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A NEW LOGARITHMIC RATIO TYPE ESTIMATOR OF POPULATION MEAN FOR SIMPLE RANDOM SAMPLING: A SIMULATION STUDY

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Abstract. *This paper provides a logarithmic ratio type estimator of a finite population mean using an auxiliary variable in simple random sampling. The expression for the mean square error (MSE) of the proposed estimator and some selected estimators are calculated along with the theoretical conditions. Further the numerical comparisons and simulation study based on different probability distributions supports the claim that the proposed estimator performs better as compared to the ones used in this study.*

Keywords: *Logarithmic-type estimators; mean square error; efficiency; simple random sampling.*

1. INTRODUCTION

Among all characteristics of the population, arithmetic mean is the simplest one. It performs better if the underlying population is homogeneous. Till date many efficient ratio, product and exponential type estimates of population mean have been developed by the researchers in simple random sampling scheme, such as Singh and Tailor [1] suggested efficient estimators of mean using the known values of the coefficient of variation while Kadilar and Cingi [2] utilized the information of coeffient of correlation for efficient estimation of population mean. Singh and Vishwakarma [3] developed a modefied ratio and product exponential type estimator of mean in double sampling. An improved family of ratio type estimators is suggested by Haq and Shabbir [4] while Singh and Solanki [5] developed an efficient class of the mean using auxiliary information. Using know value of the median of auxilary information Subramani and Kumarpandiyan [6] developed a modified ratio type estimator of the population mean. In Simple as well as stratified random sampling Muneer et al. [7] developed estimator of population mean using two auxiliary variables. Iznobi and Onyeka [8] suggested efficient logarithmic ratio and product-type estimators of population mean in simple random sampling.

Consider a finite population $\psi = \{\psi_1, \psi_2, ..., \psi_N\}$ consisting of N units. We assume the notations y_i in respect of the study variable and x_i for the auxiliary variable while $\bar{Y}$ and $\bar{X}$ as their population means respectively. Using the procedure of without replacement in simple random sampling a sample having n values is drawn from ψ. For the population variances of Y and X we assume

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$$S^2_y = E(s^2_y)$$

and

$$S^2_x = E(s^2_x),$$

where

$$S^2_y = \frac{\sum_{i=1}^{N}(Yi-\bar{Y})2}{N-1},\ s^2_y = \frac{\sum_{i=1}^{n}(yi-\bar{y})2}{n-1},\ S^2_x = \frac{\sum_{i=1}^{N}(Xi-\bar{X})2}{N-1}$$

and

$$s^2_x = \frac{\sum_{i=1}^{n}(xi-\bar{x})2}{n-1}$$

respectively. The coefficients of variation of X and Y are represented by $C_x^2 = \frac{S_x^2}{\bar{X}^2}$ and $C_y^2 = \frac{S_y^2}{\bar{Y}^2}$ respectively, b = $\frac{S_{xy}}{S_x^2}$, Further the coefficient of the correlation between X and Y is represented by $\rho = \frac{S_{xy}}{S_x S_y}$ and $\varphi_{2(y)}$ and $\varphi_{2(x)}$ represents the kurtosis coefficients of the study and auxiliary variables respectively. Further we assume $\Delta_0 = \frac{\bar{y}-\bar{Y}}{\bar{Y}}$ and $\Delta_1 = \frac{\bar{x}-\bar{X}}{\bar{X}}$ such that

$$E(\Delta_0)=E(\Delta_1)=0,\ E(\Delta_0^2) = \frac{1-f}{n} C_y^2\ ,\ E(\Delta_1^2) = \frac{1-f}{n} C_x^2$$

and

$$E(\Delta_0\ \Delta_1) = \frac{1-f}{n}\rho\ C_x C_y.$$

Let's examine some selected estimators that already exist:

1. The classical ratio estimator of the finite population mean is $\bar{y}$ and its variance is given by

$$\bar{y}_R = \bar{y}\ (\bar{\psi}/\bar{x}) \tag{1}$$

$$\mathrm{Var}(\bar{y}_R) = \lambda\ \bar{\Psi}^2\ [\ C_x^2 + C_y^2 - 2\ C_x C_y\ \rho)] \tag{2}$$

where $\lambda = \frac{1-f}{ц}$.

