ORIGINAL PAPER

SOME FORMS OF STRONG FUNCTIONS IN SOFT TOPOLOGICAL SPACES

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Abstract. *In this paper we have introduced two stronger functions namely Strongly Soft Jc open(closed) maps and Strongly Soft Jc homeomorphism. Further we have studied some of their properties in detail*
Keywords: *Soft Jc Closed; Soft Jc open; Strongly Soft Jc open(closed) maps; Soft Jc homeomorphism and Strongly Soft Jc homeomorphism.*

1. INTRODUCTION

The concept of Soft set theory was introduced by Molodtsov [1], Shabir and Naz [2] introduced the notion of Soft topological spaces with fixed set of parameters. The concept Soft closed map plays a vital role in the development of the nature of Soft topological spaces. These studies not only have the potential to serve as the theoretical foundation for more topological applications on Soft sets, but they may also help advance the development of information systems and different technical specialties. Topology heavily relies on the idea of homeomorphism. Homeomorphisms play in topology the same role that linear isomorphisms play in algebra, or that biholomorphic mappings play in function theory. Other authors [3] established the concept of a mapping on S_{soft} classes and investigated numerous features of S_{soft} set pictures and inverse images. In this paper it has introduced a new class of closed set namely Soft Jc Closed set [4] and Soft Jc Homeomorphism. In addition, we have introduced two stronger functions namely Strongly Soft Jc open (closed) maps and Strongly Soft Jc homeomorphism. Further we have studied some of their properties in detail.

2. PRELIMINARIES

Definition 2.1. [3] A soft $\hat{g}$-closed set if cl(A,E) $\tilde{\subseteq}$ (U,E) whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is Soft semi open in (X,τ,E); the complement of a $\hat{g}$-closed set is called a Soft $\hat{g}$-open set.

Definition 2.2. [5] A *Soft- Jc-closed* if $S\alpha cl(A,E) \tilde{\subseteq} Int(U,E)$ whenever $(A,E) \tilde{\subseteq} (U,E)$ and (U,E) is Soft ĝ-open in (X, τ, E); the complement of Soft Jc closed set is called a Soft Jc-open set.

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Definition.2.3. [6] A function $f : (X,\tau,E) \rightarrow (Y,\sigma,K)$ is said to be *Soft irresolute* if f $^{-1}$(V,K) is Soft -closed set in(X,τ,E) for every Soft closed set (V,K) of (Y,σ,K).

Definition 2.4. A Soft map $f : (X,\tau,E) \rightarrow (Y,\sigma,K)$ is said to be a *Soft Jc-closed map* (*Soft Jc-open map*) if the image $f\ (A,E)$ is Soft Jc-closed (Soft Jc-open) in (Y,σ,K) for each Soft closed (Soft open) set (A,E) in (X,τ,E).

Definition 2.5. [7] A bijection $f : (X,\tau,E) \rightarrow (Y,\sigma,K)$ is called *Soft homeomorphism* if f is both Soft continuous and Soft open.

Definition 2.6. [8] A bijection $f : (X,\tau,E) \rightarrow (Y,\sigma,K)$ is called *Soft Jc homeomorphism* if f is both Soft Jc continuous and Soft Jc open.

Proposition 2.7. [8] Every Soft Clopen Set is Soft Jc open.

3. STRONGLY SOFT Jc OPEN MAP

Definition 3.1. A map $f: (X,\tau,E) \rightarrow (Y,\sigma,K)$ is known to be *Strongly Soft Jc open map* if the image of each one of the Soft Jc open in (X,τ,E) is Soft Jc open in (Y,σ,K).

Definition 3.2. A map $f: (X,\tau,E) \rightarrow (Y,\sigma,K)$ is known to be *Strongly Soft Jc closed map* if the image of each one of the *Soft Jc closed* set in (X,τ,E) is *Soft Jc closed* in (Y,σ,K).

Example 3.3. Let $X = \{a,b\}, Y = \{c,d\}, E = \{e_1,e_2\}, K = \{k_1,k_2\}$. Define $u : X \rightarrow Y$ and $v : E \rightarrow K$ as $u(a) = c, u(b) = d$ and $v(e_1) = k_1, v(e_2) = k_2$.

