ORIGINAL PAPER

MHD FLOW PAST OVER OSCILLATING VERTICAL PLATE THROUGH POROUS MEDIA IN THE PRESENCE OF HALL EFFECT AND CHEMICAL REACTION

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Abstract. *MHD flow past over oscillating vertical plate with variable temperature and mass diffusion through a porous medium in the presence of Hall current effect and chemical reaction is studied here. The MHD fluid model contains partial differential equations of momentum, energy and diffusion equations. The Laplace-transform technique used to find the exact solution of equations involved in this paper. The resultant velocity profile is discussed with the help of graphs drawn for different parameters like, inclination of magnetic field, chemical reaction parameter, phase angle parameter and permeability parameter, and the numerical values Sherwood number have been also tabulated. It is noticed that the values obtained for velocity, concentration and temperature profiles are in concurrence with the actual flow of the unsteady MHD fluid.*

Keywords: *MHD flow; oscillating surface; Hall current; inclined magnetic field.*

1. INTRODUCTION

The study of MHD fluid flow behaviers in the presence of chemical reaction with variable surface temperature and mass diffusion through porous medium has play an important role in different areas of science and technology. MHD effect on heat transfer over stretching sheet embedded in porous medium with variable viscosity, viscous dissipation and heat source or sink was analyzed by Dessie and Naikoti [1]. They solved the flow model by using Runge-Kutta fourth order numerical method, and observed that if the heat sink parameter is increased then the thermal boundary layer increases, whereas the thermal boundary layer decreases with heat source. Chemically reacting MHD boundary layer flow of heat and mass transfer over a moving vertical plate with suction was investigated by Ibrahim and Makinde [2]. Mythreye et al. [3] have proposed chemical reactions on unsteady MHD convective heat and mass transfer past a semi infinite vertical permeable moving plate with heat absorption. They solved the governing equations by using perturbation technique, and explained the presence of heat absorption effect caused reductions in the fluid temperature which resulted in decrease in the fluid velocity. Kumar et al. [4] have studied chemical reaction effect on MHD flow past an impulsively started vertical cylinder with variable temperature and mass diffusion. Raptis and Kafousias [5] have developed MHD free convection flow and mass transfer through a porous medium bounded by an infinite vertical porous plate with constant heat flux. Rajput and Kumar [6] have worked on unsteady MHD flow past an impulsively started inclined plate with variable temperature and mass diffusion in

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the presence of Hall current. Further Rajput and Kumar [7] have worked on effects of radiation and chemical reaction on MHD flow past a vertical plate with variable temperature and mass diffusion. Sahin and Chamkha [8] have studied the effects of chemical reaction, heat and mass transfer and radiation on the MHD flow along a vertical porous wall in the presence of an induced magnetic field. Samad et al. [9] have developed MHD free convection and mass transfer flow through a vertical oscillatory porous plate with Hall, ion-slip currents and heat source in a rotating system. Samad et al. [9] have used explicit finite difference methods to solve the coupled non-linear partial differential equations. They examined that the shear stress and Sherwood numbers are decreased with the increase in heat source parameter, and Nusselt number is increased with increase in heat source. Hall Effect on unsteady MHD natural convection flow of a heat absorbing fluid past an accelerated moving vertical plate with ramped wall temperature was investigated by Seth et al. [10]. The model considered by Seth et al. [10] was solved by Laplace transform technique. In their study they observed that velocity and temperature of fluid near the vertical plate decreases when heat absorption parameter increases. VeeraKrishna et al. [11] have worked on Hall effects on unsteady MHD oscillatory free convective flow of second grade fluid through porous medium between two vertical plates. Youn [12] has considered heat and mass transfer in MHD micropolar flow over a vertical moving porous plate in a porous medium. In the present paper, chemical reaction and Hall current effects on unsteady MHD flow along vertical plate with variable temperature and mass diffusion in the presence of porous medium is considered. The MHD flow model of this paper is solved by Laplace transform technique. The results are shown with the help of graphs and tables.

