ORIGINAL PAPER

A MATHEMATICAL MODEL TO ADDRESS OUT-OF-SCHOOL CHILDREN MENACE FOR ACTUALIZATION OF SUSTAINABLE DEVELOPMENT IN NIGERIA

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Abstract.*The rising number of out-of-school children (OOSC) constitutes a major obstacle to growth and development in Nigeria. Despite various institutional frameworks and policy initiatives, Nigeria accounts for the highest number of OOSC worldwide with one out of every five OOSC globally residing in Nigeria. In an attempt to characterize dynamics of OOSC and how it could be tackled to fount sustainable development in Nigeria, a new mathematical model was formulated. The validity of the model was examined using some mathematical theorems and the model equilibria were derived. The inclusive schooling ratio, an analytic parameter that quantified the extent to which the rising OOSC was being tackled to fount development, was computed. The stability properties of the model were studied via stability theory of differential equations based on the derived inclusive schooling ratio. Sensitivity analysis was conducted for some major parameters following the normalized forward sensitivity index approach to examine the relative importance of the model parameters to OOSC expansion and contraction. Numerical simulation was later conducted to justify the theoretical results and the results of the simulation showed that efforts to fount development through minimization of OOSC were fruitful if the inclusive schooling ratio was greater than one otherwise the menace of OOSC persisted. The policy implication of the result is that tackling the menace of OOSC to fount sustainable development in Nigeria is a long-term process and any policies designed to pursue the course must be sustained.*

Keywords: *growth and development; out-of-school children; model and equilibria; inclusive schooling ratio; simulation.*

1. INTRODUCTION

Human capital is the bedrock of development. It is the accumulation of knowledge, habits, personality and social peculiarities manifested in the capacity to function in order to create economic values [1]. Development and expansion of human capacity are anchored on training and education and the essence is to increase productivity as well as the quality of goods produced. Every nation invests heavily on manpower as it is the bedrock of development. It is argued in [2] that qualitative education is imperative for influencing population growth, empowering women, alleviating poverty, defending children against sexual exploitation and exploitative labor, safeguarding the environment and promoting

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democracy and human rights. Development is a process that generates progress, growth and definite change. The consequence of development is the increase in quality of life as well as the creation of employment opportunities and expansion of income without impairing environmental resources [3].

The bedrock of human development is childhood development. Childhood development is important not only to secure a fulfilled and productive life for every child but also to stimulate progress of a nation. Studies have indicated that more than half of an individual intellectual capability is formed before six years of age and a timely intervention is capable of having a lifelong impact on personality, intellectual capacity and social behavior [1]. Functionally, a child, based on the definition given in [4], is someone in the middle of birth and puberty. The kind of future expected from a child is often prepared during the period of childhood. The implications of this reality brought about Education For All (EFA) declaration of 1990, Millenium Development Goals (MDGs) of 2000 and Sustainable Development Goals (SDGs) of 2015 – that identified education as essential instrument for preparing children for a bright future [5]. Education is regarded by all the movements (EFA, MDGs and SDGs) as an important vehicle for development that cannot be achieved when many children are out of school.

The need for purposeful life encourages school enrollment, but the rising number of out-of-school children (OOSC) remains a major challenge confronting Nigeria. The term OOSC, in the context of Nigeria, includes all dropouts as well as children who have never enrolled in school or those who are receiving Qu'ranic education [6-7]. In many parts of northern Nigeria, a good number of Muslim children enroll in Qu'ranic schools, where emphasis is placed only on recitement and memorization of the holy Qu'ran, but not on literacy, numeracy and other skills required to perform adequately in the present world [5]. The government of Nigeria regards children receiving such education to be officially out of school [7]. Nigeria remains the headquarters of OOSC worldwide with one out of every five OOSC residing in Nigeria [8]. The figure of OOSC for Nigeria (with 201 million population) is ahead of India (with 1.4 billion population), Indonesia (with 270.6 million population) and Pakistan (with 216.6 million population) [8].The country has maintained its lead for the figure of OOSC globally and the number has continued rising since 2014 up until 16 million in 2019 [8]. The notable composition of individuals in the OOSC set include people who abandon schooling due to rising unemployment of graduates, children of migrant fishermen, nomads and beggars, Almajiri, children with disabilities and children from conflict zones whose parents have been displaced through various forms of violence and attacks [8].

Many factors complicate OOSC issues in Nigeria but the main factors are unemployment, poverty and insecurity. First, the unemployment rate has been increasing in Nigeria. It reached 23.13% during the third quarter of 2018 and 40% in 2022 [9-11]. The existence of teeming unemployed graduates is enough reason for massive dropout. Many youths are discouraged and are abandoning schooling because they look at a future that does not give them hope. Poverty is another serious issue that is promoting the menace of OOSC in Nigeria as the country has become the headquarters of poverty where the largest number of poor people reside globally [12]. According to [12], the population of individuals living in extreme poverty in Nigeria is estimated at 89.5 million in 2020, a population that is more than the population of Germany. Olaniyi [6] observed that the major factor influencing OOSC in Nigeria is economic factors. This observation is corroborated by a certain report that indicated that 9 million out of 10.5 million OOSC in Nigeria were children of migrant fishermen, beggars, nomads and other poor individuals in the society [11].

