15,60)	**7**	0.2507	0.0486	0.0681	**0.0445**	0.0651
90	**(15,30,45)**	**8**	0.2290	**0.0294**	0.0411	0.0443	0.0558
	(30,30,30)	**9**	0.2751	**0.0168**	0.0204	0.0449	0.0502
	(20,20,80)	**10**	0.2490	0.0466	0.0528	**0.0451**	0.0591
120	**(20,40,60)**	**11**	0.2423	**0.0334**	0.0390	0.0480	0.0559
	(40,40,40)	**12**	0.2705	**0.0170**	0.0201	0.0460	0.0478
	(25,25,100)	**13**	0.2484	**0.0486**	0.0545	0.0488	0.0584
150	**(25,50,75)**	**14**	0.2423	**0.0324**	0.0366	0.0481	0.0516
	(50,50,50)	**15**	0.2641	**0.0185**	0.0194	0.0461	0.0505

Table 5. The empirical type-1 error rates of the homogeneity tests of variance for the Logistic distribution.

N	(n_1,n_2,n_3)	No. of Sample	ML	BF	TML	FK	NPL
	(5,5,20)	**1**	0.0681	0.0221	0.0658	**0.0172**	0.1144
30	**(5,10,15)**	**2**	0.0648	0.0244	0.0611	**0.0216**	0.0885
	(10,10,10)	**3**	0.0649	0.0319	0.0564	**0.0316**	0.0500
60	**(10,10,40)**	**4**	0.0618	0.0408	0.0558	**0.0395**	0.0770

	(10,20,30)	**5**	0.0579	0.0398	0.0533	**0.038**	0.0667
	(20,20,20)	**6**	0.0592	0.0418	0.0544	**0.0401**	0.0496
	(15,15,60)	**7**	0.0545	**0.0367**	0.0502	0.0379	0.0634
90	**(15,30,45)**	**8**	0.0521	0.0377	0.0483	**0.0359**	0.0527
	(30,30,30)	**9**	0.0574	0.0437	0.0542	**0.0432**	0.0498
	(20,20,80)	**10**	0.0529	0.041	0.0491	**0.0404**	0.0568
120	**(20,40,60)**	**11**	0.0547	0.0427	0.0518	**0.0415**	0.0577
	(40,40,40)	**12**	0.0530	0.0411	0.0495	**0.0408**	0.0507
	(25,25,100)	**13**	0.0503	**0.0416**	0.0491	0.0423	0.0592
150	**(25,50,75)**	**14**	0.0523	0.0448	0.0502	**0.0437**	0.0523
	(50,50,50)	**15**	0.0482	**0.0406**	0.0474	0.0416	0.0481

Table 6. The empirical type-1 error rates of the homogeneity tests of variance for the Chi-square distribution.

N	**(n_1,n_2,n_3)**	**No. of Sample**	**ML**	**BF**	**TML**	**FK**	**NPL**
	(5,5,20)	**1**	0.2479	**0.0335**	0.1957	0.0631	0.116
30	**(5,10,15)**	**2**	0.2679	**0.0387**	0.1873	0.0829	0.0912
	(10,10,10)	**3**	0.286	**0.0517**	0.1535	0.1345	0.0553
	(10,10,40)	**4**	0.2513	**0.0442**	0.1211	0.1524	0.0741
60	**(10,20,30)**	**5**	0.2646	**0.0489**	0.126	0.1603	0.0634
	(20,20,20)	**6**	0.2689	**0.0506**	0.1293	0.1609	**0.0506**
	(15,15,60)	**7**	0.2428	**0.0410**	0.1252	0.1528	0.0670
90	**(15,30,45)**	**8**	0.2542	**0.0448**	0.1261	0.1686	0.0570
	(30,30,30)	**9**	0.2591	**0.0477**	0.1164	0.1771	0.0501
	(20,20,80)	**10**	0.2456	**0.0485**	0.1092	0.1943	0.0583
120	**(20,40,60)**	**11**	0.2545	**0.0495**	0.1134	0.2046	0.0565
	(40,40,40)	**12**	0.2477	**0.0476**	0.1118	0.2114	0.0515
	(25,25,100)	**13**	0.2421	**0.0432**	0.1135	0.2007	0.0583
150	**(25,50,75)**	**14**	0.2517	**0.0507**	0.1183	0.2197	0.0554
	(50,50,50)	**15**	0.2587	0.0507	0.1185	0.2408	**0.0495**

Table 7. The empirical type-1 error rates of the homogeneity tests of variance for the Gamma distribution.

N	**(n_1,n_2,n_3)**	**No. of Sample**	**ML**	**BF**	**TML**	**FK**	**NPL**
30	**(5,5,20)**	**1**	0.1848	**0.0283**	0.1529	0.0394	0.1147
	(5,10,15)	**2**	0.1928	**0.0353**	0.1454	0.0512	0.0899
	(10,10,10)	**3**	0.1874	**0.0424**	0.1148	0.0703	0.0501
60	**(10,10,40)**	**4**	0.1732	**0.0446**	0.0958	0.0851	0.0721
	(10,20,30)	**5**	0.1768	**0.0443**	0.0978	0.0870	0.0632
	(20,20,20)	**6**	0.1865	**0.0448**	0.1053	0.0870	0.0518
90	**(15,15,60)**	**7**	0.1773	**0.0467**	0.1078	0.0861	0.0581
	(15,30,45)	**8**	0.1759	**0.0464**	0.0994	0.0928	0.0612
	(30,30,30)	**9**	0.1795	**0.0447**	0.0925	0.0949	0.0529
120	**(20,20,80)**	**10**	0.1702	**0.0478**	0.0902	0.098	0.0576
	(20,40,60)	**11**	0.1768	**0.0452**	0.0935	0.1026	0.058
	(40,40,40)	**12**	0.1729	**0.0496**	0.0903	0.1072	0.0533
150	**(25,25,100)**	**13**	0.1756	**0.0484**	0.0968	0.1086	0.0602
	(25,50,75)	**14**	0.1722	**0.0443**	0.0938	0.1042	0.0501
	(50,50,50)	**15**	0.1761	**0.0457**	0.0902	0.1094	0.0484

Table 8. The empirical type-1 error rates of the homogeneity tests of variance for the Lognormal distribution.

N	**(n_1,n_2,n_3)**	**No. of Sample**	**ML**	**BF**	**TML**	**FK**	**NPL**
30	**(5,5,20)**	**1**	0.2431	**0.0419**	0.2093	0.0502	0.1195
	(5,10,15)	**2**	0.2520	**0.0349**	0.1688	0.0581	0.0938
	(10,10,10)	**3**	0.2602	**0.0399**	0.1229	0.0979	0.0532

N	(n_1,n_2,n_3)	No. of Sample	ML	BF	TML	FK	NPL
60	**(10,10,40)**	**4**	0.2344	**0.0517**	0.1048	0.1125	0.0750
	(10,20,30)	**5**	0.2482	**0.0413**	0.1017	0.1086	0.0645
	(20,20,20)	**6**	0.2545	**0.0365**	0.0937	0.1090	0.0499
90	**(15,15,60)**	**7**	0.2382	**0.0461**	0.108	0.1055	0.0588
	(15,30,45)	**8**	0.2492	**0.0431**	0.1024	0.1113	0.0582
	(30,30,30)	**9**	0.2562	**0.0412**	0.092	0.1234	0.0515
120	**(20,20,80)**	**10**	0.2301	**0.0446**	0.0844	0.1241	0.0630
	(20,40,60)	**11**	0.2372	**0.0433**	0.087	0.1335	0.0533
	(40,40,40)	**12**	0.2425	**0.0387**	0.0833	0.1275	0.0490
150	**(25,25,100)**	**13**	0.2305	**0.0448**	0.0906	0.1264	0.0546
	(25,50,75)	**14**	0.2386	**0.0437**	0.0865	0.1331	0.0504
	(50,50,50)	**15**	0.2504	**0.0404**	0.0818	0.1414	0.0521

Table 9. The empirical type-1 error rates of the homogeneity tests of variance for the Laplace square distribution.

N	(n_1,n_2,n_3)	No. of Sample	ML	BF	TML	FK	NPL
30	**(5,5,20)**	**1**	0.3159	**0.0465**	0.2596	0.0863	0.1154
	(5,10,15)	**2**	0.3570	**0.0404**	0.2286	0.1181	0.0881
	(10,10,10)	**3**	0.3854	**0.041**	0.156	0.2008	0.0549
60	**(10,10,40)**	**4**	0.3062	**0.0491**	0.1161	0.2221	0.068
	(10,20,30)	**5**	0.3382	**0.043**	0.1174	0.2311	0.0606
	(20,20,20)	**6**	0.3453	**0.0372**	0.1101	0.2284	0.0472
90	**(15,15,60)**	**7**	0.3057	**0.0465**	0.1288	0.2189	0.0605
	(15,30,45)	**8**	0.3307	**0.0438**	0.1223	0.2522	0.0572
	(30,30,30)	**9**	0.3360	**0.0376**	0.0984	0.2681	0.0525
120	**(20,20,80)**	**10**	0.3099	**0.0467**	0.0997	0.2838	0.0552
	(20,40,60)	**11**	0.3195	**0.0392**	0.0947	0.3054	0.0535
	(40,40,40)	**12**	0.3341	**0.0375**	0.0948	0.3077	0.0506
150	**(25,25,100)**	**13**	0.3026	**0.0486**	0.1079	0.2906	0.0607
	(25,50,75)	**14**	0.3140	**0.0403**	0.0986	0.3212	0.0549
	(50,50,50)	**15**	0.3324	**0.0405**	0.0919	0.3440	0.0495

As can be seen from Table 1 presented as a result of the conducted simulation studies, the FK test gave the best result with the lowest type-1 error rates for normal distribution. The type-1 error rates of the BF test are very close to that of the FK test, and for a few cases, even the BF test has a lower type-1 error rate than the FK test. The highest type-1 error rate was given by the ML test in balanced group sizes and the NPL test in unbalanced group sizes.

It can be said from Table 2 that for the t distribution, the BF test generally has the lowest type-1 error rate. However, when the unbalanced group sizes ($n_1 / n_2 / n_3 = 1/1/4$) are concerned, the type-1 error rate of the FK test is lower than that of the BF test, except for one case ($n_1, n_2, n_3 = 10,10,40$). Also, it is observed that the ML test gives the highest type-1 error rates for the t distribution.

According to the simulation results for the Laplace distribution given in Table 3, BF and FK tests type-1 error rates are generally very close to each other and lower than the values of the other homogeneity of variance tests. In cases where the sample size is small (N = 30 and 60) and unbalanced ($n_1 / n_2 / n_3 = 1/1/4$ and $1/2/3$) NPL is the highest type-1 error. In other cases, the ML test has the largest type-1 error values.

The BF test generally exhibits the best performance with the lowest type-1 error rate for the Cauchy distribution as can be seen from Table 4. The FK test, on the other hand, has lower type-1 error rate values than the BF test for a few cases in unbalanced group sizes (

$n_1, n_2, n_3 = 5,5,20$,15,15,60 and $20,20,80$). The test with the highest type-1 error rate for the Cauchy distribution is the ML test.

It can be said from the type-1 error rates of the homogeneity of variances presented in Table 5, that the test with the lowest type-1 error rate is usually the FK test for the logistic distribution. The BF test has a lower error value than the FK test for a few cases ($n_1, n_2, n_3 = 15,15,60 ; 25,25,100 ; 50,50,50$), and the FK and BF tests have similar type-1 error rates in most cases. The ML test gave the highest type-1 error rate for balanced group sizes and the NPL test for unbalanced group sizes for the logistic distribution.

When looking at the simulation results shown in Table 6-10 for the asymmetric distributions which are chi-square, gamma, log-normal, and Laplace-square, it can be seen that the BF test has the lowest type-1 error rates for all cases except for group sizes of $n_1, n_2, n_3 = 20,20,20$ and $50,50,50$ of the Chi-square distribution and ML test have the highest type-1 error rates for all cases except for group sizes of $n_1, n_2, n_3 = 25,50,75$ and $50,50,50$ in the Laplace square distribution.

To better understand the simulation results, the simulation results of type-1 error rates related to the homogeneity of variance tests are visualized in Figure 2 according to the 15 different combinations of group size. From this figure, the results mentioned above can be also seen.

