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ESTHETICAL LOCALITY AND GLOBALITY

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Abstract. *The locality is the behavior or the structural self-organization of a collectivity around an origin. The globality is the behavior or the structural self-organization of a collectivity around a property. The interconnected collectivities are our models to esthetical behaviors. We have begun to model esthetical behavior (reception) introducing esthetical locality. We have exercised the model based on esthetical locality on an abstract painting of Mondrian. The works of art can be estimated by the properties of symmetry/asymmetry and, therefore, can be estimated by esthetical symmetrical globality. The esthetical symmetrical globality is a measure of appreciation of the works of art, of organizing them on the basis of a property and, finely, of understanding them.*

Keywords: *locality, globality, esthetical locality, symmetry, esthetical symmetrical globality.*

1. INTRODUCTION. COLLECTIVITY, STRUCTURE AND REPRESENTATION

One of the properties of the nature is the association in collectivities. The characterization of the collectivity deduces from the definition of the set. We name the collectivity only the sets selected or build helping the relations. So, we exclude the sets selected by the membership (the general definition of a set) [1]. The perception of a collectivity means firstly the perception of the self-organization of the collectivity or the perception of the relations which organizes the collectivity. What properties are at the back of the relations who organize the collectivities? What properties are at the back of the relations who associate the collectivities? It can be gravity, symmetry, survival instinct or, maybe, an esthetical property? In a word, there is a structural self-organization. The structural self-organization is based on the structural relations (not depending of time) between the structural entities. Self-organization can be functional or structural.

A basic concept in my works is the structure one. Let us shortly explain it. A first meaning is that of the reciprocal relation of the parts or the constitutive elements of a whole, determining its nature, its organization [2]. At the end of the nineteenth century it begins to appear a new sense of the structure concept. It will begin to represent not a static organization, but a whole made by solidary elements, in which everyone depends on all other ones and cannot be what it is than in and through them. The connection between parts (the first meaning) is something less necessary than the total interconnection system of each part with all other parts (the second meaning). The first meaning is a sum, the second is a whole. In our days the both senses unified focalizing, depending on the conceptual necessities, on one of the both faces of the term: the coherent, coagulated globality and the relations between local parts or, in short, the globality and the locality.

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We differentiate the structure of its representation or its image. The structure is a concept, with its sides the globality and the locality, while the representation or the image are intuitions (that is the knowledge of the happened reality, the perception of the reality). The function is opposite to the structure but to the intuition (to the image). The esthetical structures characterize by significant intuitive representations. The perception of the structural self-organization of a work of art is, finally, an intuition. ”The result of a work of art (the conception but the reception, my note) is an intuition” [3]. The representation, according to Croce, is an intuition that detaches and emphasizes on the psychic background of sensations. The representation is the elaboration of new sensations and, therefore, is an intuition.

The esthetical structures are esthetical collectivities, i.e. sets build helping the esthetical relations resulted from the esthetical properties. An esthetical relation is a relation that spiritually expresses the connections between the entities of the collectivity on the basis of the esthetical properties (synthesized, e.g., by the binomials beautiful-ugly or asymmetric-symmetric). The esthetical relations are, by definition, structural. The (esthetical) expressions are representations or images of an esthetical structure (a work of art) perceived in a succession not depending of time. The time in which it can be perceived the representation (the image) of an expression is not a time of a function evolution, but is a structural time, if we can say. Therefore, the perception of an esthetical structure is atemporal one. The structural self-organization of a work of art means a spiritual esthetical synthesis or an esthetical expression. Thus, the esthetical functionality is replaced by the esthetical structural process whose essence is the expression.

The structure of an esthetical collectivity can be, as any structure, self-organized locally and globally. Locality is the behavior or the structural self-organization of an esthetical collectivity around an origin. The origins can be multiples, spatial and/or temporal. The definition of the locality refers to the first meaning of the structure concept. The globality is the behavior or the structural self-organization of an esthetical collectivity around an esthetical property (e.g. symmetry or asymmetry). The esthetical properties can be multiples (e.g. symmetry by reflection, symmetry by rotation). The definition of the globality refers to the second meaning of the structure concept.

2. ESTHETICAL INTERCONNECTED COLLECTIVITIES

A complexity system modeling means firstly the perception of a *self-organization* of the system and then the proper modeling. *To perceive a complex*, said Wittgenstein in [4], *means to perceive the relations of its constituent parts in a determined way*. The interconnecting of N nodes by L links models a collectivity, in the sense given by Wittgenstein to the perception of the structural self-organization. The nodes are the collectivity members interconnecting by links. We named these collectivities, *interconnected collectivities*. The interconnected collectivities will not limit at the sets with the same type of nodes and/or at the sets with the same type of links. What is certain is that structural entities forming the collectivity are *interconnected one way or another*. We should limit, without losing too much of generality, to the *orthogonal interconnections* or *orthogonal collectivities*. Any number of interconnection nodes, N, can be represented as a product of whole numbers, $N=m_r \cdot m_{r-1} \cdot \ldots m_1$. On the basis of this representation, to each node of an interconnection we can associate an address X with r digits, $0 \leq X \leq N-1$. Further, we present some basic orthogonal interconnections [5, 11] as collectivities, i.e. sets selected or built by *relations*.

