ORIGINAL PAPER

SOLUTION OF THE SECOND ORDER ORDINARY DIFFERENTIAL EQUATIONS USING ADOMIAN DECOMPOSITION METHOD

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Abstract: *In this paper, the Adomian Decomposition Method (ADM) is employed in solving second order ordinary differential equation. Numerical algorithm was developed. The decomposition method provides a solution as an infinite series in which terms can easily be determined. It is observed that the method is practically suited for initial value problems. The method is effective and easy to implement. The results were presented in both tabular and graphical forms.*

Keywords: *Adomian Decomposition Method; differential equations; order; initial value problem.*

1. INTRODUCTION

Adomian Decomposition Method (ADM) is an effective method of solving singular initial value problems (IVPs) and very useful in various ranges of mathematical problems which are challenging in nature. There has been considerable deal of researches in applying Adomian decomposition method for solving deferential and integral equations, linear and nonlinear, homogeneous and non-homogeneous. Application of Adomian Decomposition Method (ADM) in [1,2] shows a rapid convergent series alongside an excellent computable terms.

Application of Adomian Decomposition method has been on wide class of functional equations, the method gives the series as an infinite series converging to an accurate solution. Abbaoui and Cherruault [3] applied the standard Adomian Decomposition on basic iteration method to solve the equation $f(x)=0$, where $f(x)$ a non-linear function, and shows effectiveness in the convergence of the series solution.

Obviously the method can be used to solve various classes of linear and nonlinear differential equations, both ordinary and partial [4,5]. Recently, the solution of fractional ordinary differential equations has been obtained through the Adomian Decomposition method [6,7]. Also, El-Shahed and Salem [8] obtained the generalized classical Navier-Stokes equations by replacing the time derivative by fractional derivative of order $\alpha, 0<\alpha\leq 1$

Wazwaz [9] extended the method to include the solution of Volterra integral equation and the boundary value problems for higher order integro-differential equations. Recently, a new modification of Adomian decomposition method (NMADM) for finding exact solution of

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linear integral equations is presented by Hossein et al. [10]. A new reliable modification of the ADM is proposed and applied for the solution of the Volterra and Fredholm integral equations in Bakodah et al.[11]. In recent years, Olayiwola et al. [12], Rabbani & Zarali [13], Hendi &Bakodah [14], Manafianheris [15] and Alao et al. [16] did some work on the solutions of Volterra-Fredholm integro-differential equations.

2. ADOMIAN DECOMPOSITION METHOD

Adomian decomposition method is a semi-analytical method for solving differential equations.

It is assumed that $y = f(x)$ is differentiable where

$$y' = f(x, y); y(a) = y_o \tag{1}$$

Equ. (1) exists and satisfies the Lipchitz condition, where $L = \frac{d}{dx}$ from the inverse operator that is; L^{-1} defined as the integral operator such that:

$$L^{-1} = \int_0^x (.)dx \tag{2}$$

For first order differential equation we obtain $L = \frac{d^2}{dx^2}$ with integral operator defined by

$$L^{-1} = \int_0^x \int_0^x (.)dxdx \tag{3}$$

For the second order of ordinary differential equations, we obtained numerical solution such that:

$$y'' - f(x, y)$$

given by

$$y(x) = y_o + y_1(x) + L^{-1}[f(x, y)] \tag{4}$$

where $y_1 = y^1(x_o)$

In General we obtain:

$$\sum_{n-0}^{\infty} y_n(x) = y(x) \tag{5}$$

The series of Adomian Decomposition Method is obtained as a result of the infinite sum of :

$$y(x)=\sum_{n=0}^{\infty}y_n x \tag{6}$$

3. NUMERICAL EXAMPLES

Here, we concentrate on the application of Adomian decomposition Method to generate numerical results. The first ten or more terms of the decomposition series are used in computing the results.

