PRANAV-POWER SERIES DISTRIBUTION: PROPERTIES AND APPLICATIONS TO SURVIVAL AND WAITING TIME DATA

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Abstract. *This research article presents a new Pranav power series class of distributions which is obtained by compounding one parameter Pranav and power series distributions. Various special cases of the new model have been unfolded. Numerous statistical properties of the proposed model are investigated including closed-form expressions for the density function, cumulative distribution function, survival function, hazard rate function, the moments of order statistics and MLEs. Finally, the flexibility and potentiality of the PPS distribution has been demonstrated by means of two real life data sets.*
Keywords: *power series distribution; order statistics; estimation; real life applications.*

1. INTRODUCTION

The interval to the happening of event of interest is known as lifetime or survival time or failure time in reliability analysis. The event may be failure of a piece of equipment, expiry of a person, advancement (or remission) of symptoms of disease, health code destruction**.** In many, if not all, applied sciences such as medicine, engineering, insurance and finance, modeling and analyzing lifetime data are crucial. Recently, attempts have been made to define new families of probability distributions, which extend well-known families of distributions and at the same time provide great flexibility in modeling data in practice. One such class of distributions generated is by compounding well-known lifetime distributions such as exponential, Weibull, generalized exponential, exponentiated Weibull, etc., with some discrete distributions such as binomial, geometric, zero truncated Poisson, logarithmic, and the power series distributions in general. A lot of lifetime distributions have been proposed over the years for analyzing and modeling lifetime data. Some prominent compound distributions introduced recently are: exponential-logarithmic (EL) distribution by Tahmasbi and Rezaei [1], exponential-power series (EPS) distribution by Chahkandi and Ganjali [2], Weibull-power series (WPS) distribution by Morais and Barreto-Souza [3], complementary Weibull geometric distribution by Tojeiro et al. [4], Burr XII negative binomial distribution by Ramos et al. [5], compound class of extended Weibull-power series distributions by Silva et al. [6], compound class of linear failure rate-power series distributions by Mahmoudi and Jafari [7], exponentiated Weibull Poisson distribution by Mahmoudi and Sepahdar [8], generalized modified Weibull power series distribution by Bagheri et al. [9], exponentiated power Lindley geometric distribution by Alizadeh et al. [10], exponentiated Burr XII power Series distribution by Nasir et al. [11], Ailamujia power series class model by Rashid et al. [12], the odd log-logistic power series family of distributions by Goldoust et al. [13], Dagum

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power series distribution by Makubate et al. [14], exponentiated power generalized Weibull power series family of distributions by Aldahlan et al. [15].

The present paper is outlined as follows: In section (2), the PPS distribution is presented. Its density, survival, hazard rate, reverse hazard functions and some other important properties are derived in section (3). Moment generating function of proposed model is attained in section (4). Order statistics, their moments and parameter estimation are given correspondingly in section (5) and (6). Special cases that include new lifetime distributions have been given in section (7). Ultimately, the potency of the proposed model and conclusion about new results are respectively given in section (8) and (9).

2. PRANAV POWER SERIES (PPS) CLASS OF DISTRIBUTIONS

Suppose an independent and identically distributed (i.i.d) random variables $X_1, X_2, ..., X_v$ follow one parameter lifetime distribution known as Pranav distribution whose density is given by

$$g(x;\alpha) = \frac{\alpha^4}{\alpha^4+6}\left(\alpha + x^3\right)e^{-\alpha x} \quad ; x>0, \alpha>0 \tag{1}$$

Here a sample size V is considered which is a discrete random variable and follows zero truncated power series distribution with probability mass function given by

$$P(V=v) = \frac{a_v \delta^v}{C(\delta)}, v = 1,2,...$$

where $a_v \geq 0$ relies only on v, $C(\delta) = \sum_{v=1}^{\infty} a_v \delta^v$ and $\delta > 0$ is fixed in such a way that $C(\delta)$ is finite.

If a random variable $X = \min(X_1, X_2, ..., X_v)$, then the conditional cumulative distribution function of $(X|V=v)$ is given by

$$G_{X|V=v}(x) = 1 - \left[\overline{G}(x)\right]^v$$

where $G(x)$ is cdf of Pranav distribution.

$$G_{X|V=v}(x) = 1 - \left[\left(1 + \frac{\alpha x\left(\alpha^2 x^2 + 3\alpha x + 6\right)}{\alpha^4+6}\right)e^{-\alpha x}\right]^v$$

and

$$P(X \leq x, V=v) = \frac{a_v \delta^v}{C(\delta)}\left\{1 - \left[\left(1 + \frac{\alpha x\left(\alpha^2 x^2 + 3\alpha x + 6\right)}{\alpha^4+6}\right)e^{-\alpha x}\right]^v\right\}; \; x>0, v \geq 1$$

The marginal cdf of X defines the Pranav power series (PPS) family of distributions as

$$F_{PPS}(x;\alpha,\delta)=\sum_{v=1}^{\infty}\frac{a_v\delta^v}{C(\delta)}\left\{1-\left[\left(1+\frac{\alpha x(\alpha^2x^2+3\alpha x+6)}{\alpha^4+6}\right)e^{-\alpha x}\right]^v\right\};\ x>0, v\geq 1$$

$$F_{PPS}(x;\alpha,\delta)=\sum_{v=1}^{\infty}\frac{a_v\delta^v}{C(\delta)}-\sum_{v=1}^{\infty}\frac{a_v\left[\left(1+\frac{\alpha x(\alpha^2x^2+3\alpha x+6)}{\alpha^4+6}\right)\delta e^{-\alpha x}\right]^v}{C(\delta)} \tag{2}$$

$$F_{PPS}(x;\alpha,\delta)=1-\frac{C\left[\left(1+\frac{\alpha x(\alpha^2x^2+3\alpha x+6)}{\alpha^4+6}\right)\delta e^{-\alpha x}\right]}{C(\delta)}$$

A random variable X with cdf (2) shall be expressed by $X \sim PPS(\alpha,\delta)$.