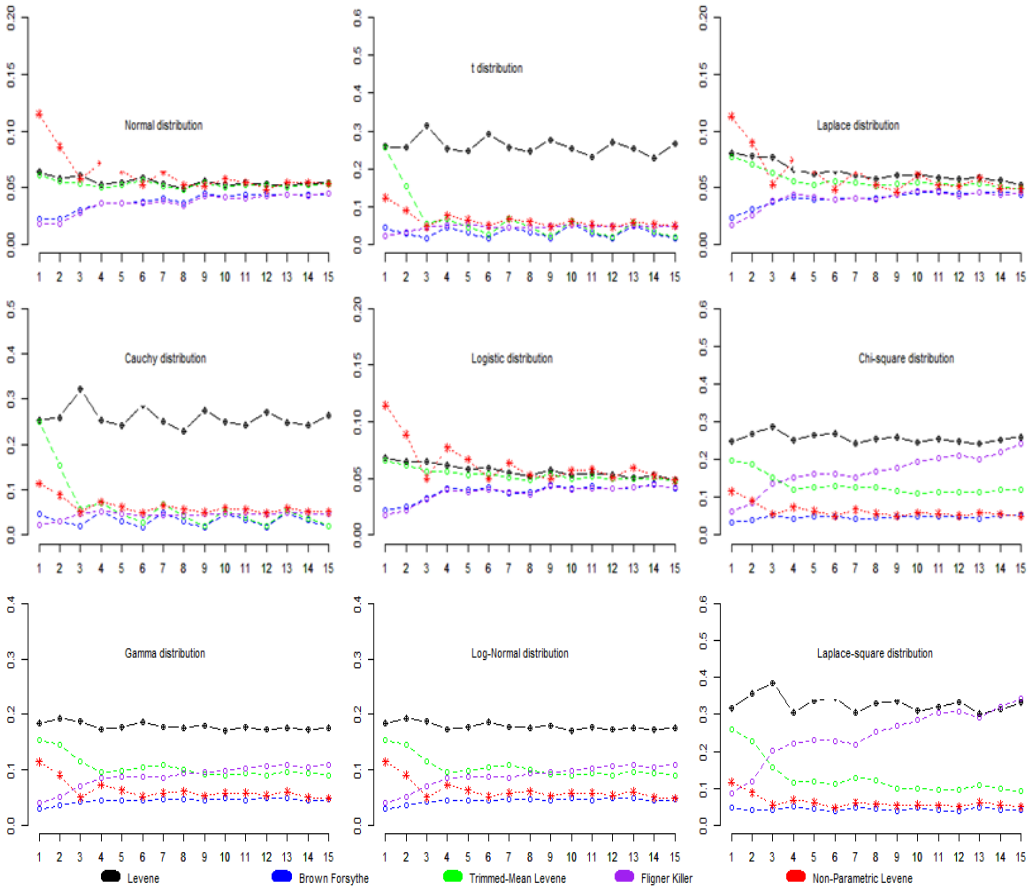


Figure 2. Empirical type -1 error rates plot for homogeneity of variance test according to different group sample sizes.

The powers of tests for homogeneity of variance were computed for nine distributions according to eight different variance ratios and different sample sizes, and the results are given in Table 10-18, respectively. The best results are in bold font.

Table 10. The power of the homogeneity tests of variance for the Normal distribution.

			Population variance ratio							
N	(n_1,n_2,n_3)	Test	1/1/4	1/4/4	1/1/2	1/2/2	2/2/1	2/1/1	4/4/1	4/1/1
		ML	0.3125	0.1716	0.1128	0.0862	0.1995	0.1483	0.5372	**0.3931**
		BF	0.2398	0.0851	0.0720	0.0396	0.0594	0.0465	0.2432	0.1998
	(5,5,20)	TML	0.2977	0.1603	0.1041	0.0808	**0.1966**	**0.1456**	**0.5364**	0.3908
		FK	0.2822	0.0878	0.0816	0.0398	0.0257	0.0228	0.0982	0.0841
		NPL	**0.4874**	**0.372**	**0.2441**	**0.1878**	0.0935	0.1174	0.1698	0.1707
		ML	**0.4496**	0.1598	0.1497	0.0902	**0.1781**	**0.1460**	**0.5114**	**0.3957**
		BF	0.3259	0.084	0.0813	0.0449	0.0786	0.0473	0.3062	0.1966
30	**(5,10,15)**	TML	0.4366	0.1463	0.1438	0.083	0.1685	0.1437	0.4967	0.3915
		FK	0.2983	0.079	0.0688	0.042	0.0612	0.0263	0.2306	0.0801
		NPL	0.3831	**0.3758**	**0.1799**	**0.1941**	0.1123	0.0890	0.2319	0.1460
		ML	**0.4829**	**0.3895**	**0.1651**	**0.1499**	**0.1407**	**0.1679**	**0.3703**	**0.4818**
		BF	0.3606	0.2452	0.0989	0.086	0.0791	0.0997	0.2347	0.3665
	(10,10,10)	TML	0.4531	0.3549	0.1520	0.1376	0.1294	0.1549	0.3418	0.4584
		FK	0.2982	0.2413	0.0883	0.0804	0.0723	0.0882	0.2283	0.3047
		NPL	0.2528	0.3337	0.1051	0.1152	0.1117	0.1061	0.313	0.2501
		ML	0.7422	0.4217	0.2300	0.137	**0.3113**	0.2228	**0.8272**	**0.6621**
		BF	0.7098	0.3491	0.2015	0.1061	0.2301	0.1651	0.7477	0.5772
	(10,10,40)	TML	0.7393	0.4105	0.2278	0.1341	0.2934	0.21	0.8102	0.6404
		FK	0.7081	0.367	0.2037	0.1099	0.1859	0.134	0.6337	0.4594
		NPL	**0.7464**	**0.5798**	**0.2895**	**0.2064**	0.1200	0.1094	0.3737	0.2516
		ML	**0.8396**	0.4194	**0.3018**	0.1343	**0.3183**	**0.2287**	**0.8460**	**0.6635**
		BF	0.8055	0.3437	0.2562	0.102	0.2400	0.1704	0.7749	0.5823
60	**(10,20,30)**	TML	0.8314	0.4079	0.2939	0.1299	0.3023	0.2155	0.8305	0.6426
		FK	0.7687	0.3565	0.2438	0.1022	0.2112	0.1370	0.7093	0.4616
		NPL	0.6984	**0.5914**	0.2546	**0.2059**	0.1706	0.0988	0.5656	0.2518
		ML	**0.8198**	**0.7449**	**0.2957**	**0.2537**	**0.2541**	**0.3056**	**0.7564**	**0.8244**
		BF	0.7719	0.6688	0.2415	0.1965	0.1948	0.2472	0.6771	0.7758
	(20,20,20)	TML	0.8107	0.7307	0.287	0.2441	0.2443	0.2958	0.7415	0.8136
		FK	0.7003	0.6433	0.2175	0.1846	0.1827	0.2178	0.6491	0.7023
		NPL	0.5347	0.6575	0.1791	0.2056	0.204	0.1861	0.6631	0.5265
		ML	**0.9287**	**0.6726**	0.3552	0.2045	**0.4368**	**0.3122**	**0.9463**	**0.8307**
		BF	0.9238	0.629	0.341	0.1765	0.342	0.2468	0.9166	0.7821
	(15,15,60)	TML	0.9274	0.6662	0.3539	0.2012	0.4249	0.3035	0.9433	0.8245
		FK	0.9205	0.6378	0.3435	0.1777	0.2889	0.2054	0.8583	0.688
		NPL	0.9022	0.762	**0.3759**	**0.2626**	0.1863	0.1384	0.6213	0.3946
		ML	**0.9622**	0.6683	**0.4452**	0.2061	**0.4591**	**0.3199**	**0.9623**	**0.8237**
		BF	0.9526	0.6255	0.4075	0.1799	0.388	0.2558	0.9422	0.7784
90	**(15,30,45)**	TML	0.9608	0.6606	0.4395	0.2028	0.4481	0.3137	0.9581	0.817
		FK	0.9354	0.6329	0.3887	0.1792	0.3533	0.209	0.9142	0.6777
		NPL	0.8732	**0.7705**	0.3493	**0.2666**	0.2697	0.1373	0.8049	0.4036
		ML	**0.9498**	**0.9274**	**0.4349**	**0.376**	**0.3671**	**0.4302**	**0.9321**	**0.949**
		BF	0.9364	0.9002	0.3912	0.3205	0.3146	0.384	0.9059	0.9334
	(30,30,30)	TML	0.9478	0.9218	0.427	0.3654	0.3585	0.4226	0.9272	0.945
		FK	0.9015	0.8832	0.3543	0.3088	0.2984	0.352	0.8875	0.9002
		NPL	0.749	0.8537	0.265	0.3005	0.2929	0.2614	0.8634	0.7535
		ML	**0.9858**	0.8461	**0.4792**	0.2831	**0.5621**	**0.4074**	**0.9867**	**0.9238**
		BF	0.9832	0.8252	0.4666	0.2597	0.5018	0.3567	0.9798	0.9045
120	**(20,20,80)**	TML	0.9849	0.8429	0.4805	0.282	0.5479	0.3948	0.9855	0.9185
		FK	0.9795	0.8249	0.4617	0.2599	0.4527	0.3155	0.9601	0.8491
		NPL	0.964	**0.8798**	0.4591	**0.3212**	0.2694	0.1832	0.7911	0.5475

			Population variance ratio							
N	(n_1,n_2,n_3)	Test	1/1/4	1/4/4	1/1/2	1/2/2	2/2/1	2/1/1	4/4/1	4/1/1
		ML	**0.9937**	0.8465	**0.5755**	0.278	**0.5797**	**0.393**	**0.9932**	**0.9211**
		BF	0.9916	0.8242	0.5515	0.2522	0.5257	0.3455	0.9898	0.900
	(20,40,60)	TML	0.9933	0.843	0.5729	0.2744	0.5691	0.3823	0.9928	0.9155
		FK	0.9864	0.8231	0.53	0.2495	0.495	0.3029	0.9819	0.8436
		NPL	0.9512	**0.8881**	0.4466	**0.3264**	0.3674	0.1792	0.9274	0.5519
		ML	**0.9868**	**0.984**	**0.5496**	**0.4934**	**0.4921**	**0.5424**	**0.9828**	**0.9879**
		BF	0.9837	0.9774	0.5135	0.4503	0.4487	0.5057	0.9778	0.9839
	(40,40,40)	TML	0.9863	0.9822	0.5432	0.4876	0.4838	0.5353	0.9815	0.9871
		FK	0.9719	0.9701	0.4787	0.4352	0.4327	0.4719	0.9687	0.9717
		NPL	0.8783	0.9488	0.347	0.3982	0.3972	0.3432	0.9487	0.8804
		ML	**0.9971**	0.9376	**0.5922**	0.3638	**0.6548**	**0.4746**	**0.9969**	**0.9651**
		BF	0.9969	0.9296	0.5907	0.3429	0.5994	0.4292	0.995	0.9532
	(25,25,100)	TML	0.997	0.9363	0.5932	0.3613	0.6458	0.4676	0.9965	0.9623
		FK	0.9961	0.9282	0.5849	0.3435	0.5498	0.3865	0.9873	0.9165
		NPL	0.9891	**0.942**	0.5482	**0.388**	0.3452	0.2237	0.8959	0.6658
		ML	**0.9989**	0.9344	**0.6868**	**0.3514**	**0.6825**	**0.4796**	**0.9987**	**0.9649**
		BF	0.9985	0.9259	0.6694	0.3299	0.6383	0.4295	0.9983	0.9555
150	**(25,50,75)**	TML	0.9988	0.9331	0.6866	0.3485	0.6741	0.4722	0.9984	0.9626
		FK	0.9974	0.9251	0.6496	0.3327	0.6104	0.3835	0.9963	0.923
		NPL	0.9859	**0.9417**	0.5427	0.3876	0.4627	0.2244	0.9713	0.6694
		ML	**0.9981**	**0.9976**	**0.6477**	**0.6063**	**0.5966**	**0.6545**	**0.9971**	**0.9978**
		BF	0.9975	0.9961	0.6222	0.574	0.5626	0.6281	0.9964	0.9972
	(50,50,50)	TML	0.998	0.9975	0.6441	0.601	0.5914	0.6496	**0.9971**	0.9974
		FK	0.9927	0.9939	0.5892	0.5555	0.5422	0.5959	0.9936	0.9935
		NPL	0.9452	0.984	0.4238	0.4978	0.4851	0.44	0.9848	0.9455

Table 11. The power of the homogeneity tests of variance for the t distribution.