2. The estimator of mean developed by Upadhyaya and Singh [9] is as follows:

$$\bar{y}_{\mathrm{US}} = [\bar{y}\ \frac{(\bar{\psi}\ \beta_{2x}+C_x\ )}{(\bar{x}\ \beta_{2x}+C_x\ )}] \tag{3}$$

The expression for its MSE is as follows.

$$\mathrm{MSE}(\bar{y}_{\mathrm{US}}\ ) = \lambda\ \bar{\Psi}^2\ \ [(C_y^2 + (\frac{\bar{\psi}\ \beta_{2x}}{\bar{\psi}\ \beta_{2x}+C_x})^2\ C_x^2 - 2\ (\frac{\bar{\psi}\ \beta_{2x}}{\bar{\psi}\ \beta_{2x}+C_x})\ \ \rho C_x C_y] \tag{4}$$

3. The estimators proposed by Kadilar and Cingi [10] is as follows:

$$\bar{y}_{\mathrm{CK1}} = [\bar{y} + \mathrm{b}\ (\ \bar{\psi}\ - \bar{x})]\ (\ \bar{\psi}\ /\bar{x}\ ) \tag{5}$$

$$\bar{y}_{\mathrm{CK2}} = [\bar{y} + \mathrm{b}\ (\ \bar{\psi}\ - \bar{x})]/\ [(\ \bar{\psi}\ + C_x)/(\bar{x} + C_x)] \tag{6}$$

$$\bar{y}_{\mathrm{CK3}} = [\bar{y} + \mathrm{b}\ (\ \bar{\psi}\ - \bar{x})]/\ [(\ \bar{\psi} + \beta_{2x}\ )/(\bar{x} + \beta_{2x})] \tag{7}$$

$$\bar{y}_{\mathrm{CK4}} = [\bar{y} + \mathrm{b}\ (\ \bar{\psi}\ - \bar{x})]/\ [(\ \bar{\psi}\ \beta_{2x} + C_x\ )/(\bar{x}\ \beta_{2x} + C_x)] \tag{8}$$

$$\bar{y}_{\text{CK5}} = [\bar{y} + \text{b}\,(\bar{\psi} - \bar{x})]/\,[(\bar{\psi}\, C_x + \beta_{2\text{x}})/(\bar{x}\, C_x + \beta_{2\text{x}})] \quad (9)$$

The expressions for their MSE are as follows:

$$\text{MSE}(\bar{y}_{\text{CK1}}) = \lambda\, \bar{\Psi}^2\, [\, C_x^2 + C_y^2(1 - \rho^2)] \quad (10)$$

$$\text{MSE}(\bar{y}_{\text{CK2}}) = \lambda\, \bar{\Psi}^2\, [(\frac{\bar{\psi}}{\bar{\psi} + C_x})^2\, C_x^2 + C_y^2(1 - \rho^2)] \quad (11)$$

$$\text{MSE}(\bar{y}_{\text{CK3}}) = \lambda\, \bar{\Psi}^2\, [(\frac{\bar{\psi}}{\bar{\psi} + \beta_{2\text{x}}})^2\, C_x^2 + C_y^2(1 - \rho^2)] \quad (12)$$

$$\text{MSE}(\bar{y}_{\text{CK4}}) = \lambda\, \bar{\Psi}^2\, [(\frac{\bar{\psi}\,\beta_{2\text{x}}}{\bar{\psi}\,\beta_{2\text{x}} + C_x})^2\, C_x^2 + C_y^2(1 - \rho^2)] \quad (13)$$

$$\text{MSE}(\bar{y}_{\text{CK5}}) = \lambda\, \bar{\Psi}^2\, [(\frac{\bar{\psi}\, C_x}{\bar{\psi}\, C_x + \beta_{2\text{x}}})^2\, C_x^2 + C_y^2(1 - \rho^2)] \quad (14)$$

4. The logarithmic ratio type estimator introduced by Izunobi and Onyeka [8] is as follows:

$$\bar{y}_{IO} = \bar{y}\, (\frac{\ln(\bar{\psi})}{\ln(\bar{x})}) \quad (15)$$

Its MSE is as follows:

$$\text{MSE}(\bar{y}_{IO}) = \lambda\, \bar{\Psi}^2\,\, [(C_y^2 + (\frac{1}{\ln(\bar{\psi})})^2\, C_x^2 - 2\, (\frac{1}{\ln(\bar{\psi})})\,\, \rho C_x C_y] \quad (16)$$

2. ESTIMATORS AND COMPARISIONS

2.1. SUGGESTED ESTIMATORS

For efficient estimation of the population mean using simple random sampling scheme, we will use the idea of Iznobi and Onyeka [8] and Upadhyaya and Singh [9] to develop new logarithmic ratio estimator as follows:

$$\bar{y}_{AS} = \bar{y}\, [\frac{\ln(\beta_{2\text{x}}\, \bar{\psi} + C_x)}{\ln(\beta_{2\text{x}}\, \bar{x} + C_x)}] \quad (17)$$

where $\beta_{2\text{x}}$ and C_x are the coefficient of kurtosis and coefficient of variation of the ancillary variable. Expressing $\bar{y}_{AS}$ in terms of Δ's as follows:

$$\bar{y}_{AS} = \bar{\Psi}\,(1 + \Delta_0)\, [\frac{ln(\beta_{2\text{x}}\, \bar{\psi} + C_x)}{ln(\beta_{2\text{x}}\,(\bar{\psi}\,(1 + \Delta 1)) + C_x)}] \quad (18)$$