Example 1: We consider the second order differential equation of the form:

$$\begin{aligned} &y''=y-x \\ &y(0)=1, y'(0)=0 \end{aligned} \tag{7}$$

$$y_0=1$$

$$y_1=1+\frac{x^2}{2!}-\frac{x^3}{3!}$$

$$y_2=1+\frac{x^2}{2!}-\frac{x^3}{3!}+\frac{x^4}{4!}+\frac{x^5}{5!}$$

$$y_3=1+\frac{x^2}{2!}-\frac{x^3}{3!}+\frac{x^4}{4!}-\frac{x^5}{5!}+\frac{x^6}{6!}-\frac{x^7}{7!}$$

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$$\begin{aligned} y_{10}=1&+\frac{x^2}{2!}-\frac{x^3}{3!}+\frac{x^4}{4!}-\frac{x^5}{5!}+\frac{x^6}{6!}-\frac{x^7}{7!}+\frac{x^8}{8!}-\frac{x^9}{9!} \\ &+\frac{x^{10}}{10!}-\frac{x^{11}}{11!}+\frac{x^{12}}{12!}-\frac{x^{13}}{13!}+\frac{x^{14}}{14!}-\frac{x^{15}}{15!}+\frac{x^{16}}{16!}-\frac{x^{17}}{17!}+\frac{x^{18}}{18!}-\frac{x^{19}}{19!} \\ &+\frac{x^{20}}{20!}-\frac{x^{21}}{21!} \end{aligned}$$

With the exact solution of the form:

$$y(x)=e^{-x}+x \tag{8}$$

The numerical solution obtained was compared with the exact solution. The detail of the results is given in the Table 1 and Fig. 1 below.

Table 1.Comparison of the ADM Solution with the exact when n=10

x	y- approximate	y-exact	Error
0	1.0000000000E+00	1.0000000000E+00	0.0000000000E+00
0.1	1.0048374180E+00	1.0048374180E+00	2.2204460493E-16
0.2	1.0187307531E+00	1.0187307531E+00	2.2204460493E-16
0.3	1.0408182207E+00	1.0408182207E+00	0.0000000000E+00

x	y- approximate	y-exact	Error
0.4	1.0703200460E+00	1.0703200460E+00	2.2204460493E-16
0.5	1.1065306597E+00	1.1065306597E+00	0.0000000000E+00
0.6	1.1488116361E+00	1.1488116361E+00	3.1086244690E-15
0.7	1.1965853038E+00	1.1965853038E+00	3.1308289294E-14
0.8	1.2493289641E+00	1.2493289641E+00	2.3003821070E-13
0.9	1.3065696597E+00	1.3065696597E+00	1.3375967001E-12
1	1.3678794412E+00	1.3678794412E+00	6.4523941745E-12

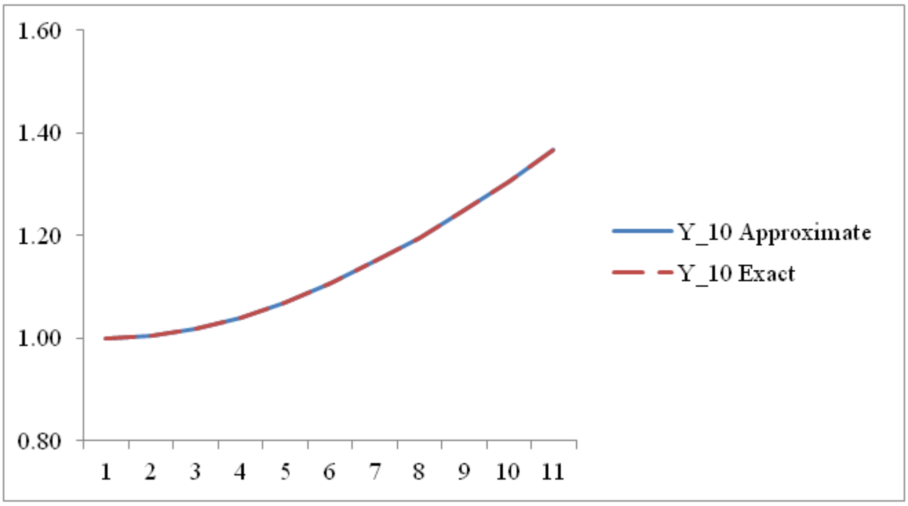

Figure 1. Graphical representation of Table 1.

