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AN EFFICIENT FAMILY OF RATIO TYPE ESTIMATORS FOR SIMPLE RANDOM SAMPLING

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Abstract. *The study provides an enhanced estimation of the population mean using known information on an auxiliary variable. An enhanced class of estimators is suggested for the same. The proposed estimator's bias and mean squared error (MSE) are calculated up to the first order of approximation. The optimum values of the characterizing constants are obtained by minimizing the MSE of the proposed estimator. The minimum MSE and the bias values are achieved by optimising the characterizing scalar. The MSE of the proposed estimator has also been compared both conceptually and empirically with the MSEs of competing estimators. Real and simulated data sets are adopted to verify the theoretical prerequisites for the proposed estimator's greater efficiency over competing estimators. The most efficient estimator is recommended for practical utility in different areas of applications and the suggested estimator filfills the requirement.*

Keywords: *Main variable; auxiliary variable; bias; mean square error; simple random sampling.*

1. INTRODUCTION

Whenever there is a large population, it is natural and cost-effective to estimate population parameters using appropriate sampling techniques. Survey sampling is widely used in, demography, education, agriculture, business management, economics, engineering, industry, medical sciences, political science, social sciences, and a variety of other fields. The primary goal of sample survey theory is to draw conclusions based on unknown population parameters such as the total population, the proportion of the population, the mean of the population $\overline{Y}$and the variance of the population σ_y^2 etc. For example, to estimate the$\overline{Y}$, sample mean is the most appropriate estimator for the estimation of population mean. Although the sample mean is an unbiased estimator of population mean, it has a significant amount of sampling variation, hence, the estimator of the parameter under study which satisfies some properties like unbiasedness, minimum variance etc. is preferred. One of the most active research areas in survey sampling is improving the efficiency of ratio, product, and regression estimators in the presence of known auxiliary data when estimating unknown population parameters for the study variable using various sample techniques. The use of auxiliary variable fulfills the requirement of searching such estimators. Sometimes in sample surveys, information on auxiliary variable X which is correlated with the main variable Y is also collected. Auxiliary information is supplied by the auxiliary variable and is collected on some additional cost of the survey. Auxiliary variable is highly negatively or positively correlated with the main variable and by the use of the auxiliary variable; the efficiency of the estimator

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is improved. Using efficient estimator, we can reduce the sampling error presented in sampling technique. Sample being a part of population, leads to sampling error which cannot be eliminated but can be minimized by using efficient estimators.

Watson [1] proposed the usual regression estimator by using highly correlated X with Y. This estimator is a biased estimator and has lesser MSE than the sample mean estimator($\bar{y}$). Cochran [2] proposed the usual ratio estimator of $\bar{Y}$ by using a positively correlated X with Y. It is a biased estimator, but it has a lower MSE than $\bar{y}$ estimator. Goodman [3] modified the usual ratio estimators so that the obtained ratio-type estimator is unbiased for the simple random sampling scheme. Chakrabarty [4] developed some ratio estimators that are more efficient than the traditional ratio estimator for estimating$\bar{Y}$, up to the first order of approximation with optimum conditions. Sahai and Ray [5] presented two families of ratio-type and product-type estimators based on simple random samples for a finite$\bar{Y}$. Sisodia and Dwivedi [6] utilized the population coefficient of variation of study and auxiliary variables and suggested an estimator of $\bar{Y}$. Using positively and negatively correlated auxiliary variables, Bahl and Tuteja [7] proposed exponential ratio-type and product-type estimators for$\bar{Y}$. Upadhyaya and Singh [8] estimated the $\bar{Y}$ using a transformed X and proposed some ratio and product type estimators of $\bar{Y}$ by using coefficient of variation and the coefficient of kurtosis of X. Kadilar and Cingi [9] investigated the chain ratio-type estimator and obtained its MSE equation, demonstrating that under certain conditions, the chain ratio-type estimator is more efficient than the existing ratio estimator. Singh [10] used the product method of estimation, that includes simple linear transformations with known coefficients of X, to estimate a finite $\bar{Y}$. The transformations provided efficient product estimators with less absolute bias than conventional product and other product type estimators. Singh et al. [11] introduced a modified ratio estimator that improved the efficiency of the ratio estimator using the prior value of the coefficient of kurtosis of X. He obtained and compared first order large sample approximations to the bias and MSE of the proposed estimator and compared with the sample mean and usual ratio type estimator. They also introduced the generalized version of the introduced variable. Singh and Tailor [12], worked on the improved ratio-cum-product estimators of $\bar{Y}$. Kadilar and Cingi [13] adjusted the estimators in Upadhyaya and Singh [8] to the estimator in Singh and Tailor [14] and presented a class of ratio estimators for the estimate of $\bar{Y}$. Koshnevisan et al. [15] proposed a general family of estimators for estimating the $\bar{Y}$ using known values of particular population parameters. Al-Omari et al. [16] introduced modified ratio estimators of the $\bar{Y}$ involving the first or third quartiles of X correlated with Y. Yan and Tian [17] utilized the given skewness coefficient of X and developed certain ratio-type estimators for$\bar{Y}$. The proposed estimator is more efficient than the traditional ratio estimator under some conditions. Pandey et al. [18] developed the estimator utilizing known auxiliary parameters. Subramani and Kumarpandiyan [19] mentioned a class of modified ratio estimators for estimating $\bar{Y}$ using a linear combination of the auxiliary variable's Coefficient of Variation and Median. They have also determined the conditions under which the proposed estimators outperform the existing modified ratio estimators. Jeelani and Maqbool [20] proposed an improved ratio type estimator of $\bar{Y}$ relying on skewness coefficient and quartile difference of X. Swain [21] suggested an alternative ratio type exponential estimator and compared with Bahl and Tuteja's ratio type exponential estimator and classical ratio estimator as regards bias and MSE with large sample approximations. Jerajuddin and Kishun [22] defined a modified ratio estimator for estimating $\bar{Y}$ using the sample size, which was drawn from the population using SRSWOR. The bias, MSE, and percent relative efficiencies of existing and proposed estimators are computed and compared to justify the superiority of the proposed estimator over the discussed estimators. Soponviwatkul and Lawson [23] proposed new ratio estimators for estimating $\bar{Y}$ based on known X. To estimate$\bar{Y}$, they used a known population coefficient of variation of X, a correlation coefficient between Y and X, and a sample regression coefficient.The bias and MSE expressions for the proposed estimators up to the first order of approximation have been obtained. The performance of the proposed estimators is compared to existing estimators. Ijaz and Ali [24] presented an efficient estimator for estimating $\bar{Y}$ using SRS scheme. They developed a revised ratio estimator with the same efficiency as a regression estimator. On the other hand, it is a well-known fact that the linear regression estimator is more efficient than the majority of ratio estimators. They found the Bias and MSE up to the first order of approximation and explained the conditions under which the proposed estimators perform better than other estimators. Yadav et al. [25] investigated the improved estimation of peppermint average production at the block level in the Barabanki district of Uttar Pradesh State (India) and suggested certain estimators for$\bar{Y}$. As a main variable, population refers to the production population, and auxiliary-variable refers to the area of field. They investigated the sampling properties of estimators, such as bias and MSE, and compared the estimates to others in the literature. They conducted a numerical study for the natural population based on primary data collected from Banikodar Block of Barabanki District in Uttar Pradesh State to support the theoretical findings. Under SRS scheme, Yadav and Baghel [26] suggested a technique for estimating $\bar{Y}$ using information pertaining to auxiliary parameters. They introduced a new class of $\bar{Y}$ estimators, as well as the class's Bias and MSE, which are deduced up to the first order of approximation. Yadav and Baghel [27] advocated for improved $\bar{Y}$ estimation using a new class of estimators that use known information on X under simple random sampling. The members of the existing $\bar{Y}$ estimators have been shown. The objective of the study is to search for a more efficient estimator of $\bar{Y}$ than the existing competing estimators by using some known auxiliary parameters. The suggested class's bias and MSE are calculated up to the first approximation order. The notations and the formulae are presented in Tables 1 and 2, respectively.

Table 1. Notations.

N: Population Size	β_2: Coefficient of Kurtosis of X
n: Sample Size	$Q_{r(x)}$: Quartile range
f: Sampling Fraction	$Q_{1(x)}$: First Quartile of X
${}^{N}Cn$: All possible samples of size n	$Q_{3(x)}$: Third Quartile of X
Y: Study variable	$Q_{a(x)}$: Quartile Average
X: Auxiliary variable	QD: Quartile Deviation
M_y: Median of the Y	TM: Tri Mean
M_x:Median of the X	$Bias(\cdot)$:Bias of the estimator
$\bar{Y}$: Population mean of Y	$V(\cdot)$: Variance of the estimator
$\bar{X}$: Population mean of X	$MSE(\cdot)$:Mean squared error of the estimator
$\bar{y}$: Sample mean of Y	C_y:Coefficient of variation of y
$\bar{x}$:Sample mean of X	C_x:Coefficient of variation of x
ρ: Correlation coefficient between X and Y	${S_y}^2$: PopulationMean Square of y
β: Regression coefficient of Y on X	${S_x}^2$: PopulationMean Square of x
β_1: Coefficient of Skewness of X	S_{yx}: Covariance between X and Y

Table 2. Formulae.

$f = \frac{n}{N}$	${S_x}^2 = \frac{1}{N-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$
$\lambda = \frac{1-f}{n}$	$V(\bar{y}) = \lambda \bar{Y}^2 {C_y}^2$

$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$	$V(\bar{x}) = \lambda\bar{X}^2{C_x}^2$
$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$	$\rho = \frac{S_{yx}}{S_x S_y}$
$C_y = \frac{S_y}{\bar{Y}}$	$QD = \frac{Q_3 - Q_1}{2}$
$C_x = \frac{S_x}{\bar{X}}$	$Q_{a(x)} = \frac{Q_1 + M_x + Q_3}{3}$
$S_{yx} = \frac{1}{N-1}\sum_{i=1}^{n}(Y_i - \bar{Y})(X_i - \bar{X})$	$Q_{r(x)} = Q_3 - Q_1$
${S_y}^2 = \frac{1}{N-1}\sum_{i=1}^{n}(Y_i - \bar{Y})^2$	$TM = \frac{Q_1 + 2M_x + Q_3}{4}$

