ORIGINAL PAPER

COMPLEMENTARY FAIR DOMINATION IN GRAPHS

SWAMINATHAN VENKATASUBRAMANIAN[1], RAMAN SUNDARESWARAN[2]

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Abstract. *In a simple, finite undirected graph G, a dominating set D is a subset of the vertex set $V(G)$ whose closed neighbourhood is $V(G)$. Many types of domination have been studied. The studies are based either on the nature of domination or the type of dominating set or the type of the complement of the dominating set. Interaction between dominating set and its complement is also considered. Fair domination is the domination where every vertex in the complement of a dominating set has equal number of neighbours in the dominating set. In this paper, a dominating set whose vertices have equal number of neighbours in the complement is the subject of study. The parameter $\gamma_{cof}(G)$ is introduced and studied.*

Keywords: *complement; domination; fair domination; co-fair domination.*
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1. INTRODUCTION

Throughout this paper, connected graphs only are considered. Let G be denote a graph with vertex set $V(G)$ and edge set $E(G)$. A subset S of $V(G)$ is called a dominating set [1] of G if every $v \in V(G)$ either belongs to S or is adjacent to a vertex in S. A total dominating set S is a subset of $V(G)$ such that every vertex $v \in V(G)$ is adjacent with some vertex of S other than v. The minimum(maximum) cardinality of a minimal dominating set of D is called the domination number of G (Upper domination number of G) and is denoted by $\gamma(G)$ $(\Gamma(G))$. A dominating set D of G is minimal if and only if for any $u \in D$, either u is an isolate of D (that is, u is not adjacent with any vertex of D) or there exists a vertex $v \in V - D$ which is adjacent with only $u \in D$. The closed neighbourhood of u, denoted by $N[u]$ is defined as $N[u] = N(u) \cup \{u\}$. For standard notations and terminologies, we refere the text book [2]. Two vertices u and v of a graph G are said to be degree equitable if $|deg(u) - deg(v)| \leq 1$. This concept is due to Prof.E.Sampahkumar. Earlier cardinality equitability of colour classes was introduced by W.Meyer in Equitable colouring in graphs [3]. A dominating set D is called a fair dominating set if $|N(u) \cap D| = |N(v) \cap D|$ for any $u, v \in V - D$. Fair domination was introduced by Yair Caro, Adriane Hansberg and Michael Henning in [4] . V. Swaminathan et al. [5-6] introduced and studied the concept of fair domiantion with respect to ourter complete and equatability in graphs.

In an administration, the decision making body can be modelled by a dominating set in the graph of the administration. A need in constituting the decision making body is to take into consideration the number of members of the decision making body whom each member of the general members is in contact and also the number of general members whom each

[1] Ramanujan Research Center in Mathematics, Saraswathi Narayan College, Madurai, India.
E-mail: swaminathan.sulanesri@gmail.com.
[2] Sri Sivasubramaniya Nadar College of Engineering, Department of Mathematics, Chennai, India.
E-mail: sundareswaranr@ssn.edu.in.

member of the decision making body knows. If there is imbalance in either of these numbers, then the decision making body may not be stable. In order to create a model for the first situation, fair domination was conceived in [1]. In this paper, a model for the second situation is presented.

The paper is organized as follows: In Section 2, definition of complementary fair domination and minimum complementary fair domination number of some known graphs are given. Some observations are presented. In Section 3, some theorems on the bounds of the parameter in regular graphs are proved. The new parameter in the case of trees is studied. Further, the relation between repetition number and the new parameter is found. In Section 4, Nordhaus Gaddum type results are derived with respect to the new parameter. In Section 5, the connection between out regular number and complementary fair domination number is found. uniform complementary fair domination is defined and studied.

2. COMPLEMENTARY FAIR DOMINATION IN GRAPHS

In this section, definition of Complementary fair domination and minimum Complementary fair domination number are given. The value of the new parameter for some classes of graphs is found. Some observations are given.

Definition 2.1. Let G be a simple, finite and undirected graph. A subset D of $V(G)$ is called a Complementary fair dominating set of G if D is a dominating set of G and for any $u, v \in V(D), |N(u) \cap (V-D)| = |N(v) \cap (V-D)|$. The minimum cardinality of a Complementary fair dominating set of H is called the Complementary fair domination number of G and is denoted by $\gamma_{cof}(G)$.