2. MATHEMATICAL ANALYSIS

The geometrical model of the MHD fluid flow problem is shown in Fig. 1.

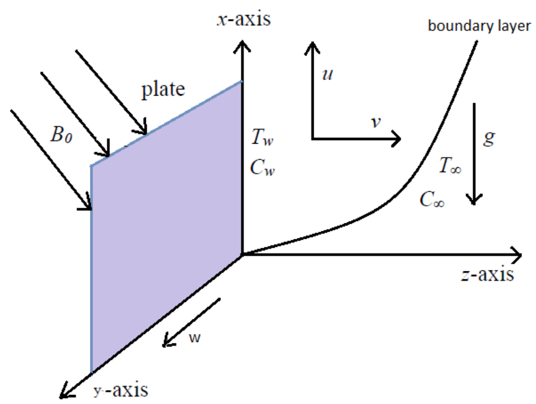

Figure 1. Physical model of the MHD fluid flow.

MHD flow past over electrically non conducting vertical plate is considered. The x axis is taken along the plate and z normal to it. The primary velocity of fluid u is along the plate and the secondary velocity of fluid v is along the z axis. An inclined magnetic field B_0 of uniform strength is applied on the flow. Initially it has been considered that the plate as well as the fluid is at the same temperature T_∞. The species concentration in the fluid is taken as C_∞. At time $t > 0$, the plate starts oscillating in its own plane with frequency ω and temperature of the plate is raised to T_w. The concentration C near the plate is raised linearly with respect to time. The fluid has a very small value of Reynolds numbers so the induced magnetic field can be neglected. So, under above assumptions, the flow model is as under.

$$\frac{\partial u}{\partial t}=\upsilon\frac{\partial^2 u}{\partial z^2}+g\beta(T-T_\infty)+g\beta^*(C-C_\infty)-\frac{\sigma B_0^2Cos^2\alpha(u+mvCos\alpha)}{\rho(1+m^2Cos^2\alpha)}-\frac{\upsilon u}{K},\tag{1}$$

$$\frac{\partial v}{\partial t}=\upsilon\frac{\partial^2 v}{\partial z^2}+\frac{\sigma B_0^2Cos^2\alpha(muCos\alpha-v)}{\rho(1+m^2Cos^2\alpha)}-\frac{\upsilon v}{K},\tag{2}$$

$$\frac{\partial C}{\partial t}=D\frac{\partial^2 C}{\partial z^2}-K_C(C-C_\infty),\tag{3}$$

$$\rho C_p\frac{\partial T}{\partial t}=k\frac{\partial^2 T}{\partial z^2}.\tag{4}$$

The initial and boundary conditions are:

$$\left.\begin{array}{l}t\le 0: u=0,\ v=0,\ T=T_\infty,\ C=C_\infty,\forall z\\ t>0: u=u_0Cos\omega t,\ v=0,\ T=T_\infty+(T_w-T_\infty)A,\ C=C_\infty+(C_w-C_\infty)A, at\ z=0.\\ u\to 0,\ v\to 0,\ T\to T_\infty,\ C\to C_\infty\ as\ z\to\infty. where\ A=\dfrac{u_0^2t}{\upsilon}.\end{array}\right\}\tag{5}$$

Here u and v are the primary and the secondary velocities along x and z respectively, ν- the kinematic viscosity, ρ- the density, C_p- the specific heat at constant pressure, k- thermal conductivity of the fluid, D- the mass diffusion coefficient, g- gravitational acceleration, β- volumetric coefficient of thermal expansion, t- time, m- the Hall current parameter, T- temperature of the fluid, β^* - volumetric coefficient of concentration expansion, C- species concentration in the fluid, T_w- temperature of the plate, C_w- species concentration, B_0- the uniform magnetic field, σ - electrical conductivity.