2. LITERATURE REVIEW OF EXISTING ESTIMATORS

To estimate the $\bar{Y}$, sample mean is very suitable estimator because it possesses almost characteristics what the whole population is having. Sample mean which is an unbiased estimator of$\bar{Y}$ is given as,

$$t_0 = \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

with the variance,

$$V(t_0) = \frac{1-f}{n}\bar{Y}^2{C_y}^2$$

Watson [1] described a technique for estimating the mean leaf area per leaf or per plant of a field crop. A large sampling is used to find the mean weight per leaf and on a small unit sample, the leaf area: leaf weight ratio and its regression on leaf weight are estimated. Alternative estimation methods based on the mean leaf weight and either unweights or weighted mean leaf area: leaf weight ratio is shown to provide positively biased estimates of mean leaf area. He suggested the estimator as,

$$t_1 = \hat{\bar{Y}}_{lr} = \bar{y} + \beta(\bar{X} - \bar{x})$$

having the MSE,

$$MSE(t_1) = \frac{1-f}{n}\bar{Y}^2{C_y}^2(1-\rho^2)$$

Cochran [2]was first to propose the classical ratio estimator of$\bar{Y}$ based on a positively correlated X and Y to enhance precision in estimating $\bar{Y}$. The classical ratio estimator is given by,

$$t_2 = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)$$

The above estimator is a biased estimator of $\bar{Y}$and its bias and MSE up to the first order of approximation are respectively,

$$Bias(t_2) = \frac{1-f}{n}\bar{Y}\left({C_x}^2 - \rho C_y C_x\right)$$

$$MSE(t_2) = \frac{1-f}{n}\bar{Y}^2\left(C_y{}^2 + C_x{}^2 - 2\rho C_y C_x\right)$$

Goodman [3] explained that when each unit in a population of N units has an x and y measurement and the population mean $\bar{X}$ of X is known, the $\bar{Y}$ is commonly estimated by taking a random sample of n units and using one of the standard biased ratio-type estimators of $\bar{Y}$, based on the ratio of $\bar{y}$ and $\bar{x}$ along with the known and auxiliary parameters. He suggested the following estimator:

$$t_3 = \bar{X}\bar{r}$$

The MSE of ratio-type estimators is,

$$MSE(t_3) = \frac{1-f}{n}\bar{Y}^2\left(C_y{}^2 + C_x{}^2 - 2\rho C_y C_x\right)$$

Chakrabarty [4] mentioned that the precision of the regression estimator is usually higher than that of the ratio estimator; the ratio estimator is widely used in large-scale sample surveys due to its simplicity. In this research, he developed some ratio-type estimators that are more efficient than the traditional ratio estimator. The developed estimator for estimating $\bar{Y}$ up to the first order of approximation and under optimum conditions is given as,

$$t_4 = (1-\alpha)\bar{y} + \alpha\bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)$$

The bias and MSE of the developed estimator are respectively,

$$Bias(t_4) = \frac{1-f}{n}\bar{Y}\left(\frac{\alpha}{2}C_x{}^2 - \alpha\rho C_y C_x\right)$$
$$MSE(t_4) = \frac{1-f}{n}\bar{Y}^2\left(C_y{}^2 + \alpha^2 C_x{}^2 - 2\alpha\rho C_y C_x\right)$$

where, $\alpha = \rho\frac{C_y}{C_x}$ which minimizes the $MSE(t_4)$.

Sahai and Ray [5] presented families of ratio-type for a finite $\bar{Y}$ based on simple random samples of observations on the variable of interest and an associated variable.

$$t_5 = \bar{y}\left\{2 - \left(\frac{\bar{x}}{\bar{X}}\right)^w\right\}$$

Using some prior knowledge, he has demonstrated that the families contain estimators with lower MSE in application than the standard ratio and sample mean estimators.

$$Bias(t_5) = \frac{1-f}{n}\bar{Y}\left(-\frac{\omega(1-\omega)}{2}C_x{}^2 - \omega\rho C_y C_x\right)$$
$$MSE(t_5) = \frac{1-f}{n}\bar{Y}^2\left(C_y{}^2 + \omega^2 C_x{}^2 - 2\omega\rho C_y C_x\right)$$

where, $\omega = \rho\frac{C_y}{C_x}$ which minimizes the $MSE(t_5)$.

Sisodia and Dwivedi [6] have proposed a modified ratio type estimator for $\bar{Y}$ by making use of coefficient of variation of X. The modified estimator is,

$$t_6 = \bar{y}\left(\frac{\bar{X} + C_x}{\bar{x} + C_x}\right)$$

The bias and MSE of the given modified estimator are respectively,

$$\mathrm{Bias}(t_6) = \frac{1-f}{n}\bar{Y}\left(R_6{}^2C_x{}^2 - R_6\rho C_yC_x\right)$$
$$\mathrm{MSE}(t_6) = \frac{1-f}{n}\bar{Y}^2\left(C_y{}^2 + R_6{}^2C_x{}^2 - 2R_6\rho C_yC_x\right)$$

where, $R_6 = \frac{\bar{X}}{\bar{X}+C_x}$.

Bahl and Tuteja [7] suggested a new ratio and product type estimator for estimating $\bar{Y}$ using a single X. These estimators are shown to be more efficient in practical situations when compared to traditional mean per unit, ratio, and product estimators. The suggested estimator is,

$$t_7 = \bar{y}\exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right)$$

The bias and mean square error of these estimators were obtained and given respectively,

$$\mathrm{Bias}(t_7) = \frac{1-f}{8n}\bar{Y}\left(3C_x{}^2 - 4\rho C_yC_x\right)$$
$$\mathrm{MSE}(t_7) = \frac{1-f}{n}\bar{Y}^2\left(C_y{}^2 + \frac{C_x{}^2}{4} - \rho C_yC_x\right)$$

Upadhyaya and Singh [8] defined a transformed X to estimate the finite $\bar{Y}$ and proposed the estimator of $\bar{Y}$ using coefficient of variation and coefficient of kurtosis of X as,

$$t_8 = \bar{y}\exp\left(\frac{\bar{X}\beta_2 - C_x}{\bar{x}\beta_2 + C_x}\right)$$

The bias and MSE of the proposed estimator have been obtained and are given respectively,

$$\mathrm{Bias}(t_8) = \frac{1-f}{n}\bar{Y}\left(R_8{}^2C_x{}^2 - R_8\rho C_yC_x\right)$$
$$\mathrm{MSE}(t_8) = \frac{1-f}{n}\bar{Y}^2\left(C_y{}^2 + R_8{}^2C_x{}^2 - 2R_8\rho C_yC_x\right)$$

where, $R_8 = \frac{\bar{X}\beta_2}{\bar{X}\beta_2 + C_x}$.

Kadilar and Cingi [9] proposed the chain ratio-type estimator and shown that, under certain conditions, the chain ratio-type estimator is more efficient than the traditional ratio estimator. The chain ratio-type estimator is given as,

$$t_9 = \bar{y}\left(\frac{\bar{X}^2}{\bar{x}^2}\right)$$

The calculated bias and MSE for the chain ratio-type estimator is as follows,

$$\mathrm{Bias}(t_9) = \frac{1-f}{n}\bar{Y}\left(3C_x^{\,2} - 2\rho C_y C_x\right)$$
$$\mathrm{MSE}(t_9) = \frac{1-f}{n}\bar{Y}^2\left(C_y^{\,2} + 4C_x^{\,2} - 4\rho C_y C_x\right)$$

Singh [10] considered a simple linear transformation of the known coefficient of variables for estimating a$\bar{Y}$ with the Ratio method of estimation.

$$t_{10} = \bar{y}\left(\frac{\bar{X}\beta_1 + S_x}{\bar{x}\beta_1 + S_x}\right)$$

The suggested transformations provide efficient ratio estimators with less absolute bias than traditional ratio and other ratio type estimators. The bias and MSE of the suggested estimator are given as,

$$\mathrm{Bias}(t_{10}) = \frac{1-f}{n}\bar{Y}\left(R_{10}^{\,2}C_x^{\,2} - R_{10}\rho C_y C_x\right)$$
$$\mathrm{MSE}(t_{10}) = \frac{1-f}{n}\bar{Y}^2\left(C_y^{\,2} + R_{10}^{\,2}C_x^{\,2} - 2R_{10}\rho C_y C_x\right)$$

where, $R_{10} = \frac{\bar{X}\beta_1}{\bar{X}\beta_2 + S_x}$.

Singh and Tailor [14] presented a modified ratio estimator using correlation coefficient to improve the efficiency of ratio estimator as,

$$t_{11} = \bar{y}\left(\frac{\bar{X} + \rho}{\bar{x} + \rho}\right)$$

The Bias and MSE of the above estimator respectively as,

$$\mathrm{Bias}(t_{11}) = \frac{1-f}{n}\bar{Y}\left(R_{11}^{\,2}C_x^{\,2} - R_{11}\rho C_y C_x\right)$$
$$\mathrm{MSE}(t_{11}) = \frac{1-f}{n}\bar{Y}^2\left(C_y^{\,2} + R_{11}^{\,2}C_x^{\,2} - 2R_{11}\rho C_y C_x\right)$$

where, $R_{11} = \frac{\bar{X}}{\bar{X}+\rho}$.

Singh et al. [11] proposed a modified ratio estimator that improved the efficiency of the ratio estimator by incorporating the prior value of the coefficient of kurtosis of X as,

$$t_{12} = \bar{y}\left(\frac{\bar{X} + \beta_2}{\bar{x} + \beta_2}\right)$$

The first order large sample approximations to the bias and MSE of the proposed estimator are obtained and given respectively as,

$$\mathrm{Bias}(t_{12}) = \frac{1-f}{n}\bar{Y}\left(R_{12}^{\,2}C_x^{\,2} - R_{12}\rho C_y C_x\right)$$
$$\mathrm{MSE}(t_{12}) = \frac{1-f}{n}\bar{Y}^2\left(C_y^{\,2} + R_{12}^{\,2}C_x^{\,2} - 2R_{12}\rho C_y C_x\right)$$

where, $R_{12} = \frac{\bar{X}}{\bar{X}+\beta_2}$.

Singh and Tailor [12] proposed the estimator of $\bar{Y}$using coefficient of variation of X as,

$$t_{13} = \bar{y}\left[\alpha\left(\frac{\bar{X} + C_x}{\bar{x} + C_x}\right) + (1 - \alpha)\left(\frac{\bar{x} + C_x}{\bar{X} + C_x}\right)\right]$$

The minimum MSE of the given estimator for $\alpha = \rho\frac{C_y}{C_x}$ is,

$$MSE(t_{13}) = \frac{1 - f}{n}\bar{Y}^2 {C_y}^2(1 - \rho^2)$$

Kadilar and Cingi [13] by adapting the estimators in Upadhyaya and Singh [8] and Singh and Tailor [14] proposed a class of ratio estimators for the estimate of $\bar{Y}$as,

$$t_{14} = \frac{\bar{y}}{\bar{x}C_x + \rho}(\bar{X}C_x + \rho)$$

The Bias and MSE of the estimator are calculated up to first order of approximation is given as,

$$Bias(t_{14}) = \frac{1 - f}{n}\bar{Y}\left({R_{14}}^2{C_x}^2 - R_{14}\rho C_y C_x\right)$$

$$MSE(t_{14}) = \frac{1 - f}{n}\bar{Y}^2\left[{C_y}^2 + {R_{14}}^2{C_x}^2 - 2R_{14}\rho C_y C_x\right]$$

where, $R_{14} = \frac{\bar{X}C_x}{\bar{X}C_x + \rho}$.