Further, many children are being kept out-of-school especially in the northern part of Nigeria through the activities of insurgents which have led to the displacement of more than 1.7 million people, 56% of whom are children [13]. The violence has also accounted for the

deaths of 2,295 teachers while more than 19,000 others have been displaced [14]. Again, a report by the United Nations Children Emergency Fund (UNICEF) claimed that over 1,400 schools in the north eastern part of Nigeria have been damaged, looted or destroyed while over 600,000 children have lost access to education through Boko Haram insurgency [14]. The violence has also instigated a series of kidnapping and abduction with about 1,000 students kidnapped from various schools by insurgents, majority of who are females [8]. A recent report by Global Coalition to Protect Education from Attack (GCPEA) indicated that there were at least 10 attacks on schools between 2020 and 2021 with more than 1,400 students abducted [15].

The menace of OOSC has been a subject of intense study both in Nigeria and around the world. Okoh et al. [5] investigated enhancing factors for OOSC and consequences for sustainable development in the north central geo-political zone of Nigeria. They discovered that the menace of OOSC is attributable to teenage pregnancy, poverty, child labor and distance to schools. They therefore recommended policy initiatives in the form of poverty alleviation and tough punishment for the parents whose children are not in school to end the scourge of OOSC in the region. Another study was conducted by [8] to examine the causes of rising OOSC in Nigeria. The researchers blamed the increasing number of OOSC in Nigeria on feeble policy and weak institutional structure, traditions and norms of the people as well as exclusion of disable children from policy consideration. They advocated for counseling and persuasion to influence people's customs and traditions as well as inclusion of disable children in policy formulation to end the challenges of OOSC in Nigeria.

At the foreign level, [16] investigated the menace of OOSC in Greece. They discovered that dropping out from secondary schools is mostly influenced by family factors such as parental professional incitement, parental school assistance, parental instigation for maximum effort, parental help in poor school performance and parental awareness of children's social life. In another study, [17] examined the reasons for the alarming rate of OOSC in Pakistan. The researchers identified three factors which are similar to the factors responsible for the increasing rate of OOSC in Nigeria – distance to school, large families and poverty. The issue of OOSC is a global phenomenon and has attracted attention of many more researchers [18-21]. Growth and development is a dream of every nation, but cannot be realized when many children are out of school. While many studies have considered factors that influence OOSC as well as identified the nexus between OOSC and development, no researcher has used a mathematical modeling approach to investigate the connection between OOSC and sustainable development. It is against this backdrop that the present study aims to employ mathematical modeling methods to study how the rising OOSC can be tackled to fount sustainable development in Nigeria.

2. MODEL FORMULATION

To conduct the study, a compartmental model is designed to examine the menace of OOSC and how it can be tackled to fount sustainable development in Nigeria. The model consists of four compartments P(t), E(t), D(t) and A(t). P(t) is the compartment for parents from diverse works of lives as well as different socio-cultural affinities and economic status. E(t) denotes proportion of children who are enrolled in schools, D(t) represents proportion of children who are out of school while A(t) is the proportion of OOSC whose parents and themselves are now seeing the reasons to enroll in school or go back to school following various policies that are implemented by the government to make schooling and school enrollment attractive such as provision of instant employment to fresh graduates, mass

employment of teeming unemployed graduates, increase in worker's salaries, poverty alleviation, increased budgetary allocation to the education sector, provison of adequate and reliable security, building of more schools especially specialized schools for the handicapp, nomadic children, Almajiri and children whose parents have been displaced through various conflicts and attacks. Compartment A(t) comes into existence because implemented policies cannot yield instant results. It will take some time before government could be taken serious by OOSC D(t) and their parents in P(t). The movement between the compartments of the model is represented in Fig. 1.

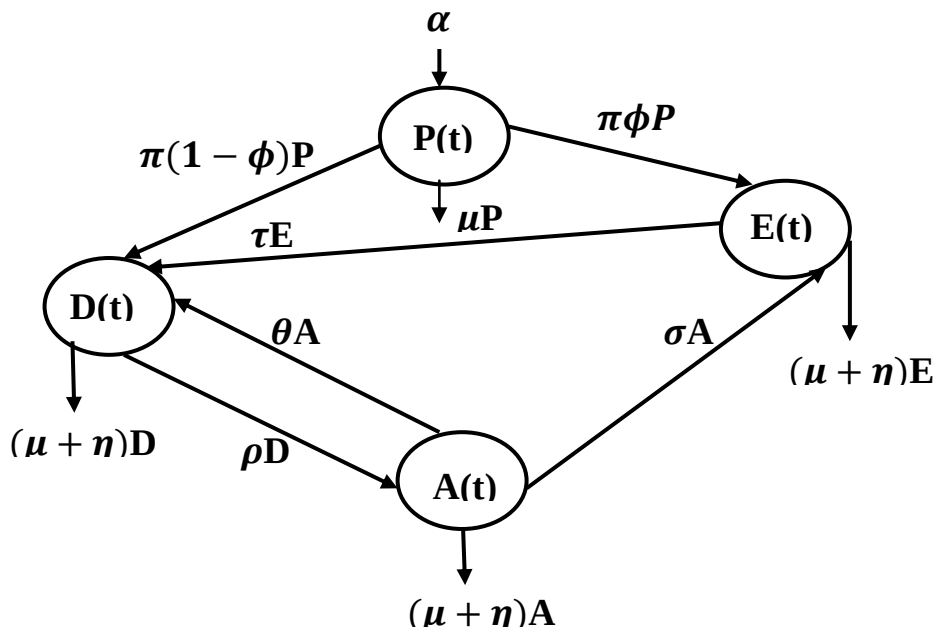

Figure 1. Transfer Diagram of the Model.

