ORIGINAL PAPER

X-INDOMINABLE GRAPHS

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Abstract. *An X-dominating set which is also X-independent is called an X-independent X-dominating set. A graph is said to be X-indominable if X(G) can be partitioned into X-independent X-dominating sets, otherwise G is called X-indominable. We define X-indominable number and give its bipartite version.*

Keywords: *X-dominating sets, X-independent sets, X-indominable number.*

1. INTRODUCTION

Bipartite theory of graphs was introduced by Stephen Hedetniemi and Renu Laskar in their two papers [3,4] in which concept in graph theory have equivalent formulations as concepts for bipartite graphs. One such formulation is the concept of X-dominating sets and X-independent sets of bipartite graphs.

Cockayne E.J and Hedetniemi S.T introduced the concept of disjoint independent dominating sets [2] in graphs. This concept was further studied in [1] by Acharya B.D. and Walikar H.B. Here, we initiate the study of X-indominable graphs. Interesting results are obtained which exhibit the method of embedding non X-indominable graphs into X-indominable graphs. We also introduce a new parameter called X-indominable number, which is bipartite version of indominable number of a graph.

2. PRELIMINARIES

A partition P of the vertex set called indomatic partition [1], if each element is independent dominating set. If $\Pi(G)$ denotes the set of all indomatic partition of G then the number b(G)=max$_{\Pi(G)}|P|$ is called indomatic number of G.

We consider only bipartite graphs G=(X, Y, E). Two vertices u and v are X-adjacent if u and v are adjacent to the same vertex y in Y. A subset S of X is called a X-dominating set[3] if every vertex in X-S is X-adjacent to a vertex of S. The minimum cardinality of a X-dominating set is called the X-domination number of a graph G and is denoted by $\gamma_X(G)$.

A subset D of X is called a X-independent set[4] if any two vertices in D are not X-adjacent. The maximum cardinality of a X-independent set is called the X-independence number of a graph G and is denoted by $\beta_X(G)$.

A X-domatic partition of G is a partition of X, all of whose elements are X-dominating sets in G. The X-domatic number of G is the maximum number of classes of a X-domatic partition of G. The X-domatic number of a graph G is denoted by $d_X(G)$.

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3. X-INDOMINABLE GRAPHS

An X-dominating set which is also X-independent is called an X-independent X-dominating set. A graph is said to be X-indominable if X(G) can be partitioned into X-independent X-dominating sets, otherwise G is called X-indominable.

Example: The partition $\{\{x_1,x_3,x_5\}, \{x_2.x_4,x_6\}\}$ are X-independent sets which are also X-dominating sets. Therefore, G = C_{12} given below is X-indominable.

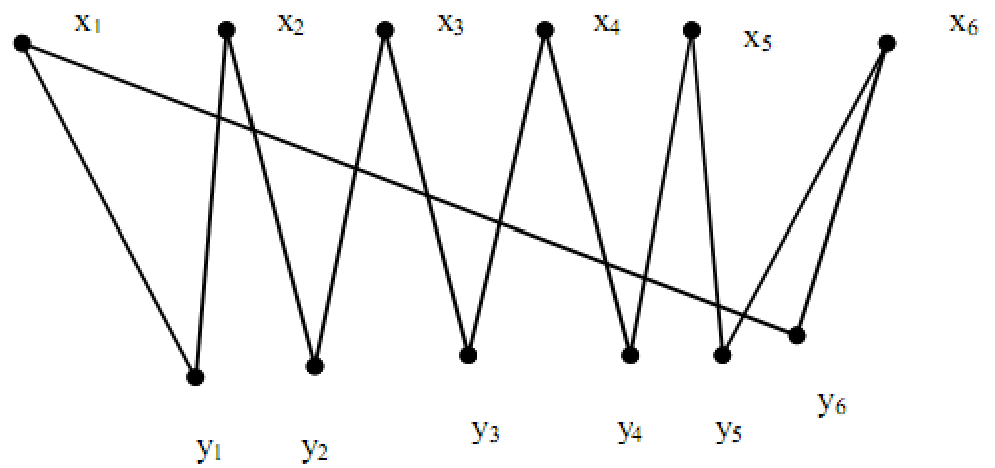


Fig. 1. Cycle C_{12}.

Example: In the graph G = C_{10} maximum X-independent set are $\{x_1,x_3\}$, $\{x_1,x_4\}$, $\{x_2,x_4\}$, $\{x_2,x_5\}$ and $\{x_3,x_5\}$. These are also X-dominating set and $\gamma_X(G) = 2$. Since $|X| = 5$, X cannot be partitioned into X-independent X-dominating sets. Therefore, G = C_{10} is non X-indominable.