Khoshnevisan et al. [15] proposed a general family of estimators for estimating the $\bar{Y}$ using known values of particular auxiliary population parameters as,

$$t_{15} = \bar{y}\left[\frac{a\bar{X} + b}{\alpha'(a\bar{x} + b) + (1 - \alpha')(a\bar{X} + b)}\right]^g$$

Up to first order of approximation, bias and MSE expressions are generated,

$$Bias(t_{15}) = \frac{1 - f}{n}\bar{Y}\left[\frac{g(g + 1)}{2}\alpha'^2{R_{15}}^2{C_x}^2 - \alpha' R_{15} g\rho C_y C_x\right]$$

where, $R_{14} = \frac{a\bar{X}}{a\bar{X} + b}$.

The minimum MSE for optimum α' 'is,

$$MSE(t_{15}) = \frac{1 - f}{n}\bar{Y}^2 {C_y}^2(1 - \rho^2)$$

Al-Omari et al. [16] used the third quartiles of an auxiliary variable that is correlated with the variable of interest are suggested as modified ratio estimators of the $\bar{Y}$of the variable of interest,

$$t_{16} = \bar{y}\left(\frac{\bar{X} + Q_3}{\bar{x} + Q_3}\right)$$

The calculated Bias and MSE for the estimator are given as,

$$Bias(t_{16}) = \frac{1 - f}{n}\bar{Y}\left({R_{16}}^2{C_x}^2 - R_{16}\rho C_y C_x\right)$$

$$MSE(t_{16}) = \frac{1-f}{n}\bar{Y}^2\left(C_y{}^2 + R_{16}{}^2C_x{}^2 - 2R_{16}\rho C_yC_x\right)$$

where, $R_{16} = \frac{\bar{X}}{\bar{X}+Q_3}$.

Yan and Tian [17] used the known skewness coefficient of the auxiliary variable, and provided some ratio-type estimators for $\bar{Y}$

$$t_{17} = \bar{y}\left(\frac{\bar{X}+\beta_1}{\bar{x}+\beta_1}\right)$$
$$t_{18} = \bar{y}\left(\frac{\bar{X}\beta_1+\beta_2}{\bar{x}\beta_1+\beta_2}\right)$$
$$t_{19} = \bar{y}\left(\frac{\bar{X}C_x+\beta_1}{\bar{x}C_x+\beta_1}\right)$$
$$t_{20} = \bar{y}\left(\frac{\bar{X}\beta_2+\beta_1}{\bar{x}\beta_2+\beta_1}\right)$$

Theoretically, for all proposed ratio estimators, the expressions of MSE up to first order of approximation were produced, which are given as,

$$Bias(t_{17}) = \frac{1-f}{n}\bar{Y}\left(R_{17}{}^2C_x{}^2 - R_{17}\rho C_yC_x\right)$$
$$Bias(t_{18}) = \frac{1-f}{n}\bar{Y}\left(R_{18}{}^2C_x{}^2 - R_{18}\rho C_yC_x\right)$$
$$Bias(t_{19}) = \frac{1-f}{n}\bar{Y}\left(R_{19}{}^2C_x{}^2 - R_{19}\rho C_yC_x\right)$$
$$Bias(t_{20}) = \frac{1-f}{n}\bar{Y}\left(R_{20}{}^2C_x{}^2 - R_{20}\rho C_yC_x\right)$$
$$MSE(t_{17}) = \frac{1-f}{n}\bar{Y}^2\left(C_y{}^2 + R_{17}{}^2C_x{}^2 - 2R_{17}\rho C_yC_x\right)$$
$$MSE(t_{18}) = \frac{1-f}{n}\bar{Y}^2\left(C_y{}^2 + R_{18}{}^2C_x{}^2 - 2R_{18}\rho C_yC_x\right)$$
$$MSE(t_{19}) = \frac{1-f}{n}\bar{Y}^2\left(C_y{}^2 + R_{19}{}^2C_x{}^2 - 2R_{19}\rho C_yC_x\right)$$
$$MSE(t_{20}) = \frac{1-f}{n}\bar{Y}^2\left(C_y{}^2 + R_{20}{}^2C_x{}^2 - 2R_{20}\rho C_yC_x\right)$$

where, $R_{17} = \frac{\bar{X}}{\bar{X}+\beta_1}$, $R_{18} = \frac{\bar{X}\beta_1}{\bar{X}\beta_1+\beta_2}$, $R_{19} = \frac{\bar{X}C_x}{\bar{X}C_x+\beta_1}$, $R_{20} = \frac{\bar{X}\beta_2}{\bar{X}\beta_2+\beta_1}$.

Pandey et al. [18] developed the estimator utilizing auxiliary information, which is given as,

$$t_{21} = \bar{y}\frac{\bar{X}}{\bar{x}}\left(1 - \frac{k\bar{x}s^2}{n\bar{x}^3}\right)^{-1}.$$

The Bias and MSE are as follows,

$$Bias(t_{21}) = \frac{1-f}{n}\bar{Y}\left(kC_x{}^2 - \rho C_yC_x\right)$$
$$MSE(t_{21}) = \frac{1-f}{n}\bar{Y}^2\left(C_y{}^2 + k^2C_x{}^2 - 2k\rho C_yC_x\right)$$

where, $k = \rho\frac{C_y}{C_x}$.

Subramani and Kumarpandiyan [19] worked with a class of modified ratio estimators that use a linear combination of the known values of the Coefficient of Variation and the Median of X to estimate $\overline{Y}$ as,

$$t_{22} = \overline{y}\left(\frac{\overline{X}C_x + M_x}{\overline{x}C_x + M_x}\right)$$

The proposed estimator's bias and MSE are calculated, which are,

$$\mathrm{Bias}(t_{22}) = \frac{1-f}{n}\overline{Y}\left(R_{22}{}^2 C_x{}^2 - R_{22}\rho C_y C_x\right)$$
$$\mathrm{MSE}(t_{22}) = \frac{1-f}{n}\overline{Y}^2\left(C_y{}^2 + R_{22}{}^2 C_x{}^2 - 2R_{22}\rho C_y C_x\right)$$

where, $k = \frac{\overline{X}C_x}{\overline{X}C_x + M_x}$.

Jeelani and Maqbool [20] worked with the linear combination of known population values of coefficient of skewness and quartile deviation of X to estimate $\overline{Y}$ as,

$$t_{23} = \overline{y}\left(\frac{\overline{X}\beta_1 + QD}{\overline{x}\beta_1 + QD}\right)$$

Up to the first degree of approximation, bias and MSE are calculated as,

$$\mathrm{Bias}(t_{23}) = \frac{1-f}{n}\overline{Y}\left(R_{23}{}^2 C_x{}^2 - R_{23}\rho C_y C_x\right)$$
$$\mathrm{MSE}(t_{23}) = \frac{1-f}{n}\overline{Y}^2\left(C_y{}^2 + R_{23}{}^2 C_x{}^2 - 2R_{23}\rho C_y C_x\right)$$

where, $R_{23} = \frac{\overline{X}\beta_{1(x)}}{\overline{X}\beta_{1(x)} + QD}$.

Swain [21] proposed an alternative ratio estimator to the ratio type exponential estimator proposed by Bahl and Tuteja and the traditional ratio estimator as,

$$t_{24} = \overline{y}\left(\frac{\overline{X}}{\overline{x}}\right)^{1/2}$$

Both conceptually and numerically, bias and MSE with large sample approximations are found,

$$\mathrm{Bias}(t_{24}) = \frac{1-f}{8n}\overline{Y}\left(3C_x{}^2 - 4\rho C_y C_x\right)$$
$$\mathrm{MSE}(t_{24}) = \frac{1-f}{n}\overline{Y}^2\left(C_y{}^2 + \frac{C_x{}^2}{4} - \rho C_y C_x\right)$$

Jerajuddin and Kishun [22] presented a modified ratio estimator for estimating the $\overline{Y}$ using the sample size picked from the population under SRSWOR.

$$t_{25} = \overline{y}\left(\frac{\overline{X} + n}{\overline{x} + n}\right)$$

The estimator's bias and MSE are calculated up to the first order of approximation,

$$\mathrm{Bias}(t_{25}) = \frac{1-f}{n}\bar{Y}\left(R_{25}{}^2C_x{}^2 - R_{25}\rho C_y C_x\right)$$
$$\mathrm{MSE}(t_{25}) = \frac{1-f}{n}\bar{Y}^2\left(C_y{}^2 + R_{25}{}^2C_x{}^2 - 2R_{25}\rho C_y C_x\right)$$

where $R_{23} = \frac{\bar{X}}{\bar{X}+n}$.

Soponviwatkul and Lawson[23] utilized a known population coefficient of variation of X , the correlation coefficient between Y and X , and the sample regression coefficient ofY on Xto estimate the $\bar{Y}$ as,

$$t_{26} = \bar{y}\left(\frac{\bar{X}+C_x}{\bar{x}+C_x}\right)^{b_1}$$
$$t_{27} = \bar{y}\left(\frac{\bar{X}+\rho}{\bar{x}+\rho}\right)^{b_1}$$

Up to the first order of approximation, equations for the bias and MSE of the proposed estimators have been obtained as,

$$\mathrm{Bias}(t_{26}) = \frac{1-f}{n}\bar{Y}\left(\frac{b_1(b_1+1)}{2}R_5{}^2C_x{}^2 - b_1R_5\rho C_y C_x\right)$$
$$\mathrm{Bias}(t_{27}) = \frac{1-f}{n}\bar{Y}\left(\frac{b_1(b_1+1)}{2}R_{11}{}^2C_x{}^2 - b_1R_{11}\rho C_y C_x\right)$$
$$\mathrm{MSE}(t_{26}) = \frac{1-f}{n}\bar{Y}^2C_y{}^2(1-\rho^2)$$
$$\mathrm{MSE}(t_{27}) = \frac{1-f}{n}\bar{Y}^2C_y{}^2(1-\rho^2)$$

Ijaz and Ali [24] suggested an effective estimator for estimating$\bar{Y}$. A modified ratio estimator with the same efficiency as a regression estimator has been developed. It is a well-known fact that the linear regression estimator beats the majority of ratio estimators, the estimator is,

$$t_{28} = \omega_1\bar{y} + (1-\omega_1)\bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)$$
$$t_{29} = \omega_2\bar{y} + (1-\omega_2)\left(\bar{y}\exp\frac{\bar{X}-\bar{x}}{\bar{x}-\bar{X}}\right)$$

Up to first order of approximation, the Bias and minimum MSE have been determined,

$$\mathrm{Bias}(t_{28}) = \frac{1-f}{n}\bar{Y}\rho C_y(C_x-\rho)$$
$$\mathrm{Bias}(t_{29}) = \frac{1-f}{n}\bar{Y}\rho C_y\left(\frac{1}{4}C_x-\rho C_y\right)$$
$$\mathrm{MSE}(t_{28}) = \mathrm{MSE}(t_{29}) = \frac{1-f}{n}\bar{Y}^2C_y{}^2(1-\rho^2)$$

Yadav et al. [25] considered the enhanced estimator of$\bar{Y}$ using known auxiliary parameters as,

$$t_{30} = \bar{y}\left(\frac{ab\bar{X} + cd}{ab\bar{x} + cd}\right)$$

The Bias and MSE of the estimator is examined and calculated as,

$$B(t_{30}) = \frac{1-f}{n}\bar{Y}\left[\theta^2 C_x^2 - \theta C_{yx}\right]$$
$$MSE(t_{30}) = \frac{1-f}{n}\bar{Y}^2\left({C_y}^2 - \frac{{C_{yx}}^2}{{C_x}^2}\right)$$

where $\theta = \frac{ab\bar{\bar{X}}}{ab\bar{X}+cd}$.