$$\bar{y}_{AS} = \bar{\Psi}\,(1 + \Delta_0)\, [\frac{ln(\beta_{2\text{x}}\, \bar{\psi} + C_x)}{ln(\beta_{2\text{x}}\, \bar{\psi} + C_x + \beta_{2\text{x}}\, \bar{\psi}\, \Delta 1)}] \quad (19)$$

$$\bar{y}_{AS}=\bar{\Psi}\,(1+\Delta_0)\left[\frac{ln(\beta_{2x}\bar{\psi}+C_x)}{ln(\beta_{2x}\bar{\psi}+C_x)(1+\frac{\beta_{2x}\bar{\psi}\Delta 1}{\beta_{2x}\bar{\psi}+C_x})}\right] \tag{20}$$

$$\bar{y}_{AS}=\bar{\Psi}\,(1+\Delta_0)\left[\frac{ln(\beta_{2x}\bar{\psi}+C_x)}{ln(\beta_{2x}\bar{\psi}+C_x)+ln(1+\frac{\beta_{2x}\bar{\psi}\Delta 1}{\beta_{2x}\bar{\psi}+C_x})}\right] \tag{21}$$

$$\bar{y}_{AS}=\bar{\Psi}\,(1+\Delta_0)\left[\frac{ln(\beta_{2x}\bar{\psi}+C_x)}{ln(\beta_{2x}\bar{\psi}+C_x)\{1+\frac{\ln\left(1+\frac{\beta_{2x}\bar{\psi}\Delta 1}{\beta_{2x}\bar{\psi}+C_x}\right)}{ln(\beta_{2x}\bar{\psi}+C_x)}\}}\right] \tag{22}$$

$$\bar{y}_{AS}=\bar{\Psi}\,(1+\Delta_0)\left[\frac{1}{1+\frac{\ln\left(1+\frac{\beta_{2x}\bar{\psi}\Delta 1}{\beta_{2x}\bar{\psi}+C_x}\right)}{ln(\beta_{2x}\bar{\psi}+C_x)}}\right] \tag{23}$$

$$\bar{y}_{AS}=\bar{\Psi}\,(1+\Delta_0)\left[1+\frac{ln\left(1+\frac{\beta_{2x}\bar{\psi}\Delta 1}{\beta_{2x}\bar{\psi}+C_x}\right)}{ln(\beta_{2x}\bar{\psi}+C_x)}\right]^{-1} \tag{24}$$

$$\bar{y}_{AS}=\bar{\Psi}\,(1+\Delta_0)\left[1-\frac{ln\left(1+\frac{\beta_{2x}\bar{\psi}\Delta 1}{\beta_{2x}\bar{\psi}+C_x}\right)}{ln(\beta_{2x}\bar{\psi}+C_x)}\right] \tag{25}$$

$$\bar{y}_{AS}=\bar{\Psi}\,(1+\Delta_0)\left[1-\frac{ln(1+\theta\Delta 1)}{K}\right] \tag{26}$$

where $\theta=\frac{\beta_{2x}\bar{\psi}}{\beta_{2x}\bar{\psi}+C_x}$ and $K=ln(\beta_{2x}\bar{\psi}+C_x)$.

Simplifying and ignoring higher order terms of $\Delta's$ we get

$$\bar{y}_{AS}=\bar{\Psi}\,(1+\Delta_0)\left[1-\frac{\theta\Delta_1}{K}+\frac{(\theta\,\Delta_1)^2}{2K}\right] \tag{27}$$

$$\bar{y}_{AS}=\bar{\Psi}\left[1+\Delta_0-\frac{\theta\Delta_1}{K}-\frac{\theta\,\Delta 0\,\Delta_1}{K}+\frac{(\theta\,\Delta_1)^2}{2K}\right] \tag{28}$$

$$\bar{y}_{AS}-\bar{\Psi}=\bar{\Psi}\left[\Delta_0-\frac{\theta\Delta_1}{K}-\frac{\theta\,\Delta 0\,\Delta_1}{K}+\frac{(\theta\,\Delta_1)^2}{2K}\right] \tag{29}$$

Squaring both sides, and ignoring higher order terms of $\Delta's$ we get

$$(\bar{y}_{AS}-\bar{\Psi})^2=\bar{\Psi}^2\left[\Delta_0{}^2+\frac{\theta^2\Delta_1{}^2}{K^2}-\frac{2\theta\Delta_0\,\Delta_1}{K}\right] \tag{30}$$

Taking expectation on both sides we get the expression for the MSE

$$\text{MSE}\,(\bar{y}_{AS})=\lambda\,\bar{\Psi}^2\,\left[C_y^2+C_x^2\frac{\theta^2}{K^2}-2\,\rho\,C_xC_y\frac{\theta}{K}\right] \tag{31}$$