A *generalized hypercube*, GHC, is an orthogonal collectivity with $N=m_r \cdot m_{r-1} \cdot \ldots m_1$ nodes interconnected in r dimensions. In every dimension i of a collectivity the m_i nodes are interconnected all by all. The relation which establishes the interconnection of N nodes all by all is: the nodes addressed by $X = (x_r\ x_{r-1} \ldots x_{i+1}\ x_i\ x_{i-1} \ldots x_1)$ are connected addressed by $X' =$

$(x_r\, x_{r\text{-}1} \ldots x_{i+1}\, x'_i\, x_{i\text{-}1} \ldots x_1)$, where $1 \le i \le r$, $0 \le x'_i \le m_i - 1$ and $x'_i \ne x_i$. The *hypercube*, HC, is a GHC with $N = m^r$. The *binary hypercube*, BHC, is a HC with $N = 2^r$ nodes, and the *completely connected structure*, CCS, is another HC with $N = m$ nodes.

A *generalized hypertorus*, GHT, is another orthogonal collectivity with $N=m_r{\cdot}m_{r\text{-}1}{\cdot}\ldots m_1$ nodes interconnected in r dimensions. In every dimension i, $1 \le i \le r$, the m_i nodes are "collectivized" in a torus. The relation which establishes the r tori of GHT collectivity is: nodes addressed by $X = (x_r\, x_{r\text{-}1}..x_{i+1}\, x_i\, x_{i\text{-}1}..x_1)$ are connected with the nearest neighbor nodes addressed by $X' = (x_r\, x_{r\text{-}1} \ldots x_{i+1}\, x'_i\, x_{i\text{-}1} \ldots x_1)$, $1 \le i \le r$, $x'_i = |\, x_i \pm 1|_{\text{modulo}\ m_i}$. *Hypertorus*, HT, is a GHT with $N = m^r$ nodes, and *torus*, T, is a HT with $N=m$ nodes. BHC can be and HT with $N=2^r$ nodes.

A *generalized hypergrid*, GHG, is, also, an orthogonal collectivity having $N=m_r{\cdot}m_{r\text{-}1}{\cdot}\ldots m_1$ nodes interconnected in r dimensions. In every dimension the m_i nodes are being collectivized in a *chain*, or, better said, every node X is connected in a *grid* with the nodes addressed by $X' = (x_r\, x_{r\text{-}1} \ldots x_{i+1}\, x'_i\, x_{i\text{-}1} \ldots x_1)$, $x'_i = x_i \pm 1|\ x_i \ne 0$ and $x_i \ne m_i - 1$; $x'_i = x_i + 1|\ x_i = 0$; $x'_i = x_i - 1|\ x_i = m_i - 1$, for $1 \le i \le r$. The *hypergrid*, HG, is a GHG with $N = m^r$ nodes. The *chain*, C, is a HG with $N=m$. A binary hypercube can be, also, a hypergrid with $N = 2^r$ nodes.

GHC, GHT and GHG are collectivities represented as *homogenous at links interconnections* or *homogenous interconnections* (the collectivities are homogenous at nodes, also). Most generally, the *non homogenous collectivities* can be represented as non homogenous (at links) interconnections. Examples of non homogenous collectivities are the collectivities represented by *generalized hyper structures*, GHS, [5].

A GHS is an orthogonal collectivity with $N=m_r{\cdot}m_{r\text{-}1}{\cdot}\ldots m_1$ nodes interconnected in r dimensions and in which every node X is collectivized (connected) in every dimension i, $1 \le i \le r$, to the nodes addressed by a *collectivizing (interconnecting) vector* $\left(\bigcup_{j=1}^{k_i} X^{ij}\right) = (x_r\, x_{r\text{-}1} \ldots x_{i+1}\, x'_i\, x_{i\text{-}1} \ldots x_1)$. $\left(\bigcup_{j=1}^{k_i} X^{ij}\right)$ specifies that a node of GHS is connected (non homogenous) by a *vector of elementary collectivizing structures* instead of a *single* structure in the homogeneous collectivities. This is non homogeneity at links of GHS specified by the collectivizing vector having, on the one hand, r elements, and on the other hand, k_i, $1 \le i \le r$, elementary collectivizing structures (homogenous) for which are specified the unions $\left(\bigcup_{j=1}^{k_i} X^{ij}\right)$, $j = 1, 2, \ldots, k_i$. So, X^{ij} are homogeneous elementary structures, like tori, grids, and chains, and must not be disjoint for a dimension.

At homogenous regular interconnections, as the GHC or HT, the origin position, "point of view", does not matter. The collectivities that they model are *spherical*. The diameter is the same, doesn't matter the point of view. At irregular networks, as GHG and other non-homogenous interconnections (e.g. GHS), it matters where the position of the origin is, it matters the point of view. The "structural" behavior around the origin at the collectivities modeled by these interconnections is not spherical anymore. Why does the origin position matter? The structural non-homogeneity of an association in a collectivity from an origin is equivalent to a "functional potential". For example, the more numerous and more varied the links in an interconnected collectivity from a point of view (an origin) are, the more sophisticated, more *adaptable* at a demand, or more *self-organized* the functions are. The interconnected collectivities, homogenous and non-homogenous, can be appreciated, at the beginning, by two general measures: *locality* and *globality*.