Example 2: We consider the second order differential equation of the form:

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = 0 \quad (9)$$

$y(0)=1, y'(0)=3$

With the exact solution:

$$y(t) = (1+5t)e^{-2t} \quad (10)$$

$y_0 = 1$

$y_1 = 1 + 3t - 2t^2$

$y_2 = 1 + 3t - 8t^2 + \frac{2t^3}{3} + \frac{2t^4}{3}$

$y_3 = 1 + 3t - 8t^2 + \frac{26t^3}{3} + 2t^4 - \frac{2t^5}{3} - \frac{4t^6}{45}$

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$$y_{15} = 1 + 3t - 8t^2 + \frac{26t^3}{3} - 6t^4 + \frac{46t^5}{15} - \frac{56t^6}{45} + \frac{44t^7}{105} - \frac{38t^8}{315} + \frac{86t^9}{2835} - \frac{32t^{10}}{4725} - \frac{212t^{11}}{155925} - \frac{116t^{12}}{467775} + \frac{4t^{13}}{96525} - \frac{272t^{14}}{42567525} + \frac{584t^{15}}{638512875} + \frac{3638t^{16}}{23648625} + \ldots$$

Table 2. Comparison of the ADM Solution with the exact for example 2 when n=15

t	y-approximate	y-exact	Error
0	1.0000000000E+00	1.0000000000E+00	0.0000000000E+00
0.1	1.2280961296E+00	1.2280961296E+00	0.0000000000E+00
0.2	1.3406400921E+00	1.3406400921E+00	1.3322676296E-15
0.3	1.3720290902E+00	1.3720290902E+00	7.8426154460E-13
0.4	1.3479868924E+00	1.3479868924E+00	8.2708506710E-11
0.5	1.2875780472E+00	1.2875780441E+00	3.1039821824E-09
0.6	1.2047769082E+00	1.2047768476E+00	6.0595289231E-08
0.7	1.1096870911E+00	1.1096863377E+00	7.5338965777E-07
0.8	1.0094893219E+00	1.0094825900E+00	6.7318997314E-06
0.9	9.0919062017E-01	9.0914388522E-01	4.6734946611E-05
1	8.1227755419E-01	8.1201169942E-01	2.6585477113E-04

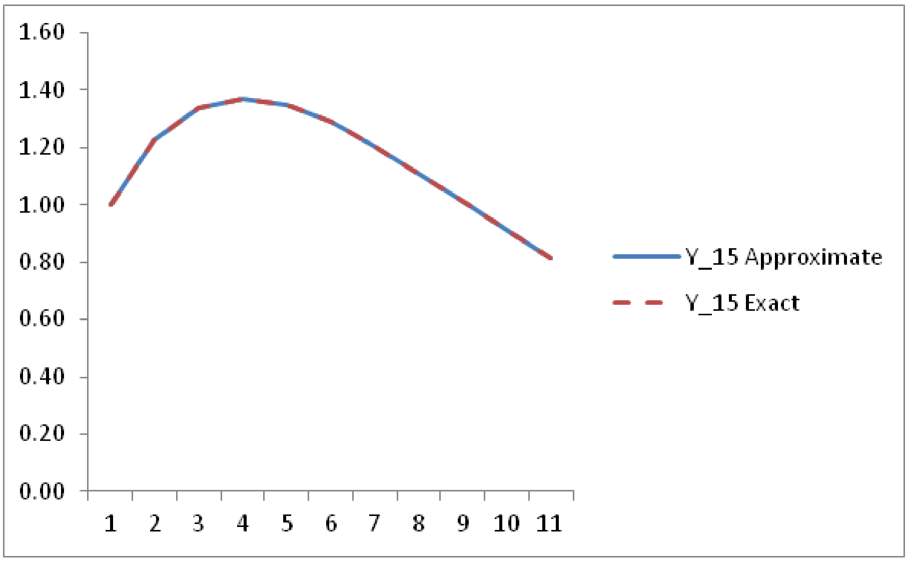

Figure 2. Graphical representation of Table 2.

