ORIGINAL PAPER

SOME INTEGRAL TRANSFORMS INVOLVING GENERALIZED BESSEL-MAITLAND FUNCTION

SAIMA BATOOL[1], GHULAM FARID[1,*], SADIA KOUSAR[1]

Manuscript received: 01.12.2022; Accepted paper: 27.07.2023;
Published online: 30.09.2023.

Abstract. *In this paper, we establish some results in terms of generalized Wright hypergeometric function by applying different integral transforms such as Laplace transform, Whittaker transform, Hankel transform, K-transform, Sumudu transform, fractional Fourier transform etc. on the generalized Bessel-Maitland function.*

Keywords: *integral transforms; special functions; generalized Bessel-Maitland function; Wright hypergeometric function.*

Mathematics Subject Classification: *44A20; 11B37; 32A17.*

1. INTRODUCTION

Integral transforms are mathematical tools that convert functions from one domain to another. Integral transforms of generalized functions have extensive applications in various domains of mathematics especially in solving complex problems having singularities, irregularities and distributions as well as in the theory of differential equations and mathematical physics. Some important properties of Bessel-Maitland function and its relationships with Fox's H-function and Wright hypergeometric function have been discussed in [1]. In [2-6], the authors have derived some interesting new integral formulae involving the generalized Bessel-Maitland function and the results have been expressed in terms of Wright hypergeometric function. In [7], many fruitful results related to Bessel-Maitland function and some properties of the newly introduced operator, by using this function as kernel, have been explored. In [8], multiple advantageous results have been derived by evaluating integral transforms of Mittag-Leffler function. In [9], some fractional calculus operators have been studied and the obtained results have been discussed in terms of Wright function and hypergeometric series. Currently, many researchers have studied integral transforms and special functions and a variety of beneficial results have been derived (see [10-19]).

The notations $\Re$, C, N, R^+, Z^+ are used for the set of real, complex, natural numbers, positive real numbers and positive integers, respectively. A brief overview of some integral transforms used throughout the work is provided here blow.

A generalized form of the classical factorial function, defined for $m \in Z^+$, as $m! = \prod_{k=1}^{m}(m-k+1)$, is the Gamma function $\Gamma(u)$ which for $\Re(u) \in R^+$, is defined (see [20]) as

$$\Gamma(u) = \int_0^\infty t^{u-1} e^{-t} dt. \tag{1}$$

[1] University of Agriculture, Department of Mathematics and Statistics, Faisalabad, Pakistan.
E-mail: saimabatool1311@gmail.com; ghulamfarid@uaf.edu.pk; bssm.14.09@gmail.com.
* *Corresponding author: ghulamfarid@uaf.edu.pk.*

Another generalization of the classical factorial function is the Pochhammer symbol $(\gamma)_m$, defined in [20], for $\gamma \in C$ and $m \in N$, is

$$(\gamma)_m = \begin{cases} \prod_{k=1}^{m} (\gamma + k - 1) & \text{for} \quad m \geq 1, \\ 1 & \text{for} \quad m = 0, \gamma \neq 0. \end{cases}$$

The Beta function $B(u,v)$, for $\Re(u) > 0$ and $\Re(v) > 0$; is defined in [20] as

$$B(u,v) = \int_0^1 t^{u-1}(1-t)^{v-1} dt. \tag{2}$$

It is easy to see that

$$\begin{gathered} (1)_m = m!, \\ \Gamma(z+1) = z\Gamma(z), \\ \Gamma(\mathrm{m}) = (\mathrm{m}-1)!, \end{gathered} \tag{3}$$

$$(\gamma)_m = \frac{\Gamma(\gamma + m)}{\Gamma(\gamma)} \tag{4}$$

and

$$B(u,v) = \frac{\Gamma(u)\Gamma(v)}{\Gamma(u+v)}. \tag{5}$$

The Bessel function $J_v(z)$ of first kind defined in [21] is

$$J_v(z) = \sum_{n=0}^{\infty} \frac{(-1)^n}{\Gamma(n+v+1)n!} \left(\frac{z}{2} \right)^{2n+v}.$$

For positive or integer *v*, it is finite at *z=0* and for negative non-integer *v*, it diverges as $z \to 0$. The Bessel function $K_v(z)$ of second kind is defined as

$$K_v(z) = \frac{J_v(z)\cos(v\pi) - J_{-v}(z)}{\sin(v\pi)}.$$

Following formulae were introduced in [22]

$$\int_0^{\infty} z^{\lambda-1} J_v(pz) dz = 2^{\lambda-1} p^{-\lambda} \frac{\Gamma\left(\frac{\lambda+v}{2}\right)}{\Gamma\left(1+\frac{v-\lambda}{2}\right)} \tag{6}$$

and

$$\int_0^{\infty} t^{\alpha-1} K_v(pz) dz = 2^{\alpha-2} p^{-\alpha} \Gamma\left(\frac{\alpha \pm v}{2}\right), \ \Re_v(p) > 0, \ \Re_v(\alpha \pm v) > 0. \tag{7}$$

The Whittaker function denoted by $W_{\alpha,\beta}(z)$ is given as

$$W_{\alpha,\beta}(z) = \frac{\Gamma(-2\alpha)}{\Gamma(\frac{1}{2} - \delta - \alpha)} M_{\delta,\alpha}(z) + \frac{\Gamma(2\alpha)}{\Gamma(\frac{1}{2} + \delta + \alpha)} M_{\delta,-\alpha}(z),$$

where $M_{\delta,\alpha}(z)$ is defined as

$$M_{\delta,\alpha}(z)=z^{\frac{1}{2}+\alpha}e^{-\frac{z}{2}}{}_1F_1(\tfrac{1}{2}+\alpha-\delta;2\alpha+1;z).$$

The Whittaker transform satisfies the following relation (see [23])

$$\int_0^\infty e^{-\frac{z}{2}}z^{\varepsilon-1}W_{a,b}(z)dz=\frac{\Gamma\left(\frac{1}{2}+b+\varepsilon\right)\Gamma\left(\frac{1}{2}-b+\varepsilon\right)}{\Gamma(1-a+\varepsilon)},\ \Re(b\pm\varepsilon)>-\frac{1}{2}. \tag{8}$$

It is easy to see that

$$\int_0^\infty t^{\upsilon-1}W_{\eta,\sigma}(t)W_{-\eta,\sigma}(t)dt=\frac{\Gamma\left(\frac{\upsilon+1}{2}\pm\sigma\right)\Gamma(\upsilon+1)}{\Gamma\left(1+\frac{\upsilon}{2}\pm\eta\right)}. \tag{9}$$