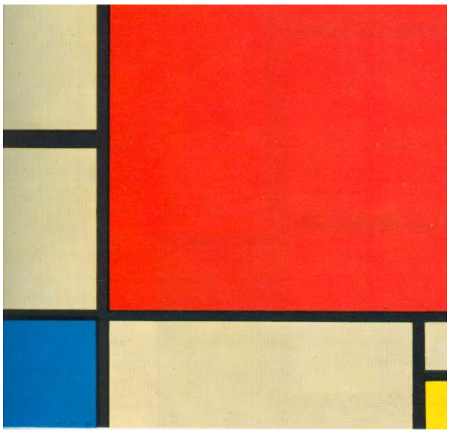

Fig. 1. Mondrian: Composition in red, blue and yellow.

In the Fig. 1 we present an interconnected collectivity from the artificial esthetical world, an *esthetical interconnected collectivity*. It is a work from 1930 of Piet Mondrian, one of the first abstractionist painters. In the beginning Mondrian knew a cubist period, working in Paris with Braque and Picasso. It wasn't long till he separated from them, because of his need to draw of cubism the "logical conclusions", which they did not draw. Regarding the object, which is still visible in cubism, it could keep *the lines, the rhythms and the colors*, and order the painting canvas with only one aim, the *creation of an autonomous composition* [6]. The Mondrian work, except the colors, may resemble with an orthogonal collectivity the nodes of which, in a first phase of study, are at the intersection of the colors. In the Fig. 2 we present the bidimensional interconnection that corresponds with the Mondrian composition from the previous figure.

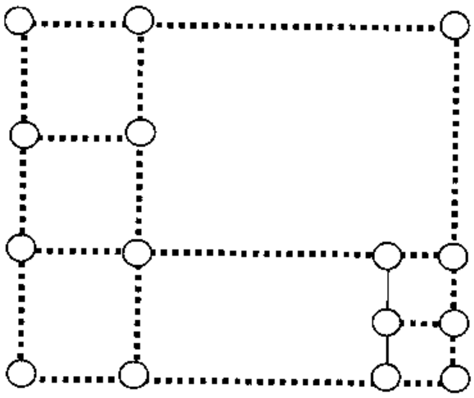

Fig. 2. Interconnected orthogonal collectivity overlapping to Mondrian composition.

3. ESTHETICAL LOCALITY. STATE OF AGGLOMERATION

The collectivities structurally modeled by the interconnections (nodes and links) may be structurally estimated, as primordial measures, by *locality* and *globality*, as we explained before. The *locality* is *the spatial behavior of a collectivity around an origin*. As in Physics, where the gravity characterizes the attraction between objects, *the locality defines a collectivity*: the *nearest* the entities that compose the collectivity are, the best communicate, the best interfere, or in the case of the interconnected collectivities, the *nearest* the nodes are, the bigger the *interconnection power* is. In the esthetical collectivities, a bigger interconnection power can mean a bigger *expression power*. Therefore, a *perception* of the structural self-organization of a work of art is bigger. Consequently, the *intuition* of the structural self-organization of a work of art is bigger, too. The *intuition of a work of art* is more intense. We name this kind of locality, *esthetical locality*. The esthetical locality helps us to understand (partially!) an esthetical collectivity.

As we have explained in the introduction, the locality definition refers to the first sense of the structure concept, the connection between entities or, in interconnected collectivities (and esthetical ones), the links between nodes. Analytically, the locality in an interconnection measures by neighborhoods, neighborhood's reserves, *Moore* reserves and, synthetically, by diameter, degree or average distances [5]. As any property which organizes the entities, the locality may be studied first *structurally* (topologically) and then *functionally*. In the present case, the esthetical functionality is replaced by the *expression*, as we have already explained [7]. Therefore, the locality of an esthetical interconnected collectivity will be defined by two partial localities: a structural locality and an *expressive locality* (which replaces the functional locality from my earlier works). The structural localities appreciate by the simplest measure: *neighborhoods*. The neighborhoods divide in *surface* (or radial) *neighborhoods* and *volume* (or spherical) *neighborhoods*. The surface neighborhood of an interconnected collectivity represents the entities, components or nodes number at the logical distance d, $SN_d(O)=N_d(O)$, where O is the arbitrary chosen origin. The volume neighborhood is $VN_d(O)= \sum_{i=1}^{d} N_d(O)$. The neighborhoods are analytical measures of the structural locality of an interconnected collectivity. But the structural locality can also be measured by synthetic measures, e.g. by the diameter: at the same number of interconnected entities, the less the diameter is, the bigger the locality (in the meaning of the agglomeration) is.

The neighborhoods and the diameters are functions on the *origin position*. At the collectivities interconnected in homogenous and regular structures, as the generalized hypercubes or hypertori are, the origin position does not matter. At the collectivities interconnected in irregular structures, as the generalized hypergrids and other non-homogenous structures (for example GHS), it does matter where the position of the origin is. The topographic model presented in some of my previous works helped us to describe and, therefore, to study the "structural" behavior of the interconnected collectivities in homogenous and, especially, non-homogenous structures. The properties of the locality can be better "read" by the *diameter contour patterns in the structural relief of an interconnected collectivity*.