3. DENSITY, SURVIVAL, HAZARD AND REVERSE HAZARD FUNCTION

The derivative of (2) gives the probability density function of PPS model as

$$f_{PPS}(x;\alpha,\delta)=\frac{-1}{C(\delta)}\left\{C'\left(\left(1+\frac{\alpha x(\alpha^2x^2+3\alpha x+6)}{\alpha^4+6}\right)\delta e^{-\alpha x}\right)\frac{d}{dx}\left(\left(1+\frac{\alpha x(\alpha^2x^2+3\alpha x+6)}{\alpha^4+6}\right)\right)\delta e^{-\alpha x}\right\}$$

$$f_{PPS}(x;\alpha,\delta)=\frac{-1}{C(\delta)}\left\{C'\left(\left(1+\frac{\alpha x(\alpha^2x^2+3\alpha x+6)}{\alpha^4+6}\right)\delta e^{-\alpha x}\right)\left[\begin{array}{l}\left(1+\frac{\alpha x(\alpha^2x^2+3\alpha x+6)}{\alpha^4+6}\right)\left(-\alpha\delta e^{-\alpha x}\right)\\+\delta e^{-\alpha x}\left(\frac{3\alpha^3x^2+6\alpha^2x+6\alpha}{\alpha^4+6}\right)\end{array}\right]\right\} \tag{3}$$

$$f_{PPS}(x;\alpha,\delta)=\frac{\alpha^4}{\alpha^4+6}\delta(\alpha+x^3)e^{-\alpha x}\frac{C'\left(\delta e^{-\alpha x}\left(1+\frac{\alpha x(\alpha^2x^2+3\alpha x+6)}{\alpha^4+6}\right)\right)}{C(\delta)};\ x,\alpha,\delta>0$$

The survival function of PPS model is

$$S_{PPS}(x)=\frac{C\left(\delta e^{-\alpha x}\left(1+\frac{\alpha x(\alpha^2x^2+3\alpha x+6)}{\alpha^4+6}\right)\right)}{C(\delta)}.$$

The hazard function of the proposed PPS model is given by

$$k_{PPS}(x)=\frac{\alpha^4}{\alpha^4+6}\delta(\alpha+x^3)e^{-\alpha x}\frac{C'\left[\delta e^{-\alpha x}\left\{1+\frac{\alpha x\left(\alpha^2x^2+3\alpha x+6\right)}{\alpha^4+6}\right\}\right]}{C\left[\delta e^{-\alpha x}\left\{1+\frac{\alpha x\left(\alpha^2x^2+3\alpha x+6\right)}{\alpha^4+6}\right\}\right]}.$$

Also, the reverse hazard function is given by

$$K_{PPS}(x)=\frac{\alpha^4}{\alpha^4+6}\delta(\alpha+x^3)e^{-\alpha x}\frac{C'\left[\delta e^{-\alpha x}\left\{1+\frac{\alpha x\left(\alpha^2x^2+3\alpha x+6\right)}{\alpha^4+6}\right\}\right]}{C(\delta)-C\left[\delta e^{-\alpha x}\left\{1+\frac{\alpha x\left(\alpha^2x^2+3\alpha x+6\right)}{\alpha^4+6}\right\}\right]}.$$

Now, some of the properties of the PPS distribution are analysed in the form of following propositions.

Proposition 1. The Pranav distribution is a limiting case of the PPS distribution when $\delta\to 0^+$.

Proof: The limit of the cumulative distribution function of PPS distribution can be obtained as

$$\lim_{\delta\to 0^+}F_{PPS}(x)=1-\lim_{\delta\to 0^+}\frac{C\left(\delta e^{-\alpha x}\left(1+\frac{\alpha x\left(\alpha^2x^2+3\alpha x+6\right)}{\alpha^4+6}\right)\right)}{C(\delta)}$$

Since, $C(\delta)=\sum_{v=1}^{\infty}a_v\delta^v$

$$\lim_{\delta\to 0^+}F_{PPS}(x)=1-\lim_{\delta\to 0^+}\frac{\sum_{v=1}^{\infty}a_v\left\{\delta e^{-\alpha x}\left(1+\frac{\alpha x\left(\alpha^2x^2+3\alpha x+6\right)}{\alpha^4+6}\right)\right\}^v}{\sum_{v=1}^{\infty}a_v\delta^v}$$

Using the L-Hospital's rule, we have

$$\lim_{\delta\to 0^+}F_{PPS}(x)=1-\lim_{\delta\to 0^+}\frac{a_1\left(1+\frac{\alpha x\left(\alpha^2x^2+3\alpha x+6\right)}{\alpha^4+6}\right)e^{-\alpha x}+\sum_{v=2}^{\infty}a_v v\delta^{v-1}\left(1+\frac{\alpha x\left(\alpha^2x^2+3\alpha x+6\right)}{\alpha^4+6}\right)^v e^{-v\alpha x}}{a_1+\sum_{v=2}^{\infty}va_v\delta^{v-1}}$$

$$=1-\left[1+\frac{\alpha x\left(\alpha^2x^2+3\alpha x+6\right)}{\alpha^4+6}\right]e^{-\alpha x}=G(x,\alpha)$$

Hence the result.

Proposition 2. The pdf of the PPS distribution can be expressed as

$$f_{PPS}(x) = \sum_{v=1}^{\infty} P(V = v) g_1(x; v)$$

where $g_1(x,v) = \min(X_1, X_2, ..., X_v)$ is the 1st order statistics of Pranav distribution.

Proof: We know that

$$C'(\delta) = \sum_{v=1}^{\infty} v a_v \delta^{v-1}$$

Using the above result in the density function obtained in (3), we have

$$f_{PPS}(x) = \frac{\alpha^4}{\alpha^4+6}\delta(\alpha + x^3)e^{-\alpha x} \frac{\sum_{v=1}^{\infty} v a_v \left(\delta e^{-\alpha x}\left(1 + \frac{\alpha x(\alpha^2 x^2 + 3\alpha x + 6)}{\alpha^4+6}\right)\right)^{v-1}}{C(\delta)}$$

$$f_{PPS}(x) = \sum_{v=1}^{\infty} \frac{a_v \delta^v}{C(\delta)} \frac{v\alpha^4}{\alpha^4+6}(\alpha + x^3)e^{-\alpha x}\left(e^{-\alpha x}\left(1 + \frac{\alpha x(\alpha^2 x^2 + 3\alpha x + 6)}{\alpha^4+6}\right)\right)^{v-1} \tag{4}$$

$$f_{PPS}(x) = \sum_{v=1}^{\infty} P(V = v) g_1(x, v)$$

where $g_1(x,v) = v\frac{\alpha^4}{\alpha^4+6}(\alpha + x^3)e^{-\alpha x}\left(e^{-\alpha x}\left(1 + \frac{\alpha x(\alpha^2 x^2 + 3\alpha x + 6)}{\alpha^4+6}\right)\right)^{v-1}$ is the 1st order statistics of Pranav distribution.