The Laplace transform $L\{g(z);s\}$, of a function $g(z)$, is defined in [24] as

$$G(s)=L\{g(z);s\}=\int_0^\infty g(z)e^{-sz}dz,\ \Re(s)>0. \tag{10}$$

It is easy to see that

$$\int_0^\infty z^{p-1}e^{-sz}dz=\frac{\Gamma(p)}{s^p},\ \Re(p)>1,\ \Re(s)>1 \tag{11}$$

and

$$\int_0^\infty z^{\beta}e^{sz}dz=\frac{(-1)^{\beta}}{(s)^{\beta+1}}\Gamma(\beta+1). \tag{12}$$

The Euler transform $E\{g(z);\theta,\vartheta\}$, of a function $g(z)$, for $\Re(\theta)>0$ and $\Re(\vartheta)>0$; is defined in [24] as

$$E\{g(z);\theta,\vartheta\}=\int_0^1(1-z)^{\vartheta-1}z^{\theta-1}g(z)dz,\ \theta,\vartheta\in C. \tag{13}$$

The Hankel transform $G(p;\nu)$, of order ν, of a function $g(z)$, is defined in [25] as

$$G(p;\nu)=\int_0^\infty(pz)^{\frac{1}{2}}J_\nu(pz)g(z)dz,\ p>0. \tag{14}$$

The K-transform $G[p;\nu]$, of a function $g(z)$, is defined (see [25]) as

$$G[p;\nu]=\Re_\nu[g(z);p]=\int_0^\infty g(z)(pz)^{\frac{1}{2}}K_\nu(pz)dz,\ \Re(\nu)>0.$$

The Varma transform $V(g,a,b;s)$, of a function $g(z)$, is defined (see [1]) as

$$V(g,a,b;s)=\int_0^\infty (sz)^{b-\frac{1}{2}}e^{-\frac{sz}{2}}W_{a,b}(sz)g(z)dz.$$

The Sumudu transform $S[g(z);p]$, of a function $g(z)$, for $z\geq 0$; is defined as [26]:

$$G(p)=S[g(z);p]=\int_0^\infty \frac{1}{p}\,e^{-\frac{z}{p}}g(z)dz,\ p\in\Re. \tag{15}$$

The fractional Fourier transform $\Im_\alpha[g](w)$, of a function $g(z)$, is defined in [27] as

$$u_\alpha(w)=\Im_\alpha[g](w)=\int_R e^{iw^{\left(\frac{1}{\alpha}\right)}z}g(z)dz, \tag{16}$$

where α denotes the order of this transform and $0<\alpha\leq 1$. One may notice that, for $\alpha=1$; the fractional Fourier transform reduces to conventional Fourier transform.

The P_v-transform $P_v[g(z);s]$, for $s\in C$ and $v>1$; is defined in [28] as

$$P_v[g(z);s]=G_{P_v}(s)=\int_0^\infty [1+(v-1)s]^{-\frac{z}{v-1}}g(z)dz. \tag{17}$$

The P_v-transform of the power function can be written as

$$P_v[z^{\alpha-1};s]=\left(\frac{v-1}{\ln[1+(v-1)s]}\right)^\alpha\Gamma(\alpha) \tag{18}$$

or

$$P_v[z^{\alpha-1};s]=\left(\frac{1}{\chi(v;s)}\right)^\alpha\Gamma(\alpha). \tag{19}$$

If $g(z)$ is a function with real values and $z\in\Re$, then the two-sided Laplace transform $G(g)(s)$ defined in [8], is

$$G(g)(s)=\int_{-\infty}^\infty e^{-sz}g(z)dz. \tag{20}$$

The relationship between two-sided Laplace transform and Laplace transform is given as

$$G(g)(s)=L(g(z))(s)+L(g(-z))(-s). \tag{21}$$

A new generalization of the generalized Bessel-Maitland function defined in [4] is given as follows:

$$J_{\theta,\eta,\sigma,\tau}^{\alpha,\beta,\delta,\gamma}(z)=\sum_{i=0}^\infty \frac{(\eta)_{\beta i}(\tau)_{\gamma i}(-z)^i}{\Gamma(\theta+1+\alpha i)(\sigma)_{\delta i}}, \tag{22}$$

where $\alpha,\theta,\eta,\sigma,\tau\in C,\ \Re(\theta)\geq -1,\ \Re(\alpha)>0,\ \Re(\eta)>0,\ \Re(\sigma)>0,\ \Re(\tau)>0,\ \beta,\gamma,\delta\geq 0,$ $\beta,\delta>\Re(\alpha)+\gamma$.

The generalized Wright hypergeometric function, for $h\in C,\ \ \eta_n,k_m\in C,$ $\delta_n,\kappa_m\in\Re/\{0\},\ m=1,2,3,...,\mu\ ,\ n=1,2,3,...,\omega$; is defined as (see [29]):

$$ {}_{\mu}\psi_{\omega}(h) = {}_{\mu}\psi_{\omega}\left[\begin{matrix}(k_m,\kappa_m)_{1,\mu}\\(\eta_n,\delta_n)_{1,\omega}\end{matrix}\,|\,h\right] = \sum_{i=0}^{\infty}\frac{\prod_{m=1}^{\mu}\Gamma(k_m+\kappa_m i)}{\prod_{n=1}^{\omega}\Gamma(\eta_n+\delta_n i)}\frac{h^i}{i!}. \tag{23} $$

For the study topics related to special functions and integral transforms, we refer at [30-35].

2. MAIN RESULTS

We are going to discuss some integral transforms of generalized Bessel-Maitland function in terms of generalized Wright hypergeometric function.

Theorem 2.1. Let $a,b,\tau,\theta,\alpha,\sigma,\eta\in C$, $\Re(a)>0$, $\Re(\theta)\geq -1$, $\Re(\sigma)>0$, $\Re(\eta)>0$, $\Re(\alpha)>0$, $\Re(\tau)>0$, $\Re(b)>0$, $\beta,\gamma,\delta\geq 0$ and $\beta,\delta>\Re(\alpha)+\gamma$. Then we have

$$ E\left\{J_{\theta,\eta,\sigma,\tau}^{\alpha,\beta,\delta,\gamma}(xz^{\vartheta});a,b\right\} = \frac{\Gamma(b)\Gamma(\sigma)}{\Gamma(\eta)\Gamma(\tau)}\,{}_4\psi_3\left[\begin{matrix}(\eta,\beta),(\tau,\gamma),(a,\vartheta),(1,1)\\(\theta+1,\alpha),(\sigma,\delta),(a+b,\vartheta)\end{matrix};-x\right]. \tag{24} $$