Remark 2.2. If D is a minimum Complementary fair dominating set of H, then G has no isolate vertex. For, if G has an isolate say u, then u belongs to D and hence $|N(u) \cap (V-D)| = 0$ and hence $|N(v) \cap (V-D)| = 0$ for any $v \in D$. Since D is a dominating set of G, $V-D$ is empty. That is, $|D| = n$. If $v \in D$ is not an isolate of D, then $D - \{v\}$ is a Complementary fair dominating set of G, a contradiction since D is a minimum Complementary fair dominating set of H. Hence, $= \overline{K_n}$. Thus, if $\gamma_{cof}(G) \neq n$, then $\delta(G) \geq 1$. If G has a full degree vertex, then $\gamma_{cof}(G) = 1$. $\gamma_{cof}(G)$ for some classes of graphs 2.3:

1. $\gamma_{cof}(K_n) = 1$.
2. $\gamma_{cof}(K_{1,n}) = 1$
3. $\gamma_{cof}(K_{m,n}) = \begin{cases} 2 + |m-n|, if\ m \neq n, m,n \geq 2 \\ 2\ , if\ m = n\ and\ m,n \geq 2 \end{cases}$
4. $\gamma_{cof}(P_n) = \lceil \frac{n}{3} \rceil$
5. $\gamma_{cof}(P_n) = \gamma(P_n) = \lceil \frac{n}{3} \rceil$
6. $\gamma_{cof}(C_n) = \gamma(C_n) = \lceil \frac{n}{3} \rceil$
7. $\gamma_{cof}(W_n) = 1$
8. $\gamma_{cof}(P) = 4$ ({1,9,10,3} is a minimum co fair dominating set.)
9. $\gamma_{cof}(K_{n_!,n_2,\dots n_r}) = \begin{cases} 2 + |n_1 - n_2|, \text{if } n_1 \leq n_2 \leq n_3 \leq \cdots \leq n_r \\ n_i \geq 2, \forall i, 1 \leq i \leq r \\ 1, n_i = 1 \end{cases}$
10. $\gamma_{cof}(K_{m(a_1,a_2,\dots a_m)}) = |a_1 + a_2 + \cdots + a_m|$

11. $\gamma_{cof}(D_{r,s}) = 2 + |r - s|$

Observation 2.3.

1. $\gamma(G) \leq \gamma_{cof}(G)$
2. $\gamma_{cof}(G) = 1$ if and only if G has a full degree vertex.
3. $\gamma_{cof}(G) = n$ if and only if $G = K_n$.
4. $\gamma_{cof}(G) = n - 1$ if and only if $G = K_2$. For, suppose $\gamma_{cof}(G) = n - 1$.
5. Let D be a γ_{cof}- set of G. Then, $V - D$ is a singleton say $\{v\}$. Every vertex in D is adjacent with v. Hence v is a full degree vertex. That is, $\gamma_{cof}(G) = 1 = n - 1$.Therefore, $n = 2$ and $G = K_2$. The converse is obvious.
6. If $n \geq 3$, and $\gamma_{cof}(G) \neq n$, then $\gamma_{cof}(G) \leq n - 2$.
7. For, suppose $n \geq 3$. Then $\gamma_{cof}(G) \neq n - 1$. Also, $\gamma_{cof}(G) \neq n$. Hence, $\gamma_{cof}(G) \leq n - 2$.

Observation 2.4. There exist many graphs for which $\gamma_{cof}(G) = n - 2$.

Proof: Consider $K_{m,2}$ where $m \geq 2$. $\gamma_{cof}(K_{m,2}) = 2 + m - 2 = m = m + 2 - 2 = n - 2$.

3. BOUNDS OF γ_{cof}

In this section, we derive few bounds of γ_{cof} for regular graphs and trees.

Theorem 3.1. Let G be a r −regular graph on n vertices with $r \geq 1$. Let $t = min\{|S|: S$ is a maximal independent set of G }. Then $\gamma_{cof}(G) \leq t$.

Proof: Let S be a maximal independent set of H with $|S| = t$. Then S is a dominating set of G. Also, $|N(v) \cap (V - S)| = r$ for every $v \in S$. Hence S is a complementary fair dominating set and so $\gamma_{cof}(G) \leq t$.

Definition 3.2 [4] Let H be a simple graph. The repetition number of H denoted by $rep(H)$ is the maximum repetition in the degree sequence of H.

