ORIGINAL PAPER

OPTIMUM SEPARATE RATIO -TYPE EXPONENTIAL ESTIMATOR FOR COMPUTATION OF POPULATION MEAN IN STRATIFIED RANDOM SAMPLING

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Abstract. *In this manuscript, we study the problem of separate type exponential ratio estimator for estimation of population mean with their properties. The MSE and Bias up to the first degree of approximation for the suggested estimator are computed. The suggested estimator is proven to be more efficient than estimators mentioned in the literature under stratified random technique. An empirical investigation was done to assess the suggested estimator. Also, the percent relative efficiency is to be remarkable for the proposed estimator.*

Keywords: *finite population mean; stratified random sampling; exponential estimator; auxiliary variable; mean squared error; percent relative efficiency.*
Subject classification codes: *62D05; 94A17; 62F07.*

1. INTRODUCTION

The utilisation of supplementary data helps estimators work more efficiently. The proposed ratio estimation approach and utilised supplementary data during the estimation stage. This method generates a ratio estimator based on the assumption that the supplementary variate's population mean is known. When a study and a supplementary variable are positively related, the ratio estimator performs well [1]. In [2], the product approach, which was self-sufficiently given by [3], is utilised when these variates are negatively associated. The estimated population mean of a study variate using the coefficient of variation of a supplementary variate [4]. In [5] it is employed a coefficient of variation of a supplementary variate, based on the work of [4]. Whereas in [7] they are used both coefficients used both the coefficients of variation and the auxiliary variate kurtosis. In [8] and [9] the estimators for stratified random sampling are well-defined estimators for stratified random sampling. Separate ratio-type estimators along the lines of Kadilar and Cingi [8] are improved in this study. Consider a population of size N, which is made up of units. Assume that *x* and *y* are the supplementary and study variables, respectively. If population U is divided into L homogeneous strata of sizes n_h *(h = 1, 2, 3,..., l)* and a sample of size n is taken from the h^{th} stratum, the typical separate ratio estimator for population mean Y is stated as

$$\hat{\bar{Y}}_{RS} = \sum_{h=1}^{L} W_h \, \bar{y}_h \left(\frac{\bar{X}_h}{\bar{x}_h}\right)$$

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The sample mean of the auxiliary variate in stratum h is $\bar{x}_h$, while the sample mean of a research variate of interest in stratum h is $\bar{y}_h$. The bias and mean squared of the traditional distinct ratio estimator are, upto a first degree of approximation,

$$B(\hat{\bar{Y}}_{RS}) = \sum_{h=1}^{L} W_h \gamma_h \bar{y}_h \left(C_{xh}^2 - \rho_{yxh} C_{xh} C_{yh}\right) \tag{1}$$

and

$$MSE(\hat{\bar{Y}}_{RS}) = \sum_{h=1}^{L} {W_h}^2 \gamma_h \left(S_{yh}^2 + {R_h}^2 S_{xh}^2 - 2R_h S_{yxh}\right) \tag{2}$$

2. ESTIMATOR IN LITERATURE

A ratio-type estimator was defined in [5] utilising the coefficient of variation C_x of the supplementary variate x as

$$\hat{\bar{Y}}_{SD} = \bar{y}\left(\frac{\bar{X} + C_x}{\bar{x} + C_x}\right) \tag{3}$$

Kadilar and Cingi, in [8] proposed a separate ratio-type estimator based on the coefficient of variation C_{xh} of the supplementary variate in the h^{th} stratum, as inspired by [5].

$$\hat{\bar{Y}}_{RS}^{SD} = \sum_{h=1}^{L} W_h \bar{y}_h \left(\frac{\bar{X}_h + C_{xh}}{\bar{x}_h + C_{xh}}\right) \tag{4}$$

$$B(\hat{\bar{Y}}_{RS}^{SD}) = \sum_{h=1}^{L} W_h \gamma_h \bar{y}_h \left(\lambda_{1h}^2 C_{xh}^2 - \lambda_{1h} \rho_{yxh} C_{xh} C_{yh}\right) \tag{5}$$

$$MSE(\hat{\bar{Y}}_{RS}^{SD}) = \sum_{h=1}^{L} {W_h}^2 \gamma_h \left(S_{yh}^2 + \lambda_{1h}^2 {R_h}^2 S_{xh}^2 - 2R_h \lambda_{1h} S_{yxh}\right) \tag{6}$$

$$\lambda_{1h} = \frac{\bar{X}_h}{\bar{X}_h + C_{xh}}\ ,\ R_h = \frac{\bar{Y}_h}{\bar{X}_h} \text{ and } \rho_{yxh} = \frac{S_{yxh}}{S_{xh} S_{yh}}$$