4. MOMENT GENERATING FUNCTION (MGF)

The moment generating function of PPS family of distributions can be attained from (4) as

$$L_X(t) = \sum_{v=1}^{\infty} P(V = v) L_{X_{(1)}}(t)$$

where $L_{X_{(1)}}(t)$ represents the mgf of 1st order statistics of Pranav distribution.

$$L_{X_{(1)}}(t) = \int_0^{\infty} e^{tx} v \frac{\alpha^4}{\alpha^4+6}(\alpha + x^3)e^{-\alpha x}\left[\left(1 + \frac{\alpha x(\alpha^2 x^2 + 3\alpha x + 6)}{\alpha^4+6}\right)e^{-\alpha x}\right]^{v-1} dx$$

$$=\frac{v\alpha^4}{\alpha^4+6}\int_0^\infty e^{-(v\alpha-t)x}(\alpha+x^3)\left[\left(1+\frac{\alpha x\left(\alpha^2x^2+3\alpha x+6\right)}{\alpha^4+6}\right)\right]^{v-1}dx$$

$$=\sum_{j=0}^{v-1}\sum_{k=0}^{v-1-j}\sum_{l=0}^{v-1-j-k}\binom{v-1}{j}\binom{v-1-j}{k}\binom{v-1-j-k}{l}\frac{6^k3^l v\alpha^{3v-3j-2k-l+1}}{\left(\alpha^4+6\right)^{v-j}}\left[\frac{\alpha\Gamma(3v-3j-2k-l-2)}{(v\alpha-t)^{3v-3j-2k-l-2}}+\frac{\Gamma(3v-3j-2k-l+1)}{(v\alpha-t)^{3v-3j-2k-l+1}}\right]$$

and it follows that

$$L_X(t)=\sum_{v=1}^{\infty}\frac{\alpha_v\delta^v}{C(\delta)}\sum_{j=0}^{v-1}\sum_{k=0}^{v-1-j}\sum_{l=0}^{v-1-j-k}\binom{v-1}{j}\binom{v-1-j}{k}\binom{v-1-j-k}{l}\frac{6^k3^l v\alpha^{3v-3j-2k-l+1}}{\left(\alpha^4+6\right)^{v-j}}\left[\frac{\alpha\Gamma(3v-3j-2k-l-2)}{(v\alpha-t)^{3v-3j-2k-l-2}}+\frac{\Gamma(3v-3j-2k-l+1)}{(v\alpha-t)^{3v-3j-2k-l+1}}\right]$$

Using (4), the s^{th} moment of PPS distribution about origin can be obtained as

$$E_{PPS}\left(X^s\right)=\sum_{v=1}^{\infty}P(V=v)\int_0^\infty x^s g_1(x,v)dx$$

$$E_{PPS}\left(X^s\right)=\sum_{v=1}^{\infty}P(V=v)\int_0^\infty x^s\left[E\left(X_{(1)}^s\right)\right]dx$$

Now,

$$E_{PPS}\left(X_{(1)}^s\right)=\int_0^\infty x^s g_1(x)dx=\int_0^\infty x^s v\frac{\alpha^4}{\alpha^4+6}(\alpha+x^3)e^{-\alpha x}\left[\left(1+\frac{\alpha x\left(\alpha^2x^2+3\alpha x+6\right)}{\alpha^4+6}\right)e^{-\alpha x}\right]^{v-1}dx$$

$$=\sum_{j=0}^{v-1}\sum_{k=0}^{v-1-j}\sum_{l=0}^{v-1-j-k}\binom{v-1}{j}\binom{v-1-j}{k}\binom{v-1-j-k}{l}\frac{6^k3^l v\alpha^{3v-3j-2k-l+1}}{\left(\alpha^4+6\right)^{v-j}}$$

$$\left[\frac{\alpha\Gamma\left(s+3v-3j-2k-l-2\right)}{(v\alpha)^{3v-3j-2k-l-2}}+\frac{\Gamma\left(s+3v-3j-2k-l+1\right)}{(v\alpha)^{3v-3j-2k-l+1}}\right]$$

Hence we have

$$E_{PPS}\left(X^s\right)=\sum_{v=1}^{\infty}\frac{a_v\delta^v}{C(\delta)}\sum_{j=0}^{v-1}\sum_{k=0}^{v-1-j}\sum_{l=0}^{v-1-j-k}\binom{v-1}{j}\binom{v-1-j}{k}\binom{v-1-j-k}{l}\frac{6^k3^l v\alpha^{3v-3j-2k-l+1}}{\left(\alpha^4+6\right)^{v-j}}$$
$$\times\left[\frac{\alpha\Gamma\left(s+3v-3j-2k-l-2\right)}{(v\alpha)^{3v-3j-2k-l-2}}+\frac{\Gamma\left(s+3v-3j-2k-l+1\right)}{(v\alpha)^{3v-3j-2k-l+1}}\right] \quad (5)$$

5. ORDER STATISTICS AND THEIR MOMENTS

Let $X_1, X_2, ..., X_n$ be a random sample of size n from PPS distribution with cdf (2) and pdf (3). Let $X_{1:n}, X_{2:n}, ..., X_{n:n}$ denote the corresponding order statistics. Then, the pdf of $X_{i:n}; i = 1, 2, 3...$ is given by

$$f_{i:n}(x) = \frac{n!}{(n-i)!(i-1)!} f_{PPS}(x)\left[F_{PPS}(x)\right]^{i-1}\left[1 - F_{PPS}(x)\right]^{n-i}$$

$$f_{i:n}(x) = \frac{n!\,f_{PPS}(x)}{(n-i)!(i-1)!}\left[1 - \frac{C\left\{\left(1 + \frac{\alpha x\left(\alpha^2 x^2 + 3\alpha x + 6\right)}{\alpha^4 + 6}\right)\delta e^{-\alpha x}\right\}}{C(\delta)}\right]^{i-1}\left[\frac{C\left\{\left(1 + \frac{\alpha x\left(\alpha^2 x^2 + 3\alpha x + 6\right)}{\alpha^4 + 6}\right)\delta e^{-\alpha x}\right\}}{C(\delta)}\right] \quad (6)$$

where F(x) and f(x) are the cdf and pdf of the PPS distribution respectively and equation (6) can be written as

$$f_{i:n}(x) = \frac{n!}{(n-i)!(i-1)!}\sum_{k=0}^{n-i}\binom{n-i}{k}(-1)^k f_{PPS}(x)\left[F_{PPS}(x)\right]^{k+i-1} \quad (7)$$

In view of the fact that

$$f_{PPS}(x)\left[F_{PPS}(x)\right]^{k+i-1} = \left(\frac{1}{k+i}\right)\frac{d}{dx}\left[F_{PPS}(x)\right]^{k+i}$$