Besides the contour patterns, we have also introduced a measure that helps us to estimate this structural relief from the locality point of view: the *state of agglomeration*. The structural localities of an interconnected collectivity are more or less *agglomerated* and can be read by the help of the diameter contour patterns, as we have explained in the previous paragraph. The depth of the *valley* (minimum diameter) informs us about the *maximum agglomerated locality*, and the height of the *peak* (maximum diameter) about the *minimum agglomerated locality*. Thus, the *structural state of agglomeration of a node (entity) of an interconnected collectivity is given by the interconnection diameter computed with the origin in the corresponding node*. The contour patterns of the structural states of agglomeration constitute a map with the *structural relief of the interconnected collectivity*.

The surface and volume neighborhoods, on the one hand, and the diameter or the degree, on the other hand, are analytical and synthetic evaluation means of the interaction capacity of an interconnected collectivity, measuring the *structural locality*. By the expressive neighborhoods and, synthetically, by the expressive average distance express which part of the structural locality is used in the esthetical process implemented on an esthetical collectivity. The expressive neighborhoods and the expressive average distances express the *expressive locality* of the esthetical collectivities.

Let us come back at the bidimensional esthetical collectivity of Fig. 1 and let us address the nodes corresponding to a mixed radix number system. From Fig. 3 results a "logical" GHG interconnection (logical because it does not take into consideration physical distances). GHG of Fig. 3 is an interconnected collectivity with $N=m_1\times m_2=4\times5$ nodes, from which 5 are intersection points (nodes) "false", "non visible". The network is a kind of "logical" raster of Mondrian work specifying the visible and non visible "nodes" (the intersection points of the colors). The generalized hypergrid, GHG, is a non homogenous (non spherical) network, the structure of which is not the same, regarding each node as an origin. In brackets are written with bolds the diameters depending on the origin position or on the "point of view".

The structural relief is like a valley or, better said, a *doline* in a karst areas and it is drawn in the Fig. 4. The maximum agglomeration (the bottom of the doline having the minimum diameter) is in the middle of the "logical" network where there are the two nodes with diameter 4. We notice that the two nodes are not invisible. Coming next, rising up towards the doline edge, there are six nodes (from which two are false) having the diameter 5, eight nodes (from which three are false) having diameter 6 and, finally, the corners of the network with diameter 7.

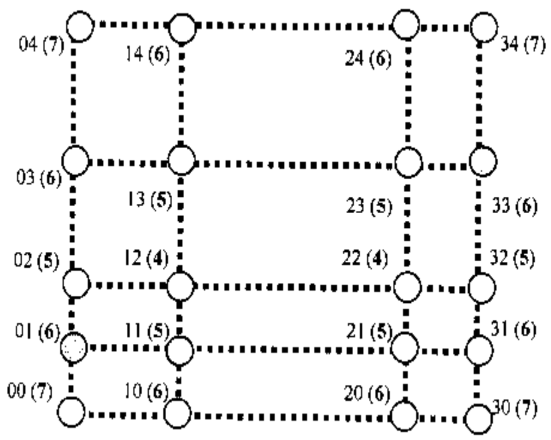

Fig. 3. GHG interconnection corresponding to Fig. 1.

Let us comment this distribution of states of agglomeration on the GHG collectivity corresponding to the Mondrian work [7, 8, 14]. The maximum agglomeration (the minimum diameter, 4), an inverse "ridge" with two visible nodes (intersections of colors), is placed between the two of the most interconnected areas, on the left side and on the right side of the painting, in the "logical" middle of the interconnected collectivity. Climbing up to the doline edges, we come across a contour pattern with diameter 5 that have the invisible nodes *asymmetrically* arranged (an invisible node in the left colors intersections and an invisible node in the right colors intersections).

The asymmetry of the invisible nodes increases at the contour pattern with diameter 6 towards the right-top side, the asymmetrical part of the painting. Mondrian leaves us, towards the right-top side, only with the painting edge, the red square, the biggest one. Mondrian painting is an asymmetrical work "as far as it is devoted to the worship of the *Imperfection*, deliberately leaving some things unfinished to complete by the play of the imagination" [9].

In this way, the Asymmetry is a *structural communication*, a kind of a *structural dynamism* in the *physical* collectivity representing Mondrian painting and in which there are two areas of *local* importance, the nodes {00, 02, 03, 04, 10, 12, 13, 14} and {20, 21, 22, 30, 31, 32}, placed asymmetrically and non homogenously.

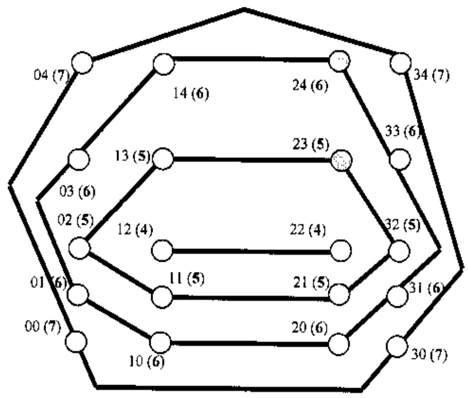

Fig. 4. Structural relief of Mondrian painting modeled with an interconnected collectivity GHG.