			Population variance ratio							
N	(n_1,n_2,n_3)	Test	1/1/4	1/4/4	1/1/2	1/2/2	2/2/1	2/1/1	4/4/1	4/1/1
		ML	0.1615	0.2126	**0.1997**	**0.2248**	**0.3331**	**0.291**	**0.427**	**0.3551**
		BF	0.0166	0.0339	0.0264	0.0315	0.0706	0.0589	0.1325	0.0973
	(5,5,20)	TML	0.1504	0.205	0.1928	0.2209	0.331	0.2882	0.4268	0.3522
		FK	0.1209	0.0587	0.0502	0.0345	0.0215	0.0216	0.0452	0.036
		NPL	**0.3017**	**0.2300**	0.1844	0.155	0.0931	0.113	0.1109	0.13
		ML	0.2207	0.2154	**0.2208**	**0.2361**	**0.318**	**0.2879**	**0.3975**	**0.3493**
		BF	0.0289	0.0165	0.0228	0.0205	0.0452	0.0468	0.0793	0.0833
30	**(5,10,15)**	TML	0.1356	0.1025	0.1322	0.1214	0.1948	0.1952	0.2514	0.26
		FK	0.1145	0.0679	0.0488	0.0398	0.0502	0.0321	0.1045	0.0426
		NPL	**0.2282**	**0.2195**	0.1363	0.1356	0.0854	0.0804	0.1312	0.108
		ML	**0.3864**	**0.3641**	**0.3345**	**0.3241**	**0.3292**	**0.3267**	**0.3689**	**0.3793**
		BF	0.0468	0.0344	0.0247	0.0235	0.0223	0.0247	0.0341	0.0518
	(10,10,10)	TML	0.0949	0.0841	0.0665	0.0633	0.0604	0.0653	0.0818	0.0992
		FK	0.1398	0.1265	0.0719	0.0684	0.0688	0.0691	0.1353	0.1403
		NPL	0.1349	0.1445	0.0723	0.0792	0.0745	0.0719	0.1497	0.1368
		ML	0.1613	0.2086	**0.1886**	**0.2196**	**0.3392**	**0.3013**	**0.4491**	**0.3619**
		BF	0.0207	0.0396	0.0225	0.0396	0.0968	0.0727	0.1835	0.1284
	(10,10,40)	TML	0.03	0.0547	0.0377	0.0555	0.1265	0.1005	0.2211	0.1566
		FK	0.2599	0.1667	0.094	0.0763	0.0996	0.0784	0.2579	0.18
		NPL	**0.3892**	**0.267**	0.1696	0.1345	0.0791	0.0805	0.1811	0.1479
		ML	0.2235	0.2074	**0.2104**	**0.2236**	**0.3096**	**0.2889**	**0.4112**	**0.3605**
60		BF	0.0426	0.0194	0.0247	0.0228	0.0579	0.0597	0.118	0.1171
	(10,20,30)	TML	0.0553	0.0272	0.037	0.0342	0.0783	0.078	0.1463	0.1392
		FK	0.2886	0.1571	0.0985	0.0636	0.1078	0.08	0.2717	0.1746
		NPL	**0.3587**	**0.2657**	0.1411	0.121	0.0976	0.0738	0.2484	0.1435
		ML	**0.3761**	**0.3485**	**0.3087**	**0.3173**	**0.3054**	**0.3058**	**0.3665**	**0.3788**
	(20,20,20)	BF	0.075	0.0478	0.0294	0.0241	0.0247	0.0267	0.0554	0.0766

			Population variance ratio							
N	(n_1,n_2,n_3)	Test	1/1/4	1/4/4	1/1/2	1/2/2	2/2/1	2/1/1	4/4/1	4/1/1
		TML	0.0916	0.064	0.0413	0.0373	0.0351	0.0391	0.0743	0.0926
		FK	0.2669	0.2409	0.091	0.0871	0.0901	0.0974	0.2489	0.2713
		NPL	0.2739	0.2862	0.0976	0.1017	0.1072	0.1034	0.2863	0.2766
		ML	0.1653	0.2149	0.1904	**0.2264**	**0.3339**	**0.3015**	**0.4506**	**0.3726**
		BF	0.0261	0.0379	0.0245	0.0384	0.0973	0.0803	0.2023	0.1452
	(15,15,60)	TML	0.0351	0.0538	0.0393	0.0587	0.1308	0.1069	0.2431	0.1744
		FK	0.4049	0.2399	0.1298	0.0943	0.0995	0.0862	0.3416	0.2327
		NPL	**0.5129**	**0.3401**	**0.1973**	0.1459	0.0945	0.0926	0.299	0.218
		ML	0.2381	0.2021	**0.2061**	**0.2169**	**0.3146**	**0.2817**	**0.4277**	**0.3607**
		BF	0.0592	0.0209	0.027	0.0216	0.0689	0.0602	0.1344	0.1363
90	**(15,30,45)**	TML	0.0681	0.0292	0.0367	0.0313	0.0857	0.077	0.1566	0.1607
		FK	0.4487	0.2381	0.1377	0.0882	0.1251	0.0837	0.4105	0.2335
		NPL	**0.5147**	**0.3477**	0.1794	0.1386	0.1238	0.0858	0.4178	0.2152
		ML	0.3888	0.3552	**0.3009**	**0.2929**	**0.2963**	**0.2974**	0.3564	0.3868
		BF	0.0896	0.0558	0.0284	0.0303	0.0263	0.0319	0.0584	0.0952
	(30,30,30)	TML	0.0978	0.0634	0.0342	0.0348	0.0318	0.0367	0.0682	0.1026
		FK	0.3927	0.3703	0.1227	0.1225	0.1151	0.1305	0.3799	0.409
		NPL	**0.409**	**0.4276**	0.1341	0.1376	0.1314	0.1393	**0.4316**	**0.4175**
		ML	0.1692	0.2072	0.1876	**0.2158**	**0.3395**	**0.3075**	**0.4603**	**0.3843**
		BF	0.0294	0.0414	0.0228	0.0407	0.1083	0.081	0.2161	0.1581
	(20,20,80)	TML	0.0328	0.0451	0.0262	0.0455	0.1164	0.0901	0.2287	0.1667
		FK	0.5209	0.3185	0.1583	0.1125	0.1538	0.1096	0.4913	0.3408
		NPL	**0.6246**	**0.4192**	**0.2247**	0.1596	0.1245	0.1021	0.4336	0.3017
		ML	0.2442	0.2037	0.2125	**0.2138**	**0.323**	**0.2853**	0.4405	0.3652
		BF	0.0662	0.0219	0.0296	0.0208	0.0685	0.0675	0.1495	0.1438
120	**(20,40,60)**	TML	0.0702	0.0235	0.0328	0.0239	0.0752	0.0727	0.1607	0.1522
		FK	0.5753	0.3168	0.1781	0.1056	0.1688	0.1103	0.5471	0.3357
		NPL	**0.6435**	**0.4227**	**0.2155**	0.1532	0.1682	0.1027	**0.5553**	**0.3008**
		ML	0.3975	0.3642	**0.2969**	**0.2959**	**0.3014**	**0.2974**	0.3732	0.3949
		BF	0.1001	0.0728	0.038	0.0313	0.0276	0.0363	0.0706	0.1052
	(40,40,40)	TML	0.1055	0.0794	0.0416	0.035	0.0316	0.0409	0.0773	0.1113
		FK	0.5206	0.4925	0.1588	0.1534	0.1531	0.1552	**0.4925**	0.5276
		NPL	**0.543**	**0.5494**	0.1708	0.1741	0.1723	0.1701	0.5569	**0.5448**
		ML	0.1752	0.2098	0.1883	**0.2192**	**0.3417**	**0.2991**	0.4699	0.3901
		BF	0.036	0.0432	0.0227	0.0356	0.1079	0.0842	0.2245	0.1731
	(25,25,100)	TML	0.0373	0.0474	0.0257	0.0402	0.1184	0.0914	0.2385	0.1837
		FK	0.626	0.3954	0.1961	0.1307	0.1724	0.1247	0.5827	**0.4153**
		NPL	**0.721**	**0.4954**	**0.2634**	0.1751	0.1579	0.124	**0.5534**	0.3861
		ML	0.2616	0.218	0.2176	**0.2139**	**0.3246**	**0.2839**	0.4486	0.3718
		BF	0.0775	0.0263	0.0299	0.0241	0.074	0.0696	0.1593	0.1543
150	**(25,50,75)**	TML	0.0818	0.0276	0.0332	0.0266	0.0804	0.0757	0.1683	0.1618
		FK	0.6819	0.3963	0.2124	0.1267	0.2025	0.1194	0.6542	0.4075
		NPL	**0.7445**	**0.5052**	**0.2565**	0.1765	0.2056	0.1135	**0.6774**	**0.3821**
		ML	0.404	0.379	**0.3047**	**0.2985**	**0.2994**	**0.2967**	0.3702	0.3993
		BF	0.1129	0.0775	0.04	0.0341	0.0359	0.0414	0.0807	0.1112
	(50,50,50)	TML	0.1171	0.0813	0.0427	0.0359	0.0389	0.0442	0.0842	0.1152
		FK	0.623	0.5969	0.1873	0.1805	0.1831	0.1973	0.6022	0.633
		NPL	**0.6481**	**0.6606**	0.1999	0.2031	0.2083	0.2082	**0.6692**	**0.6592**

Table 12. The power of the homogeneity tests of variance for the Laplace distribution.

			Population variance ratio							
N	(n_1,n_2,n_3)	Test	1/1/4	1/4/4	1/1/2	1/2/2	2/2/1	2/1/1	4/4/1	4/1/1
		ML	0.1332	0.108	0.0545	0.0664	**0.2224**	**0.1608**	**0.4835**	0.3483
		BF	0.0873	0.0475	0.0261	0.0243	0.0815	0.0583	0.2368	0.1693
30	**(5,5,20)**	TML	0.1164	0.0989	0.0475	0.0633	0.22	0.1584	0.4814	**0.3454**
		FK	0.1816	0.0705	0.0532	0.0312	0.0261	0.0214	0.0754	0.0584

			Population variance ratio							
N	(n_1,n_2,n_3)	Test	1/1/4	1/4/4	1/1/2	1/2/2	2/2/1	2/1/1	4/4/1	4/1/1
		NPL	**0.374**	**0.2924**	**0.2073**	**0.168**	0.0974	0.1085	0.1327	0.1409
		ML	0.2822	0.1072	0.1052	0.0711	**0.1793**	**0.1669**	**0.4157**	**0.3521**
		BF	0.1808	0.056	0.0521	0.033	0.0794	0.0674	0.2275	0.1787
	(5,10,15)	TML	0.2615	0.0921	0.0944	0.0599	0.1641	0.1584	0.3833	0.3465
		FK	0.2026	0.0756	0.0592	0.0398	0.0583	0.0308	0.1737	0.0659
		NPL	**0.2894**	**0.2836**	**0.1461**	**0.1528**	0.0898	0.0849	0.1659	0.125
		ML	**0.3539**	**0.2665**	**0.1433**	**0.1321**	**0.1292**	**0.1413**	**0.2767**	**0.3527**
		BF	0.2385	0.1529	0.0828	0.0665	0.0678	0.0794	0.1562	0.2389
	(10,10,10)	TML	0.3102	0.2254	0.1204	0.1103	0.1051	0.1182	0.2327	0.3119
		FK	0.2142	0.1859	0.0827	0.0728	0.0716	0.078	0.1801	0.2161
		NPL	0.1775	0.2128	0.0825	0.0946	0.0857	0.0847	0.2044	0.1753
		ML	0.3992	0.2096	0.1024	0.0877	**0.2817**	**0.2107**	**0.6888**	**0.5295**
		BF	0.3586	0.1576	0.0845	0.0609	0.1944	0.1484	0.5812	0.4407
	(10,10,40)	TML	0.3874	0.19	0.0964	0.0757	0.2446	0.1868	0.6454	0.488
		FK	0.4681	0.2474	0.1262	0.0843	0.1443	0.1129	0.4717	0.3337
		NPL	**0.5409**	**0.387**	**0.2107**	**0.1577**	0.0903	0.0959	0.2486	0.1878
		ML	**0.5817**	0.2142	0.1722	0.0886	**0.2488**	**0.1931**	**0.6644**	**0.5257**
		BF	0.5258	0.1598	0.1383	0.0618	0.1775	0.1363	0.5681	0.4319
60	**(10,20,30)**	TML	0.5589	0.1955	0.1603	0.0777	0.2201	0.1704	0.6211	0.4847
		FK	0.5381	0.2368	0.1506	0.0796	0.1501	0.1083	0.5165	0.3271
		NPL	0.4991	**0.398**	**0.1809**	**0.1478**	0.1199	0.084	0.3697	0.1769
		ML	**0.6242**	**0.5184**	**0.2096**	**0.1728**	**0.1785**	**0.2083**	**0.52**	**0.6234**
		BF	0.5585	0.4244	0.1608	0.1243	0.1272	0.1629	0.4241	0.5571
	(20,20,20)	TML	0.595	0.4805	0.1895	0.1529	0.157	0.1898	0.4806	0.5965
		FK	0.5038	0.448	0.1441	0.1247	0.1304	0.1461	0.4516	0.5016
		NPL	0.3766	0.4396	0.1299	0.1341	0.1397	0.1233	0.4412	0.3717
		ML	0.6508	0.3664	0.173	0.1203	**0.342**	**0.2476**	**0.8326**	**0.6684**
		BF	0.637	0.3193	0.1601	0.0978	0.2642	0.1917	0.7671	0.6045
	(15,15,60)	TML	0.6428	0.35	0.1656	0.1108	0.3187	0.2297	0.8139	0.6493
		FK	0.7121	0.4203	0.209	0.1238	0.1894	0.141	0.6589	0.48
		NPL	**0.7142**	**0.5293**	**0.2632**	**0.1857**	0.1168	0.1091	0.4251	0.2782
		ML	**0.7975**	0.3578	**0.2528**	0.1244	**0.3239**	**0.2398**	**0.8343**	**0.6662**
		BF	0.7749	0.3123	0.2257	0.1012	0.2625	0.1861	0.7847	0.606
90	**(15,30,45)**	TML	0.7895	0.3382	0.2438	0.1153	0.3	0.224	0.8167	0.6485
		FK	0.768	0.4165	0.234	0.1278	0.225	0.1388	0.738	0.4841
		NPL	0.697	**0.5331**	0.2329	**0.1913**	0.1755	0.1044	0.5957	0.2866
		ML	**0.8029**	**0.7139**	**0.2878**	**0.239**	**0.2387**	**0.2858**	**0.7168**	**0.7976**
		BF	0.7678	0.6603	0.2509	0.1983	0.1967	0.2473	0.6625	0.7649
	(30,30,30)	TML	0.7868	0.692	0.2719	0.2207	0.2204	0.2689	0.6918	0.7841
		FK	0.7211	0.6688	0.2279	0.1945	0.1987	0.2244	0.6735	0.7128
		NPL	0.5707	0.6333	0.1795	0.1902	0.1897	0.1771	0.6352	0.5535
		ML	0.8238	0.5112	0.2407	0.1551	**0.4178**	**0.2882**	**0.918**	**0.7792**
		BF	0.8207	0.4779	0.2356	0.1372	0.354	0.2438	0.891	0.7433
	(20,20,80)	TML	0.8248	0.497	0.2386	0.1492	0.389	0.2687	0.9059	0.7631
		FK	**0.8415**	0.5737	0.2768	0.171	0.286	0.1935	0.8334	0.6503
		NPL	0.8294	**0.6534**	**0.3153**	**0.2184**	0.1692	0.128	0.6044	0.3864
		ML	**0.9163**	0.5235	0.3352	0.1478	**0.4022**	**0.285**	0.927	**0.7759**
		BF	0.9079	0.491	0.3145	0.1315	0.3496	0.2392	0.9031	0.7384
120	**(20,40,60)**	TML	0.9132	0.5123	**0.3267**	0.141	0.3807	0.2646	**0.915**	0.7591
		FK	0.8959	0.5746	0.3161	0.1596	0.3139	0.191	0.8751	0.653
		NPL	0.8316	**0.6594**	0.2933	**0.209**	0.232	0.1243	0.7615	0.3988
		ML	**0.9047**	**0.8554**	**0.3624**	**0.307**	**0.3086**	**0.3596**	**0.8559**	**0.9025**
		BF	0.8938	0.8317	0.3308	0.2705	0.272	0.3249	0.8286	0.8854
	(40,40,40)	TML	0.8994	0.8456	0.3495	0.2928	0.2918	0.3462	0.8457	0.8942
		FK	0.8554	0.8294	0.3038	0.2705	0.2745	0.2913	0.8302	0.8463
		NPL	0.7142	0.7859	0.2347	0.2539	0.2549	0.2255	0.7864	0.7139