A modified ratio estimator based on the coefficient of kurtosis $\beta_2(x)$ of an supplementary variate x was defined in [9].

$$\hat{\bar{Y}}_{SE} = \bar{y}\left(\frac{\bar{X} + \beta_2(x)}{\bar{x} + \beta_2(x)}\right) \tag{7}$$

The suggested estimator employing the coefficient of kurtosis $\beta_2(x)$ of supplementary variate x in the h^{th} stratum is motivated by [8] and [9].

$$\hat{\bar{Y}}_{RS}^{SE} = \sum_{h=1}^{L} W_h \, \bar{y}_h \left(\frac{\bar{X}_h + \beta_{2h}(x)}{\bar{x}_h + \beta_{2h}(x)} \right) \tag{8}$$

$$B(\hat{\bar{Y}}_{RS}^{SE}) = \sum_{h=1}^{L} W_h \gamma_h \, \bar{y}_h \left(\lambda_{2h}^2 C_{xh}^2 - \lambda_{2h} \rho_{yxh} C_{xh} C_{yh} \right) \tag{9}$$

$$MSE(\hat{\bar{Y}}_{RS}^{SE}) = \sum_{h=1}^{L} {W_h}^2 \gamma_h \left(S_{yh}^2 + \lambda_{2h}^2 {R_h}^2 S_{xh}^2 - 2 R_h \lambda_{2h} S_{yxh} \right) \tag{10}$$

$$\lambda_{2h} = \frac{\bar{X}_h}{\bar{X}_h + \beta_{2h}(x)}$$

Upadhyaya and Singh (in[7]) Proposed two ratio-type estimators based on the coefficient of variation and coefficient of kurtosis as parameters.

$$\hat{\bar{Y}}_{US1} = \bar{y} \left(\frac{\bar{X} \beta_2(x) + C_x}{\bar{x} \beta_2(x) + C_x} \right) \tag{11}$$

and

$$\hat{\bar{Y}}_{US2} = \bar{y} \left(\frac{\bar{X} C_x + \beta_2(x)}{\bar{x} C_x + \beta_2(x)} \right) \tag{12}$$

The suggested separate ratio-type estimator using coefficients of kurtosis and variance in the h^{th} stratum is based on [7] and [8].

$$\hat{\bar{Y}}_{RS}^{US1} = \sum_{h=1}^{L} W_h \, \bar{y}_h \left(\frac{\bar{X}_h \beta_{2h}(x) + C_{xh}}{\bar{x}_h \beta_{2h}(x) + C_{xh}} \right) \tag{13}$$

and

$$\hat{\bar{Y}}_{RS}^{US2} = \sum_{h=1}^{L} W_h \, \bar{y}_h \left(\frac{\bar{X}_h C_{xh} + \beta_{2h}(x)}{\bar{x}_h C_{xh} + \beta_{2h}(x)} \right) \tag{14}$$

The bias and mean squared error of the suggested distinct ratio-type estimators up to the first degree of approximation are determined using the conventional approach for obtaining the bias and mean squared errors stated previously:

$$B(\hat{\bar{Y}}_{RS}^{US1}) = \sum_{h=1}^{L} W_h \gamma_h \, \bar{y}_h \left(\lambda_{3h}^2 C_{xh}^2 - \lambda_{3h} \rho_{yxh} C_{xh} C_{yh} \right) \tag{15}$$

$$MSE(\hat{\bar{Y}}_{RS}^{US1}) = \sum_{h=1}^{L} {W_h}^2 \gamma_h \left(S_{yh}^2 + \lambda_{3h}^2 {R_h}^2 S_{xh}^2 - 2 R_h \lambda_{3h} S_{yxh} \right) \tag{16}$$

$$B(\hat{\hat{Y}}_{RS}^{US2}) = \sum_{h=1}^{L} W_h \gamma_h \bar{y}_h \left(\lambda_{4h}^2 C_{xh}^2 - \lambda_{4h}\rho_{yxh}C_{xh}C_{yh}\right) \tag{17}$$

$$MSE(\hat{\hat{Y}}_{RS}^{US2}) = \sum_{h=1}^{L} {W_h}^2 \gamma_h \left(S_{yh}^2 + \lambda_{4h}^2 {R_h}^2 S_{xh}^2 - 2R_h\lambda_{4h}S_{yxh}\right) \tag{18}$$

$$\lambda_{3h} = \frac{\bar{X}_h \beta_{2h}(x)}{\bar{X}_h \beta_{2h}(x) + C_{xh}}$$
$$\lambda_{4h} = \frac{\bar{X}_h C_{xh}}{\bar{X}_h C_{xh} + \beta_{2h}(x)}$$