Observation 3.3. There are graphs with $\gamma_{cof}(H) = n - 2$ and $rep(H)$ arbitrarily large. For, consider $K_{m,2}$ where m is arbitrarily large. $\gamma_{cof}(K_{m,n}) = |V(K_{m,n})| - 2$ and $rep(K_{m,n}) = m$.

Definition 3.4. [3]. Let k be a positive integer. Then the k-fair domination number of H denoted by $fd_k(H)$, is the minimum cardinality of a dominating set D which has the property that for any $u, v \in V - D, |N(u) \cap D| = |N(v) \cap D| = k$. A dominating set D with this property is called a k −fair dominating set of H. The fair domination number of a non-empty graph H, denoted by $fd(H)$ is the minimum cardinality of a k −fair dominating set for some positive integer k.

Remark 3.5. Note that in the case of fair domination when

$$fd(G) = n - 2\,, 2 \leq rep(G) \leq 4.$$

A strong support vertex of a tree is a vertex that is adjacent to at least two pendent vertices. If T has strong support vertices which support different number of of pendent vertices, then any complementary fair dominating set can contain all strong support vertices provided pendent vertices more than 2 at any strong support vertex are included in the complementary fair dominating set. In this case, the set of all pendent vertices is the minimum complementary fair dominating set. Hence, if every strong support vertex supports the same number of pendents, then the set of strong support vertices is a minimum complementary fair dominating set.

Observation 3.6. If T is the corona of a tree and has order n, then $\gamma_{cof}(T) = \frac{n}{2}$ and $V(T)$ can be partitioned into two γ_{cof} –sets.

*Proof: O*bvious.

Observation 3.7. If T is a tree on $n \geq 3$ vertices, then $\gamma_{cof}(T) \leq l$ where l is he number of leaves.

In [3], the following definition is given. The out – regular set abbreviated as OR-set of a graph H that is non-empty is a set Q of vertices such that $|N(u) \cap (V - Q)| = |N(v) \cap (V - Q)| > 0$ for any two vertices $u, v \in Q$. The out-regular number of a non – empty graph H, denoted by $\mathcal{E}_{or}(H)$, is the maximum cardinality of a OR-set of H. Any complementary fair dominating set is an OR-set. But a maximum OR-set need not be a minimum complementary fair dominating set. For example, in P_5, with vertex set $\{v_1, v_2, v_3, v_4, v_5\}, \{v_1, v_3, v_4\}$ is a maximum OR-set but it is not a minimum complementary fair dominating set. Also, given a positive integer k, there exists a connected graph H with $\mathcal{E}_{or}(H) - \gamma_{cof}(H) = k$ namely, $K_{k+2\,,k+2}$.

Problem: Does there exist a maximum OR-set which is not a complementary fair dominating set?

Observation 3.8. Let $H \neq \overline{K_n}$. Then, $fd(H) + \gamma_{cof}(H) \leq n$.

Proof: Let $H \neq \overline{K_n}$. Let D be a minimum complementary fair dominating set of H. Consider $V - D$. Let u, v belong to $D = V - (V - D)$. From the property of $D, |N(u) \cap (V - D)| = |N(v) \cap (V - D)| > 0, V - D$ is a fair dominating set of H. Hence $fd(H) \leq |V - D| = n - \gamma_{cof}(H)$. That is, $fd(G) + \gamma_{cof}(G) \leq n$.

Observation 3.9. In $K_{n,n}, \geq 3, fd(H) + \gamma_{cof}(H) = 2 < n$. In $P_4, fd(H) + \gamma_{cof}(H) = 4$.

Theorem 3.10. Let H be a simple graph such that there exists a $\mathcal{E}_{or}(H)$ - set in H which is also a minimum Complementary fair dominating set of H. Then

$$\gamma_{cof}(H) \geq \frac{n}{\left(\bar{d}(H) + 1\right)\Delta(H)}$$

where $\bar{d}(H)$ denotes the average degree of H. If further, H is an r – regular graph, then $\gamma_{cof}(H) \geq clogn$ for some constant $c > 0$. If H is a maximal outer planar graph, then $\gamma_{cof}(H) > \frac{2n}{19}$.

Proof: Follows from Proposition 7(a), 8 and Theorem 22 of [3] and from the observations $\gamma_{cof}(H) = \mathcal{E}_{or}(H) = n - fd(H)$.

Remark 3.11. In $K_{18,20}$, $\gamma_{cof} = 4$ and $\frac{2n}{19} = \frac{2\times 38}{19} = 4$.

