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AUTOMATED ASSESSMENT AND OPTIMIZATION OF FUZZY LOGIC IN METRIC SPACE APPLICATIONS

MD ARMAN AHMAD[1], AYAZ AHMAD [1], AYHAN ESI [2]

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Abstract*. In this paper, for fuzzy metric spaces we will attempt to demonstrate some novel fixed-point theorems that is common. To support our findings with evidence, we employ the concept of compatibility. The previous review found several useful mutual fixedpoint theorems for metric and fuzzy metric spaces. Our results greatly expand and strengthen these theorems. Reconsideration of fuzzy metric spaces is done by this study so that fuzzy scalars are utilized to construct fuzzy metric spaces rather than real numbers or fuzzy numbers. In contrast to the preceding work, which defined fuzzy metric spaces using fuzzy or real numbers, this definition uses fuzzy numbers. It has been demonstrated that any complete regular metric space can, under certain conditions, complete fuzzy metric space are given. In addition, we provide evidence that the fuzzy metric spaces generate fuzzy topology that is specified in this study and is consistent with previously presented fuzzy topologies. In the coming years, these findings will pave the way for additional research into imperfect optimization and pattern recognition.*

Keywords: *Fuzzy topology; fuzzy metric space; fuzzy closed set; fuzzy mathematics.*

1. INTRODUCTION

An idea of a metric space that is fuzzy is defined as the distance among any fuzzy non-negative real number represented by two points, and then proceed to investigate some of this space's properties. In subsequent research, it was demonstrated that in fuzzy metric spaces multi-valued & single-valued mappings yielded more significant results. Among these theorems were fixed point theorems, numerous coincidence theorems, and others.

It is widely believed that the traditional metric space can be considerably enhanced by employing the fuzzy metric space. As part of this investigation, In fuzzy metric spaces we construct number of fixed-point theorems that is common for innovative contractive mappings. Previous research has influenced these theorems.

It is important to remember that fuzzy scalars, not actual numbers or fuzzy numbers, are used for estimating the distance among 2 fuzzy points. This is a primary distinction between the new definitions provided in this study and those previously offered. Using fuzzy scalars, distance calculations between fuzzy coordinates are conducted for the very first time. This is a first-time occurrence for me. This paper aims to illustrate a broad variety of additional characteristics associated with fuzzy metric spaces. These characteristics include completeness and induced fuzzy topology, among others. These distinctive characteristics will be examined in greater depth. In the following paragraphs, we will investigate some

[1] Department of Mathematics National Institute of Technology, Patna, India E-mails: Mda.ph21.ma@nitp.ac.in ; ayaz@nitp.ac.in.
[2] Malatya Turgut Ozal University, Faculty of Engineering, Department of Basic Engineering Sciences, 44040 Malatya, Turkey. E-mail: aesi23@hotmail.com; ayhan.esi@ozal.edu.tr

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fundamental concepts that are connected to hazy points and implications in an effort to clarify and make the material easier to comprehend. The following items are referred to as fuzzy elements and are included in fuzzy sets:

$$x_{\lambda}(y) = \{\lambda_{y,x} = 0, (\forall_y \, \mathcal{E} \, X)\}$$

where, X is non-empty set and $\lambda \in X$.

In this study, the notation (x) typically represents fuzzy points, whereas the notation (λ, x) represents the summation of altogether fuzzy points specified on X. Fuzzy points are sometimes referred to as fuzzy scalars, particularly when X = R, and the collection of fuzzy scalars is represented by the symbol P_F(X). In addition, fuzzy scalars and fuzzy points are frequently used interchangeably. A fuzzy set, denoted by the letter A, could be observed either as a fuzzy points collection associated with it, as indicated by the equation, A = {(x, λ)|A(x) = λ} or as fuzzy points collection that is located on it, as indicated by the equation A = {(x, λ)|A(x) = λ}.

The following is a rundown of the organizational structure of this study: Not only does Definition 2 include the definitions of strong fuzzy metric spaces, fuzzy linear normed spaces, & fuzzy metric spaces but it also includes instances illustrating the existence of these various types of spaces to show that they do in fact exist. In the third definition, both the fullness of fuzzy induced metric spaces and convergent behavior of sequences of fuzzy points are taken into consideration. This is done so that everything is clear and understandable. It is proved in the section that serves as a conclusion this illustrates that fuzzy topology that is produced by fuzzy metric spaces is congruent with the concept of topology that was described before. This finding provides evidence that metric spaces with fuzzy metrics that were well-defined in the section that came before it are helpful in some way.