2.2. EFFICIENCY COMPARISONS

A comparison of the proposed logarithmic type estimators has been made with the competing estimators. The efficiency conditions have also been mentioned.
i) From (2) and (37), we have

$$\mathrm{MSE}(\bar{y}_{\mathrm{AS}}) < \mathrm{MSE}\ (\bar{y}_{\mathrm{R}}) \tag{32}$$

$$\Lambda\,\bar{\Psi}^2\ [C_y^2 + C_x^2\frac{\theta^2}{K^2} - 2\,\rho\, C_x C_y \frac{\theta}{K}] < \lambda\,\bar{\Psi}^2\,[\,C_x^2 + C_y^2 - 2\, C_x C_y\,\rho)] \tag{33}$$

$$C_x^2\frac{\theta^2}{K^2} - C_x^2\ - 2\,\rho\, C_x C_y \frac{\theta}{K} + 2\, C_x C_y\,\rho < 0 \tag{34}$$

$$C_x^2\left(\frac{\theta^2}{K^2} - 1\right) - 2\, C_x C_y\,\rho\left(\frac{\theta}{K} - 1\right) < 0 \tag{35}$$

ii) From (4) and (37), we have

$$\mathrm{MSE}(\bar{y}_{\mathrm{AS}}) < \mathrm{MSE}\ (\bar{y}_{\mathrm{US}}) \tag{36}$$

$$\Lambda\,\bar{\Psi}^2\ [C_y^2 + C_x^2\frac{\theta^2}{K^2} - 2\,\rho\, C_x C_y \frac{\theta}{K}] < \lambda\,\bar{\Psi}^2\ [(C_y^2 + (\frac{\bar{\psi}\,\beta_{2\mathrm{x}}}{\bar{\psi}\,\beta_{2\mathrm{x}} + C_x})^2\, C_x^2 - 2\,(\frac{\bar{\psi}\,\beta_{2\mathrm{x}}}{\bar{\psi}\,\beta_{2\mathrm{x}} + C_x})\ \rho C_x C_y] \tag{37}$$

$$C_x^2\frac{\theta^2}{K^2} - 2\,\rho\, C_x C_y \frac{\theta}{K} < (\frac{\bar{\psi}\,\beta_{2\mathrm{x}}}{\bar{\psi}\,\beta_{2\mathrm{x}} + C_x})^2\, C_x^2 - 2\,(\frac{\bar{\psi}\,\beta_{2\mathrm{x}}}{\bar{\psi}\,\beta_{2\mathrm{x}} + C_x})\ \rho C_x C_y \tag{38}$$

$$C_x^2\theta^2\left(\frac{1}{K^2} - 1\right) - 2\, C_x C_y\,\rho\,\theta\,(\frac{1}{K} - 1) < 0 \tag{39}$$

iii) From (10) and (31), we have

$$\mathrm{MSE}(\bar{y}_{\mathrm{AS}}) < \mathrm{MSE}\ (\bar{y}_{\mathrm{KC1}}) \tag{40}$$

$$\Lambda\,\bar{\Psi}^2\ [C_y^2 + C_x^2\frac{\theta^2}{K^2} - 2\,\rho\, C_x C_y \frac{\theta}{K}] < \lambda\,\bar{\Psi}^2\ [C_x^2 + C_y^2(1-\rho^2)] \tag{41}$$

$$C_y^2\rho^2 + C_x^2\left(\frac{\theta^2}{K^2} - 1\right) - 2\, C_x C_y\,\rho\ \frac{\theta}{K} < 0 \tag{42}$$

iv) From (11) and (31), we have

$$\mathrm{MSE}(\bar{y}_{\mathrm{AS}}) < \mathrm{MSE}\ (\bar{y}_{\mathrm{KC2}}) \tag{43}$$

$$\lambda\,\bar{\Psi}^2\ [C_y^2 + C_x^2\frac{\theta^2}{K^2} - 2\,\rho\, C_x C_y \frac{\theta}{K}] < \lambda\,\bar{\Psi}^2\,[(\frac{\bar{\psi}}{\bar{\psi} + C_x})^2\, C_x^2 + C_y^2(1 - \rho^2)] \tag{44}$$

$$C_y^2\rho^2 + C_x^2(\frac{\theta^2}{K^2} - \left(\frac{\bar{\psi}}{\bar{\psi} + C_x}\right)^2)\ 2\, C_x C_y\,\rho\ \frac{\theta}{K} < 0 \tag{45}$$

v) From (12) and (31), we have

$$\text{MSE}(\bar{y}_{\text{AS}}) < \text{MSE}\ (\bar{y}_{\text{KC3}}) \tag{46}$$