4. NORDHAUS –GADDUM-TYPE RESULTS

Nordhaus Gaddum type results are derived with respect to the new parameter.

(i) Let H be a connected graph with at least 5 vertices. If $\overline{H}$ has at least two edges, then, $\gamma_{cof}(H) \leq n - 2$ and $\gamma_{cof}(\overline{H}) \leq n - 2$.

Hence,

(ii) $\gamma_{cof}(H) + \gamma_{cof}(\overline{H}) \leq 2n - 4$ and $\gamma_{cof}(H).\ \gamma_{cof}(\overline{H}) \leq (n-2)^2$. If $\bar{G}$ has exactly one edge, then $\gamma_{cof}(\mathrm{G}) = \mathrm{n} - 1$ and $\bar{G} = \overline{K_{n-2}} \cup \mathrm{K}_2$. Therefore $G = K_n - e$. $\gamma_{cof}(G) = 1$.

Hence, $\gamma_{cof}(G) + \gamma_{cof}(\bar{G}) \leq n \leq 2n - 4$ $(since\ n \geq 5)$. Also, $\gamma_{cof}(G).\gamma_{cof}(\bar{G}) \leq n - 1 \leq (n-2)^2$ since $n \geq 4$. Suppose $m(\bar{G}) = 0$. Then $\bar{G} = \overline{K_n}$ and $G = K_n$. Hence $\gamma_{cof}(G) = 1$ and $\gamma_{cof}(\bar{G}) = n$. Therefore,

$$\gamma_{cof}(G) + \gamma_{cof}(\bar{G}) = n + 1 \leq 2n - 4\ when\ n \geq 5\ and\ \gamma_{cof}(G).\gamma_{cof}(\bar{G}) = n \leq (n-2)^2\ for\ n \geq 4.$$

Let $G = K_{m,n}$ where $m \geq 4$ and $n = 2$. Therefore, $m + n \geq 6$. $\gamma_{cof}(G) = 2 + |m - n| = m, \gamma_{cof}(G) = K_m \cup K_2$. $\gamma_{cof}(\bar{G}) = m - n + 2 = m$. $\gamma_{cof}(G) + \gamma_{cof}(\bar{G}) = 2m = 2(m+n) - 4$.

Also, $\gamma_{cof}(G) + \gamma_{cof}(\bar{G}) = m^2 = (m + n - 2)^2$. When $n = 4$, take $G = C_4 . \gamma_{cof}(G) = 2\ and\ \gamma_{cof}(\bar{G}) = 2$.

Hence, $\gamma_{cof}(G) + \gamma_{cof}(\overline{H}) = 4 = 2(4) - 4 = 2n - 4. \gamma_{cof}(G) + \gamma_{cof}(\bar{G}) = 4 = (4-2)^2 = (n-2)^2$.

When $n = 5$, take $G = K_{3,2}$. $\gamma_{cof}(G) = 3$ and $\gamma_{cof}(\bar{G}) = 3$.

Hence, $\gamma_{cof}(G) + \gamma_{cof}(\overline{H}) = 6 = 2 \times 5 - 4 = 2n - 4$. $\gamma_{cof}(G) + \gamma_{cof}(\bar{G}) = 9 = (5-2)^2 = (n-2)^2$.

Suppose $\gamma_{cof}(G) = 1$. Then G has a full degree vertex. Hence $\overline{H}$ is not connected. Therefore, $\gamma_{cof}(\bar{G}) \geq 2$.

This implies that $\gamma_{cof}(G) + \gamma_{cof}(\bar{G}) \geq 3$ and $\gamma_{cof}(G) + \gamma_{cof}(\bar{G}) \geq 2$. If $\gamma_{cof}(G) \geq 2$, then $\gamma_{cof}(\bar{G})$ being ≥ 1, we obtain $\gamma_{cof}(G) + \gamma_{cof}(\bar{G}) \geq\geq 3$ and $\gamma_{cof}(G) + \gamma_{cof}(\bar{G}) \geq 2$.

When $G = K_2, \gamma_{cof}(G) + \gamma_{cof}(\bar{G}) = 2$.

Hence, $\gamma_{cof}(G) + \gamma_{cof}(\bar{G}) == 3$ *and* $\gamma_{cof}(G) + \gamma_{cof}(\bar{G}) = 2$. Thus, the lower bounds are sharp.