The following non-dimensional quantities are used to transform equations (1), (2), (3) and (4) into dimensionless form:

$$\left.\begin{array}{l}\bar{z}=\dfrac{zu_0}{\upsilon},\ \bar{u}=\dfrac{u}{u_0},\ \bar{v}=\dfrac{v}{u_0},\ \theta=\dfrac{(T-T_\infty)}{(T_w-T_\infty)},\ S_c=\dfrac{\upsilon}{D},\quad P_r=\dfrac{\mu C_p}{k},\\ K_0=\dfrac{\upsilon K_C}{u_0^2},G_r=\dfrac{g\beta\upsilon(T_w-T_\infty)}{u_0^{\ 3}},M=\dfrac{\sigma B_0^2\upsilon}{\rho u_0^2},\mu=\rho\upsilon,\\ G_m=\dfrac{g\beta^*\upsilon(C_w-C_\infty)}{u_0^{\ 3}},\ \overline{C}=\dfrac{(C-C_\infty)}{(C_w-C_\infty)},\ \bar{t}=\dfrac{tu_0^2}{\upsilon},\varpi=\dfrac{\omega\upsilon}{u_0^2}.\end{array}\right\}\tag{6}$$

The symbols in dimensionless form are as under: θ- the temperature, $\overline{C}$ - the concentration, G_r - thermal Grashof number, $\bar{u}$ - the primary velocity, $\bar{v}$ - the secondary velocity, , μ- the coefficient of viscosity, P_r- the Prandtl number, S_c- the Schmidt number, $\bar{t}$ - time, G_r- mass Grashof number, $\overline{K}$ - permeability parameter of the medium, M- the magnetic parameter.

The dimension less flow model becomes

$$\frac{\partial\bar{u}}{\partial\bar{t}}=\frac{\partial^2\bar{u}}{\partial\bar{z}^2}+G_r\,\theta+G_m\,\overline{C}-\frac{MCos^2\alpha\ (\bar{u}+m\bar{v}Cos\alpha)}{(1+m^2Cos^2\alpha)}-\frac{1}{\overline{K}}\bar{u},\tag{7}$$

$$\frac{\partial \bar{v}}{\partial \bar{t}} = \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} + \frac{M\,Cos^2\alpha\,(m\bar{u}Cos\alpha - \bar{v})}{(1+m^2Cos^2\alpha)} - \frac{1}{\bar{K}}\bar{v}, \tag{8}$$

$$\frac{\partial \bar{C}}{\partial \bar{t}} = \frac{1}{S_c}\frac{\partial^2 \bar{C}}{\partial \bar{z}^2} - K_0\bar{C}, \tag{9}$$

$$\frac{\partial \theta}{\partial \bar{t}} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial \bar{z}^2}. \tag{10}$$

The corresponding boundary conditions become

$$\left.\begin{array}{l} \bar{t} \le 0: \bar{u} = 0,\ \bar{v} = 0,\ \theta = 0,\ \bar{C} = 0, \text{ for all } \bar{z} \\ \bar{t} > 0: \bar{u} = Cos\bar{\omega}\bar{t},\ \bar{v} = 0,\ \theta = \bar{t},\ \bar{C} = \bar{t}, \text{ at } \bar{z} = 0 \\ \bar{u} \to 0,\ \bar{v} \to 0,\ \theta \to 0,\ \bar{C} \to 0, \text{ at } \bar{z} \to \infty. \end{array}\right\} \tag{11}$$

Dropping bars in the above equations, we get

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial z^2} + G_r\theta + G_mC - \frac{MCos^2\alpha(u + mvCos\alpha)}{(1+m^2Cos^2\alpha)} - \frac{1}{K}u, \tag{12}$$

$$\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial z^2} + \frac{MCos^2\alpha(muCos\alpha - v)}{(1+m^2Cos^2\alpha)} - \frac{1}{K}v, \tag{13}$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c}\frac{\partial^2 C}{\partial z^2} - K_0C, \tag{14}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial z^2}. \tag{15}$$