The respective cdf of $f_{i:n}(x)$ denoted by $F_{i:n}(x)$ can be obtained as

$$F_{i:n}(x) = \frac{n!}{(n-1)!(i-1)!}\sum_{k=0}^{n-i}\frac{\binom{n-i}{k}(-1)^k}{(k+i)}\left[1 - \frac{C\left(\delta e^{-\alpha x}\left(1 + \frac{\alpha x\left(\alpha^2 x^2 + 3\alpha x + 6\right)}{\alpha^4 + 6}\right)\right)}{C(\delta)}\right]^{k+i} \quad (8)$$

Expression (8) can also be written as

$$F_{i:n}(x) = 1 - \frac{n!}{(n-i)!(i-1)!}\sum_{k=0}^{i-1}\frac{\binom{i-1}{k}(-1)^k}{(k+n-i+1)}\left[\frac{C\left(\delta e^{-\alpha x}\left(1 + \frac{\alpha x\left(\alpha^2 x^2 + 3\alpha x + 6\right)}{\alpha^4 + 6}\right)\right)}{C(\delta)}\right]^{k+n-i+1}$$

The expression for the s^{th} moment of $X_{i:n}$ with cdf (8) can be determined by using the following result of Barakat et al. [16] as given

$$E\left(X_{i:n}^{s}\right)=s\sum_{k=n-i+1}^{n}(-1)^{k-n+i}\binom{k-1}{n-i}\binom{n}{k}\int_{0}^{\infty}x^{s-1}S_{PPS}(x)^{k}\,dx$$

Where S_{PPS} is the survival function of PPS distribution. Thus, we have

$$E\left(X_{i:n}^{s}\right)=s\sum_{k=n-i+1}^{n}\frac{(-1)^{k-n+i-1}}{C(\delta)^{k}}\binom{k-1}{n-i}\binom{n}{k}\int_{0}^{\infty}x^{s-1}\left[\frac{C\left(\delta e^{-\alpha x}\left(1+\frac{\alpha x\left(\alpha^{2}x^{2}+3\alpha x+6\right)}{\alpha^{4}+6}\right)\right)}{C(\delta)}\right]^{i}dx$$

Where, $s=1,2,\ldots\ldots$ and $i=1,2,\ldots n$.

6. ESTIMATION OF THE MODEL PARAMETERS

Let $X_1, X_2, \ldots, X_n$ be a random sample with observed values $x_1, x_2, \ldots x_n$ from PPS distribution with parameter vector $\Theta=(\alpha,\delta)$. The logarithm likelihood function is given by

$$\ell_n=\ell_n(x,\Theta)=4n\log\alpha+n\log\delta+\sum_{i=1}^{n}\log\left(\alpha+x_i^{3}\right)-\alpha\sum_{i=1}^{n}x_i-n\log\left(\alpha^{4}+6\right)-n\log C(\delta)$$
$$+\sum_{i=1}^{n}\log C'\left(\delta e^{-\alpha x_i}\left(1+\frac{\alpha x_i\left(\alpha^{2}x_i^{2}+3\alpha x_i+6\right)}{\alpha^{4}+6}\right)\right)$$

The maximum likelihood estimators of α,δ say $\hat{\alpha},\hat{\delta}$ are obtained by setting the first partial derivatives of $\ell_n(x,\Theta)$ equal to zero. The first partial derivatives for logarithm likelihood function with respect to α,δ are

$$\frac{\partial\ell_n}{\partial\alpha}=\frac{4n}{\alpha}+\sum_{i=1}^{n}\left(\frac{1}{\alpha+x_i^{3}}\right)-\sum_{i=1}^{n}x_i-\frac{4n\alpha^{3}}{\alpha^{4}+6}-\delta\alpha^{3}\sum_{i=1}^{n}\frac{C''\left(\delta e^{-\alpha x_i}\left(1+\frac{\alpha x_i\left(\alpha^{2}x_i^{2}+3\alpha x_i+6\right)}{\alpha^{4}+6}\right)\right)}{C'\left(\delta e^{-\alpha x_i}\left(1+\frac{\alpha x_i\left(\alpha^{2}x_i^{2}+3\alpha x_i+6\right)}{\alpha^{4}+6}\right)\right)}$$
$$\times\left(\frac{\alpha^{5}+\alpha^{4}x_i^{3}+4\alpha^{3}x_i^{2}+12\alpha^{2}x_i+6x_i^{3}+30\alpha}{\left(\alpha^{4}+6\right)^{2}}\right)\left(x_i e^{-\alpha x_i}\right)$$

$$\frac{\partial \ell_n}{\partial \delta} = \frac{n}{\delta} - \frac{nC'(\delta)}{C(\delta)} + \sum_{i=1}^{n} \frac{C''\left(\delta e^{-\alpha x_i}\left(1+\frac{\alpha x_i\left(\alpha^2 x_i^2+3\alpha x_i+6\right)}{\alpha^4+6}\right)\right)}{C'\left(\delta e^{-\alpha x_i}\left(1+\frac{\alpha x_i\left(\alpha^2 x_i^2+3\alpha x_i+6\right)}{\alpha^4+6}\right)\right)}\left(1+\frac{\alpha^2 x_i^2+3\alpha x_i+6}{\alpha^4+6}\right).$$

7. SUB-MODELS OF PPS DISTRIBUTION

In this section, some particular cases of PPS distribution: Pranav Poisson (PP), Pranav Logarithmic (PL), Pranav Geometric (PG) and Pranav binomial (PB) distributions will be investigated and we plot their pdf and hrf plots for specific values of parameters.

7.1. PRANAV- POISSON DISTRIBUTION

The Pranav Poisson (PP) distribution is a particular case of PPS distribution for $a_v = \frac{1}{v!}$ and $C(\delta) = e^{\delta} - 1$. Therefore the associated cdf, pdf, survival, hazard and reverse hazard functions are

$$F_{PP}(x) = 1 - \frac{e^{\delta e^{-\alpha x}\left\{1+\frac{\alpha x\left(\alpha^2 x^2+3\alpha x+6\right)}{\alpha^4+6}\right\}} - 1}{e^{\delta} - 1} \; ; x > 0$$

$$f_{PP}(x) = \frac{\alpha^4}{\alpha^4+6}\delta\left(\alpha + x^3\right)e^{-\alpha x} e^{\delta e^{-\alpha x}\left(1+\frac{\alpha x\left(\alpha^2 x^2+3\alpha x+6\right)}{\alpha^4+6}\right)}\left(e^{\delta}-1\right)^{-1}$$

$$S_{PP}(x) = \frac{e^{\delta e^{-\alpha x}\left(1+\frac{\alpha x\left(\alpha^2 x^2+3\alpha x+6\right)}{\alpha^4+6}\right)} - 1}{e^{\delta} - 1}$$