When $G = K_4, \gamma_{cof}(G) + \gamma_{cof}(\bar{G}) = 5 > 2 \times 2 - 4 = 2n - 4$.

When $G = K_3, \gamma_{cof}(G).\gamma_{cof}(\bar{G}) = 3 > (n-2)^2 = 1$.

Thus, the conditions on the order of n can not be relaxed.

Theorem 4.1. Let G_i , $1 \leq i \leq k$, be graphs and let G be the disjoint union of G_i. Let order of G be n. Then $\sum_{i=1}^{k} \gamma_{cof}(G) \leq -\sum_{i=1}^{k} \gamma_{cof}(G_i) + \frac{(k-1)(n-k)}{k} + 2(n - fd(G))$.

Proof: If $k = 1$, then both sides become equal. Let $k \geq 2$. Let $|V(G_i)| = n_i, 1 \leq i \leq k$ and let without loss of generality, $n_1 \leq n_2 \leq \cdots \leq n_k$. So, $n_k \geq \frac{n}{k}$ and hence $n - n_k \leq \frac{(k-1)n}{k}$. Let S_i be a γ_{cof}-set of G_i. Then $\gamma_{cof}(G_i) \leq n_i - fd(G_i)$. Therefore, $\sum_{i=1}^{k} \gamma_{cof}(G_i) \leq \sum_{i=1}^{k} n_i - \sum_{i=1}^{k} fd(G_i) = n - \sum_{i=1}^{k} fd(G_i) \leq n + \frac{(k-1)(n-k)}{k} - fd(G)$. [Theorem 24 [4]]. Thus, $\gamma_{cof}(H) + \sum_{i=1}^{k} \gamma_{cof}(G_i) \leq n + \frac{(k-1)(n-k)}{k} fd(H) + n - fd(G)$. That is, $\gamma_{cof}(G) \leq -\sum_{i=1}^{k} \gamma_{cof}(G_i) + n + \frac{(k-1)(n-k)}{k} - fd(G) + n - fd(G). = -\sum_{i=1}^{k} \gamma_{cof}(G_i) + 2n + \frac{(k-1)(n-k)}{k} - 2fd(G)$.

Observation 4.2. Let H be a k −regular graph with $k \geq 3$. Let D be a maximal independent set of G. Then D is a complemenatry fair dominating set of H and $V - D$ is a fair dominating set of G. Hence $\gamma_{cof}(G) \leq |D|$ and $fd(G) \leq |V - D| = n - |D| \leq n - \gamma_{cof}(G)$. That is, $\gamma_{cof}(G) + fd(G) \leq n$. If D is a maximum independent set of H, then $|D| = \beta_0(G). fd(G) \leq n - |D| = n - \beta_0(G) \leq n - \frac{n}{\bar{d}(G)+1}$ (For every graph H of order $n, \beta_0(G) \geq \frac{n}{\bar{d}(H)+1}$ [7 − 8]) .

5. STRONG (UNIFORM) FAIR/COMPLEMENTARY FAIR DOMINATION

The connection between out regular number and complementary-fair domination number is studied in this section. B.D. Acharya introduced the concept of strong domination in [9]. The strong domination number of a graph G is the smallest positive integer k such that every subset of $V(G)$ of cardinality k is a dominating set of G. $k = \gamma(G)$ is possible if and only if G is either $G = K_n$ or $\overline{K_n}$ or $\overline{\left(\frac{n}{2}\right) K_2}$. The concept of strong domination takes the name of uniform domination in [10].

Definition 5.1. The least positive integer k such that every k −element subset of $V(G)$ is a fair (Complementary fair) dominating set of G is called the uniform / strong fair (Complementary fair) domination number of H and is denoted by $\gamma_{fd}^{u}(G)$ / $\gamma_{fd}^{s}(G)$ ($\gamma_{cof}^{u}(G)$ / $\gamma_{cof}^{s}(G)$).

Proposition 5.2.

1. $\gamma_{fd}^{u}(K_n) = 1$; $\gamma_{cof}^{u}(K_n) = 1$.
2. $\gamma_{fd}^{u}(K_{1,n}) = n$; $\gamma_{cof}^{u}(K_{1,n}) = n + 1$.
3. $\gamma_{fd}^{u}(K_{m,n}) = m + n$; $\gamma_{cof}^{u}(K_{m,n}) = m + n$
4. $\gamma_{fd}^{u}(C_n) = n - 2$; $\gamma_{cof}^{u}(C_n) = n$
5. $\gamma_{fd}^{u}(P_n) = n - 1$; $\gamma_{cof}^{u}(P_n) = n$
6. $\gamma_{fd}^{u}(W_n) = n - 3$; $\gamma_{cof}^{u}(W_n) = n - 1$

Theorem 5.3. For any graph $H, \gamma_{fd}^{u}(G),\ \gamma_{cof}^{u}(G) \geq \gamma_u(G)$.