Yadav and Baghel [26] in instances where a Simple Random Sampling Scheme is used, in their work, they proposes a method for improving $\bar{Y}$ estimation by utilizing information from an X and suggested a new class of estimators of $\bar{Y}$ as,

$$t_{31} = \alpha\bar{y} + (1-\alpha)\bar{y}\left[\frac{ab\bar{X} + cd}{ab\bar{x} + cd}\right]$$

The Bias and MSE are calculated up to the first order of approximation. For the optimum value of the characterizing scaler, the least value of the MSE for the specified class of estimators is also calculated as,

$$B(t_{31}) = \theta^2 C_x^2 - \bar{Y}\left[\alpha\left(\theta C_{yx} - \theta^2 C_x^2\right) + \left(\theta^2 C_x^2 - \theta C_{yx}\right)\right]$$
$$MSE(t_{31}) = \frac{1-f}{n}\bar{Y}^2\left({C_y}^2 - \frac{{C_{yx}}^2}{{C_x}^2}\right)$$

where $\theta = \frac{ab\bar{\bar{X}}}{ab\bar{X}+cd}$.

Yadav and Baghel [27] advocated for improved $\bar{Y}$ estimation using a new class of estimators that use known information on an X under the SRS Scheme as,

$$t_{32} = k\bar{y}\left[\frac{ab\bar{X} + cd}{ab\bar{x} + cd}\right]$$

The suggested class's Bias and MSE are calculated up to the first order of approximation. By optimizing the characterizing scalar, the values of Bias and the minimal MSE can be achieved as,

$$Bias(t_{32}) = \bar{Y}\left(\frac{A^2}{B} - 1\right)$$
$$MSE(t_{32}) = \bar{Y}^2\left(1 - \frac{A^2}{B}\right)$$

where, $A = 1 - \theta\lambda C_{yx} + \theta^2\lambda {C_x}^2$ and$B = 1 + \lambda {C_x}^2 + 3\theta^2\lambda {C_x}^2 - 4\theta\lambda C_{yx}$.

3. PROPOSED CLASS OF ESTIMATOR

Inspired by the literature of improved estimators, we suggest an improved class of ratio type estimators for the estimation $\overline{Y}$ using the known auxiliary parameters as,

$$t_p = k_1\bar{y} + k_2\bar{y}\left[\frac{ab\overline{X} + cd}{ab\bar{x} + cd}\right]; \quad k_1+k_2 \neq 1 \tag{1}$$

where, k_1 and k_2 are characterizing constant and a, b, c and d are either constants or the known parameter of the auxiliary variable. The values of k_1 and k_2 are chosen such that the MSE of the suggested estimator is minimum. Some special cases.

Case 1. If $k_1 = 0$ and $k_2 = 1$, then MSE of the proposed estimator is same as that of [25].

Case 2. If $k_1 = 1$ and $k_2 = 0$, then MSE of the proposed estimator is same as that of mean per unit estimator.

Case 3. If $k_1 = 0$, then MSE of the proposed estimator is same as that of [27].

Case 4. If $k_2 = 0$, then MSE of the proposed estimator is same as that of [28].

Case 5. If $k_1 + k_2 = 1$, then MSE of the proposed estimator is same as that of [26].

3.1. SOME MEMBERS OF THE PROPOSED FAMILY OF ESTIMATORS

- $t_{p(1)} = k_1\bar{y} + k_2\bar{y}\left[\frac{n\overline{X}+\rho C_x}{n\bar{x}+\rho C_x}\right]$
- $t_{p(2)} = k_1\bar{y} + k_2\bar{y}\left[\frac{\beta_{1(x)}M_x\overline{X}+\rho}{\beta_{1(x)}M_x\bar{x}+\rho}\right]$
- $t_{p(3)} = k_1\bar{y} + k_2\bar{y}\left[\frac{\beta_{1(x)}M_x\overline{X}+\rho C_x}{\beta_{1(x)}M_x\bar{x}+\rho C_x}\right]$
- $t_{p(4)} = k_1\bar{y} + k_2\bar{y}\left[\frac{\beta_{1(x)}M_x\overline{X}+\rho C_{yx}}{n\beta_{1(x)}M_x+\rho C_{yx}}\right]$
- $t_{p(5)} = k_1\bar{y} + k_2\bar{y}\left[\frac{\beta_{2(x)}M_x\overline{X}+\rho C_x}{\beta_{2(x)}M_x\bar{x}+\rho C_x}\right]$

3.2. BIAS AND MSE OF THE PROPOSED ESTIMATOR

The following approximations are used to obtain the Bias and MSE of the suggested estimator,

$$\bar{y} = \overline{Y}(1 + e_0)$$

and

$$\bar{x} = \overline{X}(1 + e_1)$$

such that

$$E(e_0) = E(e_1) = 0$$

and

$$E({e_0}^2) = \lambda {C_y}^2,\ E({e_1}^2) = \lambda {C_x}^2,\ E(e_0 e_1) = \lambda C_{yx}$$

Now, expressing the t_p in terms ofe_0 and e_1, we have,

$$t_p = k_1\overline{Y}(1+e_0) + k_2\overline{Y}(1+\ e_0)\left[\frac{ab\overline{X}+cd}{ab\overline{X}(1+e_1)+cd}\right]$$

Simplifying the above equation, we get,

$$t_p = k_1\overline{Y}(1+e_0) + k_2\overline{Y}(1+\ e_0)\left[\frac{ab\overline{X}+cd}{ab\overline{X}+ab\overline{X}e_1+cd}\right]$$
$$t_p = k_1\overline{Y}(1+e_0) + k_2\overline{Y}(1+\ e_0)\left[\frac{1}{1+\frac{ab\overline{X}e_1}{ab\overline{X}+cd}}\right] \tag{2}$$
$$t_p = k_1\overline{Y}(1+e_0) + k_2\overline{Y}(1+\ e_0)(1+\theta e_1)^{-1}$$

where $\theta = \frac{ab\overline{X}}{ab\overline{X}+cd}$.

Expanding the equation (2), simplifying, and retaining the terms up to the first order of approximation, we get,

$$\begin{aligned} t_p &= k_1\overline{Y}(1+e_0) + k_2\overline{Y}(1+\ e_0)(1-\ \theta e_1+\ \theta^2{e_1}^2) \\ t_p &= k_1\overline{Y}(1+e_0) + k_2\overline{Y}(1-\ \theta e_1+\ \theta^2{e_1}^2+e_0+\theta e_0 e_1) \\ t_p &= \overline{Y}[k_1(1+e_0)+k_2(1-\ \theta e_1+\ \theta^2{e_1}^2+e_0+\theta e_0 e_1)] \\ t_p-\overline{Y} &= \overline{Y}[k_1(1+e_0)+k_2(1-\ \theta e_1+\ \theta^2{e_1}^2+e_0+\theta e_0 e_1)-1] \end{aligned} \tag{3}$$

Taking expectation on the both sides of the equation (3),

$$\begin{aligned} E(t_p-\overline{Y}) &= \overline{Y}[k_1(1+E(e_0)) \\ &\quad + k_2(1-\ \theta E(e_1)+\ \theta^2 E({e_1}^2)+E(e_0)-\theta E(e_0e_1)-1] \\ \mathrm{Bias}(t_p) &= \overline{Y}[k_1+k_2(1+\ \theta^2\lambda{C_x}^2-\theta\lambda C_{yx})-1] \end{aligned} \tag{4}$$

Squaring the equation (3), taking the expectation, and retaining the terms up to the first order of approximation, we get,

$$E(t_p-\overline{Y})^2 = \overline{Y}^2\, E[k_1(1+e_0)+k_2(1-\ \theta e_1+\ \theta^2{e_1}^2+e_0-\theta e_0 e_1)-1]^2$$

On expanding the above equation, we get,

$$\begin{aligned} E(t_p-\overline{Y})^2 &= \overline{Y}^2\, E[{k_1}^2(1+e_0)^2+{k_2}^2(1-\ \theta e_1+\ \theta^2{e_1}^2+e_0-\theta e_0 e_1)^2+1 \\ &\quad + 2\{k_1k_2(1+e_0)(1-\theta e_1+\theta^2{e_1}^2+e_0-\theta e_0 e_1) \\ &\quad - k_2(1-\theta e_1+\theta^2{e_1}^2+e_0-\ \theta e_0 e_1)+k_1(1+e_0)\}] \end{aligned}$$

$$\begin{aligned} E(t_p-\overline{Y})^2 &= \overline{Y}^2\, E[{k_1}^2(1+e_0)^2+{k_2}^2\{1+\theta^2{e_1}^2+{e_0}^2+2(-\theta e_1+\theta^2{e_1}^2+e_0 \\ &\quad -\ \theta e_0 e_1-\ \theta e_0 e_1)\}+1 \\ &\quad + 2\{k_1k_2(1-\theta e_1+\theta^2{e_1}^2+e_0-\theta e_0 e_1+e_0-\ \theta e_0 e_1+\ {e_0}^2) \\ &\quad -\ k_2(1-\theta e_1+\theta^2{e_1}^2+e_0-\theta e_0 e_1)+k_1(1+e_0)\}] \end{aligned} \tag{5}$$

$$E\left(t_p-\overline{Y}\right)^2=\overline{Y}^2\Big[k_1{}^2\left(1+\lambda C_y{}^2\right)+k_2{}^2\left(1+\theta^2\lambda C_x{}^2+\lambda C_y{}^2+2\left(\theta^2\lambda C_x{}^2-2\theta\lambda C_{yx}\right)\right)$$
$$+\ 2\left\{k_1k_2\left(1+\theta^2\lambda C_x{}^2-\theta\lambda C_{yx}-\theta\lambda C_{yx}+\lambda C_y{}^2\right)\right.$$
$$\left.-k_2\left(1+\theta^2\lambda C_x{}^2+\theta\lambda C_{yx}\right)-\ 2k_1\right\}\Big]$$

$$MSE\left(t_p\right)=\overline{Y}^2\Big[k_1{}^2\left(1+\lambda C_y{}^2\right)+k_2{}^2\left(1+3\theta^2\lambda C_x{}^2+\lambda C_y{}^2-4\theta\lambda C_{yx}\right)+1$$
$$+2k_1k_2\left(1+\ \theta^2\lambda C_x{}^2+\lambda C_y{}^2-2\theta\lambda C_{yx}\right)-2k_2\left(1+\theta^2\lambda C_x{}^2-\theta\lambda C_{yx}\right)$$
$$-2k_1\Big]$$

Let,

$A=1+\lambda C_y{}^2$

$B=1+3\theta^2\lambda C_x{}^2+\lambda C_y{}^2-4\theta\lambda C_{yx}$

$C=1+\theta^2\lambda C_x{}^2+\lambda C_y{}^2-2\theta\lambda C_{yx}$

$D=1+\theta^2\lambda C_x{}^2-\theta\lambda C_{yx}$

Rewriting the equation (5) in the terms of A, B, C and D, we get,

$$MSE\left(t_p\right)=\overline{Y}^2\left[Ak_1{}^2+Bk_2{}^2+2Ck_1k_2-2k_1-2Dk_2+1\right] \qquad (6)$$