Chouhan in [10] researched a separate ratio type exponential estimator for the population mean, which is expressed in a separate version.

$$T_c = \sum_{h=1}^{L} W_h \bar{y}_h \left(\frac{\bar{X}_h + \bar{x}_h}{\bar{X}_h + \bar{x}_h}\right) \tag{19}$$

$$MSE(\hat{\hat{T}}_c) = \sum_{h=1}^{L} {W_h}^2 \gamma_h \left(S_{yh}^2 + \frac{1}{4}{R_h}^2 S_{xh}^2 - R_h S_{yxh}\right) \tag{20}$$

Lone and Tailor [11] developed a separate ratio type estimator based on the coefficient of variation of each stratum's supplementary variate.

$$T_L = \sum_{h=1}^{L} W_h \bar{y}_h \left(\frac{\bar{X}_h + C_{xh}}{\bar{X}_h + C_{xh}}\right) \tag{21}$$

$$MSE(\hat{\hat{T}}_L) = \sum_{h=1}^{L} {W_h}^2 \gamma_h \left(S_{yh}^2 + \lambda_{5h}^2 {R_h}^2 S_{xh}^2 - 2R_h\lambda_{5h}S_{yxh}\right) \tag{22}$$

$$\lambda_{5h} = \frac{\bar{X}_h}{\bar{X}_h + C_{xh}}$$

3. SUGGESTED ESTIMATOR

To get a better and efficient estimate of population mean $\bar{Y}$ when $\bar{X}$ is known, we suggest a difference-cum-exponential type estimator as

$$T_s = \sum_{h=1}^{L} W_h \left[k\bar{y}_h + t(\bar{X}_h - \bar{x}_h)\left\{1 - exp\delta\left(\frac{\bar{X}_h - \bar{x}_h}{\bar{X}_h + \bar{x}_h}\right)\right\}^{-1}\right] \tag{23}$$

To achieve the bias and mean squared error of the suggested separate ratio type estimator T_s

$$\bar{y}_h = \bar{Y}_h(1 + \varepsilon_{0h})$$

$$\bar{x}_h = \bar{X}_h(1 + \varepsilon_{1h})$$

Such that,

$$E(\varepsilon_{0h}) = E(\varepsilon_{1h}) = 0$$

$$E(\varepsilon_{0h})^2 = \left(\frac{1}{n_h} - \frac{1}{N_h}\right) C_{yh}^2 = \gamma C_{yh}^2$$

$$E(\varepsilon_{1h})^2 = \left(\frac{1}{n_h} - \frac{1}{N_h}\right) C_{xh}^2 = \gamma C_{xh}^2$$

$$E(\varepsilon_{0h}\varepsilon_{1h}) = \left(\frac{1}{n_h} - \frac{1}{N_h}\right) \gamma_h \rho_{yxh} C_{xh} C_{yh}$$

Express T_s in terms of errors, Eq. (23) becomes

$$T_s = k\bar{Y}(1 - \varepsilon_{0h}) - t\bar{X}_h \varepsilon_{1h} \left[1 - exp\delta\left(\frac{-\varepsilon_{1h}}{2 + \varepsilon_{1h}}\right)\right]^{-1}$$

$$T_s = k\bar{Y}(1 - \varepsilon_{0h}) - t\bar{X}_h \varepsilon_{1h} \left[1 - exp\left\{-\frac{\delta\varepsilon_{1h}}{2}\left(1 + \frac{\varepsilon_{1h}}{2}\right)^{-1}\right\}\right]^{-1}$$

Expand expression in terms of $\varepsilon's$ and ignoring terms having $\varepsilon's$ degree greater than two to get first order approximation, we have

$$T_s = \left[k\bar{Y}_h(1 - \varepsilon_{0h}) - \frac{2t\bar{X}_h}{\delta}\right] + k\bar{Y}_h\varepsilon_{0h} - t\bar{X}_h\left(\frac{1}{\delta} + \frac{1}{2}\right)\varepsilon_{1h} - \frac{t\bar{X}_h\delta}{8}\varepsilon_{ih}^2 \quad (24)$$