Proof: Let $\gamma_{fd}^{u}(G) = k$ $(\gamma_{cof}^{u}(G) = k)$. Then any k- subset of $V(G)$ is a γ_{fd}-set of $H((\gamma_{cof}) - set\ of\ G)$. Hence any k-subset of G is a dominating set of G. Therefore, $\gamma_u(G) \leq k = \gamma_{fd}^{u}(G)$ $(\gamma_{cof}^{u}(G))$.

Remark 5.4. There are graphs for which $\gamma_u(G) < \gamma_{fd}^{u}(G)$ $(\gamma_{cof}^{u}(G))$. For example, $\gamma_u(K_{m,n}) = max(m,n) < \gamma_{fd}^{u}(H)$ $(\gamma_{cof}^{u}(G)) = m + n$.

Corollary 5.5. $\gamma_{fd}^{u}(G)\gamma_{cof}^{u}(G)) \geq n - \delta(H)$. (Since $\gamma_u(G) = n - \delta(G)$). But there are graphs like $K_{m,n}$ $(m, n \geq 3)$ for which $\gamma_{fd}^{u}(H)$ $(\gamma_{cof}^{u}(G)) > n - \delta(H)$.

Theorem 5.6. Let H be a k-regular graph in which for some vertex $u, N(u)$ is independent. Then $\gamma_{fd}^{u}(G) = \gamma(G)$ if and only if $H = K_n$ or $\overline{K_n}$ or $\left(\frac{n}{2}\right)K_2$ where n is even (that is, $K_{2n} - X$ where X is a 1-factor in K_{2n}) .

Proof: Suppose G is k-regular. Let for some vertex u in $G, N(u)$ be independent. Let $S = V - N(u)$. Then S is a fair dominating set of G. Hence, $\gamma_{fd}^{u} \leq |S| = n - k$. But, $\gamma_{fd}^{u}(G) \geq n - \delta(G) = n - k$. Hence $\gamma_{fd}^{u}(G) = n - k$. Moreover, $\gamma_u(H) = n - \delta = n - k$. Let $\gamma_{fd}^{u}(G) = \gamma(G)$. Then $\gamma(G) = n - k = \gamma_u(G)$. Therefore, by Theorem \cite{aru} and Theorem 3.4 of [10], $G = K_n$ or $\overline{K_n}$ or $\left(\frac{n}{2}\right)K_2$ (that is, $K_{2n} - X$ where X is a 1-factor in K_{2n}.) The converse is obvious.

Remark 5.7. $K_{m,n}$ $m \geq 3$, is regular and there are vertices u for which $N(u)$ is independent. But $\gamma_{fd}^{u}(K_{m,m}) = m$ and $\gamma(K_{m,n}) = 2 \neq \gamma_{fd}^{u}(K_{m,m})$. But $\gamma_u(K_{m,m}) = m = \gamma_{fd}^{u}(K_{m,m})$. Thus, even if $\gamma_{fd}^{u}(K_{m,m}) = \gamma_u(K_{m,m}), K_{m,m}, m \geq 3$, is not any of $K_n\ or\ \overline{K_n}$ or $\left(\frac{n}{2}\right)K_2$.

PROBLEMS FOR FUTURE STUDY:

(i) Find the necessary and sufficient conditions that G should satisfy that $\gamma_{cof}(G) = n - 2$.

(ii) When $fd(G) = \gamma_{cof}(G)$?

(iii) Study the concepts of $fd(G)$ and $\gamma_{cof}(G)$ for total / connected / equitable domination.

(iv) Find an upper bound for $\gamma_{cof}(G)$ for maximal outer planar graphs.

6. CONCLUSION

The study of dominating set has many applications in both social and communication networks Identification of complement of a dominating set has important in study of construction of stable networks. Fair dominating set deals with the neighbours in the complement of a dominating set in graphs. In this paper, we introduced complementary fair dominating sets and the bounds of $\gamma_{cof}(G)$ are studied.

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