Example 3: We will consider the second order differential equation of the form

$$y'' = t - y \tag{11}$$

$y(0) = 1, \qquad y' = 2$

$y(t) = \sin t + \cos t + t$

$y_0 = 1$

$y_1 = 1 + 2t - \frac{t^2}{2!} + \frac{t^3}{3!}$

$y_2 = 1 + 2t - \frac{t^2}{2!} - \frac{t^3}{3!} + \frac{t^4}{4!} - \frac{t^5}{5!}$

$y_3 = 1 + 2t - \frac{t^2}{2!} - \frac{t^3}{3!} + \frac{t^4}{4!} + \frac{t^5}{5!} - \frac{t^6}{6!} + \frac{t^7}{7!}$

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$$y_{10} = 1 + 2t - \frac{t^2}{2!} - \frac{t^3}{3!} + \frac{t^4}{4!} + \frac{t^5}{5!} - \frac{t^6}{6!} - \frac{t^7}{7!} + \frac{t^8}{8!} + \frac{t^9}{9!} - \frac{t^{10}}{10!}$$
$$- \frac{t^{11}}{11!} + \frac{t^{12}}{12!} + \frac{t^{13}}{13!} - \frac{t^{14}}{14!} + \frac{t^{15}}{15!} + \frac{t^{16}}{16!} - \frac{t^{17}}{17!} - \frac{t^{18}}{18!} + \frac{t^{19}}{19!}$$
$$+ \frac{t^{20}}{20!} - \frac{t^{21}}{21!}$$

Table 3. Comparison of the approximate Solution with the exact for example 3 when n=15

t	y-approximate	y-exact	Error
0	1.0000000000E+00	1.0000000000E+00	0.0000000000E+00
0.1	1.1948375819E+00	1.1948375819E+00	0.0000000000E+00
0.2	1.3787359086E+00	1.3787359086E+00	0.0000000000E+00
0.3	1.5508566958E+00	1.5508566958E+00	2.2204460493E-16
0.4	1.7104793363E+00	1.7104793363E+00	0.0000000000E+00
0.5	1.8570081005E+00	1.8570081005E+00	0.0000000000E+00
0.6	1.9899780883E+00	1.9899780883E+00	2.2204460493E-16
0.7	2.1090598745E+00	2.1090598745E+00	4.4408920985E-16
0.8	2.2140628002E+00	2.2140628002E+00	4.4408920985E-16
0.9	2.3049368779E+00	2.3049368779E+00	4.4408920985E-16
1	2.3817732907E+00	2.3817732907E+00	0.0000000000E+00

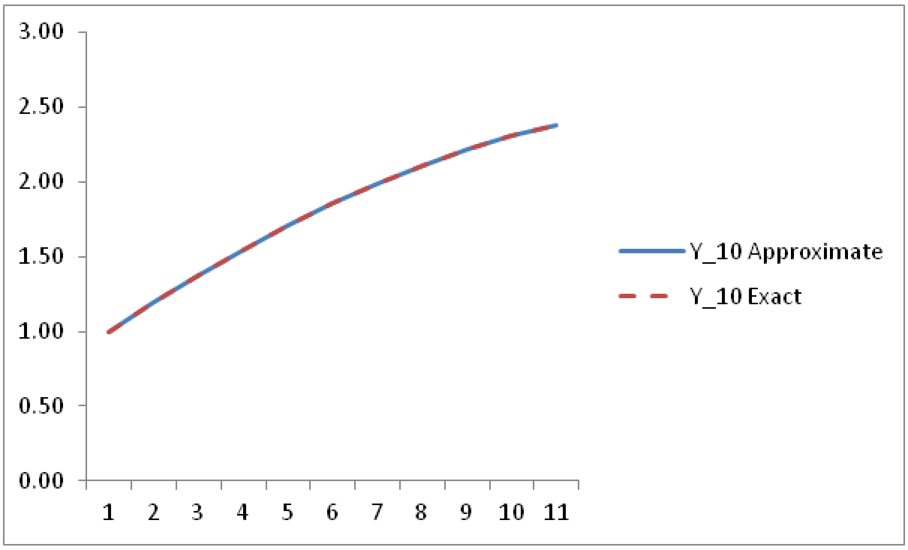

Figure 3. Graphical representation of Table 3.

4. CONCLUSION

From the tables and graphs above, one can easily deduce that Adomian Decomposition Method (ADM) gives numerical results with insignificant error when compared with the exact solution. Also, the results presented here indicate that the method is reliable, accurate and durable. It was observed that better accuracy can be obtained by accommodating more terms in our decomposition series and that the solutions of the presented equations are stable and consistent in the given intervals.

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