In Fig. 1, individuals become parents at rate α. Generally, children who are in school and those who are out of school are from parents. It is assumed that relatives or government becomes parents for the orphans. Therefore, if ϕ is the proportion of children who are in school, $(1-\phi)$ becomes the proportion of children who are out of school. Hence, ϕP and $(1-\phi)$P move to E(t) and D(t) respectively. With the implementation of various policies to promote schooling and school enrollment, individuals in D(t) moves to A(t) at rate ρ. If these programs are sustained then individuals in A(t) finally go back or enroll in school E(t) at rate σ. If the programs are not sustained, individuals in A(t) moves back to D(t) at rate θ. Also, individuals in E(t) are discouraged and begins to join D(t) at rate τ. Asides, natural death which occurs for individuals in all compartments at the same rate μ, child deaths due to childhood diseases occur at rate η in children compartments D(t), E(t) and A(t). Child death is a major issue that cannot be ignored in any studies concerning children in Nigeria as the country recorded the second highest number of child death globally in 2017 [22]. The menace of OOSC is becoming a fount to development process if more and more individuals are moving from D(t) to A(t) and subsequently from A(t) to E(t) so that $D(t) \to 0$ otherwise the polices that are put in place to tackle the menace of OOSC are not sustained and the menace persists. Considering these assumptions as well as the transfer diagram, the menace of OOSC and how it can be tackled to fount sustainable development in Nigeria are characterized by the following set of equations.

$$\frac{dP}{dt} = \alpha - \pi\phi P - \pi(1-\phi)P - \mu P \tag{1}$$

$$\frac{dD}{dt} = \pi(1-\phi)P + \theta A + \tau E - \rho D - (\mu+\eta)D \tag{2}$$

$$\frac{dA}{dt} = \rho D - \theta A - \sigma A - (\mu+\eta)A \tag{3}$$

$$\frac{dE}{dt} = \pi\phi P + \sigma A - \tau E - (\mu+\eta)E \tag{4}$$

subject to

$$P(0) = P_0,\ D(0) = D_0,\ A(0) = A_0,\ E(0) = E_0.$$

The parameters definitions are contained in Table 1.

Table 1. Nomenclature for the parameters of the model.

Parameters Description	**Symbol**
Rate of becoming parents	α
Rate at which children move out from their parents	π
Proportion of children who are in school	ϕ
Rate at which OOSC are convinced of the implemented policies	ρ
Rate at which convinced OOSC go back or enroll in school	σ
Rate at which convinced OOSC are discouraged from going back or enrolling in school due to discontinuity of the implemented policies	θ
Rate at which children who are in school are dropping out due to discontinuity of the implemented policies	τ
Death rate due to natural causes	μ
Death rate due to childhood diseases	η

2.1. BASIC FEATURES OF THE MODEL

2.1.1. Existence of solutions

The fundamental tool to establish that there exist solutions for the proposed model is existence of solution theorem. If there exists solutions, the menace of OOSC and how it can be tackled to fount sustainable development in Nigeria can be predicted. The famous Derrick and Grossman theorem, which has been explored in various studies, see [10] and [23] for example, shall be employed to establish existence of solutions for the proposed model.

Lemma 2.1. [23] Let Ω represents a region

$$|t - t_0| \le m,\ \|x - x_0\| \le n, x = (x_1, x_2, \cdots, x_n),\ x_0 = (x_{10}, x_{20}, \cdots, x_{n0}),$$

and assuming the Lipschitzian criterion $\|f(t,x_1) - f(t,x_2)\| \le k\|x_1 - x_2\|$ is fulfilled by $f(t,x)$, whenever (t,x_1) and (t,x_2) are in Ω, where k is nonnegative. Then, there exists a unique continuous vector solution $x(t)$ in the interval $t - t_0 \le \delta$ such that $\delta > 0$.

It is necessary to consider that the condition is fulfilled based on the requirement that $\frac{\partial f_i}{\partial x_j}$ for i, j = 1, 2, 3, . . . are continuous and bounded in Ω.

Theorem 2.2. The model solutions with initial conditions P(0)≥ 0, D(0)≥ 0, A(0)≥ 0, E(0)≥ 0 exist as well as unique in $\mathbb{R}_+^4$ for all $t \geq 0$.

Proof: We can write the RHS of the model as:

$$f_1 = \alpha - \pi\phi P - \pi(1-\phi)P - \mu P$$

$$f_2 = \pi(1-\phi)P + \theta A + \tau E - \rho D - (\mu+\eta)D$$

$$f_3 = \rho D - \theta A - \sigma A - (\mu+\eta)A$$

$$f_4 = \pi\phi P + \sigma A - \tau E - (\mu+\eta)E.$$

It can be shown that $\frac{\partial f_i}{\partial x_j}$ are continuous and deduce that $\left|\frac{\partial f_i}{\partial x_j}\right| < \infty$ for i, j = 1, 2, ..., 4, where x_1 = P, x_2 = D, x_3 = A, and x_4 = E. Therefore, by Lemma 2.1, the model has unique solutions.