4. SYMMETRICAL GLOBALITY

The globality is the behavior or the structural self-organization of an esthetical collectivity around an *esthetical property*. For example, the works of art can be estimated by the aid of symmetrical or asymmetrical properties. If the *Asymmetry* means *structural communication*, structural dynamics or structural lack of balance, the Symmetry means structural ”quiet”, structural statics or *structural balance*. One of the important *properties* of any structure in *physical space* is the projection on the axis *Symmetry-Asymmetry*. The *plastic space* (the fine arts) is included in the physical space or, at least, has an intersection with the physical space. Therefore, the Symmetry-Asymmetry is an *esthetic (plastic) property* that can be expressed by a projection on the axis *Beautiful-Ugly*. The axes, Symmetry-Asymmetry and Beautiful-Ugly, are not equivalent, at least, as definitions. The first axis defines mathematically and the second axis defines esthetically. *The globality concept belongs to the intersection of the two sets (axes, spaces)*.

The mathematical definition of the Symmetry is connected to the notion of automorphism. The *transformation* that keeps the structure of the space is named *automorphism*. Giving a space configuration, a structure, a form, a *collectivity*, we can emphasize a set of space automorphisms, which leave unchangeable this configuration. The emphasizing automorphisms form a *group* (or a *subgroup*), which describes precisely the *symmetry* of giving configuration.

The amorphous space has a *total symmetry* corresponding to the group of all automorphisms. The symmetry of an *interconnected collectivity* will be described, as we have told, by a (sub)group of automorphisms. The total symmetry of the space defined by *n* points (nodes, permutations) will be described by $S_{n!}$, while a *partial symmetry* is expressed by a subgroup (of permutations) *included* in $S_{n!}$. Therefore, *symmetrical groups* $S_{n!}$ model an *architectural space totally symmetric* defined by *n* points (nodes, entities, permutations) and inversely. The total symmetry of a space is represented by a total interconnected collectivity, which is a *completely connected structure* with *n!* nodes.

As an example, the plane figures with two dimensions have as constitutive symmetries only the identity, rotation, translation, reflection and reflection-translation. It is known that a rectangle has the following four symmetries: the identity transformation, *I*; the two reflections S_1 and S_2 vs. non-parallel sides perpendicular bisectors, A_{S1} and A_{S2}; the rotation with 180°, *R*. The four automorphisms can be represented by an interconnection, the vertexes of which will be noted 1, 2, 3 and 4. With this, we equate the symmetries of the rectangle with following *permutations* (*generators*): I=(1 2 3 4), S_1=(2 1 4 3), S_2=(4 3 2 1) and R=(3 4 1 2). The four rectangle symmetries form a commutative group to the composition operation but, equating them with permutations, we notice that these symmetries form only a subgroup of the symmetrical group of order 4, $S_{4!}$. In this way, we can examine quantitatively the symmetry properties of plane figures dividing the symmetrical groups $S_{n!}$ in various subgroups. Let us note G_{Sn} the symmetry groups (subgroups) dividing the symmetrical group $S_{n!}$.

We defined *the globality as the behavior or structural self-organization of a collectivity around a property* [10]. How we can measure the globality of a plane figure versus the symmetry? A quantitative appreciation of the globality of the plane figures versus the symmetry, Γ_n, or *symmetrical globality*, is given by the ratio of the symmetries group (subgroup) order and the symmetrical group order: $\Gamma_n = \left| G_{Sn} \right| / \left| S_{n!} \right|$. The inverse of Γ_n we denominated in the other works group locality, L_n [5, 11].

The collectivity globalities compares for the same number of points defining the space where are built the collectivities, that is for the same *architectural space* $S_{n!}$. For example, symmetrical globalities of a tetragon and a rectangle can be the same because they refer to the same symmetrical group, $S_{4!}$, while we cannot say anything about the globalities of the isosceles triangle and the square because they refer at two *different architectural spaces*, $S_{3!}$ and $S_{4!}$. *Maximum symmetrical globality* obtains when $G_{Sn}/S_{n!}$=1. Let us give another example of three bidimensional figures, an isosceles triangle, a trigon and an equilateral triangle, all referring to $S_{3!}$. The isosceles triangle has two symmetries, I and S. The symmetrical globality is $G_{S3}/S_{3!}$=1/3. The trigon has three symmetries, I, R_1 and R_2. The symmetrical globality is bigger, 1/2. The equilateral triangle has 6 symmetries, I, R_1, R_2, S_1, S_2 and S_3. The symmetrical globality is the biggest, 1. The structural self-organization of this triangle is most complex.

The *locality* (the behavior around an origin) is an analytic estimation mean based on logic distances between the collectivity entities. The *globality* (the behavior around a property) is an architectural principle, a synthetic and constructive principle, based on one of a characteristic property of a collectivity. There is *more localities* function of the number of origins and there is *more globalities* function of the number of properties taken in consideration.

