ORIGINAL PAPER

ON t^*-SET AND B^*-SET IN NANO TOPOLOGICAL SPACES

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Abstract. *We investigate the class of nano locally **b-closed sets in this paper. The purpose of this work is to introduce several novel sets, such as nano closed sets,nano t*-sets, nano *b-closed sets, nano B*-set and nano **b-closed sets, and to investigate the properties of these sets. We also introduced the concept of nano **b-open sets and nano locally **b-open sets, as well as examined some of their features.*

Keywords: *nano**b-open sets; nano t*-set; nano B*-set; nano locally closed sets.*

1. INTRODUCTION AND PRELIMINARIES

The concept of continuity is central to topology. More precisely, a function is continuous if it can guarantee arbitrarily small changes in its output by restricting it to sufficiently small changes in its input. In 2013, M. Lellis Thivagar [1] proposed the idea of topology.

We investigate the class of nano locally **b-closed sets in this paper. The purpose of this work is to introduce several novel sets, such as nano closed sets,nano t*-sets, nano *b-closed sets, nano B*-set and nano **b-closed sets, and to investigate the properties of these sets. We also introduced the concept of nano **b-open sets and nano locally **b-open sets, as well as examined some of their features.

Thus $\tau_R(X)$ is topology on U called the nano toplogical space with respect to X and $(U, \tau_R(X))$ is called the nano topological spaces. The elements of $\tau_R(X)$ are called nano-open sets (resp. n-open sets). The complement of a n-open set is called n-closed.

Throughout the paper, we denote a nano topological space by (U, N), where $N = \tau_R(X)$. The nano-interior and nano-closure of a subset A of U are denote by $I_n(A)$ and $C_n(A)$. respectively.

Definition 1.1. A subset A of a nano topological space $(U, \tau_R(X))$ is called

1. nano *b-open (resp. n*b-open) [2] if$A \subseteq C_n(I_n(A)) \cap I_n(C_n(A))$.
2. nano locally closed (resp. nl-closed) [3] if A = K $\cap$ H, where K is n-open and H is n-closed.

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2. ON t^*-SET AND B^*-SET IN NANO TOPOLOGICAL SPACES

Definition 2.1. A space X's subset A is known as
1. Nano t^*-set (resp. nt^*-set) if $C_n(A) = C_n(I_n(A))$.
2. Nano B^*-set (resp. nB^*-set) if A = U∩V, where U is n-open and V is a $\boldsymbol{nt}^*$-set.
3. Nano locally *b-closed (resp. nl*b-closed) if A = U∩V, where U is n-open and V is n*b-closed.
4. Nano locally **b-closed (resp. nl**b-closed) if A = U∩V, where U is n-open and V is n**b-closed.

Remark 2.2.
1. Every open set isnt*-set .
2. Every open set is nB*-set .

Example 2.3. Let U = {a, b, c, d, e} with U/R = {{a}, {b, c}, {d, e}} and X = {a, b}. Then the nano topological space N = {ϕ, {a}, {b, c}, {a, b, c}, U}. Clear that
1. {d} is nt*-set but not n-open
2. {a, b, c, d} is nB*-set but not n-open.

Definition 2.4. A nano topological space (U, N) is said to be nano extremally disconnected if the n-closure of every n-open set of U is n-open in U.

For any subset A of an-extremally disconnected space, $C_n(I_n(A))= I_n(C_n(A))$.

Definition 2.5. A subset A of X is n-locally closed iff $A = U \cap C_n(A)$ for some n-open.

Theorem 2.6. For a subset A of n-extremally disconnted space (U, N) , the following are equivalent:
1. A is open.
2. A is n**b-open and n-locally closed.

Proof:

(1) ⇒ (2) : This is obvious from the Definitions.

(2) ⇒ (1) : Let A be n**b-open and nl-closed.

Then $A \subseteq I_n(C_n(I_n(A)))\cap C_n(I_n(C_n(A)))$ and$A = G \cap C_n(A)$ where G is n-open $A \subseteq I_n(C_n(I_n(A)))\cap C_n(I_n(C_n(A)))$ and $A \subseteq G \Rightarrow A \subseteq G \cap [I_n(C_n(I_n(A))) \cap C_n(I_n(C_n(A)))] = [G\cap I_n(C_n(I_n(A)))]\cap C_n(I_n(C_n(A))) = [I_n(G \cap (C_n(I_n(A)))]\cap C_n(C_n(I_n(A))) = I_n[U \cap I_n(C_n(A))] \cap C_n(I_n(A)) = I_n(G \cap C_n(A))\cap (I_n(C_n(A))=I_n(G \cap C_n(A))=I_n(A) \Rightarrow$ A is n-open.

Theorem 2.7. Let H be a subset of (U, N), H is nl**b-closed if and only if there exist a n-open set $G \subseteq U$ such that $H = G \cap {}^{**}bC_n(H)$.

Proof:

Let H be a subset of U and H be nl**b-closed.There exists an open set G ⊆ U∋ H $= G \cap{}^{**}bC_n(H)$. Since H being nl**b-closed.Then H = G ∩ F , where G is n-open and F is n**b-closed. So $H \subseteq G$ and $H \subseteq F$. Then $H \subseteq {}^{**}bC_n(H) \subseteq {}^{**}bC_n(F) = F$. Therefore $H \subseteq G \cap{}^{**}bC_n(H) \subseteq G\cap{}^{**}bC_n(F) = G \cap F = H$.Hence $H = G \cap {}^{**}bC_n(H)$.