Consider the Soft topologies

$$\tau = \{\tilde{\phi}, \tilde{X}, (F_1,E), (F_2,E)\}$$

where (F_1,E) and (F_2,E) are described this way:

$$F_1(e_1) = \phi, F_1(e_2) = \{a\}, F_2(e_1) = \{a\},$$

$$F_2(e_2) = \{a\}$$

and

$$\sigma = \{\tilde{\phi}, \tilde{Y}, (H,K)\}$$

where (H,K) is described this way:

$$H(k_1) = \{c,d\}, H(k_2) = \{c\}.$$

Precisely the mapping $f: (X,\tau,E) \rightarrow (Y,\sigma,K)$ is Strongly Soft Jc open (Closed) map.

Proposition 3.4. Each one of the Strongly Soft Jc open map is Soft Jc open.

Proof: The argument proceeds from Proposition 2.4.

Remark 3.5. It is observed from the subsequent illustration that the reverse implication is incorrect.

Example 3.6. Let

$$X = \{x_1, x_2\}, Y = \{y_1, y_2\},$$

$$E = \{e_1, e_2\},$$

$$K = \{k_1, k_2\}.$$

Define $u : X \to Y$ and $v : E \to K$ as $u(x_1) = y_2, u(x_2) = y_1$ and $v(e_1) = k_1, v(e_2) = k_2$. Consider the Soft topologies

$$\tau = \{\tilde{\phi}, \tilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$$

where $(F_1, E), (F_2, E)$and (F_3, E) are described this way:

$$F_1(e_1) = \{x_2\}, F_1(e_2) = \phi,$$

$$F_2(e_1) = \phi, F_2(e_2) = \{x_1\},$$

$$F_3(e_1) = \{x_2\}, F_1(e_2) = \{x_1\}$$

and

$$\sigma = \{\tilde{\phi}, \tilde{Y}, (H_1, K), (H_2, K)\}$$

where (H_1, K) and (H_2, K) are described this way:

$$H_1(k_1) = \{y_2\},$$

$$H_1(k_2) = \{y_1\},$$

$$H_2(k_1) = \{y_2\},$$

$$H_2(k_2) = \{y_1, y_2\}.$$

Precisely the mapping $f: (X, \tau, E) \to (Y, \sigma, K)$ is Soft Jc open but not Strongly Soft Jc open, since $f\{(e_1, x_1, x_2), (e_2, \phi)\} = \{(k_1, y_1, y_2)(k_2, \phi)\}$ is not Soft Jc open in (Y, σ, K).

Theorem 3.7. If a Soft map $f: (X, \tau, E) \to (Y, \sigma, K)$ is Strongly Soft Jc Closed and assume that $SJcO((Y, \sigma, K))$ is Soft Closed under Soft union then $SJcCl(f(A, E)) \tilde{\subseteq} f(SCl(A, E))$ for every Soft subset (A, E) of (X, τ, E).

Proof: Suppose f is a Strongly Soft Jc Closed map and (A, E) is a Soft Jc Closed set of (X, τ, E).Then $f(SCl(A, E))$ is a Soft Jc Closed set in (Y, σ, K). There on $f(A, E) \tilde{\subseteq} f(SCl(A, E))$ and $SJcCl(f(A, E)) \tilde{\subseteq} SJcCl(f(SCl(A, E))) = f(SCl(A, E))$.

Theorem 3.8. If a Soft map $f: (X, \tau, E) \to (Y, \sigma, K)$ is Strongly Soft Jc Closed if and only if for each Soft subset (A, K) of (Y, σ, K) and for each Soft Jc open set (U, E) containing

$f^{-1}(A,E)$ there is a Soft Jc open set (V,K) of (Y,σ,K) such that $f(U,E) \tilde{\subseteq} (V,K)$ and $f^{-1}(V,K) \tilde{\subseteq} (U,E)$.

Proof:
Necessity: Suppose $f:(X,\tau,E) \to (Y,\sigma,K)$ is Strongly Soft Jc Closed. Let (A,K) be any Soft subset of (Y,σ,K) and (U,E) a Soft Jc open set of (X,τ,E) containing $f^{-1}(A,K)$. Put $(V,K) = (f((U,E)^C))^C$. Then (V,K) is Soft Jc open in (Y,σ,K) containing (A,K) and $f^{-1}(V,K) \tilde{\subseteq} (U,E)$.
Sufficiency: Let (B,E) be any Soft Closed set of (X,τ,E). Then $f^{-1}(f(B,E)^C) \tilde{\subseteq} ((B,E)^C)$ and $(B,E)^C$ is Soft open. By assumption there exists a Soft Jc open set (V,K) of (Y,σ,K) such that $f(B,E)^C \tilde{\subseteq} (V,K)$ and $f^{-1}(V,K) \tilde{\subseteq} ((B,E)^C)$ and so $(B,E) \tilde{\subseteq} (f^{-1}(V,K))^C$. Hence $(V,K)^C \tilde{\subseteq} f(B,E) \tilde{\subseteq} f(f^{-1}(V,K)^C) \tilde{\subseteq} (V,K)^C$ which implies $f(B,E) = (V,K)^C$. Since $(V,K)^C$ is Soft Jc Closed. Therefore $f(B,E)$ is Strongly Soft Jc Closed and therefore f is Soft Jc Closed .