2.1.2. Invariant region

The boundedness feature of epidemic models shall be employed to show the invariant región of the proposed model. Here,

$$N(t) = P(t) + D(t) + A(t) + E(t)$$

such that

$$\begin{aligned}\frac{dN}{dt} &= \frac{dP}{dt} + \frac{dD}{dt} + \frac{dA}{dt} + \frac{dE}{dt} \\ &= \alpha - \mu N(t) - \eta(D + A + E), \\ &\leq \alpha - \mu N(t).\end{aligned}$$

The solution for the above inequality, following Standard Comparison Theorem [24], can be expressed as

$$N(t) \leq \frac{\alpha}{\mu} + \left(N(0) - \frac{\alpha}{\mu}\right)e^{-\mu t}.$$

$N(0)$ is the initial value, hence, $N(t) = N(0)$ at $t = 0$.

Further, it is established that $N(t) \to \frac{\alpha}{\mu}$ as $t \to \infty$. Therefore, $N(t)$ remains bounded as $0 \leq N(t) \leq \frac{\alpha}{\mu}$. Hence, the invariant region or feasibility región for the proposed model is positive and is given as

$$\Omega = \left\{(\mathrm{P}, \mathrm{D}, \mathrm{A}, \mathrm{E}) \in \mathbb{R}_+^4 : N(t) \leq \frac{\alpha}{\mu}\right\}.$$

Theorem 2.3. The solutions of the system preserve positivity.

Proof: Assuming $\{P(t), D(t), A(t), E(t)\}$ represent the solutions of the model variables for $t \geq 0$ with nonnegative initial conditions $\{A(0) > 0, G(0) > 0, E(0) > 0, I(0) > 0\}$. From (1),

$$\begin{aligned} \frac{dP}{dt} &= \alpha - \pi\phi P - \pi(1-\phi)P - \mu P, \Rightarrow \\ \frac{dP}{dt} &\geq -(\mu+\pi)P, \Rightarrow \\ \frac{dP}{P} &\geq -(\mu+\pi)dt. \Rightarrow \\ \int \frac{dP}{P} &\geq -\int (\mu+\pi)\, dt, \Rightarrow \\ P(t) &\geq P_0 e^{-(\mu+\pi)t} \geq 0. \end{aligned} \tag{5}$$

Also, consider (2),

$$\begin{aligned} \frac{dD}{dt} &= \pi(1-\phi)P + \theta A + \tau E - \rho D - (\mu+\eta)D, \Rightarrow \\ \frac{dD}{dt} &\geq -(\mu+\rho+\eta)D, \Rightarrow \\ \frac{dD}{D} &\geq -(\mu+\rho+\eta)dt, \Rightarrow \\ \int \frac{dD}{D} &\geq -\int (\mu+\rho+\eta)dt, \Rightarrow \\ D(t) &\geq D_0 e^{-(\mu+\rho+\eta)t} \geq 0. \end{aligned} \tag{6}$$

In (3),

$$\begin{aligned} \frac{dA}{dt} &= \rho D - \theta A - \sigma A - (\mu+\eta)A, \Rightarrow \\ \frac{dA}{dt} &\geq -(\mu+\theta+\sigma+\eta)E, \Rightarrow \\ \frac{dA}{A} &\geq -(\mu+\theta+\sigma+\eta)dt, \Rightarrow \\ \int \frac{dA}{A} &\geq -\int (\mu+\theta+\sigma+\eta)dt, \Rightarrow \\ A(t) &\geq A e^{-(\mu+\theta+\sigma+\eta)t} \geq 0. \end{aligned} \tag{7}$$

Lastly, in (4),

$$\begin{aligned} \frac{dE}{dt} &= \pi\phi P + \sigma A - \tau E - (\mu+\eta)E, \Rightarrow \\ \frac{dE}{dt} &\geq -(\mu+\tau+\eta)I, \Rightarrow \\ \frac{dE}{E} &\geq -(\mu+\tau+\eta)dt, \Rightarrow \\ \int \frac{dE}{E} &\geq -\int (\mu+\tau+\eta)dt, \Rightarrow \\ E(t) &\geq E e^{-(\mu+\tau+\eta)t} \geq 0. \end{aligned} \tag{8}$$

Since$e^k > 0$ for all k then, the solutions $P(t), D(t), A(t)$ and $E(t)$ of the model are positive.