Theorem 3.1: If G is not X-indominable then there exists a X-indominable graph H containing G as an induced subgraph.

Proof: Let G be a non X-indominable graph with $|X| = n$. Let X=$\{u_1,u_2,\ldots,u_n\}$. Add vertices $v_1,v_2,\ldots,v_n$ to X such that every vertex u_i is X-adjacent to vertices v_j, $i \neq j$, $1 \leq i \leq n$. $\{u_i,v_i\}$ i = 1 to n forms X-independent X-dominating sets. Therefore, H is X-indominable and X(H) can be partitioned into X-independent X-dominating sets.

Corollary 3.2: The class of X-indominable graphs cannot be characterized by a family of forbidden subgraphs.

Definition 3.3: A partition P = $\{D_1,D_2,\ldots,D_k\}$ of the vertex set X(G) of G is called an X-indomatic partition, if D_i is an X-independent X-dominating set, for each i = 1,2,3,…,k. If $\Pi_{id}(G)$ denotes the set of all X-indomatic partition of G, then the number $b_X(G) = \max_{P\in\Pi_{id}(G)}|P|$ is called X-indomatic number of G.

Observation: Any X-indomatic partition is an X-domatic partition. Therefore, $b_X(G) \leq d_X(G)$.

Bipartite theory of Graphs-I and II [3,4] suggests some constructions of bipartite graphs. Let G= (V,E) be a graph. The bipartite graph VE(G) = (V, E, F) is defined as the graph with vertex $V \cup E$ and the edge set F={(u,e): e = (u,v)$\in$E}. The bipartite graph VV(G) = (V, V^1, E^1) , V^1 is a copy of the vertices of V(G). E^1={(u,v^1): (u,v) $\in$E}. The bipartite graph VV^+(G) = (V, V^1, E^{11}) contains the edges E^1 of the graph VV(G) together with the edges {(u,u^1): u$\in$V}. The bipartite graph EV(G)=(E,V,J) is defined by the edges J={(e,u): e=(u,v) $\in$E}.

Given a graph G, graphs G2 and G^2 have the same vertex set as G, with two vertices u and v adjacent in G2 if and only if they have a common neighbor in G and two vertices u and v are adjacent in G^2 if and only if the distance d(u,v) $\leq$2 in G.

Theorem 3.4: For any graph G, b(G) = b_X(VE(G)).

Proof: Let b(G)=k. There exists a partition of V(G) into independent dominating sets of cardinality k. Let $V_1,V_2,\ldots,V_k$ be the partition of V(G) into independent dominating sets. In the graph VE(G) = (X,Y, E^1) , $V_1,V_2,\ldots,V_k$ is a partition of X into X-independent X-dominating sets. Therefore, b(G) $\leq$ b_X(VE(G)).

Conversely, let α be the X-indomatic number of VE(G). The partition X_1, $X_2,\ldots,X\alpha$ is a partition of X into X-independent X-dominating set. In G, X_1, $X_2,\ldots,X\alpha$ forms a indomatic partition of G. Therefore, b(G) $\geq$ b_X(VE(G)). Hence, b(G) = b_X(VE(G)).

Theorem 3.5: For any graph G, b^1(G) = b_X(EV(G)).

Proof: Let b^1(G)=k. There exists a partition of edges into independent dominating sets of cardinality k. Let $E_1,E_2,\ldots,E_k$ be the partition of E(G) into independent dominating sets. In the graph EV(G) = (X,Y, E^1) , $E_1,E_2,\ldots,E_k$ is a partition of X into X-independent X-dominating sets. Therefore, b^1(G) $\leq$ b_X(EV(G)).

Conversely, let α be the X-indomatic number of EV(G). The partition X_1, $X_2,\ldots,X\alpha$ is a partition of X into X-independent X-dominating set. In G, X_1, $X_2,\ldots,X\alpha$ forms an indomatic partition of G. Therefore, b(G) $\geq$ b_X(EV(G)). Hence, b(G) = b_X(EV(G)).

Theorem 3.6: For any graph G, b(G2) = b_X(VV(G)).

Proof: Let b(G2)=k. There exists a partition of V(G) into independent dominating sets of cardinality k. Let $V_1,V_2,\ldots,V_k$ be the partition of V(G2) into independent dominating sets. In the graph VV(G) = (X,Y, E^1) , $V_1,V_2,\ldots,V_k$ is a partition of X into X-independent X-dominating sets. Therefore, b(G) $\leq$ b_X(VV(G)).