On simplifying, we get

$$T_s - \bar{Y}_h = \left[\bar{Y}_h(k - 1) - \frac{2t\bar{X}_h}{\delta}\right] + k\bar{Y}_h\varepsilon_{0h} - t\bar{X}_h\left(\frac{1}{\delta} + \frac{1}{2}\right)\varepsilon_{1h} - \frac{t\bar{X}_h\delta}{8}\varepsilon_{ih}^2 \quad (25)$$

To obtain the bias of T_s to the first order of approximation, take expectation on both sides of Eq. (25), we have

$$Bias(T_s) = \sum_{h=1}^{L} W_h\gamma_h\left[\bar{Y}_h(k - 1) - \frac{2t\bar{X}_h}{\delta}\right] - \frac{t\bar{X}_h\delta}{8}\gamma_h C_{xh}^2 \quad (26)$$

By squaring Eq. (25) on both sides and terminating the terms having $\varepsilon's$ degree more than two, we have

$$
\begin{aligned}
(T_s - \bar{Y}_h)^2 = {} & \left[\bar{Y}_h(k-1) - \frac{2t\bar{X}_h}{\delta}\right]^2 + k^2\bar{Y}_h^2 \in_1^2 - kt\bar{X}_h\left(1+\frac{2}{\delta}\right)\in_1\in_2 \\
& + \left[\left(\frac{3}{4}+\frac{1}{\delta}+\frac{1}{\delta^2}\right)t^2 - \frac{\delta}{4}t(k-1)R\right]\bar{X}_h^2 \in_2^2
\end{aligned}
\tag{27}
$$

To get the MSE of T_s to the first order of approximation, take expectation on both sides of Eq. (27), we have

$$
\begin{aligned}
MSE(T_s) = {} & \sum_{h=1}^{L} {W_h}^2 \left[\left[\gamma_h(k-1) - \frac{2t\bar{X}_h}{\delta}\right]^2 + k^2\gamma_h\gamma_h C_{yh}^2 \right. \\
& - kt\bar{X}_h\gamma_h\left(1+\frac{2}{\delta}\right)\gamma_h\rho_{yxh}C_{xh}C_{yh} \\
& \left. + \left[\left(\frac{3}{4}+\frac{1}{\delta}+\frac{1}{\delta^2}\right)t^2 - \frac{\delta}{4}t(k-1)R\right]\bar{X}_h^2\gamma_h C_{xh}^2\right]
\end{aligned}
$$

$$
\begin{aligned}
MSE(T_s) = {} & \sum_{h=1}^{L} {W_h}^2\gamma_h \left[\left[(k-1) - \frac{2t}{\delta R}\right]^2 + k^2\gamma_h C_{yh}^2 - kt\frac{1}{R}\left(1+\frac{2}{\delta}\right)\gamma_h C_{yxh} \right. \\
& \left. + \left[\left(\frac{3}{4}+\frac{1}{\delta}+\frac{1}{\delta^2}\right)t^2 - \frac{\delta}{4}t(k-1)R\right]\frac{\gamma_h C_{xh}^2}{R^2}\right]
\end{aligned}
\tag{28}
$$

$$
\begin{aligned}
MSE(T_s) = {} & \sum_{h=1}^{L} {W_h}^2\gamma_h \left[1 - 2k - k^2\left(1-\gamma_h C_{yh}^2\right) + t\left(\frac{4}{\delta R} + \frac{2\delta\gamma_h C_{xh}^2}{4R}\right) \right. \\
& + t^2\left[\frac{4}{\delta^2 R^2} + \left(\frac{3}{4}+\frac{1}{\delta}+\frac{1}{\delta^2}\right)\frac{\gamma_h C_{xh}^2}{R^2}\right] \\
& \left. - kt\left(\frac{4}{\delta R} + \frac{\gamma_h\delta C_{xh}^2}{4R} + \frac{1}{R}\left(1+\frac{2}{\delta}\right)\gamma_h C_{yxh}\right)\right]
\end{aligned}
\tag{29}
$$

$$
MSE(T_s) = \sum_{h=1}^{L} {W_h}^2\gamma_h\left[1 - 2k - k^2\varphi_{1h} + t\varphi_{2h} + t^2\varphi_{3h} - kt\varphi_{4h}\right]
\tag{30}
$$

$$
\varphi_{1h} = \left(1+\gamma_h C_{yh}^2\right)
$$

$$
\varphi_{2h} = \left(\frac{4}{\delta R} + \frac{2\delta\gamma_h C_{xh}^2}{4R}\right)
$$

$$
\varphi_{3h} = \left(\frac{4}{\delta^2 R^2} + \left(\frac{3}{4}+\frac{1}{\delta}+\frac{1}{\delta^2}\right)\frac{\gamma_h C_{xh}^2}{R^2}\right)
$$

$$
\varphi_{4h} = \left(\frac{4}{\delta R} + \frac{\gamma_h\delta C_{xh}^2}{4R} + \frac{1}{R}\left(1+\frac{2}{\delta}\right)\gamma_h C_{yxh}\right)
$$