			Population variance ratio							
N	(n_1,n_2,n_3)	Test	1/1/4	1/4/4	1/1/2	1/2/2	2/2/1	2/1/1	4/4/1	4/1/1
		ML	0.9205	0.6539	0.3171	0.1823	**0.4812**	**0.3412**	**0.9637**	**0.8554**
		BF	0.9212	0.6342	0.3165	0.1683	0.4218	0.2982	0.9501	0.8313
	(25,25,100)	TML	0.9203	0.6474	0.315	0.1786	0.4585	0.3245	0.9587	0.8474
		FK	**0.93**	0.6973	0.3538	0.2016	0.3503	0.2394	0.9083	0.7518
		NPL	0.9071	**0.7403**	**0.3714**	**0.2457**	0.2208	0.1572	0.7257	0.4991
		ML	**0.9652**	0.664	**0.4216**	0.1856	**0.4815**	**0.3501**	**0.9692**	**0.8585**
		BF	0.9615	0.6433	0.4061	0.1709	0.4367	0.3088	0.9605	0.8324
150	**(25,50,75)**	TML	0.9642	0.655	0.4151	0.179	0.4651	0.3354	0.9658	0.8493
		FK	0.9529	0.7129	0.4066	0.2074	0.3939	0.2488	0.9455	0.7545
		NPL	0.9027	**0.7599**	0.3632	**0.2491**	0.2987	0.1626	0.8679	0.5036
		ML	**0.9573**	**0.936**	**0.4365**	**0.3856**	**0.3847**	**0.435**	**0.9381**	**0.9582**
		BF	0.95	0.9245	0.4083	0.3541	0.3518	0.4093	0.9261	0.9528
	(50,50,50)	TML	0.9541	0.932	0.4253	0.3723	0.3717	0.4254	0.9334	0.9559
		FK	0.9259	0.9167	0.3732	0.3473	0.3478	0.3739	0.9233	0.9283
		NPL	0.8196	0.8766	0.287	0.3183	0.3177	0.2872	0.8843	0.8179

Table 13. The power of the homogeneity tests of variance for the Cauchy distribution.

			Population variance ratio							
N	(n_1,n_2,n_3)	Test	1/1/4	1/4/4	1/1/2	1/2/2	2/2/1	2/1/1	4/4/1	4/1/1
		ML	0.1684	0.2103	**0.1935**	**0.2218**	**0.3279**	**0.2903**	**0.4102**	**0.3512**
		BF	0.0195	0.0326	0.0216	0.0345	0.0748	0.0595	0.1315	0.095
	(5,5,20)	TML	0.1578	0.2036	0.1883	0.219	0.3259	0.2883	0.4094	0.3487
		FK	0.118	0.06	0.0447	0.0356	0.022	0.0211	0.0431	0.0406
		NPL	**0.2979**	**0.2324**	0.1795	0.159	0.0991	0.1129	0.1121	0.1351
		ML	0.2219	0.2253	**0.2217**	**0.2351**	**0.3136**	**0.3003**	**0.4023**	**0.3518**
		BF	0.0269	0.0185	0.0232	0.0222	0.0464	0.0495	0.0789	0.0875
30	**(5,10,15)**	TML	0.1327	0.1022	0.1305	0.1282	0.1887	0.2019	0.2544	0.2601
		FK	0.1147	0.0665	0.0476	0.0443	0.051	0.0313	0.1044	0.046
		NPL	**0.2306**	**0.2257**	0.1335	0.1423	0.0886	0.0806	0.1287	0.1033
		ML	**0.3697**	**0.3702**	**0.3264**	**0.3231**	**0.3345**	**0.3359**	**0.3565**	**0.3905**
		BF	0.0497	0.0379	0.0245	0.0214	0.0233	0.0256	0.0375	0.0514
	(10,10,10)	TML	0.0959	0.0869	0.0636	0.0621	0.0622	0.0657	0.0856	0.0994
		FK	0.1353	0.1303	0.0737	0.0638	0.0686	0.0731	0.1252	0.1454
		NPL	0.137	0.149	0.0737	0.0714	0.072	0.0753	0.1427	0.1459
		ML	0.1633	0.215	**0.1937**	**0.2271**	**0.3365**	**0.2971**	**0.4418**	**0.3639**
		BF	0.0221	0.0384	0.0254	0.0358	0.0934	0.0738	0.1781	0.1335
	(10,10,40)	TML	0.0318	0.0548	0.0396	0.0552	0.1223	0.0986	0.2115	0.1637
		FK	0.2681	0.1591	0.0948	0.0732	0.0997	0.0787	0.2607	0.1832
		NPL	**0.3877**	**0.2639**	0.1734	0.1268	0.0815	0.08	0.1858	0.1531
		ML	0.225	0.2142	0.213	**0.2225**	**0.3184**	**0.2815**	**0.4186**	**0.3679**
		BF	0.0441	0.0213	0.0259	0.0224	0.06	0.0574	0.1146	0.1228
60	**(10,20,30)**	TML	0.0576	0.0305	0.0379	0.0342	0.0808	0.0751	0.1449	0.1485
		FK	0.3007	0.1548	0.1003	0.0723	0.1069	0.0796	0.2786	0.1861
		NPL	**0.36**	**0.2684**	**0.1393**	0.1249	0.1002	0.0684	0.2543	0.1475
		ML	**0.3815**	**0.3634**	**0.3122**	**0.3151**	**0.3025**	**0.3065**	**0.3536**	**0.3704**
		BF	0.0707	0.0502	0.0275	0.0278	0.0256	0.03	0.0477	0.0712
	(20,20,20)	TML	0.0895	0.0659	0.0398	0.0384	0.0363	0.041	0.0655	0.0873
		FK	0.2647	0.2404	0.0993	0.0942	0.0908	0.096	0.2536	0.2638
		NPL	0.2704	0.2839	0.105	0.1081	0.1018	0.1019	0.2935	0.2732
		ML	0.1658	0.2112	0.1919	0.2176	**0.3337**	**0.3037**	**0.457**	**0.3705**
		BF	0.0222	0.0384	0.0229	0.0345	0.0973	0.0817	0.1994	0.1403
90	**(15,15,60)**	TML	0.0326	0.0545	0.0365	0.0531	0.1282	0.1079	0.2374	0.1694
		FK	0.4104	0.2372	0.1321	0.0904	0.1047	0.0829	0.3385	0.2331
		NPL	**0.5143**	**0.3375**	**0.1947**	**0.1377**	0.0987	0.0931	0.3018	0.2182
	(15,30,45)	ML	0.2315	0.212	**0.209**	**0.2192**	**0.3135**	**0.2875**	**0.4269**	**0.3615**

			Population variance ratio							
N	(n_1,n_2,n_3)	Test	1/1/4	1/4/4	1/1/2	1/2/2	2/2/1	2/1/1	4/4/1	4/1/1
		BF	0.0543	0.0242	0.0265	0.0199	0.065	0.06	0.1329	0.1303
		TML	0.0647	0.0317	0.0356	0.0293	0.0833	0.0777	0.1553	0.153
		FK	0.4388	0.2417	0.1338	0.0914	0.1279	0.0799	0.4041	0.2326
		NPL	**0.5058**	**0.3424**	0.1771	0.1413	0.1254	0.0814	0.4088	0.2198
		ML	0.388	0.3635	**0.3061**	**0.2957**	**0.3014**	**0.3018**	0.3658	0.389
		BF	0.0869	0.0614	0.0338	0.0284	0.0285	0.0314	0.0648	0.0904
	(30,30,30)	TML	0.0935	0.0708	0.0388	0.0342	0.0348	0.0374	0.0725	0.0978
		FK	0.3963	0.3622	0.1267	0.1181	0.1246	0.1177	0.3742	0.3979
		NPL	**0.4049**	**0.417**	0.1394	0.1367	0.1351	0.129	**0.4308**	**0.4076**
		ML	0.166	0.2111	0.1979	**0.2237**	**0.3415**	**0.3027**	0.4648	**0.388**
		BF	0.0301	0.0399	0.0257	0.0406	0.1091	0.0846	0.2179	0.1636
	(20,20,80)	TML	0.032	0.0443	0.0287	0.0447	0.1183	0.0923	0.2295	0.1745
		FK	0.5292	0.3221	0.1655	0.1116	0.1475	0.1156	**0.4972**	0.3454
		NPL	**0.6329**	**0.421**	**0.2307**	0.1536	0.1238	0.1067	0.4391	0.305
		ML	0.2479	0.2155	**0.2163**	**0.2181**	**0.3216**	**0.2883**	0.447	**0.3705**
		BF	0.0727	0.0232	0.028	0.0229	0.0733	0.065	0.1509	0.1449
120	**(20,40,60)**	TML	0.0767	0.0258	0.0321	0.0265	0.0812	0.0716	0.1621	0.1531
		FK	0.5895	0.3118	0.1748	0.1105	0.1672	0.1126	0.5419	0.3476
		NPL	**0.6519**	**0.4159**	0.2097	0.1546	0.1583	0.1011	**0.5498**	0.3044
		ML	0.3926	0.3736	**0.2997**	**0.2999**	**0.2944**	**0.3046**	0.3586	0.3917
		BF	0.099	0.071	0.0363	0.0312	0.0275	0.0369	0.0667	0.1084
	(40,40,40)	TML	0.1051	0.079	0.0395	0.0345	0.0306	0.0399	0.0719	0.1126
		FK	0.5255	0.492	0.1564	0.15	0.1481	0.1555	0.493	0.5253
		NPL	**0.5486**	**0.5498**	0.1725	0.1732	0.1719	0.1714	**0.5556**	**0.5407**
		ML	0.1788	0.2211	0.1903	**0.2223**	**0.3487**	**0.3**	**0.4771**	0.3829
		BF	0.0343	0.0433	0.0223	0.0408	0.1154	0.0809	0.2261	0.163
	(25,25,100)	TML	0.0368	0.048	0.0255	0.0468	0.1244	0.0884	0.2424	0.1718
		FK	0.6347	0.3938	0.1928	0.1291	0.1709	0.1251	0.5794	**0.4063**
		NPL	**0.7257**	**0.4887**	**0.2544**	0.1747	0.1554	0.1283	0.5575	0.3768
		ML	0.2502	0.2066	0.2092	**0.2143**	**0.322**	**0.2913**	0.4533	0.3805
		BF	0.0745	0.0261	0.0304	0.021	0.0698	0.0672	0.1601	0.1532
150	**(25,50,75)**	TML	0.0777	0.0285	0.0329	0.0231	0.076	0.0742	0.1705	0.1618
		FK	0.6896	0.3965	0.2162	0.1307	0.2053	0.1286	0.6652	**0.4137**
		NPL	**0.747**	**0.498**	**0.2535**	0.1772	0.2043	0.1215	**0.6867**	0.3858
		ML	0.4023	0.3722	**0.3005**	**0.3004**	**0.3**	**0.2965**	0.3816	0.4057
		BF	0.1162	0.0773	0.0362	0.0323	0.0334	0.0398	0.0819	0.118
	(50,50,50)	TML	0.1193	0.0811	0.0388	0.0343	0.0359	0.0422	0.0857	0.1209
		FK	0.627	0.6006	0.1911	0.187	0.1923	0.1917	0.598	0.6233
		NPL	**0.6505**	**0.6617**	0.2052	0.2087	0.2114	0.2064	**0.6663**	**0.6529**

