ORIGINAL PAPER

RADIATIVE HEAT AND MASS TRANSFER IN MHD FLOW OVER A STRETCHING SHEET UNDER THE EFFECT OF JOULE HEATING AND VISCOUS DISSIPATION WITH VARIABLE WALL TEMPERATURE PARAMETER

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Abstract. *In the present problem Heat and Mass transfer of a MHD flow in the effect of radiation, viscous dissipation and Joule heating is considered. The temperature at the wall is taken as power law with variable exponent. The problem is solved analytically using concept of confluent hypergeometric function. The effects of various parameters entering into the problem are discussed and presented in graphical form. Variable power of temperature parameter provides unique and very useful results for the effects over temperature.*

Keywords: *heat and mass transfer; MHD; radiation; chemical reaction; stretching sheet.*

1. INTRODUCTION

Heat-mass transfer over stretching sheet is very important in manufacturing industry of plastic film, artificial fibre materials, aerodynamic extrusion of plastic sheet and purification of molten metal to remove non-metallic intrusion for instance, the continuous casting which is strand casting also, the molten metal is harden as semi-finished billets, bloom or slab for succeeding rolling in a finishing mill. It is mostly utilized for copper, bronze and aluminium and growingly for cast iron and steel.

Erickson et al [1] presented a study of Heat and mass transfer of moving continuous plate with suction or injection. Gupta and Gupta [2] studied Heat-mass transfer on a plate which is stretching with suction or blowing, Vajravelu and Hadjinicolaou [3] proposed heat transfer over stretching plate which is affected by viscous dissipation and internal heat generation. When the temperature of neighbourhood is high the radiation plays an important role and it cannot be ignored (Modest [4], Siegel and Howell [5]), Kumar [6] investigated radiation and viscous dissipation effect over a stretching plate with variable heat flux. The reaction of radiation in steady motion is reported by numerous researchers, such as Cess [7], Arpaci [8], Cheng and Ozisik [9], Hasegawa, Echigo and Fakuda [10], Bankston, Lioyed and Novony [11], Hossain and Takhar [11, 12] and Hossain, Pop and Rees [14].

A large number of study have taken the temperature at boundary as power law for a constant power (mainly proportional to x^2), except few such as Kumar [15] and Kumar [16]. It is very interestingly and more general to study the heat-mass transfer on a stretching sheet when wall temperature is of variable power. The major motivation of the problem is to study the effect of wall temperature parameter, radiation and dissipation over the temperature.

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2. MATERIALS AND METHODS

Let us take a steady flow which is laminar, of an incompressible visco-elastic fluid (Walter's liquid B modal) generated by a stretching plate, which is positioned in quiescent fluid in the influence of radiation, viscous dissipation and Joule heating. The flow is supposed to be along x-axis that is taken towards the plate and y-axis normal to it. The plate is exuding from a thin slit at origin (0, 0). A particle on sheet is assumed to have speed proportional to distance from the slit. The equations which govern the flow, heat-mass transfer are written as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\vartheta\frac{\partial^2 u}{\partial y^2}-k_0\left\{\frac{\partial}{\partial x}\left[u\frac{\partial^2 u}{\partial x^2}\right]+\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}+v\frac{\partial^3 u}{\partial y^3}\right\}-\frac{\sigma B_0^2 u}{\rho} \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2-\rho k_0\frac{\partial u}{\partial y}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)-\frac{\partial q_r}{\partial y}+\sigma B_0^2 u^2 \tag{3}$$

$$\begin{aligned} &u = b\,x, \quad v=0, \quad \text{at} \quad y=0 \\ &u = 0, \quad \text{as} \quad \frac{\partial u}{\partial y}=0 \quad y\to\infty \end{aligned} \tag{4}$$

where c is a positive stretching constant.

Furthermore temperature boundary conditions are as follows:

$$T=T_w=T_\infty+A\,x^s \text{ at } y=0$$

$$T=T_\infty \text{ as } y\to\infty \tag{5}$$

here A be a constant whose value depend upon the effects of the fluid, T_w and T_∞ are temperature at the wall and at far from the wall.