To minimize$MSE(T_s)$, we have to differentiate $MSE(T_s)$ in Equation (28) partially with respect to *k* and *t*, and equating to zero, we get

$$2\varphi_{1h}k - t\varphi_{4h} = 2 \tag{31}$$

and

$$-k\varphi_{4h} + 2t\varphi_{3h} = -\varphi_{2h} \tag{32}$$

On solving simultaneous Equations. (31) and (32) for *k* and *t*, the optimal values of *k* and *t* are obtained as

$$k_0 = \frac{4\varphi_{3h} - \varphi_{2h}\varphi_{4h}}{4\varphi_{1h}\varphi_{3h} - \varphi_{4h}^2}\ , \qquad t_0 = \frac{2\varphi_{4h} - 2\varphi_{1h}\varphi_{2h}}{4\varphi_{1h}\varphi_{3h} - \varphi_{4h}^2} \tag{33}$$

The minimum $MSE(T_s)$ can be obtained by using the optimum values of *k* and *t* from Eq. (33) in Eq. (30), as

$$MSE_{min}(T_s) = \sum_{h=1}^{L} {W_h}^2\gamma_h\left[1 - \frac{4\varphi_{3h} + \varphi_{1h}\varphi_{2h}^2 - 2\varphi_{2h}\varphi_{4h}}{4\varphi_{1h}\varphi_{3h} - \varphi_{4h}^2}\right] \tag{34}$$

3.1 SPECIAL CASES

In this section, we do study the different estimators with minimum of $MSE(T_s)$ by assigning the different values to δ to show the variability of the proposed estimator.

For $\delta = 1$

The proposed estimator T_s becomes

$$T_{s1} = \sum_{h=1}^{L} W_h\left[k\bar{y}_h + t(\bar{X}_h - \bar{x}_h)\left\{1 - exp\left(\frac{\bar{X}_h - \bar{x}_h}{\bar{X}_h + \bar{x}_h}\right)\right\}^{-1}\right]$$

With

$$MSE_{min}(T_{s1}) = \sum_{h=1}^{L} {W_h}^2\gamma_h\left[1 - \frac{4\varphi_{31} + \varphi_{11}\varphi_{21}^2 - 2\varphi_{21}\varphi_{41}}{4\varphi_{11}\varphi_{31} - \varphi_{41}^2}\right] \tag{35}$$

where,

$$\varphi_{11} = \left(1 + \gamma_h C_{yh}^2\right)$$

$$\varphi_{21} = \left(\frac{4}{R} + \frac{2\gamma_h C_{xh}^2}{4R}\right)$$

$$\varphi_{31} = \left[\frac{4}{R^2} + \left(\frac{11}{4}\right)\frac{\gamma_h C_{xh}^2}{R^2}\right]$$

$$\varphi_{41} = \left(\frac{4}{R} + \frac{\gamma_h C_{xh}^2}{4R} + \frac{3}{R}\gamma_h C_{yxh}\right)$$

For $\boldsymbol{\delta = C_{xh}}$

The estimator T_s becomes

$$T_{s2}=\sum_{h=1}^{L}W_h\left[k\bar{y}_h+t(\bar{X}_h-\bar{x}_h)\left\{1-expC_{xh}\left(\frac{\bar{X}_h-\bar{x}_h}{\bar{X}_h+\bar{x}_h}\right)\right\}^{-1}\right]$$

with

$$MSE_{min}(T_{s2})=\sum_{h=1}^{L}{W_h}^2\gamma_h\left[1-\frac{4\varphi_{32}+\varphi_{12}\varphi_{22}^2-2\varphi_{22}\varphi_{42}}{4\varphi_{12}\varphi_{32}-\varphi_{42}^2}\right] \quad (36)$$