$$k_{PP}(x) = \frac{\alpha^4 \delta\left(\alpha + x^3\right)e^{-\alpha x} e^{\delta e^{-\alpha x}\left\{1+\frac{\alpha x\left(\alpha^2 x^2+3\alpha x+6\right)}{\alpha^4+6}\right\}}}{(\alpha^4+6)\left[e^{\delta e^{-\alpha x}\left\{1+\frac{\alpha x\left(\alpha^2 x^2+3\alpha x+6\right)}{\alpha^4+6}\right\}} - 1\right]}$$

$$K_{PP}(x)=\frac{\alpha^4\delta\left(\alpha+x^3\right)e^{-\alpha x}e^{\delta e^{-\alpha x}\left\{1+\frac{\alpha x\left(\alpha^2x^2+3\alpha x+6\right)}{\alpha^4+6}\right\}}}{(\alpha^4+6)\left[e^{\delta}-e^{\delta e^{-\alpha x}\left\{1+\frac{\alpha x\left(\alpha^2x^2+3\alpha x+6\right)}{\alpha^4+6}\right\}}\right]}.$$

For $x,\alpha>0$ and $0<\delta<\infty$, respectively.

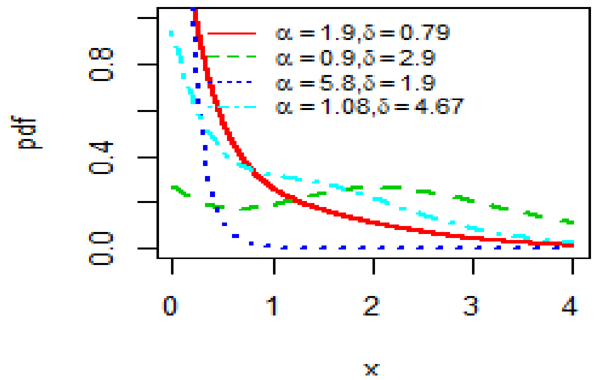

Figure 1. PDF plots of PPS Model at different values

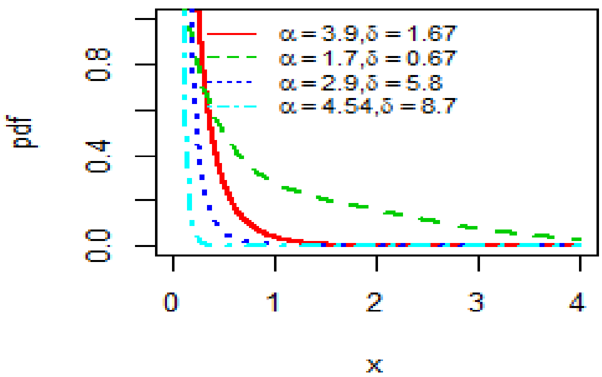

Figure 2. PDF plots of PPS Model at different values

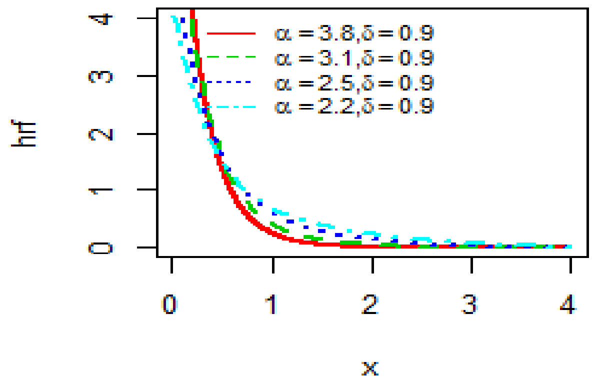

Figure 3. Hazard Rate plots

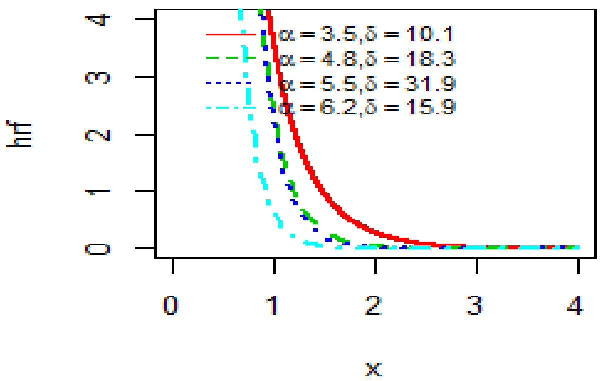

Figure 4. Hazard Rate plots

7.2. PRANAV LOGARITHMIC DISTRIBUTION

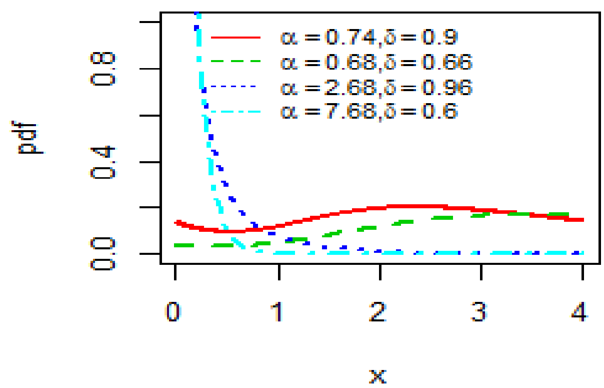

Figure 5. PDF plots of PL Model at different values

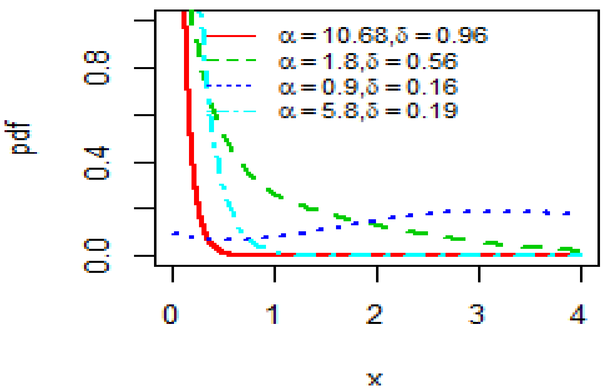

Figure 6. PDF plots of PL Model at different values

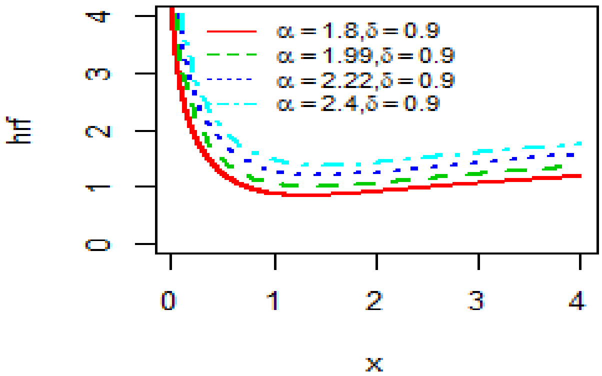

Figure 7. Hazard Rate plots

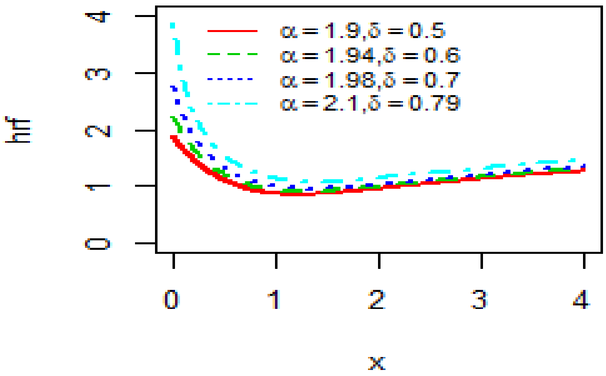

Figure 8. Hazard Rate plots

The Pranav logarithmic (PL) distribution is a particular case of the PPS distribution when $a_v = \frac{1}{v}$ and $C(\delta) = -\log(1-\delta)$. Therefore the associated cdf, pdf, survival, hazard and reverse hazard functions are

$$F_{PL}(x) = 1 - \frac{\log\left[1 - \delta e^{-\alpha x}\left(1 + \frac{\alpha x\left(\alpha^2 x^2 + 3\alpha x + 6\right)}{\alpha^4 + 6}\right)\right]}{\log(1-\delta)}, x > 0$$

$$f_{PL}(x) = \frac{\alpha^4 \delta\left(\alpha + x^3\right)e^{-\alpha x}}{\left(\alpha^4 + 6\right)\left[\delta e^{-\alpha x}\left(1 + \frac{\alpha x\left(\alpha^2 x^2 + 3\alpha x + 6\right)}{\alpha^4 + 6}\right) - 1\right]\log(1-\delta)}$$

$$S_{PL}(x) = \frac{\log\left[1 - \left(1 + \frac{\alpha x\left(\alpha^2 x^2 + 3\alpha x + 6\right)}{\alpha^4 + 6}\right)\delta e^{-\alpha x}\right]}{\log(1-\delta)}$$