Proof: By the use of (22) and (2) in (13), we have

$$ \begin{aligned} E\left\{J_{\theta,\eta,\sigma,\tau}^{\alpha,\beta,\delta,\gamma}(xz^{\vartheta});a,b\right\} &= \int_0^1 z^{a-1}(1-z)^{b-1}J_{\theta,\eta,\sigma,\tau}^{\alpha,\beta,\delta,\gamma}(xz^{\vartheta})dz \\ &= \int_0^1 z^{a-1}(1-z)^{b-1}\sum_{i=0}^{\infty}\frac{(\eta)_{\beta i}(\tau)_{\gamma i}(-xz^{\vartheta})^i}{\Gamma(\theta+1+\alpha i)(\sigma)_{\delta i}}dz \\ &= \sum_{i=0}^{\infty}\frac{(\eta)_{\beta i}(\tau)_{\gamma i}(-x)^i}{\Gamma(\theta+1+\alpha i)(\sigma)_{\delta i}}B(a+\vartheta i,b). \end{aligned} $$

Use of (3), (4) and (5) lead to the following relation

$$ E\left\{J_{\theta,\eta,\sigma,\tau}^{\alpha,\beta,\delta,\gamma}(xz^{\vartheta});a,b\right\} = \frac{\Gamma(\sigma)}{\Gamma(\eta)\Gamma(\tau)}\sum_{i=0}^{\infty}\frac{\Gamma(\eta+\beta i)\Gamma(\tau+\gamma i)\Gamma(1+i)}{\Gamma(\alpha i+\theta+1)\Gamma(\sigma+\delta i)}\frac{(-x)^i}{i!}\frac{\Gamma(a+\vartheta i)\Gamma(b)}{\Gamma(a+b+\vartheta i)}, $$

which, by the use of (23), leads to (24).

Theorem 2.2. Let $\tau,\theta,\alpha,\sigma,\eta\in C$, $\Re(s)>0$, $\Re(\theta)\geq -1$, $\Re(\sigma)>0$, $\Re(\eta)>0$, $\Re(\alpha)>0$, $\Re(\tau)>0$, $\beta,\gamma,\delta\geq 0$ and $\beta,\delta>\Re(\alpha)+\gamma$. Then we have

$$ L\left\{J_{\theta,\eta,\sigma,\tau}^{\alpha,\beta,\delta,\gamma}(xz^{\vartheta});s\right\} = \frac{\Gamma(\sigma)}{\Gamma(\eta)\Gamma(\tau)s}\,{}_4\psi_2\left[\begin{matrix}(\eta,\beta),(\tau,\gamma),(1,\vartheta),(1,1)\\(\theta+1,\alpha),(\sigma,\delta)\end{matrix};\frac{-x}{s^{\vartheta}}\right]. \tag{25} $$

Proof: Using (22) and (11) in (10), we get

$$ L\left\{J_{\theta,\eta,\sigma,\tau}^{\alpha,\beta,\delta,\gamma}(xz^{\vartheta});s\right\} = \int_0^{\infty} e^{-sz}J_{\theta,\eta,\sigma,\tau}^{\alpha,\beta,\delta,\gamma}(xz^{\vartheta})dz $$

$$=\int_0^\infty e^{-sz}\sum_{i=0}^\infty \frac{(\eta)_{\beta i}(\tau)_{\gamma i}(-xz^{\vartheta})^i}{\Gamma(\theta+1+\alpha i)(\sigma)_{\delta i}}dz \;=\sum_{i=0}^\infty \frac{(\eta)_{\beta i}(\tau)_{\gamma i}(-x)^i}{\Gamma(\theta+1+\alpha i)(\sigma)_{\delta i}}\frac{\Gamma(1+\vartheta i)}{s^{1+\vartheta i}}.$$

Applying (3) and (4) on above relation, we obtain

$$L\left\{J_{\theta,\eta,\sigma,\tau}^{\alpha,\beta,\delta,\gamma}(xz^{\vartheta});s\right\}=\frac{\Gamma(\sigma)}{\Gamma(\eta)\Gamma(\tau)s}\sum_{i=0}^\infty \frac{\Gamma(\eta+\beta i)\Gamma(\tau+\gamma i)\Gamma(1+\vartheta i)\Gamma(1+i)}{\Gamma(\theta+1+\alpha i)\Gamma(\sigma+\delta i)}\frac{\left(\frac{-x}{s^{\vartheta}}\right)^i}{i!}.$$

Using (23), the above relation gives (25).

Theorem 2.3. Let $\tau,\theta,\alpha,\sigma,\eta\in C$, $\Re(b\pm\varepsilon)>-\frac{1}{2}$, $\Re(\theta)\geq -1$, $\Re(\sigma)>0$, $\Re(\eta)>0$, $\Re(\alpha)>0$, $\Re(\tau)>0$, $\beta,\gamma,\delta\geq 0$ and $\beta,\delta>\Re(\alpha)+\gamma$. Then we have

$$\int_0^\infty e^{-\frac{z}{2}}z^{\varepsilon-1}W_{a,b}(z)J_{\theta,\eta,\sigma,\tau}^{\alpha,\beta,\delta,\gamma}(xz^{\vartheta})dz$$
$$=\frac{\Gamma(\sigma)}{\Gamma(\eta)\Gamma(\tau)}\,{}_5\psi_3\left[\begin{matrix}(\eta,\beta),(\tau,\gamma),(\frac{1}{2}+b+\varepsilon,\vartheta),(\frac{1}{2}-b+\varepsilon,\vartheta),(1,1)\\(\theta+1,\alpha),(\sigma,\delta),(1-a+\varepsilon,\vartheta)\end{matrix};-x\right]. \tag{26}$$

Proof: It follows, by using definition (22), that

$$\int_0^\infty e^{-\frac{z}{2}}z^{\varepsilon-1}W_{a,b}(z)J_{\theta,\eta,\sigma,\tau}^{\alpha,\beta,\delta,\gamma}(xz^{\vartheta})dz=\int_0^\infty e^{-\frac{z}{2}}z^{\varepsilon-1}W_{a,b}(z)\sum_{i=0}^\infty\frac{(\eta)_{\beta i}(\tau)_{\gamma i}(-xz^{\vartheta})^i}{\Gamma(\theta+1+\alpha i)(\sigma)_{\delta i}}dz$$
$$=\sum_{i=0}^\infty\frac{(\eta)_{\beta i}(\tau)_{\gamma i}(-x)^i}{\Gamma(\theta+1+\alpha i)(\sigma)_{\delta i}}\int_0^\infty e^{-\frac{z}{2}}z^{\varepsilon+\vartheta i-1}W_{a,b}(z)dz.$$

Use of the equations (3), (4) and (8) changes the above relation to

$$\int_0^\infty e^{-\frac{z}{2}}z^{\varepsilon-1}W_{a,b}(z)J_{\theta,\eta,\sigma,\tau}^{\alpha,\beta,\delta,\gamma}(xz^{\vartheta})dz$$
$$=\frac{\Gamma(\sigma)}{\Gamma(\eta)\Gamma(\tau)}\sum_{i=0}^\infty\frac{\Gamma(\eta+\beta i)\Gamma(\tau+\gamma i)\Gamma(1+i)}{\Gamma(\theta+1+\alpha i)\Gamma(\sigma+\delta i)}\frac{(-x)^i}{i!}\frac{\Gamma\left(\frac{1}{2}+b+\varepsilon+\vartheta i\right)\Gamma\left(\frac{1}{2}-b+\varepsilon+\vartheta i\right)}{\Gamma\left(1-a+\varepsilon+\vartheta i\right)},$$

from which, (26) follows with the help of (23).