where,

$$\varphi_{12}=\left(1+\gamma_h C_{yh}^2\right)$$

$$\varphi_{22}=\left(\frac{4}{C_{xh}R}+\frac{2\gamma_h C_{xh}^3}{4R}\right)$$

$$\varphi_{32}=\left[\frac{4}{C_{xh}^2R^2}+\left(\frac{3}{4}+\frac{1}{C_{xh}}+\frac{1}{C_{xh}^2}\right)\frac{\gamma_h C_{xh}^2}{R^2}\right]$$

$$\varphi_{42}=\left(\frac{4}{C_{xh}R}+\frac{\gamma_h C_{xh}^3}{4R}+\frac{1}{R}\left(1+\frac{2}{C_{xh}}\right)\gamma_h C_{yxh}\right)$$

For $\boldsymbol{\delta=\beta_{xh}}$

The estimator T_s becomes

$$T_{s3}=\sum_{h=1}^{L}W_h\left[k\bar{y}_h+t(\bar{X}_h-\bar{x}_h)\left\{1-exp\beta_{xh}\left(\frac{\bar{X}_h-\bar{x}_h}{\bar{X}_h+\bar{x}_h}\right)\right\}^{-1}\right]$$

with

$$MSE_{min}(T_{s3})=\sum_{h=1}^{L}{W_h}^2\gamma_h\left[1-\frac{4\varphi_{32}+\varphi_{12}\varphi_{22}^2-2\varphi_{22}\varphi_{42}}{4\varphi_{12}\varphi_{32}-\varphi_{42}^2}\right] \quad (37)$$

where,

$$\varphi_{13}=\left(1+\gamma_h C_{yh}^2\right)$$

$$\varphi_{23}=\left(\frac{4}{\beta_{xh}R}+\frac{2\beta_{xh}\gamma_h C_{xh}^2}{4R}\right)$$

$$\varphi_{33}=\left(\frac{4}{\beta_{xh}^2R^2}+\left(\frac{3}{4}+\frac{1}{\beta_{xh}}+\frac{1}{\beta_{xh}^2}\right)\frac{\gamma_h C_{xh}^2}{R^2}\right)$$

$$\varphi_{43}=\left(\frac{4}{\beta_{xh}R}+\frac{\gamma_h\beta_{xh}C_{xh}^2}{4R}+\frac{1}{R}\left(1+\frac{2}{\beta_{xh}}\right)\gamma_h C_{yxh}\right)$$

For $\boldsymbol{\delta=\rho_{xh}}$

The estimator T_s becomes

$$T_{s4}=\sum_{h=1}^{L}W_h\left[k\bar{y}_h+t(\bar{X}_h-\bar{x}_h)\left\{1-exp\rho_{xh}\left(\frac{\bar{X}_h-\bar{x}_h}{\bar{X}_h+\bar{x}_h}\right)\right\}^{-1}\right]$$

with

$$MSE_{min}(T_{s4}) = \sum_{h=1}^{L} {W_h}^2 \gamma_h \left[1 - \frac{4\varphi_{32} + \varphi_{12}\varphi_{22}^2 - 2\varphi_{22}\varphi_{42}}{4\varphi_{12}\varphi_{32} - \varphi_{42}^2}\right] \tag{38}$$

where,

$$\varphi_{14} = \left(1 + \gamma_h C_{yh}^2\right)$$

$$\varphi_{24} = \left(\frac{4}{\rho_{xh}R} + \frac{2\rho_{xh}\gamma_h C_{xh}^2}{4R}\right)$$

$$\varphi_{34} = \left(\frac{4}{\rho_{xh}^2 R^2} + \left(\frac{3}{4} + \frac{1}{\rho_{xh}} + \frac{1}{\rho_{xh}^2}\right)\frac{\gamma_h C_{xh}^2}{R^2}\right)$$

$$\varphi_{44} = \left(\frac{4}{\rho_{xh}R} + \frac{\gamma_h \rho_{xh} C_{xh}^2}{4R} + \frac{1}{R}\left(1 + \frac{2}{\rho_{xinh}}\right)\gamma_h C_{yxh}\right)$$

4. EFFICIENCY COMPARISONS

In this section, the performances of the suggested estimator have been demonstrated over the existing estimators as follows:

I. From (2) and (34)

$$MSE(\hat{\bar{Y}}_{RS}) - MSE_{min}(T_s) > 0$$

$$\sum_{h=1}^{L} {W_h}^2 \gamma_h \left[\tau_1 - 1 - \frac{4\varphi_{32} + \varphi_{12}\varphi_{22}^2 - 2\varphi_{22}\varphi_{42}}{4\varphi_{12}\varphi_{32} - \varphi_{42}^2}\right] > 0$$