$$k_{PL}(x) = \frac{\alpha^4 \delta\left(\alpha + x^3\right)e^{-\alpha x}}{\left(\alpha^4 + 6\right)\left[\delta e^{-\alpha x}\left(1 + \frac{\alpha x\left(\alpha^2 x^2 + 3\alpha x + 6\right)}{\alpha^4 + 6}\right) - 1\right]\log\left[1 - \delta e^{-\alpha x}\left(1 + \frac{\alpha x\left(\alpha^2 x^2 + 3\alpha x + 6\right)}{\alpha^4 + 6}\right)\right]}$$

$$K_{PL}(x) = \frac{\alpha^4 \delta\left(\alpha + x^3\right)e^{-\alpha x}\left[\delta e^{-\alpha x}\left(1 + \frac{\alpha x\left(\alpha^2 x^2 + 3\alpha x + 6\right)}{\alpha^4 + 6}\right) - 1\right]^{-1}}{\left(\alpha^4 + 6\right)\left\{\log(1-\delta) - \log\left[1 - \delta e^{-\alpha x}\left(1 + \frac{\alpha x\left(\alpha^2 x^2 + 3\alpha x + 6\right)}{\alpha^4 + 6}\right)\right]\right\}}$$

for $x, \alpha > 0$ and $0 < \delta < 1$ respectively.

7.3. PRANAV-GEOMETRIC DISTRIBUTION

The Pranav Geometric (PG) distribution is a particular case of SPS distribution when $a_v = 1$ and $C(\delta) = \delta(1-\delta)^{-1}$ .Therefore the associated cdf, pdf, survival, hazard and reverse hazard functions are

$$F_{PG}(x) = \frac{1 - \left[e^{-\alpha x} \left(1 + \frac{\alpha x\left(\alpha^2 x^2 + 3\alpha x + 6\right)}{\alpha^4 + 6} \right) \right]}{1 - \delta e^{-\alpha x} \left[1 + \frac{\alpha x\left(\alpha^2 x^2 + 3\alpha x + 6\right)}{\alpha^4 + 6} \right]}, x > 0$$

$$f_{PG}(x) = \frac{\alpha^4 (1-\delta)\left(\alpha + x^3\right) e^{-\alpha x}}{\alpha^4 + 6} \left[1 - \delta e^{-\alpha x} \left(1 + \frac{\alpha x\left(\alpha^2 x^2 + 3\alpha x + 6\right)}{\alpha^4 + 6} \right) \right]^{-2}, x > 0$$

$$S_{PG}(x) = \frac{(1-\delta)\left[1 + \frac{\alpha x\left(\alpha^2 x^2 + 3\alpha x + 6\right)}{\alpha^4 + 6} \right] e^{-\alpha x}}{1 - \delta e^{-\alpha x} \left[1 + \frac{\alpha x\left(\alpha^2 x^2 + 3\alpha x + 6\right)}{\alpha^4 + 6} \right]}$$

$$k_{PG}(x) = \frac{\alpha^4 \left(\alpha + x^3\right)}{\alpha^4 + 6 + \alpha x\left(\alpha^2 x^2 + 3\alpha x + 6\right)} \left[1 - \delta e^{-\alpha x} \left(1 + \frac{\alpha x\left(\alpha^2 x^2 + 3\alpha x + 6\right)}{\alpha^4 + 6} \right) \right]^{-1}$$

$$K_{PG}(x) = \frac{\alpha^4 \left(\alpha + x^3\right)(1-\delta)}{\alpha^4 + 6} \frac{\left[1 - \delta e^{-\alpha x} \left(1 + \frac{\alpha x\left(\alpha^2 x^2 + 3\alpha x + 6\right)}{\alpha^4 + 6} \right) \right]^{-1}}{\left[1 - \left(1 + \frac{\alpha x\left(\alpha^2 x^2 + 3\alpha x + 6\right)}{\alpha^4 + 6} \right) \right]}$$

For $x, \alpha > 0$ and $0 < \delta < 1$ respectively.