Theorem 2.4: Let $\tau,\theta,\alpha,\sigma,\eta\in C$, $\Re(\theta)\geq -1$, $\Re(\sigma)>0$, $\Re(\eta)>0$, $\Re(\alpha)>0$, $\Re(\tau)>0$, $\beta,\gamma,\delta\geq 0$ and $\beta,\delta>\Re(\alpha)+\gamma$. Then the subsequent relation holds

$$\int_0^\infty z^{\varepsilon-1}W_{a,b}(sz)W_{-a,b}(sz)J_{\theta,\eta,\sigma,\tau}^{\alpha,\beta,\delta,\gamma}(xz^{\vartheta})dz=\frac{\Gamma(\sigma)}{s^{\varepsilon}\Gamma(\eta)\Gamma(\tau)} \tag{27}$$

$$\times {}_6\psi_4\left[\begin{matrix}(\eta,\beta),(\tau,\gamma),(\frac{\varepsilon+2b+1}{2},\frac{\vartheta}{2}),(\frac{\varepsilon-2b+1}{2},\frac{\vartheta}{2}),(\varepsilon+1,\vartheta),(1,1)\\(\theta+1,\alpha),(\sigma,\delta),(\frac{2+\varepsilon+2a}{2},\frac{\vartheta}{2}),(\frac{2+\varepsilon-2a}{2},\frac{\vartheta}{2})\end{matrix};-\frac{x}{s^{\vartheta}}\right].$$

Proof: By setting $sz=t$ and using definition (22), we get

$$\int_0^\infty z^{\varepsilon-1}W_{a,b}(sz)W_{-a,b}(sz)J^{\alpha,\beta,\delta,\gamma}_{\theta,\eta,\sigma,\tau}(xz^{\vartheta})dz=\frac{1}{s^{\varepsilon}}\int_0^\infty t^{\varepsilon-1}W_{a,b}(t)W_{-a,b}(t)\sum_{i=0}^\infty\frac{(\eta)_{\beta i}(\tau)_{\gamma i}(-x(\frac{t}{s})^{\vartheta})^i}{\Gamma(\alpha i+\theta+1)(\sigma)_{\delta i}}dt$$

$$=\frac{1}{s^{\varepsilon}}\sum_{i=0}^\infty\frac{(\eta)_{\beta i}(\tau)_{\gamma i}(-\frac{x}{s^{\vartheta}})^i}{\Gamma(\alpha i+\theta+1)(\sigma)_{\delta i}}\int_0^\infty t^{\varepsilon+\vartheta i-1}W_{a,b}(t)W_{-a,b}(t)dt.$$

Using (3), (4) and (9), it becomes

$$\int_0^\infty z^{\varepsilon-1}W_{a,b}(sz)W_{-a,b}(sz)J^{\alpha,\beta,\delta,\gamma}_{\theta,\eta,\sigma,\tau}(xz^{\vartheta})dz$$

$$=\frac{\Gamma(\sigma)}{s^{\varepsilon}\Gamma(\eta)\Gamma(\tau)}\sum_{i=0}^\infty\frac{\Gamma(\eta+\beta i)\Gamma(\tau+\gamma i)\Gamma(1+i)}{\Gamma(\theta+1+\alpha i)\Gamma(\sigma+\delta i)}\frac{(-\frac{x}{s^{\vartheta}})^i}{i!}\frac{\Gamma(\frac{\varepsilon+\vartheta i+1}{2}\pm b)\Gamma(\varepsilon+\vartheta i+1)}{\Gamma(1+\frac{\varepsilon+\vartheta i}{2}\pm a)}.$$

Using (23), we obtain (27).

Theorem 2.5. Let $\tau,\theta,\alpha,\sigma,\eta\in C$, $\Re(v)>0$, $\Re(\theta)\ge -1$, $\Re(\sigma)>0$, $\Re(\eta)>0$, $\Re(\alpha)>0$, $\Re(\tau)>0$, $\beta,\gamma,\delta\ge 0$ and $\beta,\delta>\Re(\alpha)+\gamma$. Then we get

$$\int_0^\infty z^{\rho-1}K_v(pz)J^{\alpha,\beta,\delta,\gamma}_{\theta,\eta,\sigma,\tau}(xz^{\vartheta})dz=\frac{2^{\rho-2}\Gamma(\sigma)}{p^{-\rho}\Gamma(\eta)\Gamma(\tau)}$$

$$\times {}_5\psi_2\left[\begin{matrix}(\eta,\beta),(\tau,\gamma),(\frac{\rho+v}{2},\frac{\vartheta}{2}),(\frac{\rho-v}{2},\frac{\vartheta}{2}),(1,1)\\(\theta+1,\alpha),(\sigma,\delta)\end{matrix};-x(\frac{2}{p})^{\vartheta}\right]. \tag{28}$$

Proof: Using definition (22), we have

$$\int_0^\infty z^{\rho-1}K_v(pz)J^{\alpha,\beta,\delta,\gamma}_{\theta,\eta,\sigma,\tau}(xz^{\vartheta})dz=\int_0^\infty z^{\rho-1}K_v(pz)\sum_{i=0}^\infty\frac{(\eta)_{\beta i}(\tau)_{\gamma i}(-xz^{\vartheta})^i}{\Gamma(\theta+1+\alpha i)(\sigma)_{\delta i}}dz$$

$$=\sum_{i=0}^\infty\frac{(\eta)_{\beta i}(\tau)_{\gamma i}(-x)^i}{\Gamma(\alpha i+\theta+1)(\sigma)_{\delta i}}\int_0^\infty z^{\rho+\vartheta i-1}K_v(pz)dz.$$

It follows, from definitions (3), (4) and (7), that

$$\int_0^\infty z^{\rho-1}K_v(pz)J^{\alpha,\beta,\delta,\gamma}_{\theta,\eta,\sigma,\tau}(xz^{\vartheta})dz$$

$$=\frac{\Gamma(\sigma)}{\Gamma(\eta)\Gamma(\tau)}\sum_{i=0}^\infty\frac{\Gamma(\eta+\beta i)\Gamma(\tau+\gamma i)\Gamma(1+i)}{\Gamma(\theta+1+\alpha i)\Gamma(\sigma+\delta i)}\frac{(-x)^i}{i!}2^{\rho+\vartheta i-2}p^{-\rho-\vartheta i}\Gamma\left(\frac{\rho+\vartheta i\pm v}{2}\right),$$

which, by the use of (23), leads to (28).