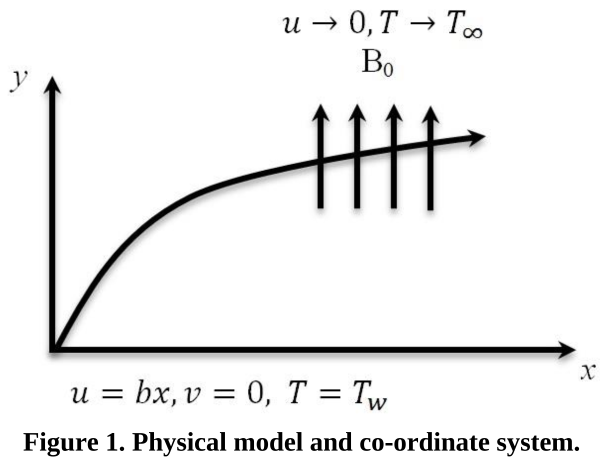

Figure 1. Physical model and co-ordinate system.

The solution of equations (1) and (2), with the boundary conditions (4) is as follows.

The stream function $u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}$,

$$\psi = \sqrt{b\vartheta}\; x\; f(\eta), \quad \eta = \sqrt{\frac{b}{\vartheta}}\, y\,,$$

$$f(\eta) = \frac{1-e^{-\alpha\eta}}{\alpha} \tag{6}$$

where

$$k_1 = \frac{k_0 b}{\vartheta},\; M_n = \frac{\sigma B_0^2}{\rho b},\; \alpha = \sqrt{\frac{1+M_n}{1-k_1}},\; 0 \le k_1 \le 1.$$

To solve equation (3) in view of boundary condition (5), the temperature in dimensionless form is defined as

$$T = T_\infty + A \;\; x^s \;\; \theta(\eta) \tag{7}$$

On assuming the approximation given by Rosseland (Brewster [17]) for radiative heat flux, as

$$q_r = -\frac{4\sigma'}{3\kappa^*}\frac{\partial T^4}{\partial y} \tag{8}$$

here κ^* is the coefficient of mean absorption and σ' is the Stefan-Boltzmann constant.

When the temperature variability within the flow are significantly small such that T^4 can be written as a linear function of the temperature, then using Taylor series for T^4 about T_∞, after ignoring HOT, is given by

$$T^4 = 4T_\infty^3 T - 3T_\infty^4\,.$$

On using eqs. (6)-(8), eq. (3) takes the following form:

$$\left(1+ 4/3N\right)\theta'' + \Pr f\,\theta' - s\Pr f'\theta = -Ec\Pr\left\{-k_1 f''(f f''' - f\; f''') + f''^2 M_n\; f'^2\right\} \tag{9}$$

the boundary conditions changes to

$$\theta = 1 \quad \text{at} \quad \eta = 0$$

$$\theta(\infty) = 0 \tag{10}$$

where,

$$N = \frac{\kappa^* k}{4\sigma\; T_\infty^3},\; \Pr = \frac{\rho\,\vartheta\, c_p}{k},\; Ec = \frac{b^2 x^2}{c_p(T_w - T_\infty)},\text{ and } k_1 = \frac{k_0\,\vartheta}{b}.$$

Taking $\left(1+ 4/3N\right) = \beta$, and to solve the eq. (9), introduce a variable ξ as follows:

$$\xi = -\frac{\Pr}{\alpha^2 \beta} e^{-\alpha\eta} \tag{11}$$

Hence, eq. (9) reduces to

$$\xi \frac{d^2\theta}{d\xi^2} + \left(1 - \frac{\Pr}{\alpha^2} - \xi\right)\frac{d\theta}{d\xi} + s\,\theta = -Q\xi \tag{12}$$

Here $Q = \dfrac{Ec\,\alpha^4 \beta}{\Pr}\left(1 - k_1 + \dfrac{M_n}{\alpha^2}\right)$

The corresponding boundary conditions are

$$\theta\left(\xi = -\frac{\Pr}{\alpha^2\beta}\right) = 1 \quad \text{and} \quad \theta(\xi = 0) = 0 \tag{13}$$