Table 14. The power of the homogeneity tests of variance for the Logistic distribution

			Population variance ratio							
N	(n_1,n_2,n_3)	Test	1/1/4	1/4/4	1/1/2	1/2/2	2/2/1	2/1/1	4/4/1	4/1/1
		ML	0.0677	0.2274	0.0632	0.0818	0.0645	**0.2009**	0.0656	**0.4974**
		BF	0.0219	0.1700	0.0189	0.0477	0.0189	0.0672	0.0242	0.2375
	(5,5,20)	TML	0.0635	0.2094	0.0612	0.074	0.0616	0.199	0.0632	0.4953
		FK	0.0185	0.2506	0.017	0.0686	0.0169	0.0257	0.021	0.0868
		NPL	**0.1132**	**0.456**	**0.1196**	**0.23**	**0.1153**	0.0954	**0.1196**	0.1588
		ML	0.0599	**0.3809**	0.0663	0.1304	0.0652	**0.177**	0.0645	**0.4643**
30		BF	0.022	0.2651	0.0241	0.0673	0.0269	0.0807	0.0233	0.2629
	(5,10,15)	TML	0.0555	0.3603	0.0608	0.1234	0.0615	0.1651	0.0596	0.4371
		FK	0.0182	0.26	0.0214	0.0652	0.0229	0.06	0.0187	0.1994
		NPL	**0.0874**	0.3556	**0.0905**	**0.1709**	**0.0888**	0.0998	**0.0842**	0.2151
		ML	**0.0684**	**0.4264**	**0.0691**	**0.1545**	**0.0634**	**0.1347**	**0.0649**	**0.3245**
	(10,10,10)	BF	0.0365	0.3134	0.0349	0.0917	0.0312	0.0735	0.0341	0.1997
		TML	0.0598	0.3948	0.0593	0.1384	0.0543	0.1155	0.058	0.2894

			Population variance ratio							
N	(n_1,n_2,n_3)	Test	1/1/4	1/4/4	1/1/2	1/2/2	2/2/1	2/1/1	4/4/1	4/1/1
		FK	0.0361	0.2659	0.0331	0.0824	0.0319	0.0713	0.0333	0.2063
		NPL	0.0582	0.2281	0.0552	0.0937	0.0537	0.099	0.057	0.2786
		ML	0.0579	0.6079	0.0578	0.1755	0.0542	**0.2992**	0.0592	**0.7671**
		BF	0.0377	0.5706	0.0395	0.153	0.0368	0.2197	0.0399	0.6766
	(10,10,40)	TML	0.0534	0.6003	0.0523	0.1719	0.0482	0.2706	0.0528	0.7383
		FK	0.0384	0.6214	0.0392	0.1807	0.0353	0.1675	0.039	0.561
		NPL	**0.0785**	**0.6819**	**0.0789**	**0.2681**	**0.0768**	**0.1054**	**0.0743**	0.3275
		ML	0.0585	**0.7416**	0.0591	**0.2376**	0.0571	**0.2851**	0.0583	**0.7804**
		BF	0.0386	0.6976	0.0413	0.1972	0.0377	0.2142	0.0409	0.7011
60	**(10,20,30)**	TML	0.0527	0.7288	0.0532	0.2276	0.0513	0.2615	0.0533	0.7571
		FK	0.0375	0.6815	0.0408	0.2015	0.0373	0.181	0.0414	0.6428
		NPL	**0.0615**	0.6268	**0.0645**	0.2242	**0.0616**	0.1495	**0.0671**	0.52
		ML	**0.0598**	**0.7537**	**0.0574**	**0.2521**	**0.0551**	**0.2181**	**0.0599**	**0.6578**
		BF	0.039	0.6983	0.04	0.2001	0.0368	0.1644	0.0398	0.5727
	(20,20,20)	TML	0.0545	0.7381	0.0522	0.2363	0.0504	0.2041	0.0547	0.6352
		FK	0.0384	0.6336	0.0395	0.1748	0.0357	0.1592	0.0381	0.5659
		NPL	0.0509	0.4953	0.0526	0.1584	0.0486	0.1831	0.0503	0.5813
		ML	0.0591	0.8497	0.0546	0.2784	0.0541	**0.3869**	0.0528	**0.912**
		BF	0.0415	0.839	0.0396	0.2618	0.0396	0.3048	0.0377	0.8691
	(15,15,60)	TML	0.0546	0.8457	0.0507	0.2742	0.0507	0.3706	0.0491	0.9021
		FK	0.0388	**0.8583**	0.038	0.2936	0.0377	0.2329	0.0384	0.7855
		NPL	**0.0649**	0.8497	**0.0623**	**0.3378**	**0.0601**	0.1593	**0.0644**	0.5663
		ML	**0.0572**	**0.9196**	0.0579	**0.3619**	0.0547	**0.4019**	0.0575	**0.9243**
		BF	0.0417	0.9074	0.0434	0.3318	0.0387	0.3343	0.0411	0.8933
90	**(15,30,45)**	TML	0.0535	0.916	0.0556	0.3579	0.0524	0.3856	0.0536	0.9166
		FK	0.0404	0.8901	0.0421	0.3276	0.0369	0.2985	0.0399	0.8576
		NPL	0.0557	0.8259	**0.0616**	0.3088	**0.0566**	0.2329	**0.0578**	0.7483
		ML	**0.0558**	**0.9031**	**0.0534**	**0.3632**	**0.0575**	**0.312**	**0.0535**	**0.8695**
		BF	0.042	0.8853	0.0414	0.3206	0.042	0.2683	0.0416	0.8345
	(30,30,30)	TML	0.0516	0.8972	0.0494	0.3521	0.0517	0.3015	0.0504	0.8579
		FK	0.0401	0.8417	0.0389	0.2912	0.04	0.2562	0.0395	0.8226
		NPL	0.0501	0.699	0.0495	0.2333	0.0517	0.2529	0.0487	0.8049
		ML	0.052	**0.9535**	0.0523	0.3805	0.056	**0.4922**	0.0531	**0.9687**
		BF	0.0416	0.9497	0.0416	0.3728	0.0456	0.4313	0.0426	0.9579
	(20,20,80)	TML	0.0491	0.952	0.0488	0.38	0.0537	0.4753	0.0503	0.9656
		FK	0.0413	0.9502	0.0422	0.392	0.0448	0.3675	0.0423	0.9263
		NPL	**0.0643**	0.9383	**0.0584**	**0.4197**	**0.0617**	0.2325	**0.0571**	0.7397
		ML	0.0537	**0.98**	**0.0541**	**0.4852**	0.0537	**0.496**	**0.0542**	**0.9794**
		BF	0.0435	0.9763	0.0448	0.4621	0.0426	0.4509	0.0455	0.9719
120	**(20,40,60)**	TML	0.051	0.9787	0.0506	0.4816	0.0507	0.4831	0.0516	0.9771
		FK	0.0412	0.9695	0.0423	0.4494	0.0437	0.4094	0.0425	0.9558
		NPL	**0.0557**	0.9265	0.0539	0.4038	**0.0572**	0.3197	0.0541	0.8867
		ML	**0.057**	**0.9691**	**0.0526**	**0.4689**	**0.0567**	**0.4057**	**0.0541**	**0.9542**
		BF	0.0469	0.9627	0.0436	0.4354	0.0468	0.3683	0.0426	0.9416
	(40,40,40)	TML	0.0529	0.9672	0.0503	0.4614	0.0539	0.3967	0.05	0.9494
		FK	0.0454	0.9425	0.0427	0.3967	0.0442	0.3595	0.0411	0.935
		NPL	0.0533	0.8412	0.049	0.3054	0.0521	0.3429	0.0528	0.9116
		ML	0.0516	**0.9854**	0.0523	0.4774	0.052	**0.5776**	0.056	**0.9895**
		BF	0.0435	0.9853	0.0425	0.4748	0.0441	0.5177	0.0453	0.9849
	(25,25,100)	TML	0.0502	0.9852	0.0494	0.4755	0.0504	0.5628	0.0535	0.989
		FK	0.0429	0.9838	0.0435	**0.4884**	0.0424	0.4526	0.0457	0.9707
150		NPL	**0.0549**	0.9751	**0.0574**	0.4859	**0.0593**	0.3023	**0.0575**	0.8639
		ML	0.0499	0.9954	0.0509	0.5741	0.0544	0.5909	0.0484	0.9945
		BF	0.0427	0.9951	0.0418	0.5579	0.0462	0.5466	0.0399	0.9925
	(25,50,75)	TML	0.0486	0.9954	0.0489	0.5708	0.0527	0.5784	0.0456	0.9935
		FK	0.0442	0.9912	0.0419	0.543	0.0445	0.5097	0.0376	0.9871

			Population variance ratio							
N	(n_1,n_2,n_3)	Test	1/1/4	1/4/4	1/1/2	1/2/2	2/2/1	2/1/1	4/4/1	4/1/1
		NPL	0.0538	0.974	0.0526	0.4784	0.0542	0.4018	0.0499	0.9545
		ML	**0.054**	**0.9894**	**0.0481**	**0.5575**	0.049	**0.5103**	**0.0536**	**0.9866**
		BF	0.0468	0.9881	0.0413	0.5305	0.0422	0.4781	0.0474	0.9832
	(50,50,50)	TML	0.0526	0.9885	0.047	0.5502	0.0476	0.4996	0.0517	0.9855
		FK	0.0451	0.9801	0.0392	0.4879	0.0413	0.4639	0.0461	0.9791
		NPL	0.0489	0.9173	0.0467	0.3752	**0.0482**	0.4288	0.051	0.9647

Table 15. The power of the homogeneity tests of variance for the Chi-square distribution