Theorem 2.6: Let $\tau,\theta,\alpha,\sigma,\eta \in C,\quad p>0,\quad \Re(\theta)\ge -1,\quad \Re(\sigma)>0,\quad \Re(\eta)>0,\quad \Re(\alpha)>0,$ $\Re(\tau)>0,\ \beta,\gamma,\delta\ge 0$ and $\beta,\delta>\Re(\alpha)+\gamma$. Then we have

$$G(p;v)=\int_0^\infty (pz)^{\frac{1}{2}} J_v(pz) J_{\theta,\eta,\sigma,\tau}^{\alpha,\beta,\delta,\gamma}(xz^{\vartheta})dz$$
$$=\frac{2^{\frac{1}{2}}\Gamma(\sigma)}{p\Gamma(\eta)\Gamma(\tau)}\,{}_4\psi_3\left[\begin{matrix}(\eta,\beta),(\tau,\gamma),(\frac{\frac{3}{2}+v}{2},\frac{\vartheta}{2}),(1,1)\\ (\theta+1,\alpha),(\sigma,\delta),(\frac{v+\frac{1}{2}}{2},-\frac{\vartheta}{2})\end{matrix};-x(\frac{2}{p})^{\vartheta}\right]. \tag{29}$$

Proof: By the use of (22), (14) becomes

$$G(p;v)=\int_0^\infty (pz)^{\frac{1}{2}} J_v(pz) J_{\theta,\eta,\sigma,\tau}^{\alpha,\beta,\delta,\gamma}(xz^{\vartheta})dz=\int_0^\infty (pz)^{\frac{1}{2}} J_v(pz)\sum_{i=0}^{\infty}\frac{(\eta)_{\beta i}(\tau)_{\gamma i}(-xz^{\vartheta})^i}{\Gamma(\theta+1+\alpha i)(\sigma)_{\delta i}}dz$$
$$=(p)^{\frac{1}{2}}\sum_{i=0}^{\infty}\frac{(\eta)_{\beta i}(\tau)_{\gamma i}(-x)^i}{\Gamma(\alpha i+\theta+1)(\sigma)_{\delta i}}\int_0^\infty z^{\vartheta i+\frac{1}{2}+1-1}J_v(pz)dz.$$

Using (3), (4) and (6), the above relation gets the form

$$G(p;v)=\frac{\Gamma(\sigma)(p)^{\frac{1}{2}}}{\Gamma(\eta)\Gamma(\tau)}\sum_{i=0}^{\infty}\frac{\Gamma(\eta+\beta i)\Gamma(\tau+\gamma i)\Gamma(1+i)}{\Gamma(\theta+1+\alpha i)\Gamma(\sigma+\delta i)}\frac{(-x)^i}{i!}2^{\vartheta i+\frac{3}{2}-1}p^{-\vartheta i-\frac{3}{2}}\frac{\Gamma\left(\frac{\vartheta i+\frac{3}{2}+v}{2}\right)}{\Gamma\left(1+\frac{v-\vartheta i-\frac{3}{2}}{2}\right)},$$

which further leads to (29) by the use of (23).

Theorem 2.7. Let $s,\tau,\theta,\alpha,\sigma,\eta\in C,\quad v>1,\quad \Re(\theta)\ge -1,\quad \Re(\sigma)>0,\quad \Re(\eta)>0,\quad \Re(\alpha)>0,$ $\Re(\tau)>0,\ \beta,\gamma,\delta\ge 0$ and $\beta,\delta>\Re(\alpha)+\gamma$. Then we get

$$P_v\left[J_{\theta,\eta,\sigma,\tau}^{\alpha,\beta,\delta,\gamma}(xz^{\vartheta});s\right]=\frac{1}{\chi(v;s)}\frac{\Gamma(\sigma)}{\Gamma(\eta)\Gamma(\tau)}\,{}_4\psi_2\left[\begin{matrix}(\eta,\beta),(\tau,\gamma),(1,\vartheta),(1,1)\\ (\theta+1,\alpha),(\sigma,\delta)\end{matrix};\frac{-x}{[\chi(v;s)]^{\vartheta}}\right]. \tag{30}$$

Proof: By the use of (22) and (18), the relation (17) becomes

$$P_v\left[J_{\theta,\eta,\sigma,\tau}^{\alpha,\beta,\delta,\gamma}(xz^{\vartheta});s\right]=\int_0^\infty [1+(v-1)s]^{-\frac{z}{v-1}}J_{\theta,\eta,\sigma,\tau}^{\alpha,\beta,\delta,\gamma}(xz^{\vartheta})dz$$
$$=\int_0^\infty [1+(v-1)s]^{-\frac{z}{v-1}}\sum_{i=0}^{\infty}\frac{(\eta)_{\beta i}(\tau)_{\gamma i}(-xz^{\vartheta})^i}{\Gamma(\alpha i+\theta+1)(\sigma)_{\delta i}}dz$$
$$=\sum_{i=0}^{\infty}\frac{(\eta)_{\beta i}(\tau)_{\gamma i}(-x)^i}{\Gamma(\alpha i+\theta+1)(\sigma)_{\delta i}}\left(\frac{v-1}{\ln[1+(v-1)s]}\right)^{\vartheta i+1}\Gamma(\vartheta i+1).$$

Using (3), (4) and (19), it follows that

$$P_v\left[J_{\theta,\eta,\sigma,\tau}^{\alpha,\beta,\delta,\gamma}(xz^{\vartheta});s\right]$$

$$=\frac{\Gamma(\sigma)}{\Gamma(\eta)\Gamma(\tau)}\sum_{i=0}^{\infty}\frac{\Gamma(\eta+\beta i)\Gamma(\tau+\gamma i)\Gamma(1+i)}{\Gamma(\theta+1+\alpha i)\Gamma(\sigma+\delta i)}\frac{(-x)^i}{i!}\left(\frac{1}{\chi(v;s)}\right)^{\vartheta i+1}\Gamma(\vartheta i+1),$$

which, by using (23), gives (30).

