

ELEMENTARY PROOFS FOR THE FERMAT'S LAST THEOREM IN Z USING ONE TRICK FOR A RESTRICTION IN Z_p

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Abstract. An elementary and short proof of Fermat's Last Theorem (FLT) is presented, which is understandable even to a student. Perhaps this proof is precisely the last proof, which could be similar to Fermat's proof. Restricting some coefficients of polynomials by value 0, except for the first term, allows to prove the Fermat's Last Theorem for domain Z, since in this case the canonical representation of p-adic numbers is limited to only one digit in the corresponding p-ary system. It was shown within the framework of elementary algebra, which corresponds to the Pythagorean theorem (PT) that the assumption of the existence of certain "Fermat's triples" (FT), as integer solutions of Fermat's Last Theorem, can not be possible in Z due to some fatal inconsistencies for the PT. Found means the PT. Some equations in Z_p were shown for n=3, 4 and 5.

Keywords: Fermat's last theorem; Pythagorean theorem; p-adic integers; numbers; integers.

1. INTRODUCTION

Theorem 1. Fermat's Last Theorem (FLT) was been proven in [1] and states: No three positive integers a, b, and c satisfy the equation aⁿ + bⁿ = cⁿ for any integer value of n greater than 2.

Anyone can find the roots for three numbers at the same time for n=3 for other numbers that are not Pythagorean Triples, PTs. For example,

$$(\dots(0)2_{47})^3 + (\dots(0)3_{47})^3 = (\dots(0)13_{47})^3 = \dots(0)35_{47} \text{ for } \mathbf{Z}_{47}.$$

There is no problem with the equations aⁿ+bⁿ=cⁿ for Z_p. For example, Pythagorean triples, PTs, i.e., the integer solutions in Z domain of the Pythagorean Theorem Equation, can give an infinite number of integer solutions for Fermat's equation for n=4 in Z_p domain:

1) In Z₆₁ for PTs 3, 4, 5, i.e., transformations of this PTs and forms of its squares, square roots and negative values from Z into Z₆₁ are shown below:

$\sqrt{16} = \dots(60)57;$	$a = \sqrt{3} = \dots0198;$	$a = \sqrt{(-3)} = \dots405427;$
$\sqrt{25} = \dots(60)56;$	$b = \sqrt{4} = \dots(60)59;$	$b = \sqrt{(-4)} = \dots273839;$
	$c = \sqrt{5} = \dots35826;$	$c = \sqrt{(-5)} = \dots64819.$

In Z₁₀₉ for PTs 3, 4, 5, i.e., transformations of this PTs and forms of its squares, square roots and negative values from Z into Z₁₀₉ are shown below:

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