4.STRONGLY SOFT Jc HOMEOMORPHISM

Definition 4.1. A Soft bijection $f:(X,\tau,E) \to (Y,\sigma,K)$ is known to be Strongly Soft Jc homeomorphism if both f and f^{-1} are Soft Jc irresolute maps.

Example 4.2. Let

$$X = \{x_1, x_2\},$$

$$Y = \{y_1, y_2\},$$

$$E = \{e_1, e_2\},$$

$$K = \{k_1, k_2\}.$$

Define $u : X \to Y$ and $v : E \to K$ as $u(x_1) = y_1, u(x_2) = y_2$ and $v(e_1) = k_1, v(e_2) = k_2$. Consider the Soft topologies

$$\tau = \{\tilde{\phi}, \tilde{X}, (F_1,E), (F_2,E)\}$$

where (F_1,E) and (F_2,E) are described this way:

$$F_1(e_1) = \{x_2\},$$

$$F_1(e_2) = \{x_1\},$$

$$F_2(e_1) = \{x_2\},$$

$$F_2(e_2) = \{x_1, x_2\}$$

and

$$\sigma = \{\tilde{\phi}, \tilde{Y}, (H_1,K), (H_2,K)\}$$

where (H_1, K) and (H_2, K) are described this way: $H_1(k_1) = \{y_2\}, H_1(k_2) = \{y_1\}, H_2(k_1) = \{y_2\}, H_2(k_2) = \{y_1, y_2\}$. Then the $S\ddot{o}ft$ identity map $f: (X, \tau, E) \to (Y, \sigma, K)$ is Strongly Soft Jc homeomorphism.

Proposition 4.3. Every Strongly Soft Jc homeomorphism is a Soft Jc homeomorphism .

Proof: Since every Soft Jc irresolute maps is Soft Jc continuous, the argument proceeds.

Remark 4.4. It is observed from the subsequent illustration that the reverse implication is incorrect.

Example 4.5. Let $X = \{x_1, x_2\}, Y = \{y_1, y_2\}, E = \{e_1, e_2\}, K = \{k_1, k_2\}$. Define $u : X \to Y$ and $v : E \to K$ as $u(x_1) = y_1, u(x_2) = y_2$ and $v(e_1) = k_1, v(e_2) = k_2$. Consider the Soft topologies $\tau = \{\tilde{\phi}, \tilde{X}, (F, E)\}$ where (F, E) is described this way: $F(e_1) = \phi, F(e_2) = \{x_2\}$ and $\sigma = \{\tilde{\phi}, \tilde{Y}, (H, K)\}$ where (H, K) and is described this way: $H(k_1) = \phi, H(k_2) = \{y_1, y_2\}$. Then the Soft identity map $f: (X, \tau, E) \to (Y, \sigma, K)$ is Soft Jc homeomorphism but not Strongly Soft Jc homeomorphism because $f\{(e_1, x_1, x_2), (e_2, \phi)\} = \{(k_1, y_1, y_2), (k_2, \phi)\}$ is not Soft Jc open in (Y, σ, K).

Theorem 4.6. The composition of two Strongly Soft Jc homeomorphisms is also a Strongly Soft Jc homeomorphism.

Proof:

1. $g \circ f$ is Soft Jc irresolute: Let (H, L) be a Soft Jc Closed set in (Z, γ, L). Since g is Soft Jc irresolute $g^{-1}(H, L)$ is Soft Jc Closed in (Y, σ, K). Since f is also Soft Jc irresolute $f^{-1}(g^{-1}(H, L)) = (g \circ f)^{-1}(H, L)$ is Soft Jc Closed in (X, τ, E). Thus $g \circ f$ is Soft Jc irresolute.
2. $(g \circ f)^{-1}$ is Soft Jc irresolute: Also for a Soft Jc open set (B, E) in (X, τ, E), $g \circ f(B, E) = g(f(B, E)) = g(C, K)$, where $f(B, E) = (C, K)$. By hypothesis (C, K) is Soft Jc openin (Y, σ, K) and, $g(C, K)$ is also Soft Jc open in (Z, γ, L). Therefore $(g \circ f)^{-1}$ is Soft Jc irresolute. Hence $g \circ f: (X, \tau, E) \to (Z, \gamma, L)$ is Strongly Soft Jc homeomorphism.