To solve the equation (12) the method of solution for a Kummer's hypergeometric equation is used, and hence the solution in terms of Kummer's function as follows (Abramowitz and Stegun [18]):

$$\theta(\eta) = C_1\; {}_1F_1\left[-s;\ 1 - \frac{\Pr}{\alpha^2};\ -\frac{\Pr}{\alpha^2\beta}e^{-\alpha\eta}\right] + C_2\left(e^{-\alpha\eta}\right)^{\frac{\Pr}{\alpha^2}}\; {}_1F_1\left[\frac{\Pr}{\alpha^2} - s;\ \frac{\Pr}{\alpha^2} + 1;\ -\frac{\Pr}{\alpha^2\beta}e^{-\alpha\eta}\right]$$
$$+ \frac{2Q\Pr}{s\,\alpha^2\beta}e^{-\alpha\eta} + \frac{Q}{s}\left(1 - \frac{\Pr}{\alpha^2}\right) \tag{14}$$

where $C_1 = -\dfrac{Q}{s}\left(1 - \dfrac{\Pr}{\alpha^2}\right)$ and the constant C_2 is given by the following equation:

$$1 = C_1\; {}_1F_1\left[-s;\ 1 - \frac{\Pr}{\alpha^2};\ -\frac{\Pr}{\alpha^2\beta}\right] + \frac{2Q\Pr}{s\alpha^2\beta} + C_2\; {}_1F_1\left[\frac{\Pr}{\alpha^2} - s;\ \frac{\Pr}{\alpha^2} + 1;\ -\frac{\Pr}{\alpha^2\beta}\right] + \frac{Q}{s}\left(1 - \frac{\Pr}{\alpha^2}\right)$$

3. RESULTS AND DISCUSSION

Radiative heat and mass transfer on a stretching plate of MHD flow are affected by viscous dissipation and joule heating with variable wall temperature parameter. The analytical solutions are derived in form of hypergeometric function, and using FORTRAN the graphs of velocity and temperature are prepared.

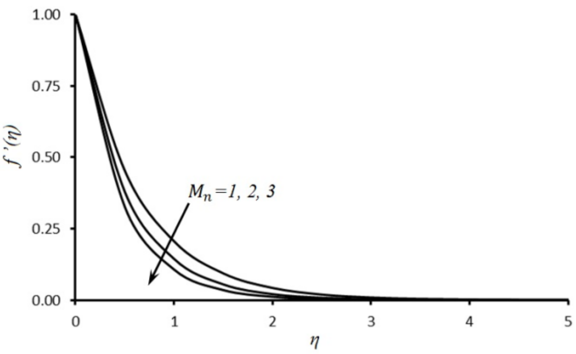

Figure 2. Velocity profile $f'(\eta)$ verses η for different values of M_n when $k_1 = 0.2$, $Ec = 1.5$, $N = 5.0$, $\Pr = 1.0$ and s=2.0

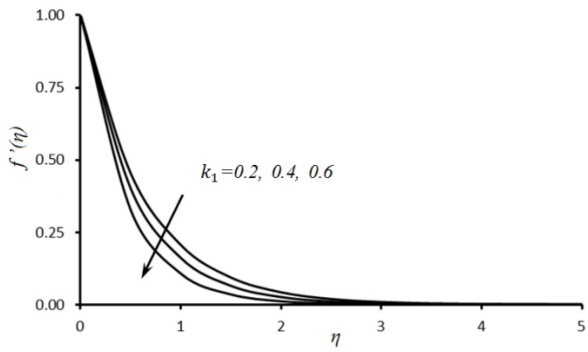

Figure 3. Velocity profile $f'(\eta)$ verses η for different values of k_1 when $M_n = 1.0$, $Ec = 1.5$, $N = 5.0$, $\Pr = 1.0$ and s=2.0

The velocity profiles are depicted verses η in Fig. 2 and Fig. 3 for various values of M_n and k_1. It is noted that $f'(\eta)$ decreases as M_n or k_1 increases. As increase in M_n represent the increase in Lorentz force which opposes the horizontal movement of fluid. The tendency of visco elastic parameter k_1 is to decrease the thickness of boundary layer and hence the velocity reduces.

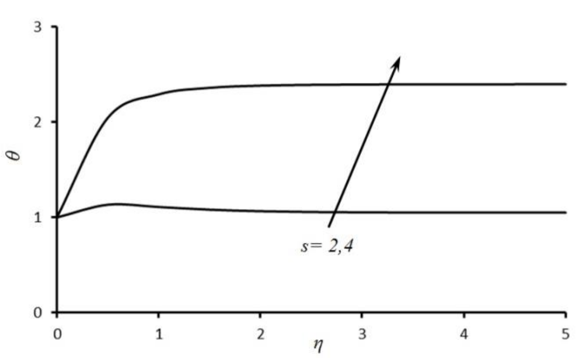

Figure 4. Dimensionless temperature $\theta(\eta)$ verses η for different values of s $(s > 0)$ when $M_n = 1.0$, $k_1 = 0.2$, $Ec = 1.5$, $\beta = 1.27$ and $\Pr = 1.0$

