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NANO JD CLOSED SETS

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Abstract. *Nano JD closed sets, a new class of Nano generalized closed sets in Nano topological spaces, relies on the g-interior and g closure operators. This paper aims to interpret and investigate its extension and connection with other existing concepts.*

Keywords: *Nano topology; Nano closed; Nano b-closed; Nano pre-closed; Nano semi-closed.*

1. INTRODUCTION

Intelligent information processing is a tricky issue in the field of information science theory and application. As the amount of information increases, the need for information analysis tools increases significantly. Rough set theory, proposed by Pawlak [1], is a momentous mathematical tool for dealing with imprecise, incompatible, curtailed information and knowledge. It is based on creating equivalence classes within the given data. All data sets that form an equivalence class are impossible to differentiate, *i.e.*, the samples are identical with respect to the attributesthat describe the data. An approximate group can be defined by a pair of identifiable groups called lower approximations and upper approximations. The lower approximation is the greatest definable set contained in the given set of objects, while the upper approximation is the smallest definable set containing the given set. The concept of nanotopology was introduced by M. Lellis Thivagar [2]. It was defined in terms of approximations and boundary region of the universe using an equivalence relation on it. Njastad [3], Levine [4], and Mashouret al [5], respectively introduced the notion of -open, semi-open, and pre-open sets. In1986, D. Andrijevic [6] introduced semi pre-open sets in topological spaces. Also, the new class of generalised sets called b-open sets was introduced by D. Andrijevic [7], which is contained in the class of semi pre-open sets and contains all semi-open andpre-open sets. In 2014, A. Dhanis Arul Mary and I. Arockiarani [8] established b-opensets in Nano Topological spaces, called Nano-b open sets and studied some of their characterisations. Recently, S. Jackson and T. Gnanapoo Denosha [9] introduced Nano JD open sets which characterize a new form of weakly open sets in Nano Topological Spaces.

In this paper, we discuss the theorems and propositions on the weaker form of Nano closed sets, namely Nano JD closed sets which was formed using the Nano generalized closure (briefly Ng-closure) operator denoted by Ncl*(A) and Nano generalised-interior (briefly Ng-interior) operator of a subset A of X, denoted by Nint*(A).

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2. PRELIMINARIES

Definition 2.1 [1] Let U be a non empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let A⊆ U

1. The Lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and is defined by

$$L_R(X) = \cup_{x\in U}\{R(x) : R(x) \subseteq X\}$$

2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as with respect to R and is defined by

$$U_R(X) = \cup_{x\in U}\{R(x) : R(x) \cap X \neq \oint \}$$

3. The boundary region of X with respect to R is the set of all objects, which can be classified by

$$B_R(X) = U_R(X) - L_R(X).$$

Definition 2.2 [8] Let U be non-empty, finite universe of objects and R be an equivalence relation on U. Let $X \subseteq U$. Let $\tau_R(X) = \{U, \varphi, L_R(X), U_R(X), B_R(X)\}$. Then $\tau_R(X)$ a topology on U, called as the Nano topology with respect to X. Elements ofthe Nano topology are known as the Nano open sets in U and $(U, \tau_R(X))$ is calledthe Nano topological space. $[\tau_R(X)]^c$ is called the Dual Nano topology on $\tau_R(X)$. Elements of $[\tau_R(X)]^c$ are called as Nano closed sets.

Definition 2.3 [10] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \varphi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(x)$ satisfies the following axioms

1. U and $\varphi \in \tau_R(X)$
2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(x)$.

The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$ .That is $\tau_R(X)$ forms a topology on U called as the Nano Topology on U with respect to X. We call $(U, \tau_R(X))$ as the Nano Topological space. The elements of $\tau_R(X)$ are called Nano open sets. If $(U, \tau_R(X))$ is a Nano Topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$ then the Nano interior of A is defined as the union of all Nano open subsets of A and it is defined as Nint(A). Nano interior is the largest open subset of A.

Definition 2.4 [8] If $\tau_R(X)$ is the Nano Topology on U with respect to X, then the set $B = \{U, L_R(X), B_R(X)\}$ is the basis for $\tau_R(x)$. The elements of $[\tau_R(X)]^c$ are called Nano closed sets with respect to $[\tau_R(X)]^c$ being called Dual Nano topologyof $\tau_R(X)$.

Definition 2.5 [10] The Nano Closure of A is defined as the intersection of all Nano closed sets containing A and it is denoted by Ncl(A). It is the smallest Nano closed set containing A.

Definition 2.6 [10] If (U, $\tau_R(X)$) is a Nano Topological space with respect to X with $A \subseteq U$. Then A is said to be

i. Nano semi-open if $A \subseteq Ncl(Nint(A))$.

ii. Nano pre-open if $A \subseteq Nint(Ncl(A))$.

iii. Nano $\alpha -$ open if $A \subseteq Nint(Ncl(Nint(A)))$.

iv. Nano $\beta -$ open if $A \subseteq Ncl(Nint(Ncl(A)))$.

v. Nano regular-open if $A = Nint(Ncl(A))$.

vi. Nano b-open if $A \subseteq Nint(Ncl(A)) \cup Ncl(Nint(A))$

NSO(U,X), NPO(U,X), N α O(U,X), N β O(U,X), NRO(U,X) and NBO(U,X) respectively denote the families of all Nano-semi open, Nano pre-open, Nano α -open, Nano β -open, Nano regular open and Nano b-open subsets of U.