2. FUZZY METRIC SPACES

This subsection's primary purpose is to define fuzzy normed linear spaces, strong fuzzy metric spaces, and fuzzy metric spaces. In addition, a concise discussion of the overall purpose of the section will follow. As a first step toward accomplishing this objective, we will begin by defining fuzzy scalars in a fundamental manner. A fuzzy metric is a type of metric created when a specific form of mapping connects two coordinates with values between 0 and 1. This value represents intuitively the degree of proximity between these points in metric spaces.

Definition 1. Suppose *(x, λ)* and *(y, γ)* are two fuzzy scalars. A series of definitions contains the following ones:
(1) We say (a, λ) (b, γ) if $a > b$ or $(a, \lambda) = (b, \gamma)$.
(2) *(a, λ)* is said to be no less than (b, γ) *if a ≥ b, denoted by* (a, λ) (b, γ) or $(b, \gamma) \prec (a, \lambda)$.
(3) (a, λ) is said to be nonnegative *if a ≥ 0.* The set of all the nonnegative fuzzy scalars is denoted by $S + F\ (R)$.

Definition 1 defines orders deemed to be partial orders, which should not come as a surprise. (R, ¬) and (R, λ) are identical to (R, ≥) when R is considered a subset of SF (R). Therefore, both can be interpreted as a generalization of the standard complete order. The sequence that is outlined in Definition 1 is superior to the sequence outlined in the Definition 1.

The concept of fuzzy metric spaces will be dissected into its component elements in the following paragraphs. It will become apparent that this definition is quite similar to that of conventional metric spaces; the only distinction is that the symbol ≤ has replaced by ≺ the symbol in the triangle inequality. This will become evident once you consider the evidence. This is due to the fact that within SF (R)+, it is impossible to find a logical order with exhaustive coverage.

Definition 2. Consider X to belong to a collection that isn't empty, and

$$dF : PF\,(X) \times PF\,(X) \rightarrow S + F\,(R)$$

is like a map. If for each $\{(x, \lambda), (y, \gamma), (z, \rho)\} \subset PF\,(X)$, dF, the following three criteria hold, then, we would say that (PF (X), dF) is a space with fuzzy metrics.

(1) Nonnegative:
$$dF\,((x, \lambda), (y, \gamma)) = 0\ iff\ x = y\ and\ \lambda = \gamma = 1.$$
(2) Symmetric:
$$dF\,((x, \lambda), (y, \gamma)) = dF\,((y, \gamma), (x, \lambda)).$$
(3) Triagle inequality:
$$dF\,((x, \lambda), (z, \rho)) \prec dF\,((x, \lambda), (y, \gamma)) + dF\,((y, \gamma), (z, \rho)).$$

It is said that dF is a fuzzy metric that is defined in $P_F(X)$, and the term "$dF\,((x, \lambda), (y, \gamma))$" refers to a distance that cannot be precisely measured that lies between two fuzzy point.

Take into consideration the fact of fuzzy metric spaces are in fact collections of fuzzy points; that is, their constituents are fuzzy points. According to the second definition, there are numerous ambiguous metric spaces. To illustrate this point, several examples are provided below.

Example 1. Assume that the space (X, d) represents is a typical metric. Using the following formula, you can calculate the distance between any two fuzzy points (x, λ), (y, γ) in PF (X):

$$dF\,((x, \lambda), (y, \gamma)) = (d(x, y), min\{\lambda, \gamma\}).$$

where the length between x & y, as specified in (X, d), is denoted by the notation $d(x, y)$. Fuzzy metric space ib denoted by the notation $(PF\,(X), dF)$.