5. MORPHOLOGICAL INTERCONNECTION AND SYMMETRICAL COLLECTIVITIES

We propose the *morphological interconnection* as a *new interconnection model* [10]. This type of interconnection assembles by certain rules in the architectural space $S_{n!}$ or in many architectural spaces $S_{n1!}$, ..., $S_{nk!}$, certain *elementary entities* named *morphems*. The elementary entities (the morphems) can be different in the same space $S_{n!}$. If it uses the architectural principle of *globality versus symmetry* we name these entities *symmetrical morphems*. By the symmetric morphems we build *symmetrical collectivities* and *symmetrical assembles* leading to *symmetrical interconnections*. In other words, the morphological (symmetrical) interconnection is resulted in symmetrical collectivities and symmetrical assembles.

The symmetrical morphems, constituent pieces of symmetrical collectivities and assembles, are bidimensional and tridimensional forms evidentiated in symmetrical group $S_{n!}$ by the *Cayley* graphs of symmetry (sub)groups G_{Sn}. These symmetry groups represent, without losing of generality, the symmetries of the plane and tridimensional figures. For example, the symmetries of the segment are the identity I=(1 2) and the reflection S=(2 1). G_{S2} has a *Cayley* graph with a transposition. The symmetrical morphem has 2 nodes and a link. The symmetries of the isosceles triangle are the same, the identity I=(1 2 3) and the reflection S=(1 3 2). The *Cayley* graph associated to the symmetries of the isosceles triangle is, also, with two nodes and a transposition (a connection), the only difference is that the symmetrical groups (the architectural spaces) on which it defines the automorphisms differ, $S_{2!}$ for the segment, $S_{3!}$ for the isosceles triangle. In the figure 5, left side, we give some symmetrical collectivities built with symmetrical morphems of the isosceles triangle. The trigon symmetries are the identity I=(1 2 3) and two rotations, R_1=(2 3 1) and R_2=(3 1 2). The symmetrical morphem of the trigon is composed of 3 nodes in a triangle with 3 links. In the figure 5, right side, we give the symmetrical collectivities built with the symmetrical morphems of the trigon. The symmetries of the equilateral triangle are the identity I=(1 2 3), the rotation with 180^o R_1=(2 3 1), the rotation with 240^o R_2=(3 1 2) and the reflections S_1=(1 3 2), S_2=(3 2 1) and S_3=(2 1 3). The symmetrical morphem of the equilateral triangle has 6 nodes but there are 2 representations, one in the shape of a prism, the other in the shape of a hexagon. The symmetrical morphem of the equilateral triangle represented by a prism has maximum globality versus symmetry, Γ=$G_{S3}/S_{3!}$=1. The morphem of the line segment is a *linear morphem*; the morphems of the triangle and the rectangle are *plane morphems* and the morphems of the pyramid or the prism are *spatial morphems*.

The first characteristic of the symmetrical collectivities appreciates the *compactity* of them. The maximal compactity of an interconnected symmetrical collectivity obtains when all symmetrical morphems of the collectivity composition have all nodes, links, surfaces and volumes interconnected [5]. There are four basic rules to interconnect the symmetrical morphems in a collectivity: *common nodes* (CN), *common links* (CL), *common surfaces* (CS) and *common volumes* (CV). The compactity is a measure of the *interconnecting degree* of the symmetrical morphems in a symmetrical collectivity. The compactity is minimal for an interconnection CN and maximal for an interconnection CV. Let us note the compactity of the symmetrical collectivities by K. K expresses function of the three types of morphems: $K_L=(\Gamma^2\times m\times n)/N_M$, $K_S=(\Gamma^3\times s\times m\times n)/(L_M\times N_M)$ and $K_V=(\Gamma^4\times v\times s\times m\times n)/(NS_M\times L_M\times N_M)$ where Γ is the globality; n is the number of overlapped nodes, $n=0...N_M/\Gamma$; m is the number of overlapped edges, $m=1...L_M/\Gamma$ ($m=1$ when no overlapped edges); s is the number of overlapped surfaces, $s=1...NS_M/\Gamma$ ($s=1$ when no overlapped surfaces); v is the number of overlapped volumes, $v=1...1/\Gamma$ ($v=1$ when no overlapped volumes); N_M is the number of nodes of the morphems; L_M is the number of edges of the morphem; NS_M is the number of surfaces of the morphem. In the Fig. 5 we give some examples of symmetrical collectivities structured in the architectural space $S_{3!}$ with linear and plane morphems and, near, the corresponding compactities.

A second characteristic of a symmetrical collectivity is the *globality vs. symmetry*. The symmetrical globality of the symmetrical collectivities of Fig. 5 is $\Gamma_L=G_L/S_{3!}=2/6=1/3$, for the collectivities on the left side, and $\Gamma_S=G_S/S_{3!}=3/6=1/2$, for the collectivities on the right side. A *total* symmetrical collectivity is formed, following from the figure 5, of *all* the morphems of a *symmetry* connected in different ways. A symmetrical collectivity can be total or partial. The globality is constant for a symmetrical collectivity indifferently of the interconnecting mode of the morphems.