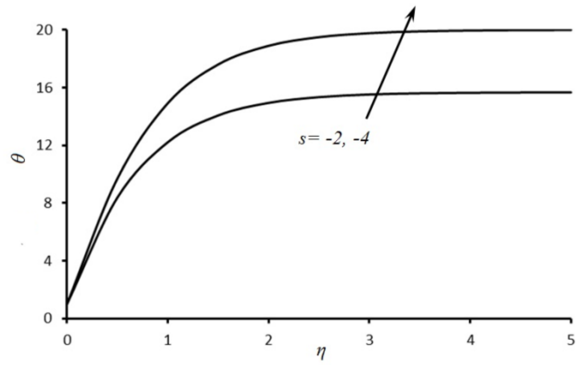

Figure 5. Dimensionless temperature $\theta(\eta)$ verses η for different values of s $(s < 0)$ when $M_n = 1.0$, $k_1 = 0.2$, $Ec = 1.5$, $N = 5.0$ and $\Pr = 1.0$

The fluid temperature is drawn verses η for different values of s in fig.4 and fig.5. It is been noted, temperature reduces with increase in s when $s < 0$, whereas temperature increases with s when $s > 0$. The reciprocity of wall temperature parameter reduces the temperature, and when wall temperature is in proportion to positive power of x then temperature of wall increases with s.

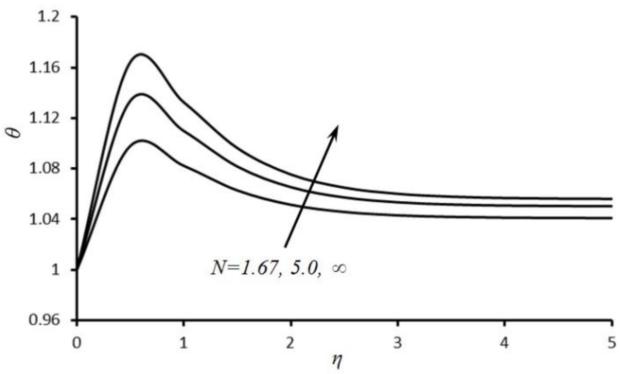

Figure 6. Dimensionless temperature $\theta(\eta)$ verses η for different values of N when $M_n = 1.0$, $k_1 = 0.2$, $Ec = 1.5$,$\Pr = 1.0$ and s=2.0

In Fig. 6 temperature plotted verses η for different values of radiation parameter. The temperature increases with radiation parameter, this is due to the fact that the thermal radiation impacts significant rise in the thermal state of the fluid, which increases the thermal boundary layer.

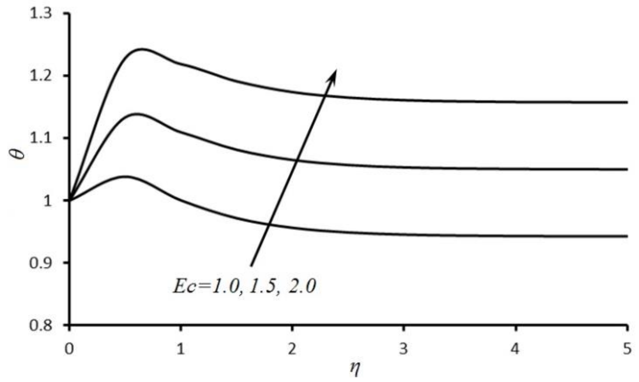

Figure 7. Dimensionless temperature $\theta(\eta)$ verses η for different values of Ec when $M_n = 1.0$, $k_1 = 0.2$, $N = 5.0$, $\mathrm{Pr} = 1.0$ and s=2.0

The temperature is drawn against η for different values of Eckert number in fig.7. The Ec rise in the flow signifies the increment in energy that induces the higher fluid temperature.

4. CONCLUSION

It is been noted that the velocity reduces when magnetic field or visco elasticity increases. When the wall temperature parameter is positive, then temperature increases with it, whereas the reverse phenomenon is observes when wall temperature is negative. The radiation or dissipation enhances the fluid temperature.

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