Theorem 2.8. Let $\tau,\theta,\alpha,\sigma,\eta\in C,\ 0<\alpha\le 1,\ \Re(\theta)\ge -1,\ \Re(\sigma)>0,\ \Re(\eta)>0,\ \Re(\alpha)>0,$ $\Re(\tau)>0,\ \beta,\gamma,\delta\ge 0$ and $\beta,\delta>\Re(\alpha)+\gamma$. Then we have

$$\Im_\alpha[J^{\alpha,\beta,\delta,\gamma}_{\theta,\eta,\sigma,\tau}(xz^\vartheta)](w)=\frac{1}{jw^{\frac{1}{\alpha}}}\frac{\Gamma(\sigma)}{\Gamma(\eta)\Gamma(\tau)}\,{}_4\psi_2\left[\begin{matrix}(\eta,\beta),(\tau,\gamma),(1,\vartheta),(1,1)\\(\theta+1,\alpha),(\sigma,\delta)\end{matrix};-x\left(\frac{-1}{jw^{\frac{1}{\alpha}}}\right)^{\vartheta}\right]. \tag{31}$$

Proof: Using (22) in (16) and setting $jw^{\frac{1}{\alpha}}z=-t$, we obtain

$$\Im_\alpha[J^{\alpha,\beta,\delta,\gamma}_{\theta,\eta,\sigma,\tau}(xz^\vartheta)](w)=\int_R e^{jw^{\frac{1}{\alpha}}z}J^{\alpha,\beta,\delta,\gamma}_{\theta,\eta,\sigma,\tau}(xz^\vartheta)dz=\int_R e^{jw^{\frac{1}{\alpha}}z}\sum_{i=0}^{\infty}\frac{(\eta)_{\beta i}(\tau)_{\gamma i}(-xz^\vartheta)^i}{\Gamma(\theta+1+\alpha i)(\sigma)_{\delta i}}dz$$

$$=\sum_{i=0}^{\infty}\frac{(\eta)_{\beta i}(\tau)_{\gamma i}(-x)^i}{\Gamma(\alpha i+\theta+1)(\sigma)_{\delta i}}\int_R e^{jw^{\frac{1}{\alpha}}z}z^{\vartheta i}dz\ =\frac{1}{jw^{\frac{1}{\alpha}}}\sum_{i=0}^{\infty}\frac{(\eta)_{\beta i}(\tau)_{\gamma i}(-x(\frac{-1}{jw^{\frac{1}{\alpha}}})^\vartheta)^i}{\Gamma(\alpha i+\theta+1)(\sigma)_{\delta i}}\int_0^\infty e^{-t}\left(t\right)^{\vartheta i+1-1}dt.$$

Using (1), (3) and (4), the above relation leads to

$$\Im_\alpha[J^{\alpha,\beta,\delta,\gamma}_{\theta,\eta,\sigma,\tau}(xz^\vartheta)](w)=\frac{1}{jw^{\frac{1}{\alpha}}}\frac{\Gamma(\sigma)}{\Gamma(\eta)\Gamma(\tau)}\sum_{i=0}^{\infty}\frac{\Gamma(\eta+\beta i)\Gamma(\tau+\gamma i)\Gamma(1+i)}{\Gamma(\theta+1+\alpha i)\Gamma(\sigma+\delta i)}\frac{(-x\left(\frac{-1}{jw^{\frac{1}{\alpha}}}\right)^{\vartheta})^i}{i!}\Gamma(1+\vartheta i),$$

which, by using (23), leads to (31).

Note: For $\alpha=1$; the result (31) also holds for the classical Fourier transform. That is;

$$\Im_1[J^{\alpha,\beta,\delta,\gamma}_{\theta,\eta,\sigma,\tau}(xz^\vartheta)](w)=\Im[J^{\alpha,\beta,\delta,\gamma}_{\theta,\eta,\sigma,\tau}(xz^\vartheta)](w)$$
$$=\frac{\Gamma(\sigma)}{\Gamma(\tau)\Gamma(\eta)\,jw}\,{}_4\psi_2\left[\begin{matrix}(\eta,\beta),(\tau,\gamma),(1,\vartheta),(1,1)\\(\theta+1,\alpha),(\sigma,\delta)\end{matrix};-x\left(\frac{-1}{jw}\right)^{\vartheta}\right].$$

Theorem 2.9. Let $\tau,\theta,\alpha,\sigma,\eta\in C,\ z\ge 0,\ p\in\Re,\ \Re(\theta)\ge -1,\ \Re(\sigma)>0,\ \Re(\eta)>0,$ $\Re(\alpha)>0,\ \Re(\tau)>0,\ \beta,\gamma,\delta\ge 0$ and $\beta,\delta>\Re(\alpha)+\gamma$. Then we have

$$S[J^{\alpha,\beta,\delta,\gamma}_{\theta,\eta,\sigma,\tau}(xz^\vartheta);p]=\frac{\Gamma(\sigma)}{\Gamma(\eta)\Gamma(\tau)}\,{}_4\psi_2\left[\begin{matrix}(\eta,\beta),(\tau,\gamma),(1,\vartheta),(1,1)\\(\theta+1,\alpha),(\sigma,\delta)\end{matrix};-xp^\vartheta\right]. \tag{32}$$

Proof: Applying definition (22) on (15) and setting $\frac{z}{p}=t,$ we get

$$S[J^{\alpha,\beta,\delta,\gamma}_{\theta,\eta,\sigma,\tau}(xz^\vartheta);p]=\int_0^\infty\frac{1}{p}\ e^{-\frac{z}{p}}J^{\alpha,\beta,\delta,\gamma}_{\theta,\eta,\sigma,\tau}(xz^\vartheta)dz=\frac{1}{p}\sum_{i=0}^{\infty}\frac{(\eta)_{\beta i}(\tau)_{\gamma i}(-x)^i}{\Gamma(\theta+1+\alpha i)(\sigma)_{\delta i}}\int_0^\infty e^{-\frac{z}{p}}z^{\vartheta i}dz$$
$$=\sum_{i=0}^{\infty}\frac{(\eta)_{\beta i}(\tau)_{\gamma i}(-xp^\vartheta)^i}{\Gamma(\theta+1+\alpha i)(\sigma)_{\delta i}}\int_0^\infty e^{-t}(t)^{\vartheta i}dt.$$

Use of (1), (3) and (4), changes above relation to the following

$$S[J^{\alpha,\beta,\delta,\gamma}_{\theta,\eta,\sigma,\tau}(xz^{\vartheta});p]=\frac{\Gamma(\sigma)}{\Gamma(\eta)\Gamma(\tau)}\sum_{i=0}^{\infty}\frac{\Gamma(\eta+\beta i)\Gamma(\tau+\gamma i)\Gamma(1+i)}{\Gamma(\theta+1+\alpha i)\Gamma(\sigma+\delta i)}\frac{(-xp^{\vartheta})^i}{i!}\Gamma(1+\vartheta i).$$

We get (32), by using (23).