The optimum value of k_1 and k_2 are obtained by,

$$\frac{\partial MSE\left(t_p\right)}{\partial k_1}=0 \text{ and } \frac{\partial MSE\left(t_p\right)}{\partial k_1}=0$$

which gives,

$$k_{1(opt)}=\frac{CD-B}{C^2-AB} \text{ and } k_{2(opt)}=\frac{C-AD}{C^2-AB}$$

The minimum MSE for the optimum value of k_1 and k_2 is given by,

$$MSE(t_p)_{min}=\overline{Y}^2\left[1+A\left(\frac{CD-B}{C^2-AB}\right)^2+B\left(\frac{C-AD}{C^2-AB}\right)^2\right.$$
$$\left.+2\left(\frac{CD-B}{C^2-AB}\right)\left(\frac{C-AD}{C^2-AB}\right)-2\left(\frac{CD-B}{C^2-AB}\right)-\ 2D\left(\frac{C-AD}{C^2-AB}\right)\right]$$
$$=\overline{Y}^2\left[1+\frac{\begin{array}{c}[A(CD-B)^2+B(C-AD)^2+2C(CD-B)(C-AD)\\ -2(C^2-AB)(CD-B)-2D(C^2-AB)(C-AD)]\end{array}}{(C^2-AB)^2}\right] \qquad (7)$$

$$MSE(t_p)_{min}=\overline{Y}^2\left[1+\frac{P}{Q^2}\right]$$

where,

$$P=[A(CD-B)^2+B(C-AD)^2+2C(CD-B)(C-AD)-2(C^2-AB)(CD-B)$$
$$-\ 2D(C^2-\ AB)(C-AD)]$$

$Q=(C^2-AB)$

4. COMPARISONS OF EFFICIENCIES

The proposed class of estimators is compared to existing ratio-type estimators of $\overline{\mathrm{Y}}$ in this section and the conditions under which it outperforms existing estimators are listed in Table 3. The following criterion must be met for the proposed class to be more efficient than existing estimators:

$$\mathrm{MSE}(\mathrm{t_i}) - \mathrm{MSE}(\mathrm{t_p}) > 0; i = 0,1,\dots,32$$

Table 3. Efficiency comparison.

Estimator	**Efficiency Condition**
$\mathbf{t_0}$Mean per unit estimator	$\lambda C_y{}^2 - \left(1+\frac{P}{Q^2}\right) > 0$
$\mathbf{t_1}$Watson [1] estimator	$\lambda C_y{}^2(1-\rho^2) - \left(1+\frac{P}{Q^2}\right) > 0$
$\mathbf{t_2}$Cochran [2] estimator	$\lambda[C_y{}^2 + C_x{}^2 - 2\rho C_y C_x] - \left(1+\frac{P}{Q^2}\right) > 0$
$\mathbf{t_3}$Goodman [3] estimator	$\lambda[C_y{}^2 + C_x{}^2 - 2\rho C_y C_x] - \left(1+\frac{P}{Q^2}\right) > 0$
$\mathbf{t_4}$Chakrabarty [4] estimator	$\lambda\left[C_y{}^2 + \alpha^2 C_x{}^2 - 2\alpha\rho C_y C_x\right] - \left(1+\frac{P}{Q^2}\right) > 0$
$\mathbf{t_5}$Sahai and Ray [5] estimator	$\lambda\left[C_y{}^2 + \omega^2 C_x{}^2 - 2\omega\rho C_y C_x\right] - \left(1+\frac{P}{Q^2}\right) > 0$
$\mathbf{t_6}$Sisodia [6]estimator	$\lambda\left[C_y{}^2 + R_6{}^2 C_x{}^2 - 2R_6\rho C_y C_x\right] - \left(1+\frac{P}{Q^2}\right) > 0$
$\mathbf{t_7}$Bahl and Tuteja [7] estimator	$\lambda\left[C_y{}^2 + \frac{C_x{}^2}{4} - \rho C_y C_x\right] - \left(1+\frac{P}{Q^2}\right) > 0$
$\mathbf{t_8}$Upadhyaya and Singh [8] estimator	$\lambda\left[C_y{}^2 + R_8{}^2 C_x{}^2 - 2R_8\rho C_y C_x\right] - \left(1+\frac{P}{Q^2}\right) > 0$
$\mathbf{t_9}$Kadilar and Cingi [9] estimator	$\lambda[C_y{}^2 + 4C_x{}^2 - 4\rho C_y C_x] - \left(1+\frac{P}{Q^2}\right) > 0$
$\mathbf{t_{10}}$Singh [10] estimator	$\lambda\left[C_y{}^2 + R_{10}{}^2 C_x{}^2 - 2R_{10}\rho C_y C_x\right] - \left(1+\frac{P}{Q^2}\right) > 0$
$\mathbf{t_{11}}$Singh and Tailor [14] estimator	$\lambda\left[C_y{}^2 + R_{11}{}^2 C_x{}^2 - 2R_{11}\rho C_y C_x\right] - \left(1+\frac{P}{Q^2}\right) > 0$
$\mathbf{t_{12}}$Singh et al. [11] estimator	$\lambda\left[C_y{}^2 + R_{12}{}^2 C_x{}^2 - 2R_{12}\rho C_y C_x\right] - \left(1+\frac{P}{Q^2}\right) > 0$
$\mathbf{t_{13}}$Singh and Tailor [12] estimator	$\lambda C_y{}^2(1-\rho^2) - \left(1+\frac{P}{Q^2}\right) > 0$
$\mathbf{t_{14}}$Kadilar and Cingi [13] estimator	$\lambda\left[C_y{}^2 + R_{14}{}^2 C_x{}^2 - 2R_{14}\rho C_y C_x\right] - \left(1+\frac{P}{Q^2}\right) > 0$
$\mathbf{t_{15}}$Khoshnevisan et al.[15] estimator	$\lambda C_y{}^2(1-\rho^2) - \left(1+\frac{P}{Q^2}\right) > 0$
$\mathbf{t_{16}}$Al-Omari et al. [16] estimator	$\lambda\left[C_y{}^2 + R_{16}{}^2 C_x{}^2 - 2R_{16}\rho C_y C_x\right] - \left(1+\frac{P}{Q^2}\right) > 0$
$\mathbf{t_{17}}$Yan and Tian [17] estimator	$\lambda\left[C_y{}^2 + R_{17}{}^2 C_x{}^2 - 2R_{17}\rho C_y C_x\right] - \left(1+\frac{P}{Q^2}\right) > 0$
$\mathbf{t_{18}}$Yan and Tian [17] estimator	$\lambda\left[C_y{}^2 + R_{18}{}^2 C_x{}^2 - 2R_{18}\rho C_y C_x\right] - \left(1+\frac{P}{Q^2}\right) > 0$
$\mathbf{t_{19}}$Yan and Tian [17]) estimator	$\lambda\left[C_y{}^2 + R_{19}{}^2 C_x{}^2 - 2R_{19}\rho C_y C_x\right] - \left(1+\frac{P}{Q^2}\right) > 0$
$\mathbf{t_{20}}$Yan and Tian [17] estimator	$\lambda\left[C_y{}^2 + R_{20}{}^2 C_x{}^2 - 2R_{20}\rho C_y C_x\right] - \left(1+\frac{P}{Q^2}\right) > 0$
$\mathbf{t_{21}}$Pandey et al. [18] estimator	$\lambda\left[C_y{}^2 + k^2 C_x{}^2 - 2k\rho C_y C_x\right] - \left(1+\frac{P}{Q^2}\right) > 0$
$\mathbf{t_{22}}$Subramani and Kumarpandiyan [19] estimator	$\lambda\left[C_y{}^2 + R_{22}{}^2 C_x{}^2 - 2R_{22}\rho C_y C_x\right] - \left(1+\frac{P}{Q^2}\right) > 0$

Estimator	Efficiency Condition
$\mathbf{t_{23}}$Jeelani et al. [20] estimator	$\lambda\left[C_y{}^2+R_{23}{}^2C_x{}^2-2R_{23}\rho C_yC_x\right]-\left(1+\frac{P}{Q^2}\right)>0$
$\mathbf{t_{24}}$Swain [21] estimator	$\lambda\left[C_y{}^2+\frac{C_x{}^2}{4}-\rho C_yC_x\right]-\left(1+\frac{P}{Q^2}\right)>0$
$\mathbf{t_{25}}$Jerajuddin and Kishun [22] estimator	$\lambda\left[C_y{}^2+R_{25}{}^2C_x{}^2-2R_{25}\rho C_yC_x\right]-\left(1+\frac{P}{Q^2}\right)>0$
$\mathbf{t_{26}}$Soponviwatkul and Lawson [23] estimator	$\lambda C_y{}^2(1-\rho^2)-\left(1+\frac{P}{Q^2}\right)>0$
$\mathbf{t_{27}}$Soponviwatkul and Lawson [23] estimator	$\lambda C_y{}^2(1-\rho^2)-\left(1+\frac{P}{Q^2}\right)>0$
$\mathbf{t_{28}}$Ijaz and Ali [24] estimator	$\lambda C_y{}^2(1-\rho^2)-\left(1+\frac{P}{Q^2}\right)>0$
$\mathbf{t_{29}}$Ijaz and Ali [24] estimator	$\lambda C_y{}^2(1-\rho^2)-\left(1+\frac{P}{Q^2}\right)>0$
$\mathbf{t_{30}}$Yadav et al. [25] estimator	$\lambda\left[C_y{}^2-\frac{C_{yx}{}^2}{C_x{}^2}\right]-\left(1+\frac{P}{Q^2}\right)>0$
$\mathbf{t_{31}}$Yadav and Baghel [26] estimator	$\lambda\left[C_y{}^2-\frac{C_{yx}{}^2}{C_x{}^2}\right]-\left(1+\frac{P}{Q^2}\right)>0$
$\mathbf{t_{32}}$Yadav and Baghel [27] estimator	$\left(1-\frac{A^2}{B}\right)-\left(1+\frac{P}{Q^2}\right)>0$

5. COMPUTATIONAL STUDY

We conducted a detailed computational analysis under two headings to demonstrate the advantages of our propositions: numerical study using real data sets and simulation study using artificially produced data sets.