Theorem 4.7. For a Soft topological space (X, τ, E), the collections of Strongly Soft Jc homeomorphism form a group under composition of functions.

Proof: Define $*: SSJcH(X, \tau, E) \times SSJcH(X, \tau, E) \to SSJcH(X, \tau, E)$ by $*(f, g) = (g \circ f)$ for every $f, g \in SSJcH(X, \tau, E)$.Then $(g \circ f) \in SSJcH(X, \tau, E)$. Hence $SSJcH(X, \tau, E)$ is Soft Closed. Since compositions of maps is associative, the Soft identity map $i: (X, \tau, E) \to (X, \tau, E)$ is a Strongly Soft Jc homeomorphism and $i \circ f = f \circ i = f$. For any $f \in SSJcH(X, \tau, E)$, there exists $f^{-1} \in SSJcH(X, \tau, E)$ such that $f \circ f^{-1} = f^{-1} \circ f = i$ and so inverse exists for each element. Therefore, $(SSJcH(X, \tau, E),*)$ is a group under the composition of maps.

Theorem 4.8. If $f: (X, \tau, E) \to (Y, \sigma, K)$ is Strongly Soft Jc homeomorphism then,

1. $SJcCl(f^{-1}(A, K)) = f^{-1}(SJcCl(A, K))$ for all $(A, K) \tilde{\in} (Y, \sigma, K)$.
2. $SJcCl(f(B, E)) = f(SJcCl(B, E))$ for all $(B, E) \tilde{\in} (X, \tau, E)$.
3. $SJc - int(f(C, E)) = f(SJc - int(C, E))$ for all $(C, E) \tilde{\in} (X, \tau, E)$.
4. $SJc - int(f^{-1}(D, K)) = f^{-1}(SJc - int(D, K))$ for all $(D, K) \tilde{\in} (Y, \sigma, K)$.

Proof:

1. Since f is Strongly Soft Jc homeomorphism, f and f^{-1} are Soft Jc irresolute. Then $SJcCl(f^{-1}(A,K)) \tilde{\subseteq} f^{-1}(SJcCl(A,K))$ and $f^{-1}(SJcCl(A,K)) \tilde{\subseteq} SJcCl(f^{-1}(A,K))$. Then the equality holds.
2. It is a corollary form of (1).
3. By combining (2) and the fact that $SJc - int(C,E) = SJcCl((C,E)^C)$ the proof follows.
4. The argument proceeds by combining the facts of (2) and (3).

Theorem 4.9. If $f:(X,\tau,E) \to (Y,\sigma,K)$ is Strongly Soft Jc homeomorphism then f induces an Soft isomorphism from the group $SSJcH(X,\tau,E)$ onto $SSJcH(X,\tau,E)$.
Proof: Using the map f, define another map $\theta: SSJcH(X,\tau,E) \to SSJcH(X,\tau,E)$ by $\theta(g) = f \circ g \circ f^{-1}$ for every $g \tilde{\in} SSJcH(X,\tau,E)$. Then by Theorem 4.6, θ is Strongly Soft Jc homeomorphism for every $g \tilde{\in} SSJcH(X,\tau,E)$.

Also for all $g_1, g_2 \in SSJcH(X,\tau,E)$,

$$\theta(g_1 \circ g_2) = f \circ (g_1 \circ g_2) \circ f^{-1} = (f \circ g_1 \circ f^{-1}) \circ (f \circ g_2 \circ f^{-1}) = \theta(g_1) \circ \theta(g_2).$$

Therefore, θ is bijective, then θ is a Soft isomorphism.

5. CONCLUSION

We study some extensions of soft topology, which are defined by reducing the stipulations of soft topology, for various purposes such as obtaining appropriate models to handle some real-life issues, or building some paradigms that demonstrate the relations among some topological notions and ideas on soft topology. To this end we have recently defined some strong functions namely Strongly soft Jc open maps and Strongly soft Jc homeomorphism.

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