Proof: It suffices to demonstrate this that dF satisfies the three criteria enumerated in Definition 2:

Nonnegative: Suppose two ambiguous points in "PF (X)" are located at (x, λ) and (y, γ), respectively. d(x, y) represents the separation between x and y, so we know that one has $d(x, y) \geq 0, dF\,((x, \lambda), (y, \gamma)) = (d(x, y), min\{\lambda, \gamma\})$ is a fuzzy scalar that cannot take on a negative value, as deduced from Definition 1. It should be evident that

$$dF\,((x, \lambda), (y, \gamma)) = 0\ iff\ d(x, y) = 0\ and\ min\,\{\lambda, \gamma\} = 1,$$

which is the same as that x = y and λ = γ = 1.

Symmetric: If "$\{(x, \lambda), (y, \gamma)\} \subset PF\,(X)$", then one obtains

$$dF\,((x, \lambda), (y, \gamma)) = (d(x, y), min\{\lambda, \gamma\}) = (d(y, x), min\{\gamma, \lambda\}) = dF\,((y, \gamma), (x, \lambda)).$$

This holds true for every "$\{(x, \lambda), (y, \gamma)\} \subset PF\,(X)$".

Triangle inequality: For any

$\{(x,\lambda),(y,\gamma),(z,\rho)\} \subset PF\ (X)$,

we have,

$$
\begin{aligned}
&dF\ ((x,\lambda),(z,\rho)) = (d(x,z), min\{\lambda,\rho\}) \\
&\prec (d(x,y) + d(y,z), min\{\lambda,\rho,\gamma\}) \\
&= (d(x,y), min\{\lambda,\gamma\}) + (d(y,z), min\{\gamma,\rho\}) \\
&= d((x,\lambda),(y,\gamma)) + d((y,\gamma),(z,\rho)).
\end{aligned}
$$

Example 2. The letter Rn denotes the conventional n-dimensional Euclidean space. Let's assume that the ambiguous linear space L specified in Rn is L. According to the notation $dF\ E((x,\lambda),(y,\gamma))$, the length between two random fuzzy points (x, λ), (y, γ) belonging to L is well-defined as follows:

$$dF\ E((x,\lambda),(y,\gamma)) = (dE(x,y), min\{\lambda,\gamma\}),$$

where dE represents the Euclidean distance standard. Consequently, the fuzzy metric space denoted by the expression (L, d_{EF}) likewise, is fuzzy metric space, where L is regarded as the collection at which fuzzy elements are included in the fuzzy set L.

Proof: Rn being a metric space within the conventional sense and L is a subset of P_F(Rn), Example 1 d_{FE} is a fuzzy metric.

As supporting evidence d_{FE} is an example of a fuzzy metric that can be derived from Example 1, given that Rn is a metric space in the conventional sense & L is capable of being considered a subdivision of P_F (Rn).

The two examples presented earlier demonstrate that a conventional (linear) metric space can produce an irregular (linear) metric space. This type of space is known as an induced (linear) metric space of the original one, & its metric is known as an induced metric of the conventional one.

Due to the fact that S+ F (R) not an exhaustive ordered sequence, the symbol in the triangle inequality used in Definition 2, ≤ has been replaced by ≺ with a much weaker variant of the original symbol. The question of whether fuzzy metric spaces satisfy inequality of triangles with partial orders greater than ≺, for instance, is a natural one. The response is affirmative; these spaces are known as robust fuzzy metric spaces.

Definition 3. Let us assume that X is not an empty set and that the mapping $dF : PF\ (X) \times PF\ (X) \rightarrow S + F\ (R)$ exists. $(PF\ (X), dF)$ is characterized as a robust fuzzy metric space. if it meets the initial two requirements in Definition 2, and for every $(x,\lambda),(y,\gamma),(z,\rho)\ in\ PF\ (X)$, one has

$$d_f\left((x,\lambda),(z,\rho)\right) \leq dF\left((x,\lambda),(y,\gamma)\right) + dF\left((y,\gamma),(z,\rho)\right).$$

Considering Definitions 2 and 3, it becomes evident that all strong fuzzy metric spaces are, in fact, fuzzy metric spaces. This is the case since Definitions 2 and 3 define robust fuzzy metric spaces. This image illustrates both the presence of robust the concept of fuzzy metric spaces distinction between these two categories of distinct spaces.