5.1. NUMERICAL STUDY

We conducted a numerical analysis with two different real populations, which are described below:

1. Data Source: Singh and Chaudhary [29]

Data Details: Study Variable: Area under wheat in a region during year 1974

Auxiliary Variable: Cultivated Area under wheat in a region during year 1973

Table 4. Parameters of the first real population

Parameter	Data Set	Parameter	Data Set
N	34	C_{yx}	0.5318817
n	5	$\beta_{1(x)}$	0.8732281
$\bar{\mathbf{Y}}$	199.4412	$\beta_{2(x)}$	5.912272
$\bar{\mathbf{X}}$	208.8824	f	0.1470588
$\mathbf{S_y}$	150.215	λ	0.1705882
$\mathbf{S_x}$	150.506	$Q_{1(x)}$	94.25
$\mathbf{C_y}$	0.7531797	$Q_{3(x)}$	275.75
$\mathbf{C_x}$	0.7205298	$Q_{r(x)}$	160.5
$\mathbf{M_y}$	142.5	$Q_{a(x)}$	1666.3333
$\mathbf{M_x}$	150	QD	80.25
$\boldsymbol{\rho}$	0.9800867	TM	162.25

The Percent Relative Efficiency (PRE) for different estimators with respect to proposed estimator is as follows:

$$PRE = \frac{MSE(t_i)}{MSE(t_{p(1)})} \times 100$$

Table 5. Comparison of ratio estimators of for real population-I

Estimator	MSE	PRE	Estimator	MSE	PRE
t_0	3849.248	2537.823	t_{19}	155.0032	102.1942
t_1	151.7761	100.0665	t_{20}	154.0141	101.542
t_2	153.8903	101.4604	t_{21}	151.7761	100.0665
t_3	153.8903	101.4604	t_{22}	1117.772	736.9511
t_4	151.7761	100.0665	t_{23}	535.4869	353.0484
t_5	151.7761	100.0665	t_{24}	1120.881	739.0008
t_6	154.5253	101.8791	t_{25}	159.8505	105.3900
t_7	1120.881	739.0008	t_{26}	151.7761	100.0665
t_8	153.9922	101.5276	t_{27}	151.7761	100.0665
t_9	3504.046	2310.230	t_{28}	151.7761	100.0665
t_{10}	951.9383	627.6163	t_{29}	151.7761	100.0665
t_{11}	154.7732	102.0425	t_{30}	151.7762	100.0666
t_{12}	161.3102	106.3524	t_{31}	151.7762	100.0666
t_{13}	151.7761	100.0665	t_{32}	152.776	100.7257
t_{14}	155.1545	102.2939	$t_{p(1)}$	151.6828	100.005
t_{15}	151.7761	100.0665	$t_{p(2)}$	151.6756	100.0003
t_{16}	1392.582	918.1343	$t_{p(3)}$	151.6755	100.0002
t_{17}	154.6699	101.9744	$t_{p(4)}$	151.6754	100.0001
t_{18}	162.7817	107.3226	$t_{p(5)}$	151.6752	100.0000

2. Data Source: Yadav et al. [25]

Data Details: Study Variable: The production (Yield) of peppermint oil in kilogram Auxiliary Variable: The area of the field in Bigha (2529.3 Square Meter)

Table 6. Parameters of the second real population

Parameter	Data Set	Parameter	Data Set
N	150	C_{yx}	0.508802
n	40	$\beta_{1(x)}$	2.801407
$\bar{Y}$	33.462	$\beta_{2(x)}$	16.44023
$\bar{X}$	4.204667	f	0.2666667
S_y	25.50316	λ	0.018333
S_x	3.080385	$Q_{1(x)}$	2
C_y	0.762153	$Q_{3(x)}$	5
C_x	0.732611	$Q_{r(x)}$	3
M_y	25	$Q_{a(x)}$	3.5
M_x	3	QD	1.5
ρ	0.911241	TM	3.25

Table 7. Comparisons of ratio estimators of population mean for real population-II

Estimator	MSE	PRE	Estimator	MSE	PRE
t_0	11.92421	591.1532	t_{19}	4.006074	198.6046
t_1	2.022821	100.2831	t_{20}	2.024702	100.3764
t_2	2.052629	101.7609	t_{21}	2.022821	100.2831
t_3	2.052629	101.7609	t_{22}	4.169214	206.6924
t_4	2.022821	100.2831	t_{23}	2.063746	102.312
t_5	2.022821	100.2831	t_{24}	4.233986	209.9036
t_6	2.125144	105.3559	t_{25}	10.03694	497.5901

Estimator	MSE	PRE	Estimator	MSE	PRE
$\mathbf{t_7}$	4.143891	205.437	$\mathbf{t_{26}}$	2.022821	100.2831
$\mathbf{t_8}$	2.041821	101.2251	$\mathbf{t_{27}}$	2.022821	100.2831
$\mathbf{t_9}$	14.21651	704.796	$\mathbf{t_{28}}$	2.022821	100.2831
$\mathbf{t_{10}}$	2.288509	113.4548	$\mathbf{t_{29}}$	2.022821	100.2831
$\mathbf{t_{11}}$	2.198031	108.9693	$\mathbf{t_{30}}$	2.022823	100.2832
$\mathbf{t_{12}}$	8.126777	402.8921	$\mathbf{t_{31}}$	2.022823	100.2832
$\mathbf{t_{13}}$	2.022821	100.2831	$\mathbf{t_{32}}$	2.019175	100.1024
$\mathbf{t_{14}}$	2.365171	117.2554	$\mathbf{t_{p(1)}}$	2.017145	100.0017
$\mathbf{t_{15}}$	2.022821	100.2831	$\mathbf{t_{p(2)}}$	2.018095	100.0488
$\mathbf{t_{16}}$	2.022821	100.2831	$\mathbf{t_{p(3)}}$	2.017808	100.0346
$\mathbf{t_{17}}$	3.3555886	166.3563	$\mathbf{t_{p(4)}}$	2.017559	100.0222
$\mathbf{t_{18}}$	5.124432	254.0482	$\mathbf{t_{p(5)}}$	2.017110	100.0000

5.2. SIMULATION STUDY

The computational procedure for comparing estimators is described in this section. For the comparison of the ratio estimators, a bivariate population with a specified correlation between the study and auxiliary variables is required. Srinivas et al. [30] described how to generate such correlated populations using a simple procedure.We have generated two symmetric populations namely Normal, Uniform and two asymmetric populations namely Beta-I, Log-Normal of size N=10000 units each using the model $\mathrm{Y} = \rho\mathrm{X} + \sqrt{1-\rho^2}\,\mathrm{X}^*;0 < \rho = 0.6, 0.7, 0.8, 0.9 < 1$ where, X and X^* are independent variates of respective parent distribution.

Table 8. Comparison of ratio estimators of population mean for Simulated Normal Population

Estimator	ρ			
	0.6	**0.7**	**0.8**	**0.9**
t_0	0.2770852	0.2512603	0.2213679	0.18734630
t_1	0.2224999	0.1773046	0.1251562	0.06605467
t_2	0.3898414	0.3106549	0.2192858	0.11573420
t_3	0.3898414	0.3106549	0.2192858	0.11573420
t_4	0.2224999	0.1773046	0.1251562	0.06605467
t_5	0.2224999	0.1773046	0.1251562	0.06605467
t_6	0.3894628	0.3103199	0.2190133	0.11555090
t_7	0.2301947	0.1794773	0.1251591	0.06998474
t_8	0.3897779	0.3105986	0.2192400	0.11570340
t_9	1.3287470	1.1818920	0.9785459	0.69656600
t_{10}	0.2769206	0.2510703	0.2211580	0.18712810
t_{11}	0.3882926	0.3089821	0.2176353	0.11438400
t_{12}	0.3703253	0.2934439	0.2053564	0.10646920
t_{13}	0.2224999	0.1773046	0.1251562	0.06605467
t_{14}	0.3761109	0.2960009	0.2050618	0.10441550
t_{15}	0.2224999	0.1773046	0.1251562	0.06605467
t_{16}	0.2224999	0.1773046	0.1251562	0.06605467
t_{17}	0.3898395	0.3106532	0.2192844	0.11573320
t_{18}	0.2730589	0.2466013	0.2162092	0.18197140
t_{19}	0.3898235	0.310639	0.2192729	0.11572550
t_{20}	0.3898411	0.3106546	0.2192856	0.11573400
t_{21}	0.2224999	0.1773046	0.1251562	0.06605467
t_{22}	0.2517396	0.2213433	0.1876804	0.15167180
t_{23}	0.2748301	0.2486541	0.2184854	0.18434620
t_{24}	0.2301946	0.1794773	0.12515910	0.06998473
t_{25}	0.2532104	0.2231283	0.18973620	0.15389510

Estimator	**ρ**			
	0.6	**0.7**	**0.8**	**0.9**
t_{26}	0.2224999	0.1773046	0.12515620	0.06605467
t_{27}	0.2224999	0.1773046	0.12515620	0.06605467
t_{28}	0.2224999	0.1773046	0.12515620	0.06605467
t_{29}	0.2224999	0.1773046	0.12515620	0.06605467
t_{30}	0.2224999	0.1773046	0.12515620	0.06605467
t_{31}	0.2224999	0.1773046	0.12515620	0.06605467
t_{32}	0.3898344	0.3106499	0.21928270	0.11573280
$\boldsymbol{t_{p(1)}}$	**0.2224983**	**0.1773033**	**0.1251554**	**0.06605416**
$\boldsymbol{t_{p(2)}}$	**0.2224983**	**0.1773034**	**0.1251554**	**0.06605423**
$\boldsymbol{t_{p(3)}}$	**0.2224983**	**0.1773033**	**0.1251554**	**0.06605417**
$\boldsymbol{t_{p(4)}}$	**0.2224983**	**0.1773033**	**0.1251554**	**0.06605416**
$\boldsymbol{t_{p(5)}}$	**0.2224983**	**0.1773033**	**0.1251554**	**0.06605425**

Table 9. PRE for simulated normal population

Estimator	**ρ**			
	0.6	**0.7**	**0.8**	**0.9**
t_0	124.5336	141.7121	176.8744	283.6249
t_1	100.0007	100.0007	100.0006	100.0006
t_2	175.2110	175.2109	175.2108	175.2108
t_3	175.2110	175.2109	175.2108	175.2108
t_4	100.0007	100.0007	100.0006	100.0006
t_5	100.0007	100.0007	100.0006	100.0006
t_6	175.0408	175.0220	174.9931	174.9333
t_7	103.4591	101.2261	100.0030	105.9504
t_8	175.1824	175.1792	175.1742	175.1642
t_9	597.1942	666.5930	781.8647	1054.536
t_{10}	124.4596	141.6049	176.7067	283.2946
t_{11}	174.5149	174.2674	173.8921	173.1668
t_{12}	166.4396	165.5038	164.0811	161.1845
t_{13}	100.0007	100.0007	100.0006	100.0006
t_{14}	169.0399	166.9460	163.8457	158.0754
t_{15}	100.0007	100.0007	100.0006	100.0006
t_{16}	100.0007	100.0007	100.0006	100.0006
t_{17}	175.2101	175.2100	175.2097	175.2093
t_{18}	122.7240	139.0844	172.7526	275.4878
t_{19}	175.2029	175.2019	175.2005	175.1977
t_{20}	175.2108	175.2107	175.2107	175.2105
t_{21}	100.0007	100.0007	100.0006	100.0006
t_{22}	113.1423	124.8387	149.9579	229.6170
t_{23}	123.5201	140.2421	174.5713	279.0830
t_{24}	103.4590	101.2261	100.0030	105.9504
t_{25}	113.8033	125.8455	151.6005	232.9829
t_{26}	100.0007	100.0007	100.0006	100.0006
t_{27}	100.0007	100.0007	100.0006	100.0006
t_{28}	100.0007	100.0007	100.0006	100.0006
t_{29}	100.0007	100.0007	100.0006	100.0006
t_{30}	100.0007	100.0007	100.0006	100.0006
t_{31}	100.0007	100.0007	100.0006	100.0006
t_{32}	175.2078	175.2081	175.2083	175.2087
$\boldsymbol{t_{p(1)}}$	**100.0000**	**100.0000**	**100.0000**	**99.99986**
$\boldsymbol{t_{p(2)}}$	**100.0000**	**100.0001**	**100.0000**	**99.99997**
$\boldsymbol{t_{p(3)}}$	**100.0000**	**100.0000**	**100.0000**	**99.99988**
$\boldsymbol{t_{p(4)}}$	**100.0000**	**100.0000**	**100.0000**	**99.99986**
$\boldsymbol{t_{p(5)}}$	**100.0000**	**100.0000**	**100.0000**	**100.0000**