II. From (6) and (34)

$$MSE(\hat{\bar{Y}}_{RS}^{SD}) - MSE_{min}(T_s) > 0$$

$$\sum_{h=1}^{L} {W_h}^2 \gamma_h \left[\tau_2 - 1 - \frac{4\varphi_{3h} + \varphi_{1h}\varphi_{2h}^2 - 2\varphi_{2h}\varphi_{4h}}{4\varphi_{1h}\varphi_{3h} - \varphi_{4h}^2}\right] > 0$$

III. From (10) and (34)

$$MSE(\hat{\bar{Y}}_{RS}^{SE}) - MSE_{min}(T_s) > 0$$

$$\sum_{h=1}^{L} {W_h}^2 \gamma_h \left[\tau_3 - 1 - \frac{4\varphi_{3h} + \varphi_{1h}\varphi_{2h}^2 - 2\varphi_{2h}\varphi_{4h}}{4\varphi_{1h}\varphi_{3h} - \varphi_{4h}^2}\right] > 0$$

IV. From (16) and (34)

$$MSE(\hat{\bar{Y}}_{RS}^{US1}) - MSE_{min}(T_s) > 0$$

$$\sum_{h=1}^{L} {W_h}^2\gamma_h\left[\tau_4 - 1 - \frac{4\varphi_{3h} + \varphi_{1h}\varphi_{2h}^2 - 2\varphi_{2h}\varphi_{4h}}{4\varphi_{1h}\varphi_{3h} - \varphi_{4h}^2}\right] > 0$$

V. From (18) and (34)

$MSE(\hat{\hat{Y}}_{RS}^{US2}) - MSE_{min}(T_s) > 0$

$$\sum_{h=1}^{L} {W_h}^2\gamma_h\left[\tau_5 - 1 - \frac{4\varphi_{3h} + \varphi_{1h}\varphi_{2h}^2 - 2\varphi_{2h}\varphi_{4h}}{4\varphi_{1h}\varphi_{3h} - \varphi_{4h}^2}\right] > 0$$

VI. From (20) and (34)

$MSE(\hat{\hat{T}}_c) - MSE_{min}(T_s) > 0$

$$\sum_{h=1}^{L} {W_h}^2\gamma_h\left[\tau_6 - 1 - \frac{4\varphi_{3h} + \varphi_{1h}\varphi_{2h}^2 - 2\varphi_{2h}\varphi_{4h}}{4\varphi_{1h}\varphi_{3h} - \varphi_{4h}^2}\right] > 0$$

VII. From (22) and (34)

$MSE(\hat{\hat{T}}_L) - MSE_{min}(T_s) > 0$

$$\sum_{h=1}^{L} {W_h}^2\gamma_h\left[\tau_7 - 1 - \frac{4\varphi_{3h} + \varphi_{1h}\varphi_{2h}^2 - 2\varphi_{2h}\varphi_{4h}}{4\varphi_{1h}\varphi_{3h} - \varphi_{4h}^2}\right] > 0$$

where,

$$\tau_1 = \sum_{h=1}^{L} {W_h}^2\gamma_h\left(S_{yh}^2 + {R_h}^2 S_{xh}^2 - 2R_h S_{yxh}\right)$$

$$\tau_2 = \sum_{h=1}^{L} {W_h}^2\gamma_h\left(S_{yh}^2 + \lambda_{1h}^2 {R_h}^2 S_{xh}^2 - 2R_h\lambda_{1h} S_{yxh}\right)$$

$$\tau_3 = \sum_{h=1}^{L} {W_h}^2\gamma_h\left(S_{yh}^2 + \lambda_{2h}^2 {R_h}^2 S_{xh}^2 - 2R_h\lambda_{2h} S_{yxh}\right)$$

$$\tau_4 = \sum_{h=1}^{L} {W_h}^2\gamma_h\left(S_{yh}^2 + \lambda_{3h}^2 {R_h}^2 S_{xh}^2 - 2R_h\lambda_{3h} S_{yxh}\right)$$

$$\tau_5 = \sum_{h=1}^{L} {W_h}^2\gamma_h\left(S_{yh}^2 + \lambda_{4h}^2 {R_h}^2 S_{xh}^2 - 2R_h\lambda_{4h} S_{yxh}\right)$$

$$\tau_6 = \sum_{h=1}^{L} {W_h}^2\gamma_h\left(S_{yh}^2 + \frac{1}{4}{R_h}^2 S_{xh}^2 - R_h S_{yxh}\right)$$

$$\tau_7 = \sum_{h=1}^{L} {W_h}^2 \gamma_h \left(S_{yh}^2 + \lambda_{5h}^2 {R_h}^2 S_{xh}^2 - 2R_h \lambda_{5h} S_{yxh} \right)$$

5. EMPIRICAL INVESTIGATION

Two natural population data sets were used to compare the performance of the suggested estimator to that of other estimators considered in this work. In table 1, the estimators based on population data are compared.