Example 3. The L-space is an irregular version of the linear Rn-space. The definition of the distance between any two arbitrary ambiguous locations on L, such as (x, λ) and (y, γ), is:

$$d_{FE}\left((x,\lambda),(y,\gamma)\right) = (d_E\ (x,y), \min\{\lambda,\gamma\})\ \ldots (1)$$

where the Euclidean distance is denoted by the symbol d_E. Then the space denoted by (L, dF E) is an example from a dense irregular (fuzzy) metric space, and the fuzzy set L contains a collection of fuzzy points. is denoted by the letter L. It's evidence. The first two requirements can be demonstrated using the same method as in Example 1. Only the third one is going to be proven here. Given a random set of three uncertain (fuzzy) points on L, with coordinates (x, λ), (y, γ) and (z, ρ), respectively because (Rn, dE) requires one to use metric space, where, exactly the sign dE is used to indicate the Euclidean distance between two points. A strong it is an irregular (fuzzy) metric space that the letters L and dF E designate. The letter L stands for the set of fuzzy points that make up the fuzzy set L. This is the evidence. It is possible to demonstrate compliance with the first two standards by employing the approach shown in Example 1. The evidence here will only support the third one. Suppose that we have a random set of three fuzzy points on L with the coordinates $(x,\lambda),(y,\gamma)\&(z,\rho)$, respectively. It is essential to take into consideration the fact that (Rn, dE) is metric space.

$$d_E\ (x,z) \leq d_E\ (y,z) + d_E\ (x,y).\ (2)$$

In the event that inequality (2) is demonstrated to be true, then it is possible to derive with one hundred percent certainty that the condition has been satisfied by referring to Definition 1(1). Under the alternative hypothesis, there has to be a function of the kind λ ∈ F such that y may be expressed as $y\ =\ (1-\lambda)x+\lambda z$. Shall we specify a minimum first, using the notation $\alpha\ =\ min\{\lambda,\rho\}$ respectively. It has been brought to our attention that $\{x,z\}\ \subset\ L\alpha.L'$ is equivalent to L. Because L is a fuzzy linear space (for further information, see the Representation Theorem for Fuzzy Linear Spaces), L' is a linear subspace of Rn. Rn itself is a fuzzy linear space. Because of this, y is less than L, which means that y is greater than L, which may be written as $y\ \in\ L\alpha, i.e., \gamma\ =\ L(y)\ \geq\ \alpha\ =\ min\{\lambda,\rho\}$. This suggests that the $min\ \{\lambda,\rho,\gamma\}\ =\ min\{\lambda,\rho\}$.

Thus, one has

$d_{FE}((x,\lambda),(z,\rho))=(d_E(x,z),\ \min\{\lambda,\rho\})$

$=(d_E(x,y)+\ d_E(y,z),\min\{\lambda,\rho,\gamma\})$

$=\ d_{FE}((x,\lambda),(y,\gamma))+\ d_{FE}((y,\gamma)+(z,\rho)).$

As a result, the condition has been satisfied.

Consider the fact that the above-described fuzzy metric space strength nothing more than a collection of fuzzy points in a fuzzy linear space. Example 2 demonstrates a fuzzy metric space comprised of fuzzy points associated with a fuzzy linear space. The change has occurred because the symbol has been substituted with the partial sequence, which is significantly more potent.

Definition 4. Consider L to be a fuzzy linear space. (L, k·k) if the mapping conforms to the fuzzy linear norm $k\cdot k\ :\ L\ \rightarrow\ S+\ F\ (R)$ satisfies the following conditions:

(a) $k(x,\lambda)k\ =\ 0$ if and only if $x\ =\ 0\ \&\ \lambda\ =\ 1$,

(b) For any "$k\ \in\ R$ and $(x,\lambda)\ \in\ L$", one has $kk(x,\lambda)k\ =\ |k|\cdot k(x,\lambda)k$,

(c) For any $\{(x,\lambda),(y,\gamma)\}\ \subset\ L$, one has $k(x,\lambda)+(y,\gamma)kk(x,\lambda)k+k(y,\gamma)k$.

The fuzzy norm of $k\cdot k\ :\ x\ \rightarrow\ kxk$ refers to the mapping that looks like this: $(L,k\cdot k)$. Take note that the elements of fuzzy linear normed space denoted by the letter L are fuzzy points that belong within the irregular (fuzzy) set L.