Theorem 2.10. Let $\tau,\theta,\alpha,\sigma,\eta\in C$, $\Re(\theta)\geq -1$, $\Re(\sigma)>0$, $\Re(\eta)>0$, $\Re(\alpha)>0$, $\Re(\tau)>0$, $\beta,\gamma,\delta\geq 0$ and $\beta,\delta>\Re(\alpha)+\gamma$. Then we have

$$\int_0^{\infty}(z)^{\alpha-1}e^{-\frac{sz}{2}}W_{a,b}(sz)J^{\alpha,\beta,\delta,\gamma}_{\theta,\eta,\sigma,\tau}(xz^{\vartheta})dz=\frac{1}{s^{\alpha}}\frac{\Gamma(\sigma)}{\Gamma(\eta)\Gamma(\tau)}$$
$$\times{}_5\psi_3\left[\begin{matrix}(\eta,\beta),(\tau,\gamma),(\frac{1}{2}+b+\alpha,\vartheta),(\frac{1}{2}-b+\alpha,\vartheta),(1,1)\\(\theta+1,\alpha),(\sigma,\delta),(\frac{1}{2}-a+\alpha,\vartheta)\end{matrix};-\frac{x}{s^{\vartheta}}\right]. \quad (33)$$

Proof: Setting $sz=t$ and using definition (22) it follows, that

$$\int_0^{\infty}(z)^{\alpha-1}e^{-\frac{sz}{2}}W_{a,b}(sz)J^{\alpha,\beta,\delta,\gamma}_{\theta,\eta,\sigma,\tau}(xz^{\vartheta})dz=\frac{1}{s^{\alpha}}\int_0^{\infty}(t)^{\alpha-1}e^{-\frac{t}{2}}W_{a,b}(t)\sum_{i=0}^{\infty}\frac{(\eta)_{\beta i}(\tau)_{\gamma i}(-x(\frac{t}{s})^{\vartheta})^i}{\Gamma(\alpha i+\theta+1)(\sigma)_{\delta i}}dt$$
$$=\frac{1}{s^{\alpha}}\sum_{i=0}^{\infty}\frac{(\eta)_{\beta i}(\tau)_{\gamma i}(-\frac{x}{s^{\vartheta}})^i}{\Gamma(\alpha i+\theta+1)(\sigma)_{\delta i}}\int_0^{\infty}(t)^{\alpha+\vartheta i-1}e^{-\frac{t}{2}}W_{a,b}(t)dt.$$

Using (3), (4) and (8), we have

$$\int_0^{\infty}(z)^{\alpha-1}e^{-\frac{sz}{2}}W_{a,b}(sz)J^{\alpha,\beta,\delta,\gamma}_{\theta,\eta,\sigma,\tau}(xz^{\vartheta})dz$$
$$=\frac{1}{s^{\alpha}}\frac{\Gamma(\sigma)}{\Gamma(\eta)\Gamma(\tau)}\sum_{i=0}^{\infty}\frac{\Gamma(\eta+\beta i)\Gamma(\tau+\gamma i)\Gamma(1+i)}{\Gamma(\theta+1+\alpha i)\Gamma(\sigma+\delta i)}\frac{\Gamma(\frac{1}{2}+b+\alpha+\vartheta i)\Gamma(\frac{1}{2}-b+\alpha+\vartheta i)}{\Gamma(\frac{1}{2}-a+\alpha+\vartheta i)}\frac{(-\frac{x}{s^{\vartheta}})^i}{i!},$$

from which, we obtain (33), by using (23).

Theorem 2.11. Let $z,\tau,\theta,\alpha,\sigma,\eta\in C$, $\Re(\theta)\geq -1$, $\Re(\sigma)>0$, $\Re(\eta)>0$, $\Re(\alpha)>0$, $\Re(\tau)>0$, $\beta,\gamma,\delta\geq 0$ and $\beta,\delta>\Re(\alpha)+\gamma$. Then we get

$$G[J^{\alpha,\beta,\delta,\gamma}_{\theta,\eta,\sigma,\tau}(xz^{\vartheta})](s)=\frac{1}{s}\frac{\Gamma(\sigma)}{\Gamma(\eta)\Gamma(\tau)}{}_4\psi_2\left[\begin{matrix}(\eta,\beta),(\tau,\gamma),(1,\vartheta),(1,1)\\(\theta+1,\alpha),(\sigma,\delta)\end{matrix};-xs^{-\vartheta}\right]$$
$$+\frac{1}{s}\frac{\Gamma(\sigma)}{\Gamma(\eta)\Gamma(\tau)}{}_4\psi_2\left[\begin{matrix}(\eta,\beta),(\tau,\gamma),(1,\vartheta),(1,1)\\(\theta+1,\alpha),(\sigma,\delta)\end{matrix};x(-s^{-1})^{\vartheta}\right]. \quad (34)$$

Proof: It follows by the use of (20) and (21), that

$$G[J^{\alpha,\beta,\delta,\gamma}_{\theta,\eta,\sigma,\tau}(xz^{\vartheta})](s)=\int_{-\infty}^{\infty}e^{-sz}J^{\alpha,\beta,\delta,\gamma}_{\theta,\eta,\sigma,\tau}(xz^{\vartheta})dz$$
$$=L[J^{\alpha,\beta,\delta,\gamma}_{\theta,\eta,\sigma,\tau}(xz^{\vartheta})](s)+L[J^{\alpha,\beta,\delta,\gamma}_{\theta,\eta,\sigma,\tau}(-xz^{\vartheta})](-s).$$

From definitions (10) and (22), we have

$$G[J_{\theta,\eta,\sigma,\tau}^{\alpha,\beta,\delta,\gamma}(xz^{\vartheta})](s)=\int_0^{\infty}e^{-sz}J_{\theta,\eta,\sigma,\tau}^{\alpha,\beta,\delta,\gamma}(xz^{\vartheta})dz+\int_0^{\infty}e^{sz}J_{\theta,\eta,\sigma,\tau}^{\alpha,\beta,\delta,\gamma}(-xz^{\vartheta})dz$$

$$=\int_0^{\infty}e^{-sz}\sum_{i=0}^{\infty}\frac{(\eta)_{\beta i}(\tau)_{\gamma i}(-xz^{\vartheta})^i}{\Gamma(\alpha i+\theta+1)(\sigma)_{\delta i}}dz+\int_0^{\infty}e^{sz}\sum_{i=0}^{\infty}\frac{(\eta)_{\beta i}(\tau)_{\gamma i}(xz^{\vartheta})^i}{\Gamma(\alpha i+\theta+1)(\sigma)_{\delta i}}dz$$

$$=\sum_{i=0}^{\infty}\frac{(\eta)_{\beta i}(\tau)_{\gamma i}(-x)^i}{\Gamma(\alpha i+\theta+1)(\sigma)_{\delta i}}\int_0^{\infty}e^{-sz}z^{\vartheta i+1-1}dz+\sum_{i=0}^{\infty}\frac{(\eta)_{\beta i}(\tau)_{\gamma i}(x)^i}{\Gamma(\alpha i+\theta+1)(\sigma)_{\delta i}}\int_0^{\infty}e^{sz}z^{\vartheta i+1-1}dz.$$