The boundary conditions are

$$\left.\begin{array}{l} t \le 0: u = 0,\ v = 0,\ \theta = 0,\ C = 0, \text{ for all z.} \\ t > 0: u = Cos\omega t,\ v = 0,\ \theta = t,\ C = t, \text{ at } z = 0 \\ u \to 0,\ v \to 0,\ \theta \to 0,\ C \to 0, \text{ as } z \to \infty. \end{array}\right\} \tag{16}$$

Combining equations (12) and (13), the model becomes

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial z^2} + G_r\theta + G_mC - \left(\frac{MCos^2\alpha}{1+m^2Cos^2\alpha}\right)(1 - imCos\alpha)q - \frac{q}{K}, \tag{17}$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c}\frac{\partial^2 C}{\partial z^2} - K_0C, \tag{18}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial z^2}. \tag{19}$$

Finally, the boundary conditions become:

$$\left.\begin{array}{l} t \le 0: q = 0,\ \theta = 0,\ C = 0, \text{ for all } z. \\ t > 0: q = Cos\omega t,\ \theta = t,\ C = t, \text{ at } z = 0. \\ q \to 0,\ \theta \to 0,\ C \to 0, \text{ as } z \to \infty. \end{array}\right\} \quad (20)$$

Here $q = u + i\,v$.

The dimensionless governing equations (17) to (19), subject to the boundary conditions (20), are solved by the usual Laplace - transform technique. The solution obtained is as under:

$$C = \frac{1}{4\sqrt{K_0}} \exp(-z\sqrt{S_c K_0})\{erfc[\frac{1}{2\sqrt{t}}(z\sqrt{S_c} - 2t\sqrt{K_0})](-z\sqrt{S_c} + 2t\sqrt{K_0})$$
$$+ \exp(2z\sqrt{S_c K_0}) erfc[\frac{1}{2\sqrt{t}}(z\sqrt{S_c} + 2t\sqrt{K_0})](z\sqrt{S_c} + 2t\sqrt{K_0})\}.$$

$$\theta = t\{(1 + \frac{z^2 P_r}{2t}) erfc[\frac{z\sqrt{P_r}}{2\sqrt{t}}] - \frac{z\sqrt{P_r}}{\sqrt{\pi}\sqrt{t}} \exp(-\frac{z^2}{4t}) P_r\}.$$

$$q = \frac{e^{-it\omega}}{4} e^{-z\sqrt{a+i\omega}} + \frac{e^{-it\omega}}{4} e^{-z\sqrt{a-i\omega}} + \frac{e^{-it\omega}}{4} e^{-z\sqrt{a+i\omega}+2it\omega} + \frac{e^{-it\omega}}{4} e^{z\sqrt{a-i\omega}+2it\omega}$$
$$- \frac{e^{-it\omega}}{4} e^{-z\sqrt{a+i\omega}} \left(erf\left[\frac{z - 2t\sqrt{a - i\omega}}{2\sqrt{t}}\right] + erf\left[\frac{z + 2t\sqrt{a - i\omega}}{2\sqrt{t}}\right]\right)$$
$$- \frac{e^{-it\omega}}{4} e^{-z\sqrt{a+i\omega}+2it\omega} \left(erf\left[\frac{z - 2t\sqrt{a + i\omega}}{2\sqrt{t}}\right] + erf\left[\frac{z + 2t\sqrt{a + i\omega}}{2\sqrt{t}}\right]\right),$$
$$+ \frac{1}{4a^2} G_r\{ z\chi_{11} + 2\exp(-\sqrt{a}z)\chi_2 P_r + 2\chi_{14}\chi_4(1 - P_r)\} + \frac{G_m}{4(a - K_0 S_c)^2}\{ z\chi_{11}$$
$$+ 2\chi_{13}\chi_5(1 - S_c) + 2\exp(-\sqrt{a}z)\chi_2 S_c(1 - tK_0) - \frac{1}{\sqrt{a}} z\exp(-\sqrt{a}z)\chi_3 K_0 S_c\}$$
$$- \frac{1}{2a^2\sqrt{\pi}} G_r\{ 2za\exp(\frac{-z^2 P_r}{4t})\sqrt{tP_r} + \sqrt{\pi}\chi_{14}(\chi_6 + \chi_7 P_r) + \sqrt{\pi}\chi_{12}(az^2 P_r - 2$$
$$+ 2at + 2P_r)\} + \frac{G_m\sqrt{S_c}}{2\sqrt{\pi}(a - K_0 S_c)^2}\{ \frac{1}{2\sqrt{K_0}} \exp(-z\sqrt{K_0 S_c})\chi_9\sqrt{\pi S_c}(S_c K_0 - az)$$
$$+ \chi_{13}\sqrt{\pi}\chi_{10}(S_c - 1) + \exp(-z\sqrt{K_0 S_c})\sqrt{\pi}\chi_8(1 - at - S_c + tK_0 S_c)\}$$