Conversly,

Assume that H = G ∩ $**bC_n$(H). H is nl**b-closed. Since $**bC_n$(H) is n**b-closed and H = G ∩$**bC_n$(H).Then H is nl**b-closed.

Remark 2.8.

1. The union of any family of n**b-open set is n**b-open.
2. The intersection of n-open set and a n**b-open set is a n**b-open set.

Example 2.9. In Example 2.3,

1. the {a} and {b, c} are n**b-open sets. {a} ∪ {b, c} = {a, b, c} is n**b-open.
2. the {b, c} is n-open and {a, b, c} is n**b-open. {b, c} ∩ {a, b, c} = {b, c} is n**b-open.

Theorem 2.10. Let A be a subset of a nano topological space U , if A is nl**b-closed, then

1. $**bC_n$(A) – A is n**b-closed.
2. A ∪ (G – $**bC_n$(A)) is n**b-open.

Proof:

1. Let A be a subset of a nano topological space U and A be nl**b-closed.Then there exist a n-open set G in U ∋ A = G ∩ $**bC_n$(A).$**bC_n$(A) – A = $**bC_n$(A) – [U ∩$**bC_n$(A)] = $**bC_n$(A)∩ [U – (G ∩$**bC_n$(A))] = $**bC_n$(A) ∩ [(U – G) ∪ (U – $**bC_n$(A)] = [$**bC_n$(A)∩ (U – G)] ∪[$**bC_n$(A)∩ (U – $**bC_n$(A))] = $**bC_n$(A) ∩ (U – G) By Theorem 2.10 $**bC_n$(A) – A is n**b-closed.

2. Since $**bC_n$(A) – A is n**b-closed.⇒ [U – ($**bC_n$(A) – A)] is n**b-open.
   Consider [U – ($**bC_n$(A) – A)] = U – [$**bC_n$(A)∩ A^c] = [U – $**bC_n$(A)]∪ [U–A^c] = (U – $**bC_n$(A))∪ A∪ (U – $**bC_n$(A)) is n**b-open.
3. It is clear that A⊆ [A ∪(U– $**bC_n$(A))] = $**bI_n$[A ∪ (U– $**bC_n$(A))].

Remark 2.11.

1. The arbitrary intersection of n**b-closed set is n**b-closed.
2. The intersection of a nl**b-closed set and nl-closed set is nl**b-closed.

Example 2.12. In Example 2.3,

1. the {b, d} and {b, e} are n**b-closed. {b, d} ∩ {b, e} = {b} is n**b-closed.
2. the {b, c, d, e} is nl**b-closed and {a, d, e} is nl-closed. {b, c, d, e} ∩ {a, d, e} = {d, e} is n**b-closed.

Definition 2.13. Let A,B ⊆ U. Then A and B are said to be separated if A∩ C_n(B) = ϕ and B ∩ C_n(A) = ϕ.

Theorem 2.14. Suppose (U, N),is n-closed under finite unions of n**b-closed sets . Let A and B be the nl**b-closed sets. If A and B are separated, then A∪ B is nl**b-closed.

Proof:

Since A and B are nl**b-closed.
Therefore, A = G ∩ $**bC_n$(A) and B = H ∩ $**bC_n$(B), where Gand H are n-open in U.
Put K = G∩ (U – C_n(B)) and V = H ∩ (U – C_n(A)). Then

$$K \cap {}^{**}bC_n(A) = G\cap (U - C_n(B))\cap {}^{**}bC_n(A) = G \cap [U - C_n(B)) \cap {}^{**}bC_n(A)] = G\cap[{}^{**}bC_n(A)\cap (U - C_n(B))] = [G \cap {}^{**}bC_n(A)]\cap (U - C_n(B)) = A\cap (U - C_n(B)) = A \cap(C_n(B))^c = A.U \cap {}^{**}bC_n(A) = A.$$

Similarly ,$V \cap {}^{**}bC_n(B) = B$.

$$U \cap {}^{**}bC_n(B)\subseteq K \cap C_n(B) = G \cap [(U - C_n(B))\cap C_n(B)]= G\cap [(C_n(B))^c \cap C_n(B)]= G\cap \phi = \phi.$$

$$U \cap {}^{**}bC_n(B) = \phi$$

Similarly , $V\cap {}^{**}bC_n(A) = \phi$.

Since K and V are n-open

$$(K \cup V)\cap {}^{**}bC_n(A \cup B) = (K \cup V) \cap ({}^{**}bC_n(A)\cup {}^{**}bC_n(B))=(K \cap {}^{**}bC_n(A))\cup (K\cap{}^{**}bC_n(B))\cup (V \cap{}^{**}bC_n(A))\cup (V \cap{}^{**}bC_n(B))= A \cup \phi \cup \phi\ B = A \cup B.$$

Hence, $A \cup B$ is n-closed.

3. CONCLUSION

This paper introduces the concept of nano **b-closed sets and investigates its significant characterizations with several theorems, assertions, and demonstrations. We anticipate that this work will serve as the foundation for a new structure, inspiring many people to contribute to the advancement of Nano continuity in mathematics.

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