Example 4. Take into consideration the linear normed space that will be defined on R as $(G, || \cdot ||G)$. An example of a defined imprecise linear (fuzzy) space by G & k·kF, is denoted by the letter L. The concept of G can be thought of as a mapping from L to "$SF\ (R+)$".

$$||(x, \lambda)||\ FG := (||x||G, \lambda),\ \forall (x, \lambda)\ \in L.$$

Consequently, the space denoted by $(L, k \cdot kF\ G)$ is fuzzy linear normed space, and its validity can be established in a manner the same as Example 2.

The following assertion, which is made without providing any proof to back up the claim, establishes the connection between fuzzy linear normative & fuzzy metric spaces.

Proposition 1. Let us imagine for a moment that the space denoted by $(L, k \cdot kF\ G)$ is an uncertain (fuzzy) linear norm space. After that, we will refer to the fuzzy metric space as (L, dF G), where dF G will be defined as

$$d_{FG}\ ((x, \lambda), (y, \gamma) := ||(x, \lambda) - (y, \gamma)||\ FG.$$

Proof: It is absent. Example 4 equation G = Rn yields the following sentence illustrates the connection between the fuzzy norm & fuzzy points' interior product.

Proposition 2. Let's say that the space denoted by the notation "$(L, k \cdot kF\ E)$" is a fuzzy linear space with a normed definition in Rn. If $(x, \lambda)\ \in\ L$, then one has

$$<\ (x, \lambda), (x, \lambda), > =\ ||(x, \lambda)||2\ FE,$$

where the definition of the core product is located, i.e.,

$$<\ (x, \lambda), (y, \gamma)\ > =\ (<\ x, y\ >, min\,\{\ \lambda, \gamma\ \}).$$

Proof: After perusing the explanation of the product of fuzzy elements' interior, one reaches this conclusion.

$$\begin{aligned} <\ (x, \lambda), (x, \gamma)\ > &=\ (<\ x, x\ >, \lambda) \\ &= (||x||2E, \lambda\ ) \\ &= (||x||E, \lambda\ ) \,.\, (||x||E, \lambda\ ) \\ &= \big||(x, \lambda)|\big|2\ FE\ , \end{aligned}$$

where, $k \cdot kE$ is Euclidean standard.

3. THE RELIABILITY OF FUZZY METRIC SPACES

This section examines the convergence of a series of fuzzy points & the completeness of fuzzy metric spaces induced by fuzzy points. The convergence of a sequence of fuzzy scalars is the first factor considered when determining the distances between fuzzy points. This is because fuzzy scalars are utilized to calculate the distance between fuzzy coordinates.

Definition 5. Use $\{(a_n, \lambda_n)\}$ to designate the sequence of fuzzy scalars. The notation $lim_{n\to\infty}(a_n, \lambda_n) = (a, \lambda)$ indicates that it converges to a fuzzy scalar $((a, \lambda), \lambda\ ) =\ 0$. If $lim_{n\to\infty}\ a_n\ =\ a, \{\lambda_i | \lambda_i\ <\ \lambda, i\ \in\ N\}$. There exists a subsequence of the set if it is finite i such that $\lim_n a_n = a$, then the statement is true. Given that we have high aspirations that the convergence degree will be greater than λ, it is only natural to require that virtually all of the $\lambda_i \in N$ satisfy the $\lambda_i \geq \lambda$ in condition. The following is A fresh explanation of the convergence of a string of fuzzy points derived from the fuzzy metric discussed in the preceding section.

Definition 6. Let's say that $(PF\ (X), dF)$ is the irregular metric space induced by (X, d), and that the notation $\{(x_n,\ \lambda_n)\}$ here denotes a series of fuzzy points in $(PF\ (X), dF)$. $\{(x_n,\ \lambda_n)\}\ )\}$ to an indistinct point is said to converge $(x, \lambda\ )$ if the expression $lim_{n\to\infty}\ dF\ ((x_n,\ \lambda_n), (x, \lambda))\ =\ 0$ for $\gamma\ \in\ (0, 1)$ in such a way $lim_{n\to\infty}\ dF\ ((x_n,\ \lambda_n)\}, (x, \gamma))\ =\ 0,$ for $\lambda\ \geq\ \gamma$. The value (x, λ) is referred to as the sequence's limit, and it is represented by the notation limn $lim_{n\to\infty}\ (x_n,\ \lambda_n) = (x, \lambda)$.