3. QUALITATIVE ANALYSIS

3.1. EQUILIBRIA

In epidemic modeling, there exists two equilibria – disease-free equilibrium (DFE) and endemic equilibrium (EE). There exists disease-free equilibrium if there is no infection in the population but the population is at endemic equilibrium if the disease is present in the population. In this analysis, enhancing developmental processes for Nigeria is anchored on reducing the population of OOSC while reducing the population of OOSC itself is anchored not only on the implementation of policies that will make schooling and school enrollment attractive but also on the sustainability of the implemented policies. Therefore, two equilibria exist as well – total schooling equilibrium (TSE) when all the children are in school and partial schooling equilibrium (PSE). As in epidemic modeling and with reference to Figure 1, it is assumed that TSE exists when every child is in school so that $\phi = 1, \rho = 0, \theta = 0, \sigma = 0$ and $\tau = 0$ while PSE exists when all children are not in school so that the inequality $0 < x < 1$ is satisfied by all the model parameters. Therefore, the system (1)-(4) permits a total schooling equilibrium (TSE) denoted by

$$\mathrm{S}_0 = (\mathrm{P}_0, \mathrm{D}_0, \mathrm{A}_0, \mathrm{E}_0) = \left(\frac{\alpha}{(\mu+\pi)}, 0, 0, \frac{\alpha\pi}{(\mu+\pi)(\mu+\eta)}\right),$$

and a partial schooling equilibrium (PSE) denoted by $\mathrm{S}^* = (\mathrm{P}^*, \mathrm{D}^*, \mathrm{A}^*, \mathrm{E}^*)$ with coordinates

$$\mathrm{P}^* = \frac{\alpha}{(\mu+\pi)},$$

$$\mathrm{D}^* = \frac{\mu+\theta+\sigma+\eta}{\rho}\, A,$$

$$\mathrm{E}^* = \frac{1}{(\mu+\tau+\eta)}\left[\frac{\alpha\phi\pi}{(\mu+\pi)} + \sigma A\right],$$

$$\mathrm{A}^* = \frac{\rho(\mu+\tau+\eta)[\alpha\pi(1-\phi)(\mu+\tau+\eta)+\tau\alpha\phi\pi]}{(\mu+\tau+\eta)(\mu+\rho+\eta)(\mu+\theta+\sigma+\eta)-[\theta\rho(\mu+\tau+\eta)+\tau\rho\sigma]}.$$

3.2. INCLUSIVE SCHOOLING RATIO, S_R

In epidemic modeling (see for instance, [25-28]), the reproduction number, R_0 measures the average effect of an infectious individual in terms of the number of the susceptibles they are able to infect throughout the period of their infectiousness [29]. If $R_0 > 1$, an infectious individual is able to infect at least one person and the disease spreads in the population but if $R_0 < 1$, the infectious individual is unable to infect a single individual and the disease dies out or fails to spread in the population [30-31]. In the present analysis, inclusive schooling ratio, S_R, is like the reproduction number, R_0. It measures the extent of sustainability of the policies implemented to encourage schooling and school enrolment in terms of the population of OOSC that are convinced to go back or enroll in school and the population of convinced OOSC that are going back or enrolling in school. Overcoming the

menace of OOSC in Nigeria is a complex process because getting more and more OOSC willing to go back or enroll in school is a function of implementation of several enabling policies while retaining children who are already in school as well as OOSC who are willing to go back or enroll in school is a function of sustainability of the enabling implemented policies. Therefore, tackling the menace of OOSC to fount development in Nigeria in terms of implemented policies depend on the rate at which OOSC are being convinced of the implemented policies (ρ), the rate at which the convinced OOSC go back or enroll in school (σ), the proportion of children who are in school (ϕ), the rate at which the convinced OOSC are being discouraged from going back or enrolling in school due to discontinuity of the implemented policies (θ) and the rate at which children who are in school are dropping out due to discontinuity of the implemented policies (τ). Hence, following [32-33], the inclusive schooling ratio S_R is defined as the ratio of the product of the proportion of children who are in school (ϕ) and the sum of rates at which the implemented policies convinced and brought OOSC to school ($\rho + \sigma$) to the sum of the rates at which the convinced OOSC and the children who are already in school are being discouraged from schooling due to discontinuity of the implemented policies ($\theta + \tau$) i.e.,

$$S_R = \frac{\phi(\rho + \sigma)}{(\theta + \tau)}. \tag{9}$$

In (9), the inclusive schooling ratio (S_R) is inversely proportional to the sum of the rates at which the convinced OOSC and the children who are already in school are being discouraged from schooling due to discontinuity in the implemented policies ($\theta + \tau$). The implication is that the rising OOSC would become a fount to actualize development in Nigeria if the enabling implemented policies are sustained so that the convinced OOSC and the children who are already in school are not discouraged from schooling. This is achievable if ($\theta + \tau$) reduces continuously. Following epidemic modeling principle as in [31], OOSC menace becomes a fount of development in Nigeria if $S_R > 1$ otherwise the enabling implemented policies are not sustained and the menace of OOSC persists (i.e., $S_R < 1$). The condition $S_R > 1$ ensures that the education sector is being adequately funded so that all the children who are in school are actually learning because schooling does not always results in learning especially when the education sector is poorly financed. A report by UNICEF indicated that children who are in school but who are learning nothing outnumbered those who are out of school in Nigeria [34].

3.3. STABILITY OF TOTAL SCHOOLING EQUILIBRIUM, S_0

Theorem 3.1. The total schooling equilibrium of the model is stable if $S_R > 1$ otherwise it is unstable.

Proof: The total schooling equilibrium is stable and $S_R > 1$ if it is established that all the eigenvalues of the variational matrix of the system (1)-(4) are negative.