$$\Lambda\,\bar{\Psi}^2\ [C_y^2+C_x^2\frac{\theta^2}{K^2}-2\,\rho\,C_xC_y\frac{\theta}{K}]<\lambda\,\bar{\Psi}^2\,[(\frac{\bar{\psi}}{\bar{\psi}+\beta_{2x}})^2\,C_x^2+C_y^2(1-\rho^2)] \tag{47}$$

$$C_y^2\rho^2+C_x^2(\frac{\theta^2}{K^2}-\left(\frac{\bar{\psi}}{\bar{\psi}+\beta_{2x}}\right)^2)\ 2\,C_xC_y\,\rho\,\frac{\theta}{K}<0 \tag{48}$$

vi) From (13) and (31), we have

$$\text{MSE}(\bar{y}_{\text{AS}}) < \text{MSE}\ (\bar{y}_{\text{KC4}}) \tag{49}$$

$$\lambda\,\bar{\Psi}^2\ [C_y^2+C_x^2\frac{\theta^2}{K^2}-2\,\rho\,C_xC_y\frac{\theta}{K}]<\lambda\,\bar{\Psi}^2\,[(\frac{\bar{\psi}\beta_{2x}}{\bar{\psi}\,\beta_{2x}+C_x})^2\,C_x^2+C_y^2(1-\rho^2)] \tag{50}$$

$$C_y^2\rho^2+C_x^2(\frac{\theta^2}{K^2}-\left(\frac{\bar{\psi}\beta_{2x}}{\bar{\psi}\,\beta_{2x}+C_x}\right)^2)\ 2\,C_xC_y\,\rho\,\frac{\theta}{K}<0 \tag{51}$$

vii) From (14) and (31), we have

$$\text{MSE}(\bar{y}_{\text{AS}}) < \text{MSE}\ (\bar{y}_{\text{KC5}}) \tag{52}$$

$$\Lambda\,\bar{\Psi}^2\ [C_y^2+C_x^2\frac{\theta^2}{K^2}-2\,\rho\,C_xC_y\frac{\theta}{K}]<\lambda\,\bar{\Psi}^2\,[(\frac{\bar{\psi}\,C_x}{\bar{\psi}\,C_x+\beta_{2x}})^2\,C_x^2+C_y^2(1-\rho^2)] \tag{53}$$

$$C_y^2\rho^2+C_x^2\left(\frac{\theta^2}{K^2}-\frac{\bar{\psi}\,C_x}{\bar{\psi}\,C_x+\beta_{2x}}\right)-2\,C_xC_y\,\rho\,\frac{\theta}{K}<0 \tag{54}$$

viii) From (16) and (31), we have

$$\text{MSE}(\bar{y}_{\text{AS}}) < \text{MSE}\ (\bar{y}_{\text{IO}}) \tag{55}$$

$$\Lambda\,\bar{\Psi}^2\ [C_y^2+C_x^2\frac{\theta^2}{K^2}-2\,\rho\,C_xC_y\frac{\theta}{K}]<\lambda\,\bar{\Psi}^2\ [(C_y^2+(\frac{1}{ln(\bar{\psi})})^2\,C_x^2-2\,(\frac{1}{ln(\bar{\psi})})\ \rho C_xC_y] \tag{56}$$

$$C_y^2\rho^2+C_x^2\left(\frac{\theta^2}{K^2}-(\frac{1}{\ln(\bar{\psi})})^2\right)-2\,C_xC_y\,\rho\,(\frac{\theta}{K}-\frac{1}{\ln(\bar{\psi})})<0 \tag{57}$$

3. RESULTS AND DISCUSSION

In this section, we use two data sets to compare the efficiency of the proposed estimator and the mentioned estimator. The descriptive statistics about used data sets are given as follows.

Data set 1: Singh and Chaudhry [11],
y = Area under wheat in 1974, x = Area under wheat in 1971,
Ŋ = 34 ц = 20 $\bar{\Psi}$ = 856.4118 $\bar{\psi}$ = 208.8824 ρ = 0.4491 S_y = 733.1407 C_y = 0.8561 S_x= 150.5060 C_x= 0.7205 β_{2x}= 0.0974

Data set 2: Murthy [12],
y = Output for 80 factories in a region, x = Fixed capital,
Ŋ = 80 ц = 20 $\bar{\Psi}$ = 51.8264 $\bar{\psi}$ = 11.2646 ρ = 0.7941 S_y = 18.3569 C_y = 0.3542 S_x= 8.4062 C_x= 0.9484 β_{2x}= 2.8664

The Table 1 shows the MSE values along with the Percentage Relative Efficiency (PRE) of all estimators with respect to the conventional ratio estimator $\bar{y}_R$. Its formula is as follows;

$$\text{PRE} = \frac{MSE(\bar{y}_R)}{MSE(\bar{y}_i)} \text{ where } i = \bar{y}_{US}, \bar{y}_{KC1}, \bar{y}_{KC2}, \bar{y}_{KC3}, \bar{y}_{KC4}, \bar{y}_{KC5}, \bar{y}_{IO}, \bar{y}_{AS} \tag{58}$$

Table 1. MSE values of the estimators along with PRE values.