Using (3), (4), (11) and (12), the above relation becomes

$$G[J_{\theta,\eta,\sigma,\tau}^{\alpha,\beta,\delta,\gamma}(xz^{\vartheta})](s)=\frac{\Gamma(\sigma)}{\Gamma(\eta)\Gamma(\tau)}\sum_{i=0}^{\infty}\frac{\Gamma(\eta+\beta i)\Gamma(\tau+\gamma i)\Gamma(1+i)}{\Gamma(\theta+1+\alpha i)\Gamma(\sigma+\delta i)}\frac{(-x)^i}{i!}\frac{\Gamma(\vartheta i+1)}{s^{\vartheta i+1}}$$

$$+\frac{\Gamma(\sigma)}{\Gamma(\eta)\Gamma(\tau)}\sum_{i=0}^{\infty}\frac{\Gamma(\eta+\beta i)\Gamma(\tau+\gamma i)\Gamma(1+i)}{\Gamma(\theta+1+\alpha i)\Gamma(\sigma+\delta i)}\frac{(x)^i}{i!}\frac{(-1)^{\vartheta i}}{s^{\vartheta i+1}}\Gamma(\vartheta i+1).$$

Using (23), we get (34).

3. CONCLUSION

In this paper, we have utilized generalized Bessel-Maitland as integrand in different integral transforms and derived results in terms of Wright hypergeometric function. The transforms evaluated in this paper hold significant utility in mathematics, applied physics and engineering.

REFERENCES

[1] Singh, M., Khan, M.A., Khan, A.H., *International Journal of Mathematics Trends and Technology*, **14**, 46, 2014.

[2] Ali, M., Khan, W.A., Khan, I.A., *Palestine Journal of Mathematics*, **9**, 991, 2020.

[3] Ghayasuddin, M., Khan, W.A., *Matematicki Vesnik*, **70**, 292, 2018.

[4] Ali, R.S., Mubeen, S., Nayab, I., Araci, S., Rahman, G., Nisar, K.S., *Discrete Dynamics in Nature and Society*, **2020**, 1, 2020.

[5] Choi, J., Agarwal, P., Mathur, S., Purohit, S.D., *Bulletin of the Korean Mathematical Society*, **51**, 995, 2014.

[6] Khan, W.A., Khan, I.A., Ahmad, M., *Journal of Applied and Pure Mathematics*, **2**, 63, 2020.

[7] Ali, M., Khan, W.A., Khan, I.A., *Journal of Fractional Calculus and Applications*, **11**, 82, 2020.

[8] Sontakke, B.R., Kamble, G.P., Ul-Haque, M.M., *International Journal of Pure and Applied Mathematics*, **108**, 327, 2016.

[9] Kilbas, A.A., Sebastian, N., *Integral Transforms and Special Functions*, **19**, 869, 2008.

[10] Ata, E., Kiymaz, I.O., *Applied Mathematics and Nonlinear Sciences*, **5**, 147, 2020.

[11] Belafhal, A., Nossir, N., Dalil-Essakali, L., Usman, T., *Analysis*, **43**, 105, 2023.

[12] Belafhal, A., El Halba, E.M., Usman, T., *International Journal of Applied and Computational Mathematics*, **6**, 1,2020.
[13] Shilin, I.A., Choi, J., *Integral Transforms and Special functions*, 1, 2023.
[14] Khan, S., Zaman, S., Arshad, M., Alhazmi, S.E., Khan, F., Park, J., *Mathematics and Computers in Simulation*, **208**, 727, 2023.
[15] Abilassan, A., Restrepo, J.E., Suragan, D., *Integral Transforms and Special Functions*, **34**, 244, 2023.
[16] Mushtt, I.Z., Kuffi, E.A., *Iraqi Journal for Computer Science and Mathematics*, **4**, 181, 2023.
[17] Fahad, H.M., Rehman, M.U., Fernandez, A., *Mathematical Methods in the Applied Sciences*, **46**, 8304, 2023.
[18] Srivastava, H.M., *International Journal of Applied Mathematics and Computer Science*, **6**, 27, 2022.
[19] Padma, A., Rao, M.G., Shimelis, B., *Research in Mathematics*, **10**, 1, 2023.
[20] Rainville, E.D., *Special Functions*, The Macmillan Company, New York, 1960.
[21] Watson, G.N., *A treatise on the theory of Bessel functions*, The University Press, 1922.
[22] Mathai, A.M., Saxena, R.K., Haubold, H.J., *The H-Function theory and application*, Springer New York Dordrecht, Heilberg, London, 2010.
[23] Whittaker, E.T., Watson, G.N., *A Course of Modern Analysis*, Cambridge University Press, Cambridge, 1996.
[24] Sneddon, I.N., *The use of Integral Transform*, New Delhi, Tata McGraw Hill, 1979.
[25] Erdelyi, A., Magnus, W., Oberhettinger, F., Tricomi, F. G., *Higher Transcendental Functions*, McGraw Hill, New York, 1954.
[26] Watugala, G.K., *International Journal of Mathematical Education in Science and Technology*, **24**, 35, 1993.
[27] Luchko, Y., Martinez, H., Trujillo, J.J., *Fractional Calculus and Applied Analysis*, **11**, 457, 2008.
[28] Kumar, D., *Journal of Mathematical Physics*, **54**, 043509, 2013.
[29] Wright, E.M., *Journal of London Mathematical Society*, **10**, 286, 1935.
[30] Srivastava, H.M., Karlson, P.W., *Multiple Gaussian Hypergeometric Series*, Ellis Horwood Ltd. Chichester, 1985.
[31] Piessens, R., *The Hankel Transform*, In Poularikas, A.D. (Ed.), *The Transforms and Applications Handbook: Second Edition*, CRC Press LLC, Boca Raton, 2000.
[32] Debnath, L., Bhatta, D., *Integral Transforms and Their Applications, 3rd Ed.*, Chapman and Hall (CRC Press), London, UK, New York, 2007.
[33] Brychkov, Y.A., Prudnikov, A.P., *Integral transforms of generalized functions*, CRC Press, 1989.
[34] Davies, B., *Integral transforms and their applications*, Springer Science and Business Media, 2002.
[35] Antimirov, M.Y., Kolyshkin, A.A., Vaillancourt, R., *Applied integral transforms*, American Mathematical Society, 2007.