Table 10. Comparison of ratio estimators of population mean for Simulated Uniform Population

Estimator	ρ			
	0.6	**0.7**	**0.8**	**0.9**
t_0	0.001518244	0.0014473490	0.001367226	0.0012789710
t_1	0.001107771	0.0008827548	0.000623121	0.0003288694
t_2	0.002227942	0.0017753910	0.001253217	0.0006614203
t_3	0.002227942	0.0017753910	0.001253217	0.0006614203
t_4	0.001107771	0.0008827548	0.000623121	0.0003288694
t_5	0.001107771	0.0008827548	0.000623121	0.0003288694
t_6	0.002139741	0.0016969800	0.001189124	0.0006181472
t_7	0.001151389	0.0008921056	0.000624305	0.0003684822
t_8	0.002208848	0.0017583840	0.001239274	0.0006519483
t_9	0.008711274	0.0078575500	0.006626538	0.0048575770
t_{10}	0.001517651	0.0014466540	0.001366447	0.0012781470
t_{11}	0.002007332	0.0015464350	0.001040107	0.0005028236
t_{12}	0.001265527	0.0009638183	0.000645379	0.0003296018
t_{13}	0.001107771	0.0008827548	0.000623121	0.0003288694
t_{14}	0.001479850	0.0010712900	0.000678485	0.0003288721
t_{15}	0.001107771	0.0008827548	0.000623121	0.0003288694
t_{16}	0.001107771	0.0008827548	0.000623121	0.0003288694
t_{17}	0.002227889	0.0017753440	0.001253178	0.0006613937
t_{18}	0.001517863	0.0014469020	0.001366725	0.0012784410
t_{19}	0.002227665	0.0017751440	0.001253015	0.0006612825
t_{20}	0.002227931	0.0017753810	0.001253209	0.0006614147
t_{21}	0.001107771	0.0008827548	0.000623121	0.0003288694
t_{22}	0.001497661	0.0014232040	0.001340113	0.0012502300
t_{23}	0.001518222	0.0014473220	0.001367196	0.0012789390
t_{24}	0.001151389	0.0008921056	0.000624305	0.0003684822
t_{25}	0.001507461	0.0014347110	0.001353045	0.0012639510
t_{26}	0.001107771	0.0008827548	0.000623121	0.0003288694
t_{27}	0.001107771	0.0008827548	0.000623121	0.0003288694
t_{28}	0.001107771	0.0008827548	0.000623121	0.0003288694
t_{29}	0.001107771	0.0008827548	0.000623121	0.0003288694
t_{30}	0.001107771	0.0008827548	0.000623121	0.0003288694
t_{31}	0.001107771	0.0008827548	0.000623121	0.0003288694
t_{32}	0.001107762	0.0008827491	0.001251787	0.0003288685
$\boldsymbol{t_{p(1)}}$	**0.001107747**	**0.0008827365**	**0.000623107**	**0.0003288631**
$\boldsymbol{t_{p(2)}}$	**0.001107767**	**0.0008827543**	**0.000623120**	**0.0003288665**
$\boldsymbol{t_{p(3)}}$	**0.001107766**	**0.0008827541**	**0.000623121**	**0.0003288672**
$\boldsymbol{t_{p(4)}}$	**0.00110776**	**0.0008827503**	**0.000623120**	**0.0003288694**
$\boldsymbol{t_{p(5)}}$	**0.001107747**	**0.0008827362**	**0.000623107**	**0.0003288625**

Table 11. PRE for simulated uniform population

Estimator	ρ			
	0.6	**0.7**	**0.8**	**0.9**
t_0	137.0569	163.9617	219.4207	388.9075
t_1	100.0022	100.0021	100.0022	100.0021
t_2	201.1237	201.1236	201.1239	201.1237
t_3	201.1237	201.1236	201.1239	201.1237
t_4	100.0022	100.0021	100.0022	100.0021
t_5	100.0022	100.0021	100.0022	100.0021
t_6	193.1615	192.2409	190.8378	187.9652
t_7	103.9397	101.0614	100.1923	112.0475
t_8	199.4000	199.1970	198.8862	198.2434
t_9	786.3956	890.1357	1063.467	1477.084
t_{10}	137.0034	163.8829	219.2957	388.6570
t_{11}	181.2085	175.1865	166.9227	152.8978
t_{12}	114.2433	109.1853	103.5743	100.2248

Estimator	ρ			
	0.6	**0.7**	**0.8**	**0.9**
t_{13}	100.0022	100.0021	100.0022	100.0021
t_{14}	133.5910	121.3602	108.8874	100.0029
t_{15}	100.0022	100.0021	100.0022	100.0021
t_{16}	100.0022	100.0021	100.0022	100.0021
t_{17}	201.1189	201.1183	201.1176	201.1156
t_{18}	137.0225	163.9110	219.3403	388.7464
t_{19}	201.0987	201.0956	201.0915	201.0818
t_{20}	201.1227	201.1225	201.1226	201.1220
t_{21}	100.0022	100.0021	100.0022	100.0021
t_{22}	135.1988	161.2264	215.0695	380.1680
t_{23}	137.0549	163.9586	219.4159	388.8978
t_{24}	103.9397	101.0614	100.1923	112.0475
t_{25}	136.0835	162.5300	217.1449	384.3403
t_{26}	100.0022	100.0021	100.0022	100.0021
t_{27}	100.0022	100.0021	100.0022	100.0021
t_{28}	100.0022	100.0021	100.0022	100.0021
t_{29}	100.0022	100.0021	100.0022	100.0021
t_{30}	100.0022	100.0021	100.0022	100.0021
t_{31}	100.0022	100.0021	100.0022	100.0021
t_{32}	100.0014	100.0015	200.8944	100.0018
$\boldsymbol{t_{p(1)}}$	**100.0000**	**100.0000**	**100.0000**	**100.0002**
$\boldsymbol{t_{p(2)}}$	**100.0018**	**100.0021**	**100.0021**	**100.0012**
$\boldsymbol{t_{p(3)}}$	**100.0017**	**100.002**	**100.0022**	**100.0014**
$\boldsymbol{t_{p(4)}}$	**100.0012**	**100.0016**	**100.0021**	**100.0021**
$\boldsymbol{t_{p(5)}}$	**100.0000**	**100.000**	**100.0000**	**100.0000**

Table 12. Comparison of ratio estimators of population mean for simulated Log-normal population

Estimator	ρ			
	0.6	**0.7**	**0.8**	**0.9**
t_0	0.0001657873	0.0001743760	0.0001844353	0.0001960627
t_1	9.181981e-05	7.316891e-05	5.164864e-05	2.725901e-05
t_2	0.0002433058	0.0001938843	0.0001368595	7.223140e-05
t_3	0.0002433058	0.0001938843	0.0001368595	7.223140e-05
t_4	9.181981e-05	7.316891e-05	5.164864e-05	2.725901e-05
t_5	9.181981e-05	7.316891e-05	5.164864e-05	2.725901e-05
t_6	0.000122758	9.077962e-05	5.807458e-05	2.738201e-05
t_7	9.525625e-05	7.338368e-05	5.296234e-05	3.713840e-05
t_8	0.0002240488	0.0001766549	0.0001226808	6.259102e-05
t_9	0.0011951460	0.0010993640	0.0009507640	0.0007244692
t_{10}	9.879226e-05	7.474506e-05	5.178201e-05	3.302812e-05
t_{11}	0.0001124595	7.949035e-05	5.17596e-05	3.198116e-05
t_{12}	0.000137751	0.0001408177	0.000146002	0.0001543792
t_{13}	9.181981e-05	7.316891e-05	5.164864e-05	2.725901e-05
t_{14}	9.369701e-05	7.367427e-05	5.978484e-05	5.417107e-05
t_{15}	9.181981e-05	7.316891e-05	5.164864e-05	2.725901e-05
t_{16}	9.181981e-05	7.316891e-05	5.164864e-05	2.725901e-05
t_{17}	0.0001014359	9.363731e-05	8.842545e-05	8.827385e-05
t_{18}	0.0001058297	9.990263e-05	9.656087e-05	9.808899e-05
t_{19}	0.0001191694	0.0001176183	0.0001185185	3.587589e-05
t_{20}	0.0001608043	0.0001215856	7.92973e-05	3.587589e-05
t_{21}	9.181981e-05	7.316891e-05	5.164864e-05	2.725901e-05
t_{22}	0.0001175534	0.0001155379	0.0001159964	0.0001207477
t_{23}	0.0001033459	9.640315e-05	9.205067e-05	9.267838e-05
t_{24}	9.525625e-05	7.338369e-05	5.296235e-05	3.713840e-05
t_{25}	0.0001655167	0.0001740573	0.0001840752	0.0001956773
t_{26}	9.181981e-05	7.316891e-05	5.164864e-05	2.725901e-05

Estimator	ρ			
	0.6	**0.7**	**0.8**	**0.9**
t_{27}	9.181981e-05	7.316891e-05	5.164864e-05	2.725901e-05
t_{28}	9.181981e-05	7.316891e-05	5.164864e-05	2.725901e-05
t_{29}	9.181981e-05	7.316891e-05	5.164864e-05	2.725901e-05
t_{30}	9.181981e-05	7.316891e-05	5.164864e-05	2.725901e-05
t_{31}	9.181981e-05	7.316891e-05	5.164864e-05	2.725901e-05
t_{32}	0.0002300046	0.0001804162	0.000120841	6.047715e-05
$\boldsymbol{t_{p(1)}}$	**9.180510e-05**	**7.315633e-05**	**5.163917e-05**	**2.725344e-05**
$\boldsymbol{t_{p(2)}}$	**9.180901e-05**	**7.316089e-05**	**5.164389e-05**	**2.725736e-05**
$\boldsymbol{t_{p(3)}}$	**9.180744e-05**	**7.315913e-05**	**5.164215e-05**	**2.725606e-05**
$\boldsymbol{t_{p(4)}}$	**9.180567e-05**	**7.315714e-05**	**5.164019e-05**	**2.725454e-05**
$\boldsymbol{t_{p(5)}}$	**9.180605e-05**	**7.315717e-05**	**5.164034e-05**	**2.725429e-05**