Estimators	Data set 1	PRE	Data set 2	PRE
$\bar{y}_R$	27271.90247	100	156.9718016	100
$\bar{y}_{US}$	10299.06594	264.7998	42.08169621	373.0168
$\bar{y}_{KC1}$	16673.80929	163.5613	95.26557601	164.7728
$\bar{y}_{KC2}$	16620.01056	164.0908	81.74120824	192.0351
$\bar{y}_{KC3}$	16666.50403	163.6330	77.48223015	202.5907
$\bar{y}_{KC4}$	16146.76108	168.9001	84.58337505	185.5823
$\bar{y}_{KC5}$	16663.67288	163.6608	76.66903976	204.7395
$\bar{y}_{IO}$	9775.684135	278.9769	43.87283275	357.7882
$\boldsymbol{\bar{y}_{AS}}$	**9203.137592**	**296.3327**	**4.854008365**	**3,233.8593**

Both data sets, Table 1 shows for $\boldsymbol{\bar{y}_{AS}}$ has minimum the MSE values and higher PRE values compared to $\bar{y}_R$, $\bar{y}_{US}$,$\bar{y}_{KC1}$, $\bar{y}_{KC2}$, $\bar{y}_{KC3}$, $\bar{y}_{KC4}$, $\bar{y}_{KC5}$ and $\bar{y}_{IO}$.

Table 2. Efficiency conditions on two data sets

Equation No.	Condition	Dataset1	Dataset2
35	$C_x^2\left(\frac{\theta^2}{K^2}-1\right)-2\,C_xC_y\,\rho\left(\frac{\theta}{K}-1\right)<0$	-0.0885<0	-0.3619<0
39	$C_x^2\theta^2\left(\frac{1}{K^2}-1\right)-2\,C_xC_y\,\rho\,\theta\,(\frac{1}{K}-1)<0$	-0.0726<0	-0.1580<0
42	$C_y^2\rho^2+C_x^2\left(\frac{\theta^2}{K^2}-1\right)-2\,C_xC_y\,\rho\,\frac{\theta}{K}<0$	-0.4947<0	-0.8717<0
45	$C_y^2\rho^2+C_x^2(\frac{\theta^2}{K^2}-\left(\frac{\bar{\psi}}{\bar{\psi}+C_x}\right)^2)\;2\,C_xC_y\,\rho\,\frac{\theta}{K}<0$	-0.4912<0	-0.8654<0
48	$C_y^2\rho^2+C_x^2(\frac{\theta^2}{K^2}-\left(\frac{\bar{\psi}}{\bar{\psi}+\beta_{2x}}\right)^2)\;2\,C_xC_y\,\rho\,\frac{\theta}{K}<0$	-0.4943<0	-0.8708<0
51	$C_y^2\rho^2+C_x^2(\frac{\theta^2}{K^2}-\left(\frac{\bar{\psi}\beta_{2x}}{\bar{\psi}\,\beta_{2x}+C_x}\right)^2)\;2\,C_xC_y\,\rho\,\frac{\theta}{K}<0$	-0.4598<0	-0.5425<0
54	$C_y^2\rho^2+C_x^2\left(\frac{\theta^2}{K^2}-\frac{\bar{\psi}\,C_x}{\bar{\psi}\,C_x+\beta_{2x}}\right)-2\,C_xC_y\,\rho\,\frac{\theta}{K}<0$	-0.4941<0	-0.8705<0
	$C_y^2\rho^2+C_x^2\left(\frac{\theta^2}{K^2}-(\frac{1}{\ln(\bar{\psi})})^2\right)-2\,C_xC_y\,\rho\,(\frac{\theta}{K}-\frac{1}{\ln(\bar{\psi})})<0$	-0.0379<0	-0.3098<0

Table 2 shows that all the efficiency conditions are satisfied on two data sets used in the analysis. Finally, we conduct simulation study compare the efficiency of the proposed estimator and the mentioned estimator based on different distributions.

- We consider sampling from bivariate normal distribution considering N= 10000 with parameters $\underline{\mu}$ = [2, 2] and

$$\Sigma = \begin{bmatrix} 1.5 & 2 \\ 2 & 3 \end{bmatrix}.$$

Table 3 lists the MSE values of the estimators for n = 100, 200, 300, 400, and 500. The number of Simulations is 5000.