The symbols involved in the above equations are mentioned in the appendix.

Sherwood number:

$$S_h=\left(\frac{\partial C}{\partial z}\right)_{z=0}=erfc[-\sqrt{tK_0}\,](-\frac{1}{4\sqrt{K_0}}\sqrt{S_c}-\frac{t\sqrt{S_cK_0}}{2})+\sqrt{S_c}\,erfc[\sqrt{tK_0}\,](\frac{1}{4\sqrt{K_0}}+t\sqrt{K_0}\,)-\frac{e^{-tK_0}\sqrt{tS_cK_0}}{\sqrt{\pi K_0}}.$$

The numerical values of S_h for different parameters are given in Table1.

3. RESULTS AND DISCUSSION

In this theoretical investigation, the solutions of the fluid velocity have two components, one along the direction of motion of the plate as primary velocity *u* and the other along the transverse direction as secondary velocity *v*, are represent in Figs. 1-8 for different parameters like, inclination of magnetic field parameter (α), chemical reaction parameter (K_0), phase angle parameter (*wt*) and permeability parameter (*K*). From Figs. 1 and 5, it is observed that the primary velocity increases and secondary velocity of fluid decreases when the angle of inclination (α) is increased. It is deduced that when permeability parameter *K* is increased then the velocities are increased (Figs. 2 and 6). This is due to an increase in *K* permeability parameter, demonstrating that there is a decrease in the resistance of the porous medium which tends to accelerate primary velocity as well as secondary velocity in the boundary layer region. It is deduced that when chemical reaction parameter K_0 is increased then the velocities are decreased (Figs. 3 and 7). It is observed from Figs. 4 and 8 that the phase angle ωt is increases then the velocities are decreased. Sherwood number is given in Table 3. The value of Sherwood number decreases with the increase in the chemical reaction parameter, Schmidt number and time.