Proposition 3. Assume that $\{(x_n,\ \lambda_n)\}$ consists of a series of fuzzy objects in (P_F (X), d_F) and that $(x, \lambda)\ \in\ (PF\ (X), dF\ ), \lambda\ 6 =\ 0$. We have established that $lim_{n\to\infty}\ \ (x_n,\ \lambda_n) = (x, \lambda)$ if and only if $lim_{n\to\infty}\ x_n\ =\ x, \{\lambda_i|\ \lambda_i\ <\ \lambda, i\ \in\ N\}$ there exists a subsequence of the set if it is finite. $\{\lambda_i\}$, designated by $\{\lambda_l\}$, such that $lim_{l\to\infty}\ \ \lambda l\ =\ \lambda$. This is the only condition under which this statement is true.

Proof: It is omitted.

Definition 7: A series of fuzzy points, denoted by $(x_n,\ \lambda_n)\ \in\ (PF\ (X), dF\ )$, a sequence is a Cauchy order if there is any λ∈(0,1) such that the sequence has the following property:

$$lim_{n\to\infty}\ \ dF\ \left((x_{m+n}, \lambda_{m+n}), (x_n,\ \lambda_n)\right) =\ 0, \forall m\ \in\ N.$$

It is important to keep in mind that each Cauchy sequence of fuzzy points that has been created the one above has a singular fuzzy point that serves as its limit. This limit is somewhat comparable to the traditional one. We are going to start thinking about how complete fuzzy metric spaces are right now.

Definition 8. It is argued that a fuzzy metric space that has been induced is exhaustive if any Cauchy sequence satisfies the condition contained within it has a limit that is only found in that space.

Theorem 1. Assume that standard metric space (X, d) has an induced fuzzy metric space (PF (X), dF) denoted by the notation. If both (X, d) are complete, then it is finished.

Proof: Necessity: It is obvious.
Sufficiency: Assume that the sequence $\{(x_n,\ \lambda_n)\}$ is a random Cauchy sequence of the form $(PF\ (X), dF)$. Since (X, d) is exhaustive and $lim_{n\to\infty}\ d(x_{m+n},\ x_n)\ =\ 0$ for every m smaller than $\in\ N$, there must be some $x\ \in$ within X for which $lim_{n\to\infty}\ x_n\ =\ x$. The index set $\{l|\lambda_l\ =\ min\{\lambda_{m+n}, \lambda_n\}, n\ =\ 1, 2\ \cdots\}$ is referred to as L_m for any value of m ∈ N that is less than N. According to according to the concept of Cauchy sequences for fuzzy points, there are several $\lambda\ \in\ (0, 1]$ therefore for any $m\ \in\ N$, the set $\{\{\lambda_l\}\ l \in L_m\}$ is finite.

Additionally, a subsequence of exists $\{\lambda_l\}\ \ l \in L$m denoted by $\{\lambda_k\}$, which is also a subsequence of n, such that $lim_{k\to\infty}\ \lambda_k\ \ =\ \lambda$. It cannot be denied that

$$\{l|\ \lambda\ _l <\ \lambda, l\ \in\ \ L_m\} \subset\ \{n|\lambda\ _n\ <\ \lambda, n\ =\ 1,2, \ldots\ \}.$$

As a direct consequence of this, $\{\lambda_n|\lambda_n < \lambda, n = 1, 2, \cdots$ is likewise considered a finite set. We may deduce that $lim_{n\to\infty}$ $((x_n, \lambda_n) = (x, \lambda)$ from the reasons presented above. It suggests that there is a limit of $\{(x_n, \lambda_n\}$ in P_F (X) function. In the following, we will demonstrate how special this is. Assume, for the sake of consistency, that the Cauchy sequence $\{(x_n, \lambda_n)\}$ has additional limits beyond those already established. Given that we are aware that x is the only possible limit of xn, we are able to indicate by (x, γ) the limit that is distinct from $\{x_n\}$, where $((x, \lambda), \gamma) = \lambda$, which we will refer to as $\gamma > \lambda$. Then we have $\{\lambda_n|\lambda_n < \gamma\}$, which is an example of a finite collection. As a result of the discussion that came before, we are aware that $lim_{k\to\infty}\lambda_k = \lambda$ and $\{\lambda_k|\lambda_k < \lambda\}$ is a finite set. Consequently, by assuming that equals plus $\rho = \lambda + \gamma$, we have