Table 3. The MSE values for n =100, 200, 300, 400, and 500.

n	$\bar{y}_R$	$\bar{y}_{US}$	$\bar{y}_{KC1}$	$\bar{y}_{KC2}$	$\bar{y}_{KC3}$	$\bar{y}_{KC4}$	$\bar{y}_{KC5}$	$\bar{y}_{IO}$	$\bar{y}_{AS}$
100	0.004611	0.002813	0.028416	0.015158	0.006042	0.022372	0.005162	0.015098	**0.002679**
200	0.002049	0.001250	0.012629	0.006737	0.002685	0.009943	0.002294	0.006710	**0.001190**
300	0.001195	0.000729	0.007367	0.003930	0.001566	0.005800	0.001338	0.003914	**0.000694**
400	0.000769	0.000469	0.004736	0.002526	0.00101	0.003729	0.000860	0.002516	**0.000446**
500	0.000512	0.000313	0.003157	0.001684	0.000671	0.002486	0.000574	0.001678	**0.000298**

- We consider sampling from bivariate normal distribution with parameters $\underline{\mu}$ = [15, 3] and

$$\Sigma = \begin{bmatrix} 1.5 & 2 \\ 2 & 3 \end{bmatrix}.$$

Table 4 lists The MSE values of the estimators for n = 100, 200, 300, 400, and 500.

Table 4. The MSE values for n =100, 200, 300, 400, and 500.

n	$\bar{y}_R$	$\bar{y}_{US}$	$\bar{y}_{KC1}$	$\bar{y}_{KC2}$	$\bar{y}_{KC3}$	$\bar{y}_{KC4}$	$\bar{y}_{KC5}$	$\bar{y}_{IO}$	$\bar{y}_{AS}$
100	0.490812	0.427296	0.654116	0.467106	0.169822	0.580421	0.090088	0.374953	**0.052681**
200	0.218139	0.1899093	0.290718	0.207603	0.075476	0.257965	0.040039	0.166646	**0.023414**
300	0.127248	0.1107804	0.169586	0.121102	0.044028	0.150480	0.023356	0.097210	**0.013658**
400	0.081802	0.071216	0.109019	0.077851	0.028304	0.096737	0.015015	0.062492	**0.008780**
500	0.054535	0.04747733	0.072680	0.051900	0.018869	0.064491	0.010010	0.041661	**0.005853**

- We consider simulation from standard uniform distribution, considering N = 10000.

Table 5 lists the MSE values of the estimators for n = 100, 200, 300, 400, and 500.

Table 5. The MSE of the estimators from standard uniform distribution.

n	$\bar{y}_R$	$\bar{y}_{US}$	$\bar{y}_{KC1}$	$\bar{y}_{KC2}$	$\bar{y}_{KC3}$	$\bar{y}_{KC4}$	$\bar{y}_{KC5}$	$\bar{y}_{IO}$	$\bar{y}_{AS}$
100	0.001589	0.001500	0.001630	0.000989	0.000801	0.001539	0.000800	0.002733	**0.000924**
200	0.000786	0.000742	0.000807	0.000489	0.000396	0.000762	0.000396	0.001353	**0.000457**
300	0.000519	0.000490	0.000533	0.000323	0.000262	0.000503	0.000261	0.000893	**0.000302**
400	0.000385	0.000364	0.000395	0.000240	0.000194	0.000373	0.000194	0.000663	**0.000224**
500	0.000305	0.000288	0.000313	0.000190	0.000154	0.000295	0.000154	0.000525	**0.000177**

- We consider simulation from exponential distribution, considering N=10000, X~ exp (0.1) and Y~exp (0.1).

Table 6 lists the MSE values of the estimators for n = 100, 200, 300, 400, and 500.

Table 6: The MSE values of the estimators from exponential distribution

n	$\bar{y}_R$	$\bar{y}_{US}$	$\bar{y}_{KC1}$	$\bar{y}_{KC2}$	$\bar{y}_{KC3}$	$\bar{y}_{KC4}$	$\bar{y}_{KC5}$	$\bar{y}_{IO}$	$\bar{y}_{AS}$
100	1.696749	1.694619	1.770398	1.625508	0.920078	1.768171	0.919817	1.035818	**0.917239**
200	0.839805	0.838751	0.876258	0.804544	0.455392	0.875155	0.455263	0.512678	**0.453987**
300	0.554157	0.553461	0.578211	0.530890	0.300497	0.577484	0.300412	0.338297	**0.299570**
400	0.411333	0.410817	0.429188	0.394063	0.223049	0.428648	0.222986	0.251107	**0.222361**
500	0.325639	0.325230	0.339773	0.311966	0.176581	0.339346	0.176531	0.198793	**0.176036**

- We consider 1000 Simulation from t- Distribution with Parameters $\underline{\mu}$ = [15, 3] and

$$\Sigma = \begin{bmatrix} 1.5 & 2 \\ 2 & 3 \end{bmatrix}.$$

Table 7 lists the MSE values of the estimators for n = 100, 200, 300, 400, and 500.