Conversely, let α be the X-indomatic number of VV(G). The partition X_1, $X_2,\ldots,X\alpha$ is a partition of X into X-independent X-dominating set. In G2, X_1, $X_2,\ldots,X\alpha$ forms an indomatic partition of G2. Therefore, b(G2) $\geq$ b_X(VV(G)). Hence, b(G) = b_X(VV(G)).

Theorem 3.7: For any graph G, b(G^2) = b_X(VV^+(G)).

Proof: Let b(G^2)=k. There exists a partition of V(G^2) into independent dominating sets of cardinality k. Let $V_1,V_2,\ldots,V_k$ be the partition of V(G^2) into independent dominating sets. In the graph VV^+(G) = (X,Y, E^1) , $V_1,V_2,\ldots,V_k$ is a partition of X into X-independent X-dominating sets. Therefore, b(G^2) $\leq$ b_X(VV^+(G)).

Conversely, let α be the X-indomatic number of VV^+(G). The partition X_1, $X_2,\ldots,X\alpha$ is a partition of X into X-independent X-dominating set. In G^2, X_1, $X_2,\ldots,X\alpha$ forms an indomatic partition of G^2. Therefore, b(G^2) $\geq$ b_X(VV^+(G)). Hence, b(G2) = b_X(VV^+(G)).

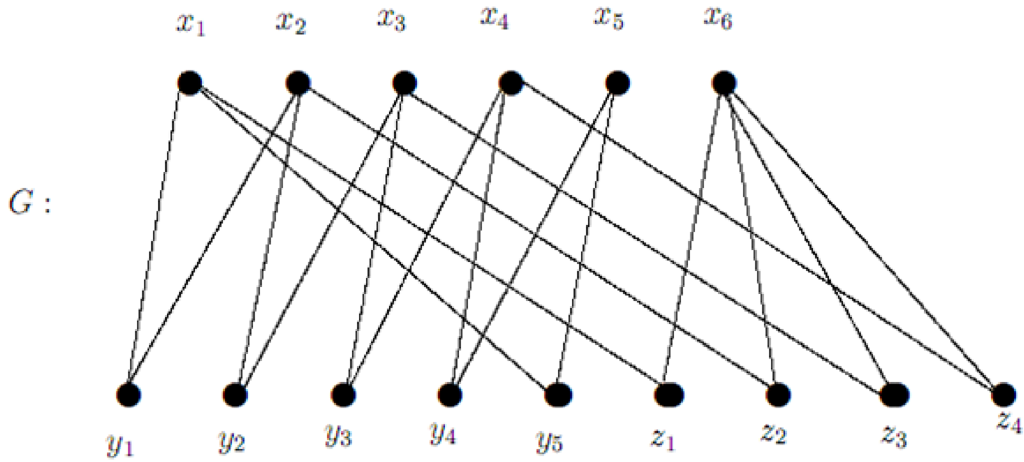
Definition 3.8: The X-indominable number of a non X-indominable graph G with respect to X-domination, denoted by IND_X(G) is defined as X(H)-X(G) where H is an X-indominable graph of least order in which G can be embedded.

Remark: $1 \leq IND_X(G) \leq n$.

Remark: $IND_X(C_{2n}) = 1$ if n is odd and $n \neq 3$.

Let $G = C_{2n}$, n is odd and $n \neq 3$. Let H be the graph obtained from G by adding a vertex x_{n+1} to X(G) and $z_1, z_2, \ldots, z_{n-1}$ to Y(G) and making x_i and x_{n+1} adjacent with z_i, $1 \leq i \leq n-1$. Then the partition $\{\{x_1, x_3, x_5, \ldots, x_{n-2}\}, \{x_2, x_4, x_6, \ldots, x_{n-1}\}, \{x_n, x_{n+1}\}\}$ is an X-indominable partition of H. Therefore, $IND_X(C_{2n}) = 1$ if n is odd and $n \neq 3$.

Example:



The graph $G = C_{10}$ is non X-indominable. We add a vertex x_6 to X and z_1, z_2, z_3 and z_4 to Y. Make x_6 adjacent to z_1, z_2, z_3 and z_4. z_1 is adjacent to x_1. z_2 is adjacent to x_2. z_3 is adjacent to x_3. z_4 is adjacent to x_4. The graph obtained by these operation gives a X-indominable graph which is given above. Hence, $IND_X(C_{10}) = 1$.

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