Linearizing (1) – (4) about S_0 (i.e., $D = A = 0$, $\phi = 1, \rho = 0, \theta = 0, \sigma = 0$ and $\tau = 0$) then

$$J(S_0)=\begin{pmatrix} -(\mu+\pi) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \pi & 0 & 0 & -(\mu+\eta) \end{pmatrix}. \tag{10}$$

In (10),

$$|J(S_0)-\lambda I|=0$$

is evaluated to $\lambda_1=-(\mu+\pi), \lambda_2=-(\mu+\eta), \lambda_3=0$ and $\lambda_4=0$.

Since all the eigenvalues of the variational matrix (10) are not negative then the total schooling equilibrium S_0 is unstable. The implication of total schooling equilibrium being unstable is that getting all the children into school is not feasible. Notice that all the children would be in school and OOSC would be completely eradicated if $\phi=1$. However, the existence of zero eigenvalues in the analysis shows that while schooling is not feasible for all children, OOSC could be reduced to the barest mínimum if $\phi \to 1$.

3.4. STABILITY OF PARTIAL SCHOOLING EQUILIBRIUM, S^*

Theorem 3.2. The partial schooling equilibrium of the model is stable if $S_R<1$ otherwise it is unstable.

Proof: The partial schooling equilibrium is stable and $S_R<1$ if it is shown that all the eigenvalues of the Jacobian matrix of the model is less than zero.

At PSE (S^*) when all the model varaibles are greater than zero and all the model parameters satisfy $0<x<1$, the variational matrix of the model is computed as

$$J(S^*)=\begin{pmatrix} -(\mu+\pi) & 0 & 0 & 0 \\ \pi(1-\phi) & -(\mu+\rho+\eta) & \theta & \tau \\ 0 & \rho & -(\mu+\theta+\sigma+\eta) & 0 \\ \pi\phi & 0 & \sigma & -(\mu+\tau+\eta) \end{pmatrix}. \tag{11}$$

One of the eigenvalues of $|J(S^*)-\lambda I|$ is $-(\mu+\pi)$ and the other eigenvalues can be obtained from submatrix A given as

$$A=\begin{pmatrix} -(\mu+\rho+\eta) & \theta & \tau \\ \rho & -(\mu+\theta+\sigma+\eta) & 0 \\ 0 & \sigma & -(\mu+\tau+\eta) \end{pmatrix}. \tag{12}$$

$$|J(A)-\lambda I|=0$$

is evaluated to

$$c_0\lambda^3+c_1\lambda^2+c_2\lambda+c_3=0, \tag{13}$$

where

$$\left.\begin{array}{c} c_0 = 1, \\ c_1 = 3(\mu+\eta)+\theta+\tau+\rho, \\ c_2 = (\mu+\rho+\eta)[(2\mu+2\eta+\theta+\sigma+\tau)]+(\mu+\theta+\sigma+\eta)(\mu+\tau+\eta)-\rho\theta, \\ c_3 = (\mu+\rho+\eta)(\mu+\theta+\sigma+\eta)(\mu+\tau+\eta)-\rho[\theta(\mu+\tau+\eta)+\sigma\tau]. \end{array}\right\} \quad (14)$$

Given (14), the stability of partial schooling equilibrium can be examined following Routh-Hurwitz stability criteria [10, 35-37]. The partial schooling equilibrium is stable if in (14), $c_1 > 0, c_3 > 0$ and $c_1 c_2 > c_3$ otherwise it is unstable [38-39]. The implication of partial schooling equilibrium being stable is that the policies implemented to encourage schooling and school enrolment are not sustained and the menace of OOSC persists.

3.5 SENSITIVITY ANALYSIS

The sensitivity analysis is conducted to quantify the relative change in the inclusive schooling ratio S_R when some system parameters change. The sustainability of the enabling implemented policies, to curtail the rising OOSC to fount development in Nigeria, is measured theoretically by computing the relative impacts of major parameters on inclusive schooling ratio S_R following the formula of normalized forward sensitivity index [32],

$$\Gamma_w^{S_R} = \frac{\partial S_R}{\partial w} \times \frac{w}{S_R} \quad (15)$$

With (15), the indices of sensitivity for the major parameters with respect to the inclusive schooling ratio S_R are derived in (16)-(20).

$$\frac{\partial S_R}{\partial \phi} \times \frac{\phi}{S_R} = \frac{(\rho+\sigma)}{(\theta+\tau)} \times \frac{\phi}{S_R} \quad (16)$$

$$\frac{\partial S_R}{\partial \rho} \times \frac{\rho}{S_R} = \frac{\rho}{(\rho+\sigma)} \quad (17)$$

$$\frac{\partial S_R}{\partial \sigma} \times \frac{\sigma}{S_R} = \frac{\sigma}{(\rho+\sigma)} \quad (18)$$

$$\frac{\partial S_R}{\partial \theta} \times \frac{\theta}{S_R} = \frac{-\theta}{(\theta+\tau)} \quad (19)$$

$$\frac{\partial S_R}{\partial \tau} \times \frac{\tau}{S_R} = \frac{-\tau}{(\theta+\tau)} \quad (20)$$

4. QUANTITATIVE ANALYSIS AND DISCUSSION

Simulation is performed to confirm the theoretical results in section 3 and to verify the stability nature of the model. It is achieved via software maple as well as a set of logical parameter values, some of which are from [40-41]. With simulations, the extent of sustainability of the policies implemented to promote schooling and school enrollment in

terms of the population of OOSC that are convinced to go back or enroll in school as well as the population of convinced OOSC that are going back or enrolling in school which has been computed in (9) is studied numerically. Further, the relative importance of the major parameters on limiting the menace of OOSC to fount development is revealed via indices of sensitivity for the key parameters. In the system (1)-(4), set $\phi = 0.65$, $\rho = 0.35$, $\sigma = 0.4$, $\theta = 0.45$, $\tau = 0.35$, $\alpha = 0.25$, $\pi = 0.1$, $\mu = 0.018$, $\eta = 0.0562$. Then, from (9), $S_R = 0.61$ and the indices of sensitivity for the major parameters in (16)-(20) are shown in Table 2.