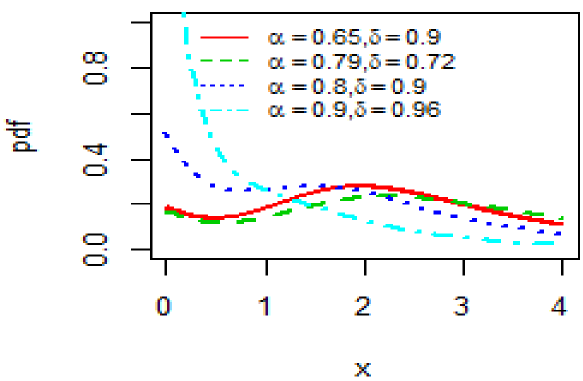

Figure 9. PDF plots of PG Model at different values

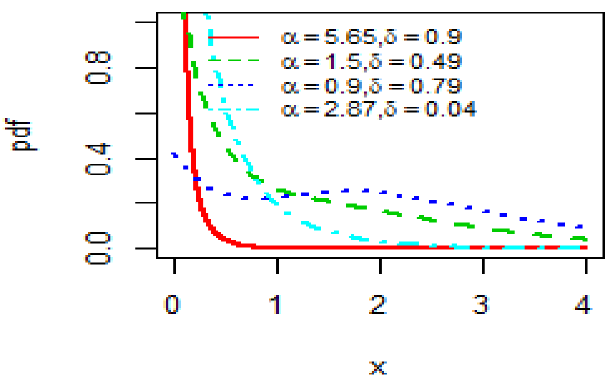

Figure 10. PDF plots of PG Model at different values

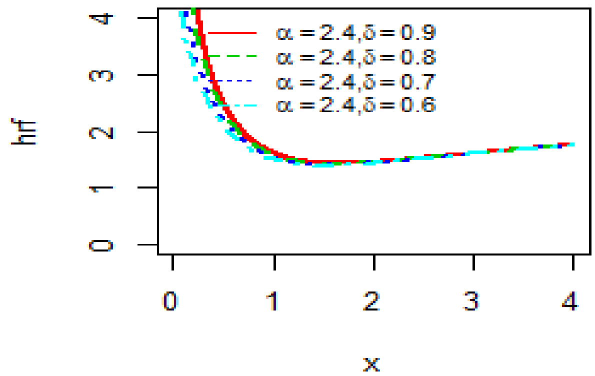

Figure 11. Hazard rate plots

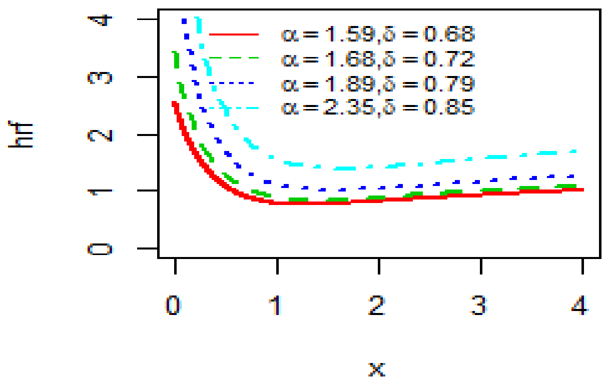

Figure 12. Hazard rate plots

7.4. PRANAV BINOMIAL DISTRIBUTION

The Pranav Binomial (PB) distribution is a particular case of PPS distribution for $a_v = \binom{m}{v}$ and $C(\delta) = (\delta+1)^m - 1$. The cdf, pdf, survival, hazard and reverse hazard functions are given as

$$F_{PB}(x) = 1 - \frac{\left[\delta e^{-\alpha x}\left(1+\frac{\alpha x\left(\alpha^2 x^2+3\alpha x+6\right)}{\alpha^4+6}\right)+1\right]^m - 1}{(\delta+1)^m - 1}, x>0$$

where m is a positive integer.

$$f_{PB}(x) = \frac{\alpha^4 m}{\alpha^4+6}\left(\alpha+x^3\right)\delta e^{-\alpha x}\left[\delta e^{-\alpha x}\left(1+\frac{\alpha x\left(\alpha^2 x^2+3\alpha x+6\right)}{\alpha^4+6}\right)\right]^{m-1}\left[(\delta+1)^m-1\right]^{-1}$$

$$S_{PB}(x) = \frac{\left[\delta e^{-\alpha x}\left(1+\frac{\alpha x\left(\alpha^2 x^2+3\alpha x+6\right)}{\alpha^4+6}\right)+1\right]^m - 1}{(\delta+1)^m - 1}$$

$$k_{PB}(x) = \frac{\alpha^4 m}{\alpha^4+6}\left(\alpha+x^3\right)\delta e^{-\alpha x}\frac{\left[\delta e^{-\alpha x}\left(1+\frac{\alpha x\left(\alpha^2 x^2+3\alpha x+6\right)}{\alpha^4+6}\right)\right]^{m-1}}{\left[\left(\delta e^{-\alpha x}\left(1+\frac{\alpha x\left(\alpha^2 x^2+3\alpha x+6\right)}{\alpha^4+6}\right)+1\right)^m-1\right]}$$

$$K_{PB}(x) = \frac{\alpha^4 m}{\alpha^4+6}\left(\alpha+x^3\right)\delta e^{-\alpha x}\frac{\left[\delta e^{-\alpha x}\left(1+\frac{\alpha x\left(\alpha^2 x^2+3\alpha x+6\right)}{\alpha^4+6}\right)\right]^{m-1}}{\left\{(\delta+1)^m-\left[\left(\delta e^{-\alpha x}\left(1+\frac{\alpha x\left(\alpha^2 x^2+3\alpha x+6\right)}{\alpha^4+6}\right)+1\right)^m-1\right]\right\}}$$

For $x,\alpha>0$ and $0<\delta<1$ respectively.

8. REAL DATA APPLICATIONS

This section demonstrates the practical applicability of some special models of PPS distribution to two real life data sets taken from the literature in order to illustrate its flexibility. The special models of the proposed distribution are compared with the base model i.e., Pranav distribution by using model selection tools such as Akaike information criterion (AIC), Schwarz information criterion (SIC). In general, the best distribution corresponds to smaller values of these model selection tools.

Data set I. The data represents the survival times of 72 guinea pigs infected with virulent tubercle bacilli observed and reported by T. Bjerkedal [17]. The data set is given as:
10, 33, 44, 56, 59, 72, 74, 77, 92, 93, 96, 100, 100, 102, 105, 107, 107, 108, 108, 108, 109, 112, 113, 115, 116, 120, 121, 122, 122, 124, 130, 134, 136, 139, 144, 146, 153, 159, 160, 163, 163, 168, 171, 172, 176, 183, 195, 196, 197, 202, 213, 215, 216, 222, 230, 231, 240, 245, 251, 253, 254, 254, 278, 293, 327, 342, 347, 361, 402, 432, 458, 555.