$$\{\lambda_k\}\cap[\lambda,\rho]\subset\{\lambda_n \mid \lambda_n<\gamma\}$$

is a set that can go on forever. This runs counter to the assertion that $\{\lambda_n|\lambda_n < \gamma\}$ is a finite set. As a result, there is only ever going to be a single limit to the Cauchy sequence.

It is important to keep in mind a robust fuzzy linear metric space not typically full. It is observable by looking at the contrasting sample that is provided below.

Example 5. Take into consideration the strongly fuzzy linear metric space denoted by (L, d_{FE}), where

$$L=\{(x,\lambda) \mid x\in R-\{0\}, \lambda=1/2\}\cup\{(0,1)\}$$

and the conventional Euclidean metric, d_E, will cause dF E to be produced. The order (1,1,2) found in L is considered to be a Cauchy sequence according to Definition 7. The limit of the series, however, which is (0,1,2) can not on the space L.

4. THE FUZZY TOPOLOGY SPACES THAT ARE BROUGHT ABOUT BY THE FUZZY METRIC SPACES

The conventional metric spaces and the fuzzy metric spaces that are discussed in this study have many characteristics. A result that is analogous to the idea that any metric space has the potential to produce topology will be proven in this part. The correlation between imprecise point distances and their inner products was discussed in the previous section. Within this context, fuzzy topology is defined in terms of fuzzy closed sets. It makes it easy to design An conventional metric space with a fuzzy topology by providing a technique for doing so. In order to accomplish this goal, initially fuzzy closed set definition in terms of fuzzy metric spaces that have been induced will be presented. Assume for the moment that (X, d) is a regular metric space. In light of the fact that a fuzzy set A in X can be interpreted as a collection of fuzzy points that are associated with it, A can be seen as a subset of PF (X), which will be referred to as Uncertain set in the induced fuzzy metric space (PF (X), dF) in the following description.

Definition 9. It is said that fuzzy set A in (PF (X), dF) is closed if and only if the limit of any subset X is contained within Cauchy order that occurs in A is a member of that fuzzy set. There is a claim that a fuzzy set A in (PF (X), dF) is open if the fuzzy closed set A0 is not. A0 is defined by the equation A0 (x)=1 − A(x), where x can be any value within $x \in X$.

The following assertion demonstrates that the recently proposed concept of fuzzy closed sets is consistent with common sense.

Proposition 4. A fuzzy set A in $(P_F(X), d_F)$ is said to be closed if and only if every α-cut set of A, falling inside the range $\alpha \in [0, 1]$, is a closed set in (X, d) in the traditional sense.

Proof: It is omitted.

In what follows, we will demonstrate that it is possible to induce a fuzzy topology using any induced fuzzy metric space. To demonstrate this, a theorem involving Cauchy orders of fuzzy points is presented here.

Lemma 1. Every subsequence of fuzzy points in a Cauchy sequence is itself a Cauchy order and possesses the same edge as the initial sequence did.

Proof: The evidence presented in Definition 7 demonstrates this without a doubt.

Theorem 2. Let's say that a metric space (X, d) has an induced fuzzy metric space that we'll refer to as (P_F (X), d_F). When this is the case, the fuzzy topology space denoted by (X, T_F) is referred to as the fuzzy topology space prompted by (P_F (X), dF). TF is well-defined as follows:

$$T_F=\{A\subset P_F(X) \mid A \text{ is a fuzzy closed set in } (P_F(X), d_F\}.$$