Table 2.Indices of sensitivity for key parameters to inclusive schooling ratio S_R.

Parameters	Sensitivity Indices
ϕ	$+1.00$
ρ	$+0.47$
σ	$+0.53$
θ	-0.56
τ	-0.44

According to the interpretation of S_R in subsection 3.2, the result $S_R = 0.61$ indicates that given the values chosen for the parameters, enhancing development for Nigeria via minimization OOSC remains a tall dream because the implemented policies to encourage schooling and school enrollment are not sustained. Considering the result $S_R = 0.61$, the failure to fount development through minimization of OOSC could be linked to discontinuity of the enabling implemented policies that instigates high rates of θ and τ, discouragement of the convinced OOSC from going back or enrolling in school and hopelessness of the children who are already in school from continuing schooling respectively.

On a serious note, the present challenges of rising OOSC in Nigeria despite various institutional frameworks and policy initiatives affirms the result $S_R = 0.61$. Successive governments have initiated various policies to tackle the menace but the policies are not sustained which makes the menace of OOSC persists in Nigeria. For instance, in 1976, the military government of Nigeria launched the Universal Primary Education (UPE) to cater for primary education of every child of six years of age with the intention to speed up economic development and to strengthen equal spread of developmental results across the regions of the country [42]. The UPE program initially recorded successes as there was an explosion in primary school enrollment but the program ran into problems when enough facilities and personnel in terms of classrooms and trained teachers were not provided to accommodate millions of admitted children [43]. The program later suffered a major setback in the 1980s from the successive governments due to poor financing occasioned by dwindling oil prices [42].

The failure of the UPE scheme of 1976 was one of the reasons for the introduction of the Universal Basic Education (UBE) in 1999 [11]. UBE was launched to provide free and compulsory primary and junior secondary education to children aged 5 to 14 years [11]. Unfortunately, however, 23 years after the introduction of UBE, millions of children are still on the streets, outside the classrooms. Aside from inadequate facilities and personnel that are frustrating the success of UBE, the problem of teeming unemployed graduates is driving many children out of school and discouraging OOSC from going back or enrolling in school. The unemployment rate at the time of launching UPE in 1976 was 4.3 % [44]. The unemployment rate remained below 10 % throughout the military era up to 1999 so the failure of UPE and the menace of OOSC before the introduction of UBE were not compounded by the high rate of unemployment [43]. However, after the introduction of UBE in 1999, the unemployment rate rose to 13.1 % in 2000, and as of today, it has risen to 40 % [11,44]. Therefore, the rising number of OOSC today is compounded not only because of the poor implementation of UBE but because of the high rate of unemployment. Poor funding of the

education sector is another factor that is militating against the success of UBE. The budgetary allocation to the education sector has been less than 10 % of the total annual budget since the introduction of the UBE program in 1999 against UNESCO's 26 % benchmark for the education sectors of the developing countries [7,45].

As regards sensitivity indices for key parameters in Table 2, parameters with positive indices are directly related to the inclusive schooling ratio (S_R). For example, the index $+1$ for ϕ indicates that stability of the enabling implemented policies to promote schooling and school enrollment increases the proportion of children who enrol in school by 1. Also, the indices +0.47 and +0.53 for ρ and σ respectively indicate that stability of the enabling implemented policies to promote schooling and school enrollment increases the rate at which OOSC are being convinced of the implemented policies and the rate at which the convinced OOSC are being gone back or enrolled in school by 47% and 53% respectively. On the other hand, parameters with negative indices are inversely related to the inclusive schooling ratio (S_R). For example, the indices -0.56 and -0.44 for θ and τ respectively indicate that instability of the enabling implemented policies to promote schooling and school enrollment increases the rate at which convinced OOSC are being discouraged from going back or enrolling in school and the rate at which children who are in school are dropping out by 56% and 44% respectively. It is therefore deduced from the sensitivity indices for the key parameters that to facilitate development by tackling the menace of OOSC, Nigerian government must stabilize policies that are capable of convincing OOSC of the intention of the government (ρ) and finally bringing them back or enrolling them in school (σ). To examine the stability properties of the system which are stated in subsections 3.3 and 3.4, the inclusive schooling ratio (S_R) is evaluated at various values of the key parameters. The outcomes of the computation are shown in Table 3.

Table 3. Results of stability for OOSC model.