Table 1. ML Estimates and goodness-of-fit criteria for Survival time data.

Model	ML Estimates	AIC	BIC
Pranav Poisson	$\hat{\alpha}=0.018,\ \hat{\delta}=1.798$	856.12	859.17
Pranav Geometric	$\hat{\alpha}=0.015,\ \hat{\delta}=0.707$	855.47	858.52
Pranav Logarithmic	$\hat{\alpha}=0.017,\ \hat{\delta}=0.812$	855.88	858.93
Pranav	$\hat{\alpha}=0.023$	856.52	858.80

Data set II. The second data set previously analyzed by Ghitnay et al. [18] and Bhat et al. [19] comprises the waiting times (in minutes) of 100 bank costumers before service. The data are as follows:
0.8, 0.8, 1.3, 1.5, 1.8, 1.9, 1.9, 2.1, 2.6, 2.7, 2.9, 3.1, 3.2, 3.3, 3.5, 3.6, 4.0, 4.1, 4.2, 4.2, 4.3, 4.3, 4.4, 4.4, 4.6, 4.7, 4.7, 4.8, 4.9, 4.9, 5.0, 5.3, 5.5, 5.7, 5.7, 6.1, 6.2, 6.2, 6.2, 6.3, 6.7, 6.9, 7.1, 7.1, 7.1, 7.1, 7.4, 7.6, 7.7, 8.0, 8.2, 8.6, 8.6, 8.6, 8.8, 8.8, 8.9, 8.9, 9.5, 9.6, 9.7, 9.8, 10.7, 10.9, 11.0, 11.0, 11.1, 11.2, 11.2, 11.5, 11.9, 12.4, 12.5, 12.9, 13.0, 13.1, 13.3, 13.6, 13.7, 13.9, 14.1, 15.4, 15.4, 17.3, 17.3, 18.1, 18.2, 18.4, 18.9, 19.0, 19.9, 20.6, 21.3, 21.4, 21.9, 23.0, 27.0, 31.6, 33.1, 38.5.

Table 2. ML Estimates and goodness-of-fit criteria for Waiting time data.

Model	ML Estimates	AIC	BIC
Pranav Poisson	$\hat{\alpha}=0.342,\ \hat{\delta}=1.474$	663.54	666.59
Pranav Geometric	$\hat{\alpha}=0.275,\ \hat{\delta}=0.787$	657.09	660.14
Pranav Logarithmic	$\hat{\alpha}=0.218,\ \hat{\delta}=0.992$	643.52	646.58
Pranav	$\hat{\alpha}=0.404$	667.91	670.52

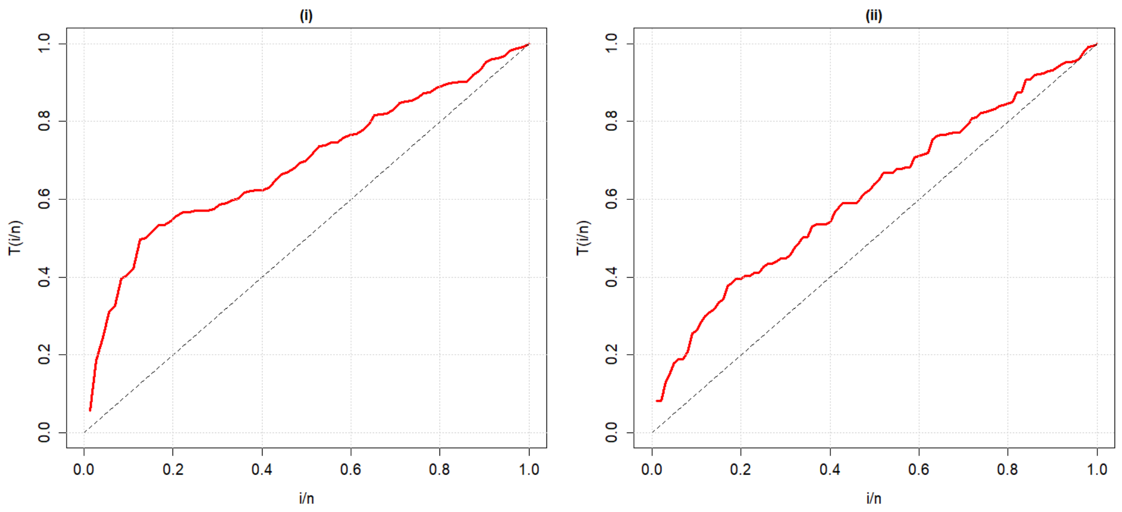

Figure 13. TTT plots for the survival and waiting time data sets.

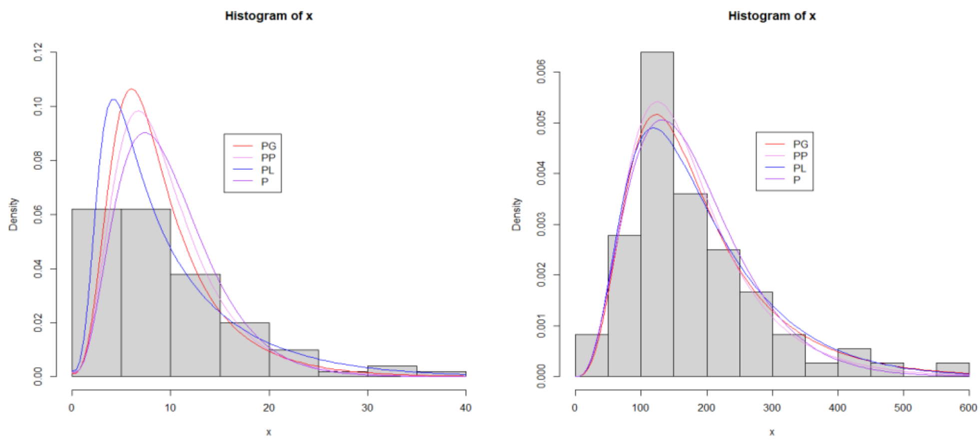

Figure 14. The estimated density plots of the survival and waiting time data sets.

9. CONCLUSION

We have developed a new two parameter class of distributions called Pranav Power Series (PPS) distribution generated by compounding Pranav and Power series distribution and unfolded several statistical properties of the PPS distribution like survival function, moment generating function and order statistics. Moreover, the unknown parameters are estimated by maximum likelihood estimation procedure. We have also illustrated the application of PPS distribution to two real data sets used by researchers earlier and compare its sub-models with the base model. The results of the real data sets indicate that all the sub-models of PPS distribution performs excellently better and can be suggested for lifetime modelling encountered in engineering, medical science, biological science and other applied sciences.

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