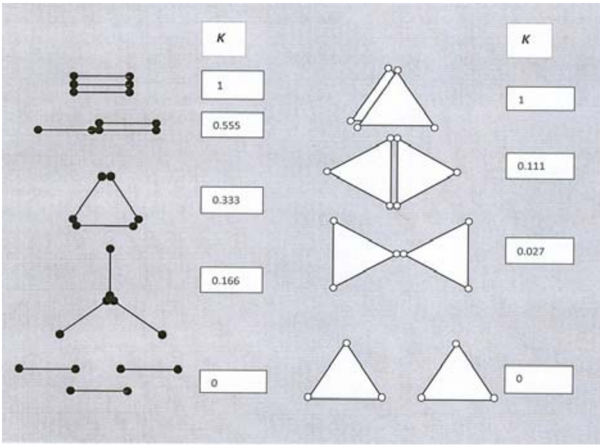

Fig. 5. Examples of symmetrical collectivities structured in $S_{3!}$ with linear and plane morphemes.

6. SYMMETRICAL ENSEMBLES

The symmetrical ensembles are symmetrical collectivities, partial or total, realized in different architectural spaces, $S_{n1!}$, ..., $S_{nk!}$, by different symmetrical morphems. For example, in the Fig. 6, we give an ensemble realized with two symmetrical morphems in $S_{3!}$ and $S_{2!}$. With the same two morphems we can realize an ensemble only in $S_{3!}$.

To measure the globality of symmetrical ensembles we introduced the notion of *compound globality*.

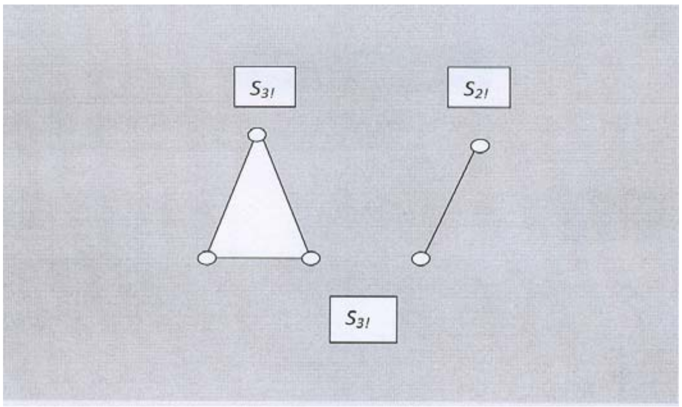

Fig. 6. A symmetrical ensemble built in $S_{3!}$ and $S_{2!}$. With the same morphems we can realize a symmetrical ensemble in $S_{3!}$.

For example, the ensemble of figure 6 built with the trigon symmetrical morphem in $S_{3!}$ and the symmetrical morphem of the line segment $S_{2!}$ will have the compound globality $\Gamma=\Gamma_1 + \Gamma_2$ where Γ_1 is the trigon globality vs. symmetry (1/2) and Γ_2 is the segment globality vs. symmetry (1). So, the ensemble will have the globality Γ=1.5. Building the ensemble only in $S_{3!}$ with a morphem of the trigon and a morphem of the isosceles triangle (globality 1/3) we obtain a less *compound globality* Γ=1/2+1/3=5/6.

7. ESTHETICAL COLLECTIVITIES AND ENSEMBLES

The symmetrical collectivities and ensembles can be *esthetical collectivities and ensembles* (works of art). We distinguish the *esthetical interconnected collectivities* (composed of nodes and links) from the *esthetical symmetrical collectivities* (composed of symmetrical morphems). The globality characterizing these esthetical symmetrical collectivities and ensembles we name *esthetical symmetrical globality*. The famous *Cézanne* expression „*traiter la nature par le cylindre, le cône et la sphère*" can be an explanation of the name: the morphems of an esthetical symmetrical collectivity or ensemble can be plane (line segment, triangle, rectangle, circle...) or, as in Cézanne' adagio, tridimensional (cylinder, cone, sphere, pyramid …). We give three works of art helping to define *esthetical symmetrical collectivities* and *ensembles* as a particular case of morphological interconnecting.

In the Fig. 7 we give a still life of Cézanne of which "strictness of elaboration, architectural solidity of the volumes and faultless organization in space of the masses remain impossibly to hide even under the cover of new and not real moving chromatically garments" [12]. In the Fig. 8, Malevich, the father of the *Suprematism*, tries "to establish the *supremacy of the pure sensibility* using only elementary and plane geometrical figures" [6]. The abstraction art, non-Kandinsky, ends with some rationalist painters, classical and anti-romantic, as Herbin (Fig. 9), which build with pure geometrical elements, in the senses given by Malevich and, especially, by Mondrian, a statically painting and extremely rationalist.

Fig. 7. Cézanne: *Still Life*.

Fig. 8. Malevich: *Suprematiste Composition*

Fig. 9. Herbin: *Veine*.

8. ESTHETICAL SYMMETRICAL GLOBALITY

The globality characterizing the esthetical symmetrical collectivities and ensembles we named *esthetical symmetrical globality*. This globality is, generally, a compound globality and it helps to appreciate quantitatively different paintings.