Table 13. PRE for simulated Log-normal population

Estimator	ρ			
	0.6	**0.7**	**0.8**	**0.9**
t_0	180.5843	238.358	357.1535	719.3829
t_1	100.0150	100.016	100.0161	100.0173
t_2	265.0215	265.0243	265.0244	265.0276
t_3	265.0215	265.0243	265.0244	265.0276
t_4	100.0150	100.0160	100.0161	100.0173
t_5	100.0150	100.0160	100.0161	100.0173
t_6	133.7145	124.0885	112.4597	100.4686
t_7	103.7581	100.3096	102.5600	136.2663
t_8	244.0458	241.4731	237.5678	229.6557
t_9	1301.816	1502.743	1841.127	2658.184
t_{10}	107.6097	102.1705	100.2743	121.1850
t_{11}	122.4968	108.6570	100.2309	117.3436
t_{12}	150.0457	192.4865	282.7286	566.4400
t_{13}	100.0150	100.0160	100.0161	100.0173
t_{14}	102.0597	100.7068	115.7716	198.7616
t_{15}	100.0150	100.0160	100.0161	100.0173
t_{16}	100.0150	100.0160	100.0161	100.0173
t_{17}	110.4893	127.9947	171.2333	323.8897
t_{18}	115.2753	136.5589	186.9873	359.9029
t_{19}	129.8056	160.7748	229.5076	131.6339
t_{20}	175.1565	166.1978	153.5569	131.6339
t_{21}	100.0150	100.0160	100.0161	100.0173
t_{22}	128.0454	157.9311	224.6236	443.0411
t_{23}	112.5698	131.7754	178.2534	340.0506
t_{24}	103.7581	100.3096	102.5600	136.2663
t_{25}	180.2895	237.9224	356.4562	717.9688
t_{26}	100.0150	100.0160	100.01610	100.0173
t_{27}	100.0150	100.0160	100.01610	100.0173
t_{28}	100.0150	100.0160	100.01610	100.0173
t_{29}	100.0150	100.0160	100.01610	100.0173
t_{30}	100.0150	100.0160	100.01610	100.0173
t_{31}	100.0150	100.0160	100.01610	100.0173
t_{32}	250.5332	246.6145	234.0050	221.8996
$\boldsymbol{t_{p(1)}}$	**99.99897**	**99.99885**	**99.99773**	**99.99688**
$\boldsymbol{t_{p(2)}}$	**100.0032**	**100.0051**	**100.0069**	**100.0113**
$\boldsymbol{t_{p(3)}}$	**100.0015**	**100.0027**	**100.0035**	**100.0065**
$\boldsymbol{t_{p(4)}}$	**99.99959**	**99.99996**	**99.99971**	**100.0009**
$\boldsymbol{t_{p(5)}}$	**100.0000**	**100.0000**	**100.0000**	**100.0000**

Table 14. Comparison of ratio estimators of population mean for simulated Beta population

Estimator	ρ

	0.6	0.7	0.8	0.9
t_0	1.600922e-05	1.525086e-05	1.438764e-05	1.342725e-05
t_1	1.166382e-05	9.294609e-06	6.560901e-06	3.462698e-06
t_2	2.170868e-05	1.72991e-05	1.221113e-05	6.444764e-06
t_3	2.170868e-05	1.72991e-05	1.221113e-05	6.444764e-06
t_4	1.166382e-05	9.294609e-06	6.560901e-06	3.462698e-06
t_5	1.166382e-05	9.294609e-06	6.560901e-06	3.462698e-06
t_6	1.437119e-05	1.098636e-05	7.328383e-06	3.559013e-06
t_7	1.195802e-05	9.332376e-06	6.605128e-06	3.97378e-06
t_8	1.958308e-05	1.540988e-05	1.067019e-05	5.413374e-06
t_9	8.261556e-05	7.488819e-05	6.358867e-05	4.716008e-05
t_{10}	1.576126e-05	1.495907e-05	1.40586e-05	1.307625e-05
t_{11}	1.219994e-05	9.315742e-06	6.783533e-06	4.779543e-06
t_{12}	1.415159e-05	1.302615e-05	1.184165e-05	1.067265e-05
t_{13}	1.166382e-05	9.294609e-06	6.560901e-06	3.462698e-06
t_{14}	1.251283e-05	1.13417e-05	1.029412e-05	9.416891e-06
t_{15}	1.166382e-05	9.294609e-06	6.560901e-06	3.462698e-06
t_{16}	1.166382e-05	9.294609e-06	6.560901e-06	3.462698e-06
t_{17}	1.893917e-05	1.484403e-05	1.02169e-05	5.121822e-06
t_{18}	1.587204e-05	1.508958e-05	1.420592e-05	1.323356e-05
t_{19}	1.443537e-05	1.103732e-05	7.362204e-06	3.570523e-06
t_{20}	2.114336e-05	1.679384e-05	1.179543e-05	6.161383e-06
t_{21}	1.166382e-05	9.294609e-06	6.560901e-06	3.462698e-06
t_{22}	1.598764e-05	1.522551e-05	1.43591e-05	1.339685e-05
t_{23}	1.599946e-05	1.52394e-05	1.437473e-05	1.34135e-05
t_{24}	1.195802e-05	9.332377e-06	6.605129e-06	3.97378e-06
t_{25}	1.600043e-05	1.524055e-05	1.437602e-05	1.341488e-05
t_{26}	1.166382e-05	9.294609e-06	6.560901e-06	3.462698e-06
t_{27}	1.166382e-05	9.294609e-06	6.560901e-06	3.462698e-06
t_{28}	1.166382e-05	9.294609e-06	6.560901e-06	3.462698e-06
t_{29}	1.166382e-05	9.294609e-06	6.560901e-06	3.462698e-06
t_{30}	1.166382e-05	9.294609e-06	6.560901e-06	3.462698e-06
t_{31}	1.166382e-05	9.294609e-06	6.560901e-06	3.462698e-06
t_{32}	1.985868e-05	1.536875e-05	1.040142e-05	5.083501e-06
$\boldsymbol{t_{p(1)}}$	**1.166341e-05**	**9.294285e-06**	**6.560770e-06**	**3.462583e-06**
$\boldsymbol{t_{p(2)}}$	**1.166375e-05**	**9.294592e-06**	**6.560900e-06**	**3.462644e-06**
$\boldsymbol{t_{p(3)}}$	**1.166372e-05**	**9.294579e-06**	**6.560901e-06**	**3.462661e-06**
$\boldsymbol{t_{p(4)}}$	**1.166357e-05**	**9.294474e-06**	**6.560860e-06**	**3.462697e-06**
$\boldsymbol{t_{p(5)}}$	**1.166344e-05**	**9.29432e-06**	**6.560715e-06**	**3.462617e-06**

Table 15. PRE for simulated Beta population

Estimator	$\boldsymbol{\rho}$			
	0.6	**0.7**	**0.8**	**0.9**
t_0	137.2598	164.0880	219.2999	387.7775
t_1	100.0033	100.0031	100.0028	100.0023
t_2	186.1259	186.1255	186.1250	186.1241
t_3	186.1259	186.1255	186.1250	186.1241
t_4	100.0033	100.0031	100.0028	100.0023
t_5	100.0033	100.0031	100.0028	100.0023
t_6	123.2157	118.2051	111.7010	102.7839
t_7	102.5257	100.4095	100.6770	114.7623
t_8	167.9014	165.7989	162.6376	156.3376
t_9	708.3293	805.7415	969.2338	1361.978
t_{10}	135.1339	160.9485	214.2846	377.6407
t_{11}	104.5998	100.2305	103.3962	138.0327
t_{12}	121.3329	140.1517	180.4933	308.2250
t_{13}	100.0033	100.0031	100.0028	100.0023
t_{14}	107.2825	122.0283	156.9055	271.9588

Estimator	ρ			
	0.6	**0.7**	**0.8**	**0.9**
t_{15}	100.0033	100.0031	100.0028	100.0023
t_{16}	100.0033	100.0031	100.0028	100.0023
t_{17}	162.3807	159.7108	155.7285	147.9177
t_{18}	136.0837	162.3527	216.5301	382.1838
t_{19}	123.7660	118.7534	112.2165	103.1163
t_{20}	181.2789	180.6893	179.7888	177.9401
t_{21}	100.0033	100.0031	100.0028	100.0023
t_{22}	137.0748	163.8152	218.8649	386.8996
t_{23}	137.1762	163.9647	219.1031	387.3804
t_{24}	102.5257	100.4095	100.6770	114.7623
t_{25}	137.1845	163.9770	219.1228	387.4203
t_{26}	100.0033	100.0031	100.0028	100.0023
t_{27}	100.0033	100.0031	100.0028	100.0023
t_{28}	100.0033	100.0031	100.0028	100.0023
t_{29}	100.0033	100.0031	100.0028	100.0023
t_{30}	100.0033	100.0031	100.0028	100.0023
t_{31}	100.0033	100.0031	100.0028	100.0023
t_{32}	170.2643	165.3564	158.5410	146.8110
$\boldsymbol{t_{p(1)}}$	**99.99974**	**99.99962**	**100.0008**	**99.99902**
$\boldsymbol{t_{p(2)}}$	**100.0027**	**100.0029**	**100.0028**	**100.0008**
$\boldsymbol{t_{p(3)}}$	**100.0024**	**100.0028**	**100.0028**	**100.0013**
$\boldsymbol{t_{p(4)}}$	**100.0011**	**100.0017**	**100.0022**	**100.0023**
$\boldsymbol{t_{p(5)}}$	**100.0000**	**100.0000**	**100.0000**	**100.0000**

6. RESULTS AND DISCUSSION

Table 3 lists the conditions in which our proposed class of estimators surpasses the existing ones. Tables 4 and 6 contains the parametric values of two real data that we used to empirically validate our results. Tables 5 and 7 contains the MSE and PRE of the existing and proposed estimator for the two populations respectively. Tables 8, 10, 12, and14 contains the MSE of simulated population of Normal, Uniform, Log-normal and Beta respectively. Tables 9, 11, 13, and 15 consist PRE of the simulated population of Normal, Uniform, Log-normal and Beta respectively. We examine the proposed class's Bias and MSE up to first order of approximation. Since Efficiency is stronger property than the unbiasedness hence as a result, we prefer the biased estimator with the lowest MSE over the unbiased estimator with a higher MSE in this case. We can easily notice that the suggested class of estimators has lower MSE and PRE, demonstrating that our proposed class of estimators is efficient enough for practical purposes.

7. CONCLUSIONS

In this manuscript, we have suggested a generalized class of estimators of population mean using known auxiliary parameters. We studied the biases and MSEs of the suggested class up to the first order of approximation. We compared the suggested family with the competing estimators of population mean and the efficiency conditions over competing estimators are obtained. These efficiency conditions are verified using both real and simulated data sets. From the results, it is observed that the suggested estimator is the best among the competing estimators. Hence, we can undoubtedly recommend the proposed class of estimator for practical utility in different fields like agriculture, medical sciences, economics, commerce, engineering etc. In the light of above results and observations, the suggested estimator is recommended for practical applications in different areas of applications.

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