			Population variance ratio							
N	(n_1,n_2,n_3)	Test	1/1/4	1/4/4	1/1/2	1/2/2	2/2/1	2/1/1	4/4/1	4/1/1
		ML	**0.2735**	**0.2662**	**0.2262**	**0.2337**	0**.3728**	0.316	**0.5453**	**0.4249**
		BF	0.0316	0.0378	0.0246	0.032	0.0787	0.0638	0.1805	0.1269
	(5,5,20)	TML	0.1334	0.175	0.1351	0.1669	0.3384	**0.2717**	0.5257	0.3931
		FK	0.2215	0.1346	0.1239	0.0882	0.0513	0.0607	0.0762	0.0717
		NPL	0.1739	0.1582	0.1363	0.1283	0.1	0.1136	0.1015	0.1122
		ML	**0.397**	**0.2908**	**0.2893**	**0.263**	**0.3505**	**0.3299**	**0.5227**	**0.4454**
		BF	0.0778	0.0509	0.0432	0.0382	0.072	0.0643	0.1563	0.1344
30	**(5,10,15)**	TML	0.2695	0.1697	0.1907	0.1642	0.2635	0.2586	0.4123	0.3892
		FK	0.2212	0.1622	0.1101	0.1145	0.1082	0.0752	0.2057	0.0955
		NPL	0.1189	0.1348	0.1023	0.1118	0.0845	0.0795	0.0867	0.0874
		ML	**0.4626**	**0.4406**	**0.3321**	**0.3255**	**0.3284**	**0.3277**	**0.4403**	**0.4704**
		BF	0.1354	0.1048	0.0676	0.0643	0.0684	0.0682	0.1093	0.1397
	(10,10,10)	TML	0.293	0.2629	0.1829	0.1781	0.1832	0.1828	0.2597	0.2936
		FK	0.2695	0.2596	0.1636	0.1623	0.1667	0.1651	0.2627	0.2698
		NPL	0.0622	0.0647	0.0515	0.0552	0.0551	0.0545	0.0663	0.0562
		ML	**0.4636**	**0.3717**	**0.2752**	**0.2692**	**0.4093**	**0.3316**	**0.6548**	**0.5391**
		BF	0.0968	0.067	0.0358	0.0432	0.1468	0.1024	0.3662	0.2675
	(10,10,40)	TML	0.2382	0.1746	0.1126	0.1228	0.2663	0.1937	0.5067	0.3946
		FK	0.4495	0.328	0.2302	0.1982	0.2205	0.1988	0.4296	0.3337
		NPL	0.1595	0.1313	0.1055	0.0925	0.0693	0.0737	0.072	0.0777
		ML	**0.5878**	**0.3815**	**0.3454**	**0.2746**	**0.4006**	**0.3535**	**0.667**	**0.5447**
		BF	0.21	0.0732	0.0736	0.0443	0.1202	0.1087	0.3082	0.2643
60	**(10,20,30)**	TML	0.3694	0.1848	0.1746	0.1234	0.2377	0.2111	0.4827	0.3917
		FK	0.4955	0.3366	0.2564	0.1942	0.2451	0.1995	0.4781	0.3421
		NPL	0.1073	0.1302	0.0795	0.0828	0.0616	0.0604	0.0838	0.0624
		ML	**0.6191**	**0.5847**	**0.373**	**0.3608**	**0.3596**	**0.3637**	**0.5839**	**0.6091**
		BF	0.2875	0.1887	0.1013	0.0856	0.085	0.1002	0.1913	0.2771
	(20,20,20)	TML	0.4359	0.3645	0.2097	0.1928	0.1901	0.2088	0.3649	0.4244
		FK	0.4684	0.4488	0.2441	0.2349	0.2407	0.2423	0.4479	0.4565
		NPL	0.0692	0.0902	0.0576	0.0583	0.0632	0.057	0.0902	0.0721
		ML	**0.6285**	**0.4789**	**0.3301**	**0.3061**	**0.4481**	**0.3755**	**0.7635**	**0.6256**
		BF	0.2109	0.1033	0.0551	0.0502	0.1766	0.1301	0.5035	0.357
	(15,15,60)	TML	0.3727	0.2508	0.1456	0.1433	0.3329	0.259	0.6726	0.5137
		FK	0.6219	0.4504	0.3009	0.2396	0.2499	0.2182	0.5458	0.4209
		NPL	0.1749	0.1445	0.0979	0.0795	0.0587	0.0663	0.0707	0.0712
		ML	**0.7364**	**0.4909**	**0.4002**	**0.3124**	**0.442**	**0.3733**	**0.7701**	**0.6254**
		BF	0.3791	0.1128	0.1026	0.0578	0.1561	0.1323	0.4569	0.3641
90	**(15,30,45)**	TML	0.5599	0.2527	0.2253	0.1485	0.2871	0.2504	0.6249	0.5137
		FK	0.6759	0.4726	0.3268	0.2629	0.3128	0.2292	0.6311	0.4329
		NPL	0.1224	0.149	0.079	0.0809	0.0629	0.0537	0.0954	0.0638
		ML	**0.7328**	**0.7027**	**0.4181**	**0.4071**	**0.404**	**0.4107**	**0.7071**	**0.7284**
		BF	0.4317	0.3159	0.1373	0.1088	0.1112	0.1329	0.3128	0.4331
	(30,30,30)	TML	0.5715	0.4908	0.2432	0.2242	0.2223	0.2388	0.4927	0.5674
		FK	0.6318	0.62	0.315	0.32	0.3162	0.3184	0.6207	0.6185
		NPL	0.0893	0.1156	0.0572	0.0648	0.0635	0.0613	0.1129	0.0825
120	**(20,20,80)**	ML	**0.7548**	**0.5841**	**0.388**	**0.3348**	**0.4908**	**0.4111**	**0.8334**	**0.6943**

			Population variance ratio							
N	(n_1,n_2,n_3)	Test	1/1/4	1/4/4	1/1/2	1/2/2	2/2/1	2/1/1	4/4/1	4/1/1
		BF	0.352	0.1527	0.0831	0.0665	0.2213	0.1542	0.6209	0.4676
		TML	0.5279	0.3172	0.1872	0.1559	0.3348	0.2555	0.7318	0.571
		FK	0.7489	0.5783	0.3845	0.3127	0.3616	0.2934	0.7098	0.5627
		NPL	0.1992	0.1571	0.1011	0.0876	0.0574	0.0624	0.0789	0.0705
		ML	**0.8385**	0.5765	**0.452**	**0.3339**	**0.4895**	**0.4042**	**0.8518**	**0.6976**
		BF	0.5394	0.1601	0.1425	0.0698	0.1955	0.152	0.5875	0.4644
	(20,40,60)	TML	0.6742	0.3204	0.257	0.1543	0.3201	0.2513	0.7243	0.5722
		FK	0.7915	**0.58**	0.4206	0.317	0.403	0.3011	0.7781	0.5754
		NPL	0.1564	0.1625	0.0859	0.0816	0.0716	0.0525	0.113	0.0694
		ML	**0.8154**	**0.7913**	**0.4524**	**0.4362**	**0.4594**	**0.4615**	**0.7903**	**0.8189**
		BF	0.5698	0.4355	0.1677	0.1322	0.1423	0.1743	0.4321	0.5661
	(40,40,40)	TML	0.6888	0.6091	0.2764	0.246	0.2636	0.2931	0.5974	0.6862
		FK	0.7414	0.7384	0.3879	0.396	0.3971	0.4004	0.7433	0.7453
		NPL	0.0965	0.1446	0.0648	0.0718	0.0707	0.0666	0.1383	0.0988
		ML	**0.842**	**0.6673**	0.4307	**0.3562**	**0.5342**	**0.4416**	**0.885**	**0.7586**
		BF	0.4915	0.2333	0.1114	0.078	0.2567	0.1857	0.7115	0.5425
	(25,25,100)	TML	0.6501	0.4115	0.2146	0.1756	0.4016	0.3071	0.8234	0.6611
		FK	0.8412	0.674	**0.4504**	0.3463	0.4076	0.3408	0.7938	0.6479
		NPL	0.2269	0.1818	0.106	0.0878	0.0623	0.059	0.0994	0.0794
		ML	**0.8935**	0.6645	**0.4913**	0.3651	**0.5341**	**0.4448**	**0.9007**	**0.7646**
		BF	0.6616	0.2333	0.1771	0.0828	0.2357	0.1882	0.6884	0.5531
150	**(25,50,75)**	TML	0.7762	0.4132	0.3081	0.1735	0.3678	0.3054	0.8055	0.6662
		FK	0.871	**0.683**	0.4763	**0.3758**	0.4627	0.3593	0.8507	0.6599
		NPL	0.1744	0.1817	0.0888	0.0826	0.0691	0.0599	0.1324	0.0715
		ML	**0.8732**	**0.8658**	**0.5061**	**0.4831**	**0.4877**	**0.5035**	**0.8639**	**0.8787**
		BF	0.6772	0.5623	0.2122	0.1683	0.174	0.2121	0.5581	0.6826
	(50,50,50)	TML	0.7726	0.7132	0.3282	0.2877	0.2963	0.3309	0.7109	0.7832
		FK	0.8308	0.8378	0.465	0.4504	0.4625	0.4663	0.8383	0.8381
		NPL	0.119	0.1707	0.0707	0.0785	0.0793	0.0738	0.1759	0.1198

Table 16. The power of the homogeneity tests of variance for the Gamma distribution

			Population variance ratio							
N	(n_1,n_2,n_3)	Test	1/1/4	1/4/4	1/1/2	1/2/2	2/2/1	2/1/1	4/4/1	4/1/1
		ML	**0.2924**	**0.2438**	**0.1772**	**0.1871**	**0.317**	**0.2585**	**0.5357**	**0.4043**
		BF	0.0854	0.0463	0.0355	0.0351	0.0726	0.059	0.1989	0.1476
	(5,5,20)	TML	0.1812	0.1783	0.1174	0.1409	0.2998	0.2337	0.5266	0.3859
		FK	0.2458	0.112	0.1009	0.0659	0.0322	0.0419	0.0743	0.0689
		NPL	0.1869	0.1792	0.1471	0.1439	0.1057	0.1215	0.117	0.1299
		ML	**0.419**	**0.2479**	**0.2347**	**0.1905**	**0.3036**	**0.2656**	**0.5213**	**0.4208**
		BF	0.1484	0.0607	0.0554	0.0406	0.0811	0.0616	0.2024	0.148
30	**(5,10,15)**	TML	0.3269	0.1612	0.1762	0.13	0.2404	0.2246	0.4382	0.3881
		FK	0.2439	0.1222	0.0949	0.0727	0.0835	0.0441	0.2055	0.0749
		NPL	0.1164	0.1687	0.1033	0.1186	0.0862	0.0887	0.0867	0.1099
		ML	**0.4676**	**0.4226**	**0.2612**	**0.2538**	**0.2527**	**0.2732**	**0.4196**	**0.4637**
		BF	0.2092	0.1463	0.0808	0.0708	0.0695	0.084	0.1515	0.2067
	(10,10,10)	TML	0.3521	0.2889	0.1665	0.1621	0.1613	0.1831	0.2918	0.3462
		FK	0.2613	0.241	0.1166	0.1095	0.1138	0.1251	0.2379	0.2551
		NPL	0.0522	0.0674	0.0513	0.0554	0.0536	0.0543	0.0699	0.047
		ML	**0.5788**	**0.3942**	**0.257**	**0.2219**	**0.3706**	**0.3087**	**0.7164**	**0.5709**
		BF	0.2699	0.1238	0.0743	0.0587	0.1723	0.1283	0.4998	0.3639
	(10,10,40)	TML	0.4078	0.2367	0.1435	0.1222	0.2684	0.2163	0.6201	0.4718
		FK	0.5423	0.328	0.1995	0.1476	0.1847	0.1487	0.4705	0.3461
60		NPL	0.1908	0.1778	0.1103	0.0989	0.0699	0.0718	0.0867	0.0905
		ML	**0.6843**	**0.4091**	**0.3242**	**0.2289**	**0.3743**	**0.3149**	**0.7246**	**0.5819**
	(10,20,30)	BF	0.4191	0.1313	0.1132	0.0588	0.1572	0.13	0.4709	0.3737
		TML	0.5504	0.2489	0.206	0.1272	0.2604	0.2148	0.6059	0.4801