Let us to appreciate the compound globality versus symmetry of Cézanne' *Still Life* (Fig. 7). In the Fig. 10 we give a representation of the painting as an esthetic ensemble. The symmetrical ensemble is an approximate drawing containing only plane figures: two line segments (LS), a rectangle (R), eight circles (C) and four isosceles triangles (IT). The globality of this ensemble is Γ= 2 x Γ_{LS} + Γ_R + 8 x Γ_C + 4 x Γ_{IT}. The globality of the line segment is Γ_{LS} = 1. The globality of the rectangle is Γ_R = 1/6. The globality of the circle is Γ_C = 1. The globality of the isosceles triangle is Γ_{IT} = 1/3. The compound globality versus symmetry of the painting of Cézanne is Γ= 2x1+1/6+8x1+4x1/3=11,5. Let us compare the globality of *Still Life* of Cézanne (figure 7) with the globality of *Suprematiste Composition* of Malevich (figure 8). The figure 8 can be read as a symmetrical ensemble composed of a square (S), four rectangles (for simplicity we approximated the blue figure with a rectangle) and a circle. The globality of this esthetic ensemble is Γ=1/3+4x1/6+1=2. The compound globality versus symmetry of the *Still Life* of Cézanne is bigger than of the *Suprematiste Composition* of Malevich. The *suprematiste composition* of Malevich deserves the name!

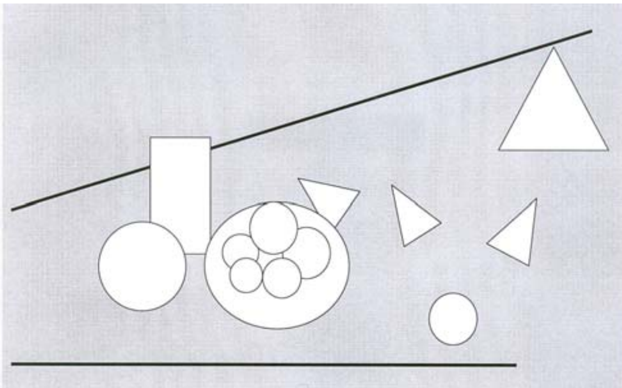

Fig. 10. The representation as symmetrical ensemble of Cézanne' *Still Life* (Fig. 7).

Let us measure the globality of the third work of art (figure 9). There is no need of a *representation* with pure geometrical figures. Herbin work is a geometrical work composed of four circles (C), three isosceles triangles (IT), four rectangles (R) and four squares (S). The square has eight symmetries: the identity (*I*), three rotations, R_1, R_2 and R_3, with 90°, 180° and 270°, the reflections *S* and *T* versus the two median perpendicular and the reflections *U* and *T* versus diagonals. So, the globality of a square is Γ_S=| G_{Sn}|/|$S_{n!}$| = 8/4!=1/3. The compound globality of the Herbin painting is, taking the constituted globalities into account (Γ_C=1, Γ_{IT}=1/3, Γ_R=1/6 si Γ_S=1/3), Γ=4x1+3x1/3+4x1/6+4x1/3=7.

In this way, using the symmetrical globality we can to *hierarch* (to organize, to understand) the presented works of art from the point of view of symmetry: Malevich' *Suprematiste Composition* (Γ=2), Herbin' *Veine* (Γ=7) and Cézanne' *Still Life* (Γ=11,5). The Symmetry/Asymmetry is one of the planes on which we can project the *expression* of a work of art. The expression projection is equivalent to a *understanding*.

9. CONCLUSIONS

The (inter)connections are the generators, the algorithms of discoveries, are "patterns of discovery" [13]. The interconnected collectivities, as we defined, are our models to *esthetical behaviors*. We have begun to model esthetical behavior (reception) by *esthetical locality*, a measure which can be estimated by neighborhoods, expressive states of agglomeration, expressive relief of the esthetical interconnected collectivity. We have exercised the esthetical model based on esthetical locality on an abstract painting of Mondrian, reading this work by another language: of the locality and of the symmetry. The esthetical locality makes the connection between the *interconnection power* and the *expression power*.

The works of art can be estimated by the properties of symmetry or asymmetry and, therefore, can be estimated by esthetical symmetrical globality. The *morphological* *interconnection*, which we proposed as a *new model of interconnecting*, assembles by certain rules, in the architectural space $S_{n!}$ or in many architectural spaces $S_{n1!}$, ..., $S_{nk!}$, certain *elementary entities* named *morphems*. If we use the architectural principle of the *globality versus symmetry* we name these entities *symmetrical morphems*. With these we build *symmetrical collectivities* and *symmetrical ensembles* leading to a *symmetrical interconnection*. The symmetrical collectivities and ensembles can be *esthetical collectivities and ensembles* (the works of art). We distinguish the esthetical interconnecting collectivities (composed of nodes and links) from the esthetical symmetrical collectivities (composed of symmetrical morphems). The globality characterizing these esthetical symmetrical collectivities and ensembles we named *esthetical symmetrical globality*. The esthetical symmetrical globality is a measure of appreciation of the works of art, of organizing them on the basis of a property and, finely, of understanding them.

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