Table 7. The MSE values of the estimators from t- distribution.

n	$\bar{y}_R$	$\bar{y}_{US}$	$\bar{y}_{KC1}$	$\bar{y}_{KC2}$	$\bar{y}_{KC3}$	$\bar{y}_{KC4}$	$\bar{y}_{KC5}$	$\bar{y}_{IO}$	$\bar{y}_{AS}$
100	0.5102498	0.4848091	0.687098	0.484858	0.048674	0.657511	0.021653	0.398939	**0.019118**
200	0.2267777	0.2154707	0.305377	0.215493	0.021633	0.292227	0.009623	0.177306	**0.008497**
300	0.132287	0.1256913	0.178136	0.125704	0.012619	0.170466	0.005613	0.103429	**0.004957**
400	0.085042	0.080802	0.114516	0.080810	0.008112	0.109585	0.003609	0.066490	**0.003186**
500	0.056694	0.053868	0.076344	0.053873	0.005408	0.073057	0.002406	0.044327	**0.002124**

Table 3 considered same mean values for both response and auxiliary variables while Table 4 shows different values of mean for both variables. Simulation results from bivariate normal population in both tables show that $\bar{y}_{AS}$ performs better as compared to $\bar{y}_R$, $\bar{y}_{US}$, $\bar{y}_{KC1}$, $\bar{y}_{KC2}$, $\bar{y}_{KC3}$, $\bar{y}_{KC4}$, $\bar{y}_{KC5}$, $\bar{y}_{IO}$.

Table 5 shows the MSE values generated from uniform distribution, Table 6 shows the MSE values generated from exponential distribution while Table 7 represents the MSE values generated from t distribution. It is obvious from the results of Tables 5-7 that for all n= 100, 200, 300, 400 and 500 the proposed estimator $\bar{y}_{AS}$ has minimum MSE values as compared to $\bar{y}_R$, $\bar{y}_{US}$, $\bar{y}_{KC1}$, $\bar{y}_{KC2}$, $\bar{y}_{KC3}$, $\bar{y}_{KC4}$, $\bar{y}_{KC5}$, $\bar{y}_{IO}$.

4. CONCLUSIONS

An improved logarithmic ratio-type estimator of $\bar{\Psi}$ using the auxiliary information in simple random sampling is proposed. The results of Tables 1 shows the MSE value of $\bar{y}_{AS}$ is minimum and the PRE is maximum as compared to$\bar{y}_R$, $\bar{y}_{US}$, $\bar{y}_{KC1}$, $\bar{y}_{KC2}$, $\bar{y}_{KC3}$, $\bar{y}_{KC4}$, $\bar{y}_{KC5}$, $\bar{y}_{IO}$. A total of eight efficiency conditions are derived and all eight are satisfied on the two data sets. The results of simulation study i.e Tables 3-7 also supports the claim that $\bar{y}_{AS}$ performs better as compared to $\bar{y}_R$,$\bar{y}_{US}$, $\bar{y}_{KC1}$, $\bar{y}_{KC2}$, $\bar{y}_{KC3}$, $\bar{y}_{KC4}$, $\bar{y}_{KC5}$, $\bar{y}_{IO}$. In future study, we hope to extend the proposed estimators presented in this article to the stratified random sampling.

REFERENCES

[1] Singh, H. P, Tailor, R., *Statistica*, **65**(3), 301, 2005.
[2] Kadilar, C., Cingi, H., *Hacettepe Journal of Mathematics and Statistics*, **35**(1), 103, 2006.
[3] Singh, H. P., Vishwakarma, G. K., *Australian Journal of Statistics*, **36**(3), 217, 2007.
[4] Haq, A., Shabbir, J., *Communications in Statistics - Theory and Methods*, **42**(5), 782, 2013.
[5] Singh, H. P., Solanki, R. S., *Communications in Statistics - Theory and Methods*, **42**(15), 2718, 2013.
[6] Subramani, J., Kumarapandiyan, G., *Pakistan Journal of Statistics and Operation Research*, **9**(2), 137, 2013.

[7] Muneer, S., Shabbir, J., Khalil, A., *Communication in Statistics Theory and Methods*, **46**(5), 2181, 2017.
[8] Izunobi, C. H., Onyeka, A. C., *International Journal of Advanced Statistics and Probability*, **7**(2), 145, 2019.
[9] Upadhyaya, L. N, Singh, H. P., *Biometrical Journal*, **41**(5), 627, 1999.
[10] Kadilar, C., Cingi, H., *Applied Mathematics Letters*, **19**(1), 75, 2006.
[11] Singh, D., Choudhary, F. S., *Theory and Analysis of Sample Survey Designs,* Wiley Eastern Limited, New Delhi, 1986.
[12] Murthy, M.N., *Sampling theory and methods, Statistical Publishing Society,* Calcutta, India, 1967.