			Population variance ratio							
N	(n_1,n_2,n_3)	Test	1/1/4	1/4/4	1/1/2	1/2/2	2/2/1	2/1/1	4/4/1	4/1/1
		FK	0.5903	0.3325	0.2175	0.144	0.2047	0.1438	0.5424	0.3535
		NPL	0.1068	0.1846	0.0818	0.0909	0.0648	0.0653	0.0766	0.0747
		ML	**0.6854**	**0.6347**	**0.3406**	**0.3236**	**0.3345**	**0.3392**	**0.6318**	**0.6831**
		BF	0.457	0.3237	0.1378	0.1107	0.1133	0.1386	0.3247	0.4569
	(20,20,20)	TML	0.575	0.482	0.2268	0.2044	0.2142	0.2293	0.4766	0.5736
		FK	0.5314	0.4914	0.1977	0.1939	0.1961	0.1979	0.4899	0.5281
		NPL	0.0637	0.1002	0.0537	0.066	0.0649	0.0561	0.0958	0.0646
		ML	**0.7578**	**0.548**	**0.3405**	**0.2698**	**0.442**	**0.3415**	**0.8343**	**0.6799**
		BF	0.4827	0.2349	0.1249	0.0808	0.2261	0.1546	0.6719	0.5018
	(15,15,60)	TML	0.604	0.3797	0.2067	0.1599	0.3596	0.2592	0.7814	0.6168
		FK	0.7409	0.5078	0.2981	0.2002	0.2273	0.1724	0.6419	0.4729
		NPL	0.2123	0.2088	0.1068	0.0989	0.0608	0.0618	0.0743	0.0759
		ML	**0.8357**	**0.5503**	**0.4085**	**0.277**	**0.4483**	**0.3584**	**0.8504**	**0.6848**
		BF	0.6458	0.2249	0.1863	0.0816	0.2099	0.1693	0.6699	0.5054
90	**(15,30,45)**	TML	0.7443	0.3706	0.2851	0.1596	0.3335	0.2742	0.7778	0.6177
		FK	0.7796	0.5022	0.3194	0.2044	0.2837	0.1922	0.7407	0.4803
		NPL	0.1275	0.2059	0.0833	0.0868	0.0667	0.058	0.0952	0.0654
		ML	**0.8268**	**0.7886**	**0.4161**	**0.3908**	**0.401**	**0.4097**	**0.7861**	**0.8248**
		BF	0.6639	0.5324	0.1995	0.1659	0.1689	0.1998	0.5336	0.6587
	(30,30,30)	TML	0.7469	0.661	0.297	0.2654	0.271	0.2913	0.6684	0.7375
		FK	0.7268	0.7039	0.291	0.281	0.2812	0.2846	0.7094	0.7239
		NPL	0.0683	0.1395	0.0604	0.0705	0.0677	0.0568	0.1418	0.0756
		ML	**0.8697**	**0.6672**	**0.4261**	**0.3214**	**0.4902**	**0.3945**	**0.9056**	**0.7859**
		BF	0.6825	0.353	0.1852	0.112	0.2842	0.211	0.8026	0.6473
	(20,20,80)	TML	0.7734	0.4977	0.2811	0.1935	0.3825	0.2894	0.8601	0.7197
		FK	0.8658	0.6453	0.3828	0.2659	0.3252	0.2543	0.8105	0.6509
		NPL	0.2551	0.2403	0.1137	0.1037	0.063	0.066	0.0791	0.069
		ML	**0.9212**	**0.6737**	**0.4841**	**0.3148**	**0.5114**	**0.3951**	**0.9285**	**0.7839**
		BF	0.8159	0.3577	0.2549	0.1049	0.2816	0.2053	0.8167	0.6448
120	**(20,40,60)**	TML	0.8661	0.5033	0.3525	0.186	0.3931	0.2888	0.8778	0.7206
		FK	0.9022	0.6551	0.4071	0.2618	0.3791	0.2575	0.8746	0.6479
		NPL	0.1484	0.2474	0.0882	0.0915	0.0655	0.0563	0.1101	0.0605
		ML	**0.902**	**0.8841**	**0.486**	**0.4589**	**0.4576**	**0.4859**	**0.8859**	**0.904**
		BF	0.793	0.7119	0.2674	0.2212	0.2213	0.2673	0.705	0.796
	(40,40,40)	TML	0.8459	0.8011	0.3651	0.3267	0.3242	0.3634	0.7989	0.8512
		FK	0.8465	0.8478	0.3746	0.3676	0.3582	0.3702	0.8496	0.8516
		NPL	0.0776	0.1902	0.0659	0.0775	0.0839	0.0674	0.1797	0.0856
		ML	**0.9342**	**0.7669**	**0.4771**	**0.3558**	**0.5637**	**0.4428**	**0.9495**	**0.8396**
		BF	0.8255	0.4761	0.2407	0.1343	0.3479	0.252	0.8844	0.7259
	(25,25,100)	TML	0.8782	0.6225	0.3288	0.2224	0.4701	0.348	0.9256	0.7934
		FK	0.936	0.7612	0.4548	0.3047	0.4085	0.2989	0.8914	0.7278
		NPL	0.2976	0.2807	0.1169	0.1048	0.0602	0.0577	0.0854	0.0757
		ML	**0.9603**	**0.767**	**0.5577**	**0.3675**	**0.5672**	**0.4405**	**0.9637**	**0.8541**
		BF	0.9017	0.4808	0.3222	0.1375	0.3366	0.2448	0.9016	0.7389
150	**(25,50,75)**	TML	0.9327	0.6182	0.4316	0.2298	0.451	0.3451	0.9373	0.8057
		FK	0.9535	0.7611	0.4959	0.3209	0.4558	0.3005	0.938	0.7443
		NPL	0.1754	0.2794	0.093	0.1012	0.0736	0.057	0.1401	0.0667
		ML	**0.9497**	**0.9415**	**0.5365**	**0.5246**	**0.5208**	**0.5469**	**0.9426**	**0.9487**
		BF	0.8902	0.8295	0.3164	0.2768	0.2793	0.3318	0.8328	0.8862
	(50,50,50)	TML	0.9234	0.8895	0.4149	0.3816	0.3864	0.4273	0.8916	0.9196
		FK	0.9238	0.9264	0.4388	0.444	0.4431	0.4561	0.9251	0.9221
		NPL	0.0885	0.2356	0.0743	0.0893	0.0852	0.0708	0.2334	0.0894

Table 17. The power of the homogeneity tests of variance for the Log-Normal distribution

			Population variance ratio							
N	(n_1,n_2,n_3)	Test	1/1/4	1/4/4	1/1/2	1/2/2	2/2/1	2/1/1	4/4/1	4/1/1
		ML	**0.2401**	**0.2469**	**0.1995**	**0.226**	**0.3383**	**0.3018**	**0.4892**	**0.4054**
		BF	0.0373	0.0386	0.0258	0.0334	0.0764	0.065	0.1657	0.1268
	(5,5,20)	TML	0.1378	0.1823	0.1389	0.1784	0.3214	0.2751	0.4791	0.3852
		FK	0.2245	0.1148	0.0948	0.0671	0.0372	0.0409	0.0645	0.066
		NPL	0.1388	0.1567	0.1237	0.1301	0.1217	0.1253	0.1457	0.1562
		ML	**0.3556**	**0.2607**	**0.263**	**0.2469**	**0.3328**	**0.3098**	**0.4857**	**0.4081**
		BF	0.0717	0.0411	0.0369	0.0342	0.0687	0.0624	0.1423	0.1215
30	**(5,10,15)**	TML	0.24	0.1401	0.1749	0.1488	0.2396	0.2414	0.368	0.351
		FK	0.2109	0.1353	0.0959	0.0872	0.0919	0.0578	0.1888	0.0777
		NPL	0.0858	0.1346	0.0835	0.0973	0.0869	0.1024	0.087	0.1427
		ML	**0.4326**	**0.4152**	**0.3049**	**0.3069**	**0.3115**	**0.3198**	**0.4058**	**0.4271**
		BF	0.1219	0.0929	0.0555	0.0537	0.0514	0.0604	0.0911	0.1259
	(10,10,10)	TML	0.2456	0.2235	0.1438	0.1464	0.1501	0.1583	0.2143	0.2496
		FK	0.2399	0.2301	0.1281	0.1289	0.1301	0.1332	0.2183	0.2382
		NPL	0.0465	0.043	0.0482	0.0546	0.0491	0.0512	0.041	0.0433
		ML	**0.3809**	**0.3031**	**0.2409**	**0.2416**	**0.3785**	**0.3269**	**0.5923**	**0.479**
		BF	0.0792	0.0569	0.0298	0.0435	0.1457	0.1076	0.3396	0.2477
	(10,10,40)	TML	0.1718	0.1306	0.089	0.0954	0.2284	0.1825	0.4455	0.3348
		FK	0.4597	0.3021	0.2032	0.1618	0.1919	0.154	0.4004	0.3066
		NPL	0.1064	0.1156	0.0812	0.084	0.0869	0.0824	0.1068	0.1192
		ML	**0.509**	**0.3275**	**0.3026**	**0.2521**	**0.3593**	**0.3282**	**0.6003**	**0.4836**
		BF	0.1833	0.0522	0.058	0.0404	0.1064	0.1007	0.2768	0.2438
60	**(10,20,30)**	TML	0.2962	0.1283	0.1252	0.098	0.1944	0.1747	0.3957	0.3343
		FK	0.498	0.2899	0.217	0.1614	0.2007	0.1629	0.4605	0.3028
		NPL	0.0615	0.1119	0.0612	0.0742	0.0647	0.0773	0.0631	0.107
		ML	**0.5536**	**0.5174**	**0.3426**	**0.3308**	**0.3308**	**0.3344**	**0.5141**	**0.5539**
		BF	0.2491	0.1661	0.0799	0.0677	0.0669	0.0824	0.1609	0.2457
	(20,20,20)	TML	0.3511	0.2839	0.1623	0.1451	0.1472	0.1601	0.2879	0.3515
		FK	0.4567	0.4225	0.1956	0.1913	0.1922	0.2015	0.4225	0.4557
		NPL	0.045	0.0446	0.0484	0.0487	0.0489	0.0494	0.0471	0.0429
		ML	0.5045	0.3948	**0.2691**	**0.2622**	**0.409**	**0.3382**	**0.6786**	**0.547**
		BF	0.1629	0.0847	0.0404	0.0421	0.1716	0.1159	0.4408	0.3193
	(15,15,60)	TML	0.263	0.1785	0.099	0.109	0.2807	0.2088	0.5731	0.4345
		FK	**0.6478**	**0.4275**	0.2677	0.1922	0.2215	0.1774	0.5342	0.4074
		NPL	0.0956	0.1091	0.0695	0.0668	0.0688	0.069	0.0857	0.1125
		ML	0.6343	0.3946	**0.3481**	**0.2731**	**0.4008**	**0.3421**	**0.6773**	**0.5628**
		BF	0.3114	0.0893	0.0812	0.0403	0.1405	0.114	0.3901	0.3284
90	**(15,30,45)**	TML	0.4271	0.1755	0.1649	0.0937	0.2325	0.2027	0.5098	0.435
		FK	**0.6904**	**0.4373**	0.2921	0.2003	0.2699	0.1809	0.6376	0.4164
		NPL	0.0482	0.1042	0.0556	0.0651	0.0554	0.0647	0.047	0.1066
		ML	**0.6428**	**0.6185**	**0.3703**	**0.3766**	**0.3596**	**0.3737**	**0.6257**	**0.6535**
		BF	0.369	0.2585	0.1117	0.0962	0.0872	0.1066	0.262	0.3686
	(30,30,30)	TML	0.4607	0.3786	0.185	0.1731	0.1585	0.1826	0.3775	0.4645
		FK	0.6254	0.6104	0.2705	0.2648	0.2547	0.2663	0.6056	0.6214
		NPL	0.0453	0.0466	0.0459	0.0506	0.0458	0.0477	0.0496	0.0476
		ML	0.6097	0.4576	0.3177	**0.2861**	**0.4289**	**0.3607**	**0.7397**	**0.616**
		BF	0.2589	0.1225	0.0621	0.0563	0.2009	0.1453	0.5553	0.4116
	(20,20,80)	TML	0.362	0.2103	0.1223	0.1073	0.2735	0.2118	0.6273	0.4865
		FK	**0.7675**	**0.5504**	**0.3367**	0.2543	0.2988	0.2384	0.7151	0.5531
		NPL	0.0841	0.11	0.059	0.0694	0.0632	0.0669	0.0878	0.1131
120		ML	0.7168	0.4668	**0.3803**	**0.2987**	**0.4357**	**0.3661**	0.7589	**0.6177**
		BF	0.4259	0.1274	0.1087	0.0524	0.1707	0.1392	0.5149	0.4008
	(20,40,60)	TML	0.5206	0.2202	0.1762	0.1038	0.2442	0.2041	0.6069	0.478
		FK	**0.8006**	**0.5563**	0.368	0.2581	0.3471	0.2451	**0.7766**	0.5551
		NPL	0.0441	0.1119	0.0535	0.0603	0.0501	0.0662	0.0444	0.1053

			Population variance ratio							
N	(n_1,n_2,n_3)	Test	1/1/4	1/4/4	1/1/2	1/2/2	2/2/1	2/1/1	4/4/1	4/1/1
		ML	0.7238	0.6896	**0.4045**	**0.3986**	**0.3942**	**0.4084**	0.6885	0.7272
		BF	0.4773	0.355	0.1448	0.1154	0.1092	0.1404	0.3592	0.4866
	(40,40,40)	TML	0.5626	0.4662	0.2133	0.189	0.1851	0.2117	0.4684	0.5677
		FK	**0.7509**	**0.7398**	0.3352	0.3267	0.3165	0.3358	**0.7409**	**0.7615**
		NPL	0.0494	0.0464	0.0481	0.0478	0.0524	0.0507	0.0505	0.0468
		ML	0.7009	0.5237	0.3481	0.2995	**0.4605**	**0.3734**	0.7919	**0.6711**
		BF	0.3655	0.1662	0.0781	0.0604	0.2325	0.1597	0.6321	0.4779
	(25,25,100)	TML	0.4644	0.2708	0.1415	0.1175	0.3262	0.2298	0.713	0.5622
		FK	**0.8523**	**0.6582**	**0.3992**	0.2848	0.3496	0.2732	**0.8072**	0.6397
		NPL	0.0902	0.1086	0.0612	0.0608	0.065	0.0641	0.0899	0.1154
		ML	0.7878	0.5396	0.4225	**0.3109**	**0.4716**	**0.3934**	0.8139	**0.6709**
		BF	0.5406	0.173	0.1365	0.0611	0.2054	0.1646	0.6061	0.475
150	**(25,50,75)**	TML	0.6245	0.2794	0.2168	0.114	0.2888	0.2354	0.6868	0.5566
		FK	**0.8841**	**0.6598**	**0.4285**	0.2879	0.4043	0.2815	**0.8685**	0.6465
		NPL	0.0435	0.1124	0.0489	0.0605	0.0494	0.0616	0.0438	0.1089
		ML	0.7895	0.7534	**0.4378**	**0.4311**	**0.4245**	**0.4328**	0.7589	0.7859
		BF	0.5804	0.4583	0.1709	0.1425	0.1378	0.1771	0.4595	0.5861
	(50,50,50)	TML	0.6492	0.558	0.2402	0.2199	0.2119	0.2448	0.5611	0.657
		FK	**0.8405**	**0.8358**	0.3952	0.393	0.3855	0.396	**0.8375**	**0.8471**
		NPL	0.0518	0.0503	0.047	0.0472	0.0489	0.0484	0.052	0.0478