S/No.	ϕ	ρ	σ	θ	τ	S_R	Nature of Stability
1	0.65	0.35	0.40	0.45	0.35	0.61	Unstable
2	0.65	0.25	0.30	0.55	0.45	0.36	Unstable
3	0.65	0.15	0.20	0.65	0.55	0.19	Unstable
4	0.65	0.05	0.10	0.75	0.65	0.07	Unstable
5	0.65	0.40	0.45	0.40	0.30	0.79	Unstable
6	0.65	0.45	0.50	0.35	0.25	1.03	Stable
7	0.65	0.50	0.55	0.30	0.20	1.37	Stable
8	0.65	0.55	0.60	0.25	0.15	1.87	Stable
9	0.65	0.60	0.65	0.20	0.10	2.71	Stable

In Table 3, S/No. 1 is the base that gives the initial result for the inclusive schooling ratio (i.e., $S_R = 0.61$). From S/No. 2 to S/No. 4, it is observed that as a decrease in the rate at which OOSC are being convinced of the implemented policies (ρ) and a decrease in the rate at which the convinced OOSC are being gone back or enrolled in school (σ), the menace of OOSC persisted. The model is unstable at these points (S/No. 2 – S/No. 4) as $S_R < 1$. The problem of OOSC is bound to persist if the policies implemented to check the menace are not sustained because OOSC who have been convinced of the policies would begin to be discouraged (θ) and those who have gone back or enrolled in school as well as those who are already in school might lose motivation and begin to drop out (τ). In S/No. 5, the menace of OOSC persisted despite an increase in (ρ) and (σ) and a decrease in (θ) and (τ) which are instigated by sustainability of the implemented policies.The persistence of OOSC menace despite sustainability of the implement policies indicates that tackling the menace of OOSC to fount sustainable development in Nigeria is a long-term process. The implemented policies might not yield instant results even if it is sustained.

On the other hand, in S/No. 6 - S/No. 9, as a continuous increase in (ρ) and (σ) is accompanied with a continuous decrease in both (θ) and (τ), the menace of OOSC is overcome as $S_R > 1$. The model becomes stable in the region within S/No. 6 and S/No. 9. The stability of the model in the region S/No. 6 – S/No. 9 is evidence that the menace of OOSC could be championed to fount sustainable development in Nigeria if the enabling implemented policies are maintained. The results of stability analysis in Table 3 complement the result of sensitivity analysis in Table 2. It is deduced from simulation in Table 3 that to tackle the menace of rising OOSC to fount development in Nigeria, the government must implement enabling policies to raise (ρ) and (σ) and at the same time, the implemented policies must be sustained to prevent increase in (θ) and (τ).

Overcoming the menace of OOSC is desireable not only to drive development in Nigeria but to overcome numerous challenges confronting the country such as illitracy, population explosion, unemployment, poverty, terrorism, banditary, kidnapping, cattle rustling, farmer-herder clashes and a host of others. This is achievable if efforts are made to operate the country educational system in the region defined within S/No. 5 and S/No. 9 in Table 3. Aside minimizing unemployment rate and improving budgetary allocation to the education sector, the issue of frequent interruption of academic calendar due to incessant strike by the teachers has to be addressed to discourage children from being out of school. It is disheartening that the public university lecturers in Nigeria embarked on industrial actions 16 times between 1999 and 2022 (i.e., 16 times in 23 years). The public university students in Nigeria have been kept out of schools for more than seven months in the year 2022 due to industrial action embarked upon by their teachers since February 14, 2022. The current strike by the university teachers has pushed the figure of out-of-school children in Nigeria to 20 million by September 2022 [46]. Incessant strikes by the teachers make the battle against OOSC menace difficult to win.

5. CONCLUSIONS

The rising number of OOSC constitutes a major hindrance to growth and development worldwide. Given the menace of OOSC in Nigeria, a mathematical model has been developed to characterize the necessary and sufficient conditions for tackling the menace to fount development in the country. The necessary condition is the implementation of enabling policies while the sufficient condition is the sustainability of the implemented policies - provision of instant employment to fresh graduates, mass employment of teeming unemployed graduates, increased budgetary allocation to the education sector, increase in worker's salaries, poverty alleviation, provision of adequate and reliable security, building of more schools especially specialized schools for the handicapp, nomadic children, Almajiri and children whose parents have been displaced through various conflicts and attacks. Ample mathematics theorems have been employed to show that the solutions for the proposed model exist, positive and bounded. The model equilibria have been derived and the analytic threshold for quantifying the extent of sustainability of the policies implemented to fount development through OOSC minimization has been computed and used to examine the stability nature of the system equilibria. The sensitivity study of some major parameters has been performed and the simulations have been conducted to determine the region where the menace of OOSC is overcome through sustainability of the implemented policies and the region where the menace persists when the implemented policies are not sustained.

While the menace of OOSC is attributable to many factors in Nigeria, the study finds unemployment and poor funding of the education sector as the roots of other factors

militating against the promotion of social development, poverty reduction, agricultural productivity, fertility and mortality reduction, national cohesion, gender parity and culture of saving among the people of Nigeria. Based on the results of the study, it is, therefore, recommended that the government should target a reduction in unemployment rate below 10 % and an increase in the budgetary allocation to the education sector to 26 % UNESCO's benchmark. If these targets are achieved and maintained for three decade, the country would begin to operate in the region within S/No. 6 – S/No. 9 in Table 3 and overcome the menace of OOSC.

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