Table 1. Population I: [3]

N=10 n=4	$n_1 = 2$	$n_2 = 2$
	$N_1 = 5$	$N_2 = 5$
	$\bar{X}_1 = 214.4$	$\bar{X}_2 = 333.8$
	$\bar{Y}_1 = 1925.8$	$\bar{Y}_2 = 3115.6$
	$\beta_{21}(x) = 1.88$	$\beta_{22}(x) = 2.32$
	$\rho_{yx1} = 0.85$	$\rho_{yx2} = 0.98$
	$C_{x1} = 0.34$	$C_{x2} = 0.19$
	$S_{x1}^2 = 5605.85$	$S_{x2}^2 = 4401.76$
	$S_{y1}^2 = 379360.16$	$S_{y2}^2 = 115860.24$
	$S_{yx1} = 39360.69$	$S_{yx2} = 22356.52$

Table 2. Population 2. [6]

N=1344 n=52	$n_1 = 14$	$n_2 = 9$	$n_3 = 12$	$n_4 = 17$
	$N_1 = 400$	$N_2 = 216$	$N_3 = 364$	$N_4 = 364$
	$\bar{X}_1 = 76.21$	$\bar{X}_2 = 58.11$	$\bar{X}_3 = 69.08$	$\bar{X}_4 = 63.71$
	$\bar{Y}_1 = 79.35$	$\bar{Y}_2 = 59.44$	$\bar{Y}_3 = 76.66$	$\bar{Y}_4 = 64.57$
	$\beta_{21}(x) = 2.22$	$\beta_{22}(x) = 2.29$	$\beta_{23}(x) = 1.96$	$\beta_{24}(x) = 2.47$
	$\rho_{yx1} = 0.76$	$\rho_{yx2} = 0.82$	$\rho_{yx3} = 0.73$	$\rho_{yx4} = 0.85$
	$C_{x1} = 0.1906$	$C_{x2} = 0.2416$	$C_{x3} = 0.201$	$C_{x4} = 0.1908$
	$S_{x1}^2 = 210.9938$	$S_{x2}^2 = 197.1041$	$S_{x3}^2 = 192.795$	$S_{x4}^2 = 147.765$
	$S_{y1}^2 = 166.70$	$S_{y2}^2 = 174.28$	$S_{y3}^2 = 226.60$	$S_{y4}^2 = 170.61$
	$S_{yx1} = 148.76$	$S_{yx2} = 161.19$	$S_{yx3} = 192.21$	$S_{yx4} = 143.83$

Table 3. Percent Relative Efficiency (PRE)

Estimators	**R.E. (%)**	
	Population I	**Population II**
$\hat{\bar{Y}}_{RS}$	100	100
$\hat{\bar{Y}}_{RS}^{SD}$	101.190	100.442
$\hat{\bar{Y}}_{RS}^{SE}$	115.059	104.011
$\hat{\bar{Y}}_{RS}^{US1}$	100.512	100.190
$\hat{\bar{Y}}_{RS}^{US2}$	182.734	110.983
$\hat{\bar{T}}_c$	137.623	111.704
$\hat{\bar{T}}_L$	157.859	127.794
$\boldsymbol{T_{s1}}$	**187.278**	**156.101**
$\boldsymbol{T_{s2}}$	**225.884**	**186.867**
$\boldsymbol{T_{s3}}$	**292.711**	**212.573**
$\boldsymbol{T_{s4}}$	**270.987**	**204.378**

CONCLUSION

The suggested ratio exponential type estimator would be better than the existing estimators, according to the efficiency comparisons carried out in section 4. Table 1 demonstrates that the conditions established in section 4 are met empirically. The suggested ratio exponential type estimators have a greater percent relative efficiency, as shown in Table 1. As a result, the suggested estimator is the most efficient and recommended for predicting the population mean.

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