Table 18. The power of the homogeneity tests of variance for the Laplace-square distribution

			Population variance ratio							
N	(n_1,n_2,n_3)	Test	1/1/4	1/4/4	1/1/2	1/2/2	2/2/1	2/1/1	4/4/1	4/1/1
		ML	**0.2481**	**0.2832**	**0.2581**	**0.2946**	**0.3956**	**0.3619**	**0.5071**	**0.4214**
		BF	0.0152	0.0378	0.0244	0.0325	0.0815	0.0681	0.1438	0.1014
	(5,5,20)	TML	0.1283	0.1963	0.1717	0.2218	0.3577	0.3137	0.4873	0.3831
		FK	0.2085	0.1495	0.1411	0.1134	0.0664	0.0813	0.0817	0.0857
		NPL	0.1456	0.1381	0.1373	0.1306	0.1074	0.1141	0.109	0.1141
		ML	**0.3752**	**0.3229**	**0.3287**	**0.3364**	**0.4057**	**0.3947**	**0.505**	**0.4597**
		BF	0.0364	0.029	0.0279	0.0324	0.0586	0.0576	0.0985	0.0993
30	**(5,10,15)**	TML	0.2181	0.1576	0.1929	0.1854	0.265	0.2769	0.3487	0.3524
		FK	0.2097	0.1849	0.1413	0.1522	0.1334	0.1082	0.195	0.1141
		NPL	0.098	0.1127	0.098	0.101	0.0804	0.0836	0.0843	0.0861
		ML	**0.4751**	**0.4577**	**0.4056**	**0.4018**	**0.3995**	**0.4029**	**0.4618**	**0.4718**
		BF	0.0721	0.0639	0.0474	0.0466	0.0463	0.0406	0.0592	0.0725
	(10,10,10)	TML	0.2171	0.2071	0.1675	0.1672	0.1643	0.1631	0.204	0.2155
		FK	0.2876	0.2836	0.2144	0.2176	0.2184	0.2108	0.2785	0.274
		NPL	0.056	0.0617	0.0507	0.0533	0.0521	0.0565	0.0515	0.0567
		ML	0.3295	0.3216	**0.2825**	**0.2966**	**0.4088**	**0.3739**	**0.5792**	**0.4679**
		BF	0.0187	0.0428	0.0209	0.0373	0.1172	0.0913	0.2479	0.1659
	(10,10,40)	TML	0.0813	0.1081	0.0765	0.101	0.2138	0.1746	0.3725	0.2668
		FK	**0.4011**	**0.3348**	0.2692	0.2461	0.2716	0.2585	0.4194	0.3391
		NPL	0.1119	0.1043	0.0911	0.0861	0.0668	0.0793	0.068	0.0731
		ML	0.4586	0.3492	0.35	0.3148	0.4231	0.3989	0.5762	0.4974
		BF	0.0627	0.0314	0.0311	0.0274	0.0826	0.0811	0.1595	0.1676
60	**(10,20,30)**	TML	0.1715	0.1071	0.1131	0.0972	0.1869	0.1685	0.301	0.2754
		FK	0.4453	0.34	0.2837	0.2563	0.2928	0.2578	0.4411	0.3554
		NPL	0.082	0.0899	0.0709	0.0729	0.0608	0.0628	0.0667	0.061
		ML	**0.5279**	**0.5075**	**0.3959**	**0.3935**	**0.3981**	**0.3934**	**0.5133**	**0.5306**
		BF	0.1194	0.0816	0.0518	0.0475	0.0465	0.0515	0.0876	0.1207
	(20,20,20)	TML	0.2436	0.2085	0.1418	0.1386	0.1408	0.1423	0.211	0.2464
		FK	0.4146	0.4164	0.2758	0.2741	0.2799	0.2782	0.4191	0.4286
		NPL	0.0567	0.0617	0.0547	0.055	0.0494	0.0534	0.0632	0.058
90	**(15,15,60)**	ML	0.4213	0.3718	0.3036	**0.3058**	**0.4446**	**0.3793**	**0.6251**	**0.517**
		BF	0.0342	0.0441	0.0226	0.0334	0.1369	0.098	0.3025	0.2141

			Population variance ratio							
N	(n_1,n_2,n_3)	Test	1/1/4	1/4/4	1/1/2	1/2/2	2/2/1	2/1/1	4/4/1	4/1/1
		TML	0.117	0.1263	0.0829	0.1044	0.2678	0.203	0.4687	0.3481
		FK	**0.5359**	**0.4051**	**0.3149**	0.2722	0.2792	0.2578	0.4661	0.3786
		NPL	0.1115	0.0948	0.0798	0.0705	0.058	0.0617	0.0655	0.0639
		ML	0.5486	0.384	**0.3756**	**0.3386**	**0.4329**	**0.39**	**0.6328**	**0.5286**
		BF	0.1161	0.0405	0.0447	0.0367	0.0996	0.0901	0.2285	0.2163
	(15,30,45)	TML	0.2513	0.1144	0.1328	0.1073	0.2082	0.1906	0.384	0.3441
		FK	**0.5768**	**0.441**	0.3383	0.3048	0.3391	0.2839	0.5555	0.4066
		NPL	0.0862	0.0941	0.0686	0.0739	0.0571	0.0538	0.0657	0.0529
		ML	**0.5902**	**0.5732**	**0.422**	**0.4095**	**0.4004**	**0.4208**	**0.5683**	**0.5809**
		BF	0.181	0.1204	0.0649	0.06	0.0603	0.0683	0.1193	0.1805
	(30,30,30)	TML	0.3015	0.2449	0.1534	0.1389	0.142	0.1552	0.2446	0.2941
		FK	0.5521	0.5533	0.3534	0.3527	0.3567	0.3551	0.5505	0.5477
		NPL	0.0613	0.0673	0.0516	0.0532	0.0533	0.0545	0.0688	0.0579
		ML	0.5031	0.4248	0.3326	0.3174	**0.4562**	**0.3986**	**0.6776**	**0.5607**
		BF	0.0602	0.059	0.0249	0.0408	0.1593	0.1119	0.3684	0.26
	(20,20,80)	TML	0.1599	0.136	0.08	0.093	0.2487	0.186	0.4737	0.3558
		FK	**0.6431**	**0.5343**	**0.3943**	**0.3522**	0.3813	0.3453	0.6171	0.4959
		NPL	0.1081	0.0965	0.0774	0.0688	0.0531	0.0605	0.0617	0.0595
		ML	0.6094	0.4379	0.3881	0.3341	**0.4374**	**0.3974**	**0.6761**	**0.5661**
		BF	0.1714	0.0508	0.051	0.0321	0.1104	0.1061	0.286	0.2589
120	**(20,40,60)**	TML	0.2933	0.1262	0.1198	0.0877	0.1958	0.1747	0.4153	0.3499
		FK	**0.6814**	**0.5347**	**0.427**	**0.371**	0.4146	0.3671	0.6668	0.5206
		NPL	0.0911	0.0911	0.0623	0.0625	0.0541	0.0534	0.0685	0.0556
		ML	0.6328	0.6173	**0.4251**	**0.4235**	**0.4284**	0.4198	0.623	0.6334
		BF	0.2439	0.1636	0.0818	0.0695	0.0688	0.0789	0.1697	0.246
	(40,40,40)	TML	0.3577	0.2915	0.161	0.1483	0.1508	0.1585	0.2983	0.3599
		FK	**0.6562**	**0.6548**	0.414	0.4161	0.4237	**0.4221**	**0.6693**	**0.6556**
		NPL	0.0671	0.0782	0.0553	0.0573	0.0568	0.0535	0.0825	0.0613
		ML	0.5652	0.4684	0.3539	0.3371	**0.4583**	**0.3992**	**0.7118**	**0.5963**
		BF	0.097	0.0647	0.0313	0.0422	0.1683	0.1197	0.4265	0.3045
	(25,25,100)	TML	0.2051	0.1553	0.0872	0.0989	0.267	0.2026	0.5545	0.4187
		FK	**0.7322**	**0.5954**	**0.4537**	**0.3779**	0.4147	0.3675	0.6937	0.5749
		NPL	0.1176	0.1043	0.0796	0.0719	0.0556	0.0533	0.0629	0.0646
		ML	0.6699	0.4716	0.4237	0.3462	0.4632	**0.4031**	0.727	**0.6066**
		BF	0.2352	0.0648	0.0663	0.0328	0.1278	0.1128	0.3498	0.3086
150	**(25,50,75)**	TML	0.3655	0.1501	0.1477	0.0867	0.2193	0.1951	0.4872	0.4146
		FK	**0.7657**	**0.6157**	**0.4822**	**0.4135**	**0.4677**	0.3909	**0.7601**	0.5869
		NPL	0.0927	0.0961	0.0681	0.0643	0.0532	0.0532	0.0733	**0.0581**
		ML	0.6897	0.6731	0.4411	0.4409	0.4374	0.4497	0.6668	0.6898
		BF	0.3153	0.2161	0.0956	0.0841	0.0825	0.0996	0.2162	0.321
	(50,50,50)	TML	0.4227	0.3471	0.1692	0.1628	0.1658	0.1798	0.3464	0.4318
		FK	**0.7435**	**0.7475**	**0.4762**	**0.4852**	**0.4802**	**0.4752**	**0.7493**	**0.7452**
		NPL	0.0682	0.0889	0.056	0.0566	0.0533	0.0553	0.0846	0.0712

Among the homogeneity of variances tests considered, it is seen from Table 10 that the ML test has the highest power in almost all cases for the normal distribution. NPL test also has high power especially in cases with small sample sizes and unbalanced group size. The ML test has the highest power in balanced group sizes in almost all cases of variance ratio 1/4/4, while the NPL test has the highest power in unbalanced group sizes. Moreover, the FK test for small sample sizes and the NPL test for large sample sizes generally have the lowest power values.

It can be said from Table 11 that the test with the highest power for the variance ratio 1/1/ 4 and 1/4/4 is NPL when the sample size is small and unbalanced, and for the other variance ratios, the test with the highest power is ML for the t distribution. While the NPL test

is the most powerful for variance ratios of 1/1/4, 1/ 4/4, 4/4/1, and 4/1/1 in large sample sizes, the ML is the most powerful for other variance ratios. BF test has the smallest power for almost all situations. For the Laplace distribution whose simulation results are given in Table 12, the ML test has the highest power value. For small and unbalanced group sizes, the NPL test has the highest power value for variance ratios of 1/1/4, 1/4/4, 1/1/2, and 1/2/2 in almost all cases, while for large sample sizes, it usually has the lowest test value. In addition, the power values of the BF test are in most cases lower than others. It can be seen from Table 13, the power of the NPL test is the highest in most cases, especially in unbalanced group sizes for 1/1/4, 1/4/4, and 1/1/2 variance ratios for the Cauchy distribution. The power values of the BF test are also lower than others in most cases. For the logistic distribution, it is observed from Table 14 that while NPL has the highest power values in unbalanced group sizes, the ML test has the highest power in balanced group sizes. Moreover, the FK test has the lowest power in almost all cases. As can be seen from Table 12-18, the ML test has the highest power values for asymmetric distributions in most cases. Also, in small sample sizes, BF and FK tests have the smallest power values, while in large sample cases, NPL often has the smallest power value.

4. CONCLUSIONS

In this study, the performance of five different non-parametric tests for homogeneity of variances which are mean-based Levene, Brown-Forsyte, trimmed-mean-based Levene, Fligner-Killeen, and nonparametric Levene tests are compared in terms of empirical type 1 error rates and powers for different sample sizes and various symmetric and asymmetric distributions by Monte Carlo simulation studies. As a result of the study, the following findings are obtained:

(i) Brown-Forsyte and Fligner-Killeen tests generally perform well with the lowest type-1 error rates, while the mean-based Levene test has the lowest performance with the largest type-1 error rates for the symmetrical distributions;

(ii) Brown-Forsyte test showed the best performance with the lowest type-1 error rate in almost all cases for asymmetric distributions;

(iii) For the symmetric distributions, the nonparametric Levene test can be used for small and unbalanced group sizes, and the mean-based Levene test can be used for other cases in terms of the power of the test;

(iv) In the asymmetric distributions, it should be recommended to use the mean-based Levene test in terms of the power of the test.

As with all simulation studies, the limitation of our study is that the results are applicable only to the conditions investigated in this study. However, given the wide variety of sample sizes and variance ratios adopted, it was determined that the mean-based Levene test widely used in the literature, performed well with respect to power while generally underperforming other considered tests in terms of type-1 error rates for both symmetric and asymmetric distributions. As a result of this study, it can be suggested that researchers should choose the most appropriate homogeneity test of variance, considering the distribution, group sample sizes, and variance ratios of data to be analyzed.

Future studies could further investigate the robustness of these tests under more diverse and complex conditions, such as data with outliers, to enhance the generalizability of the findings and provide more comprehensive guidelines for researchers.

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