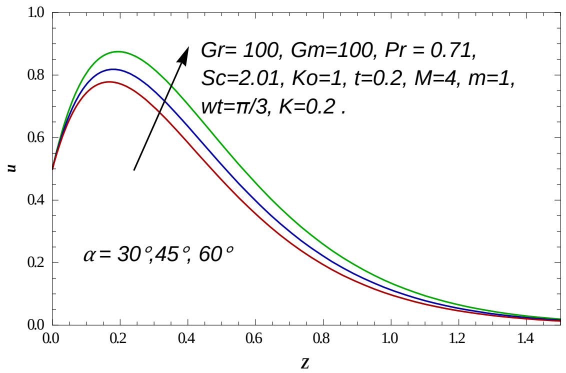

Figure 1. Primary velociy *u* for different values of α

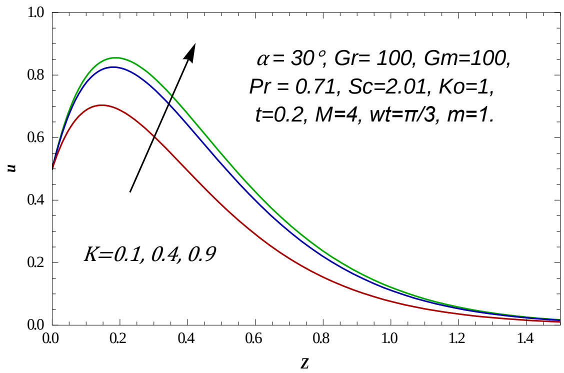

Figure 2. Primary velociy *u* for different values of *K*

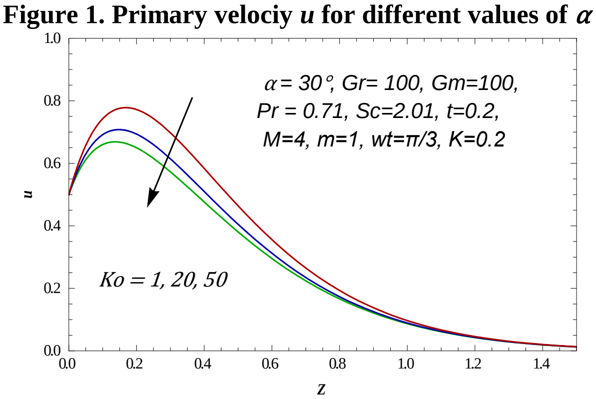

Figure 3. Primary velociy *u* for different values of *Ko*

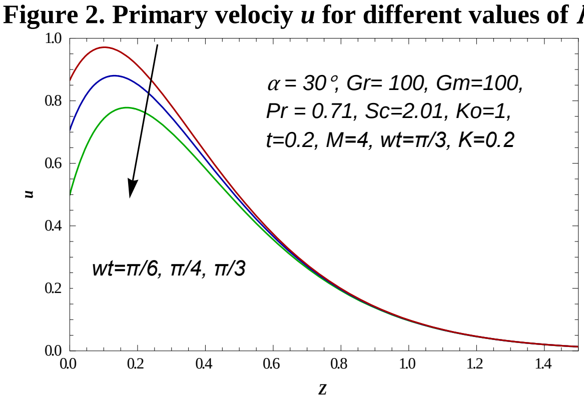

Figure 4. Primary velociy *u* for different values of *wt*

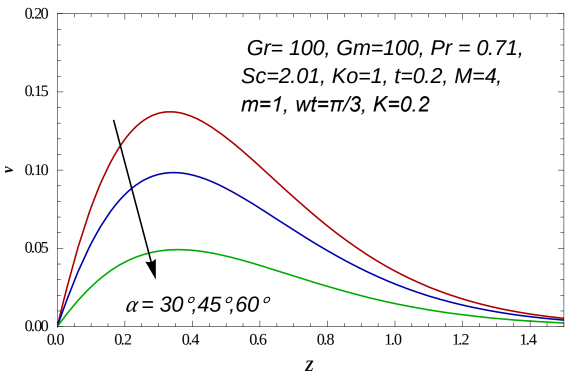

Figure 5. Secondary velociy *v* for different values of *φ*

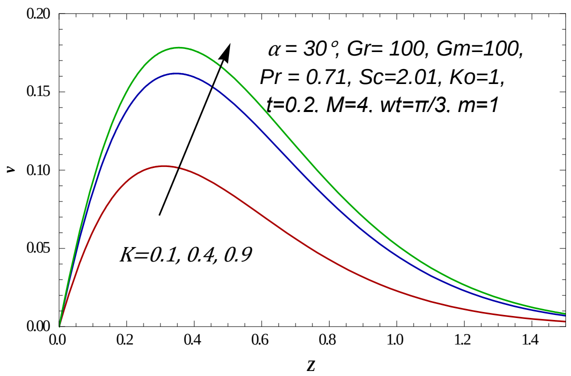

Figure 6. Secondary velociy *v* for different values of *K*

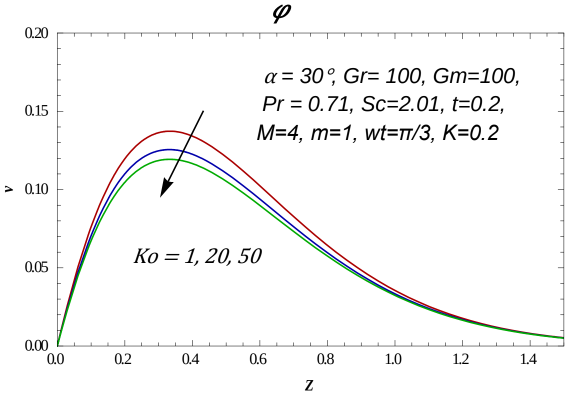

Figure 7. Secondary velociy *v* for different values of *Ko*

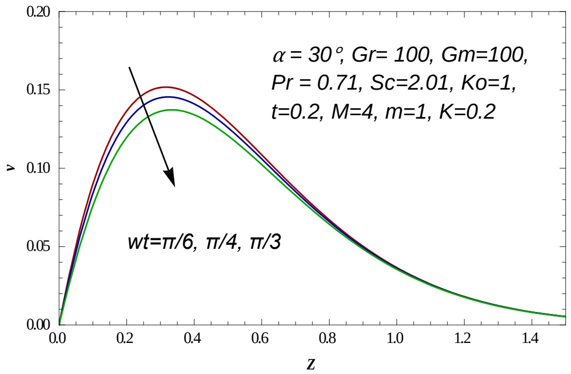

Figure 8. Secondary velociy *v* for different values of *wt*

Table1. Sherwood number for different Parameters.

K_0	Sc	t	S_h
1	2.01	0.2	-0.7622
10	2.01	0.2	-1.1182
20	2.01	0.2	-1.4264
5	3.00	0.2	-1.1399
5	4.00	0.2	-1.3162
5	2.01	0.3	-1.2602
5	2.01	0.4	-1.5814

4. CONCLUSION