Proof: To demonstrate that T_F satisfies all three characteristics outlined in the concept of fuzzy topology is all that is required.
(1) It should come as no surprise that X and ∅ are both examples of fuzzy closed sets.
(2) In the following, we will demonstrate that the statement " A ∪ B ∈ T_F " is true for any given {A,B} ⊂T_F. For a Cauchy sequence of fuzzy points to be included in A ∪ B , either A or B, let's assume A must contain a subsequence of the sequence yn, n denoted by the notation $\{(y_m, \gamma_m)\}$. According to Lemma 1, the expression " $\{(y_m,\gamma_m\}$ " is also a Cauchy sequence and, as a result, has a limit. Due to the fact that A is a closed fuzzy set, the limit of " $\{(y_m, \gamma_m)\}$ ", which is also the limit of " $\{(y_m, \gamma_m)\}$", is considered to be part of "A." As a direct result of this, the limit of $\{(y_m, \gamma_m)\}$ is accounted for in A ∪ B, which indicates that is greater than A ∪ B ∈ T_F.
(3) It is sufficient to demonstrate that T i∈I A_i ∈ T_F in order to demonstrate that any arbitrary $\{A_i\}$i∈I ⊂ T_F is equivalent to TF. We have that any Cauchy sequence in T i∈I Ai, denoted by $\{(x_n, \lambda_n)\}$, satisfies the condition that $\{(x_n, \lambda_n)\} \subset A_i$ for any i ∈ I. Due to the fact that each and every Ai is a closed fuzzy set, the limit of $\{(x_n, \lambda_n)\}$ is contained within A_i for every i ∈ I. As a result, T i∈I Ai is what's known as a closed fuzzy set, according to Definition 9 of the term. Therefore, one must conclude that T i∈I A_i ∈ T_F. It looks like the proof is finished.

As a result of the theorem that was presented earlier, we are aware that every fuzzy metric space is capable of inducing a fuzzy topology space. This indicates, in a different sense, that the fuzzy measure that was developed in this study is not only reasonable but also significant.

5. CONCLUSIONS

The purpose of this research was to design a method that makes use of fuzzy metrics to identify the sentences within a text that are the most informative and then to compare and classify the text that was created as a response to learning tasks with the reference responses for these tasks. The study was carried out to accomplish these two objectives. The fact that fuzzy logic is a strategy that is often recommended for applications that deal with unclear information can be traced to the employment of fuzzy logic in those applications. Within the context of an ambiguous metric space, several metric structures are studied. To be more specific, we provide the concrete form of the metric function concerning the metrizable topology for a fuzzy metric when two exceptional criteria are met. This is done as a way to illustrate the connection that exists between the two. Based on the conclusions that were presented in the study, intriguing new research may be carried out in the future on relevant subjects. In addition, the research approach that was taken for the purpose of this investigation offers a prospective answer to the problem that was described earlier in the context of the overall instance.

In this particular piece of research, both the existence of fixed points for nonlinear contractions in fuzzy metric spaces and the uniqueness of such fixed points were investigated. We were able to determine the relationships between fuzzy metrics and a family of quasi-metrics by making use of a procedure called level cutting. We were able to derive common fixed-point theorems for a pair of self-mappings in fuzzy metric spaces that satisfied implicit Lipschitz-type conditions with their help. They helped us, and that made this possible. Our understanding of fixed points in fuzzy metric spaces has without a doubt been broadened as a direct result of the work that has been done here. The maximum of R is the foundation for the research on fixed points in fuzzy environments that is described in this work. The question of whether or not the dialogue can be continued in the more relaxed situation (R-2) should be the focus of our research, and we should direct it toward establishing the answer to that question. The investigation of the connection that exists between fuzzy metrics and Lowen-type fuzzy topologies was the primary focus of our efforts. We propose a formulation for determining, with the assistance of fuzzy metrics, how one might go about designing a fuzzy topology. This formulation can be found in the following paragraph. By contrasting and comparing the two fuzzy topologies, we demonstrate that the induced fuzzy topology and the fuzzy topology imposed by the Lowen functor w are distinct for non-coprinciple fuzzy metric spaces. These fuzzy topologies are generated by the induced fuzzy topology and the fuzzy topology imposed by the Lowen function. We also provide a revised and improved form of continuity for fuzzy metric spaces to achieve continuity equivalence between fuzzy metric spaces and induced fuzzy topological spaces. This is done to achieve continuity and equivalence. This particular implementation of continuity applies to fuzzy metric spaces. In conclusion, we investigate the compactness of these spaces and demonstrate that the stronger form of fuzzy compactness matches the fuzzy compactness of an induced fuzzy topological space. In this report, we will share the results of our investigation.

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