In this present paper an analytical study has been done to the study of unsteady MHD flow through a porous medium past an oscillating vertical plate with variable wall temperature and mass diffusion in the presence of Hall current and chemical reaction. It has been found that the velocity in the boundary layer decreases when the chemical reaction parameter is increased. When the phase angle parameter increases, the velocities are decreased near the surface. It is observed that inclination of angle of magnetic field parameter has tendency to accelerate the primary flow while it retards secondary flow. We found that the values obtained for velocity, concentration and temperature are in concurrence with the actual flow of the MHD chemically reacting fluid.

Appendix

$$q = u + iv,\ a = \frac{(Ha)^2 Cos^2\alpha(1 - imCos\alpha)}{1 + m^2 Cos^2\alpha} + \frac{1}{K},\ \chi_1 = 1 + \chi_{16} + \exp(2\sqrt{a}z)(1 - \chi_{17}),$$

$$\chi_2 + \chi_1 = 0,\ \chi_3 + \exp(2z\sqrt{a})(1 - \chi_{17}) = 1 + \chi_{16},\ \chi_4 + 1 = \chi_{22} + \chi_{18}(\chi_{23} - 1),$$

$$\chi_5 + 1 = \chi_{24} + \chi_{19}(\chi_{25} - 1),\ \chi_6 + \chi_{26} + 1 = \chi_{18}(\chi_{27} - 1),\ \chi_7 + \chi_6 = 0,$$

$$\chi_8 + \chi_{20} + 1 = \chi_{30}(\chi_{21} - 1),\ \chi_9 = 1 + \chi_{20} + \chi_{30}(\chi_{21} - 1),\ \chi_{10} + \chi_{28} + 1 = \chi_{19}(\chi_{29} - 1),$$

$$z\chi_{11} = \exp(-\sqrt{az})(2\chi_1 + 2at\chi_2 + \sqrt{a}\chi_3),\ \chi_{12} + 1 = erf[\frac{z\sqrt{P_r}}{2\sqrt{t}}],\ \chi_{13} = \exp(\frac{at - tK_0 S_c}{S_c - 1} - z\chi_{31}\sqrt{S_c}),$$

$$\chi_{14} = \exp((\chi_{32})^2 t - z\chi_{32}\sqrt{P_r}),\ \chi_{15} = 1,\ \chi_{16} = erf[\frac{2\sqrt{a}t - z}{2\sqrt{t}}]\ \chi_{17} = erf[\frac{2\sqrt{a}t + z}{2\sqrt{t}}],$$

$$\chi_{18} = \exp(-2z\chi_{32}\sqrt{P_r}),\ \chi_{19} = \exp(-2z\chi_{31}\sqrt{S_c}),\ \chi_{20} = erf[\sqrt{tK_0} - \frac{z\sqrt{S_c}}{2\sqrt{t}}],$$

$$\chi_{21} = erf[\sqrt{tK_0} + \frac{z\sqrt{S_c}}{2\sqrt{t}}],\ \chi_{22} = erf[\frac{1}{2t}(z - 2t\chi_{32}\sqrt{P_r})],\ \chi_{23} = erf[\frac{1}{2t}(z + 2t\chi_{32}\sqrt{P_r})],$$

$$\chi_{24} = erf[\frac{1}{2t}(z - 2t\chi_{31}\sqrt{S_c})],\ \chi_{25} = erf[\frac{1}{2t}(z + 2t\chi_{31}\sqrt{S_c})],\ \chi_{26} = erf[\frac{1}{2\sqrt{t}}(2t\chi_{32} - z\sqrt{P_r})],$$

$$\chi_{27} = erf[\frac{1}{2\sqrt{t}}(2t\chi_{32} + z\sqrt{P_r})],\ \chi_{28} = erf[\sqrt{t}\chi_{31} - \frac{zS_c}{2\sqrt{t}}],\ \chi_{29} = erf[\sqrt{t}\chi_{31} + \frac{zS_c}{2\sqrt{t}}],$$

$$\chi_{30} = \exp(2z\sqrt{K_0\sqrt{S_c}}),\ (\chi_{31})^2 = \frac{(a - K_0)}{S_c - 1},\ (\chi_{32})^2 = \frac{a}{P_r - 1}$$

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REFERENCES

[1] Dessie, H., Naikoti, K., *Ain Shams Engineering Journal*, **5**, 967, 2014.
[2] Ibrahim, S.Y., Makinde, O.D., *Scientific Research and Essays*, **5**(19), 2875, 2010.
[3] Mythreye, A., Pramoda, J.P., Balamurugan, K.S., *Procedia Engineering,* **127**, 613, 2015.
[4] Kumar, G., Kumar, A., Misra, M.K., Srivastava, V., *Journal of Science and arts*, **16**(2), 513, 2016.
[5] Raptis, A., Kafousias, N.G., *Canadian Journal of Physics*, **60**(12), 1725, 1982.
[6] Rajput, U.S., Gaurav, K., *Application and Applied Mathematics: An International Journal*, **11** (2), 693, 2016.
[7] Rajput, U.S., Kumar, G., *Journal of Naval Architecture and Marine Engeneering*, **16**, 99, 2019.
[8] Sahin, A., Chamkha, A. J., *International Journal of Industrial Mathematics*, **2**(4), 245, 2010.
[9] Samad, M.A, Hossain, M.D., Alam, M.M., *Procedia Engineering*, **105**, 56, 2015.
[10] Seth, G. S., Sharma, R., Hussain, S.M., *Emirates Journal for Engineering Research*, **19**(2), 19, 2014.
[11] VeeraKrishna, M., Ahamad, N.A., Chamka, A.J., *Alexandria Engineering Journal*, **59**(2), 565, 2020.
[12] Youn, J.K., *Transport in Porous Media*, **56**(1), 17, 2004.