Definition 2.7 [8] Let (U, $\tau_R(X)$) be a Nano Topological space and $A \subseteq U$. A is said to be Nano semi - closed, Nano pre-closed, Nano α - closed, Nano β - closed, Nano regular closed and Nano b-closed if its complement is Nano semi-open, Nano pre-open, Nano -open, Nano semi pre-open, Nano regular open and Nano b-open respectively.

Definition 2.8 [10] Let (U, $\tau_R(X)$) be a Nano Topological space, a subset A of a Nano Topological Space (U, $\tau_R(X)$) is called Nano α - generalised closed set $N\alpha cl(A) \subseteq V$ where $A \subseteq V$ where V is Nano open.

Definition 2.9 [10] A subset A of a Nano Topological space X is

1. Nano generalized closed (briefly Ng-closed) if $cl(A) \subseteq U$ whenever

$A \subseteq U$ and U is Nano-open in X.

2. Nano generalized open (briefly Ng-open) if $X \setminus A$ is Nano g- closed.

The intersection of all Ng-closed sets containing A is called the Nano g-closure of A and denoted by $Ncl^*(A)$ and the Ng-interior of A is the union of all Ng-open sets contained in A and is denoted by $Nint^*(A)$.

Definition 2.10 [9] A subset A of a space X is

i. Nano semi* open if $A \subseteq Ncl^*(Nint(A))$.

ii. Nano semi* $-$ closed if $X \setminus A$ is Nano semi* $-$ open .

iii. Nano pre* $-$ open if $A \subseteq Nint^*(Ncl(A))$.

iv. Nano pre* $-$ closed if $X \setminus A$ is Nano pre*$-$ open .

v. Nano α^* $-$ open if $A \subseteq Nint^*(Ncl(Nint^*(A))$.

vi. Nano semi pre*open (β^*-open) if $A \subseteq Ncl(Nint^*(Ncl(A))$

vii. Nano JD open if $A \subseteq Nint^*(Ncl(A)) \cup Ncl^*(Nint(A))$

NS*O(U,X),NP*O(U,X), Nα * O(U,X), Nβ^* O(U,X) and NJDO(U,X) respectively denote the families of all Nano semi*open, Nano pre*open, Nano α*open, Nano β^*-open and Nano JD open subsets of U.

Proposition 2.11.

1. $Np^{*}int(S) = S \cap Nint^{*}(Ncl(S))$
2. $Ns^{*}int(S) = S \cap Ncl^{*}(Nint(S))$
3. $Np^{*}int(S) \cup Ncl^{*}(Nint(S) = Np^{*}cl(Npint(S))$
4. $Ncl_{\alpha}(Nint_{\alpha}(S) = Ncl(Nint(S))$
5. $Nint_{\alpha}((Ncl_{\alpha}(S) = Nint(Ncl(S))$

Remark 2.12 [8] In this paper, U and V are non-empty finite universes; X ⊆ U and X ⊆ V ; U/R and V/R' denote the family of equivalence classes by equivalence relations R and R' on U and V. (U, τ_R(X)) and (V, τ_R(Y)) are the Nano topological spaces with respect to X and Y respectively.

3. NANO JD CLOSED SETS

In this section we define and investigate a new class of generalised Nano closed sets called Nano JD-closed sets in Nano topological spaces. Also we introduce new theorems and propositions related to Nano JD open sets, Nano JD interior and Nano JD Closure operators which relies mainly on the Nano generalized interior and Nano generalized closure operators.

Definition 3.1. A subset A of a space X is called Nano JD- closed if **X-A** is Nano JD-open.
Thus, A is **Nano JD-closed** if and only if

$$Ncl^{*}(Nint(A)) \cap Nint^{*}(Ncl(A)) \subseteq A$$

Definition 3.2. If A is a subset of a space X, then the **Nano JD -closure** of A, denoted by NJDclA, is the smallest Nano JD closed set containing A. The **Nano JD interior** of A is denoted by NJDintA, is the largest Nano JD -open set contained in A.

Theorem 3.3.

1. NP*O(X) ∪ NS*O(X) ⊆ NJDO(X)
2. NJDO(X) ⊆ NSP*O(X)

Proof:

1. Let A ∈ NP*O(X) ∪ NS*O(X)
⇒ A ∈ NP*O(X) (or) A ∈ NS*O(X)
⇒ A ⊆ Nint*(Ncl(A)) (or) A ⊆ Ncl*(Nint(A))
⇒ A ⊆ Nint*(Ncl(A)) ∪ Ncl*(Nint(A))
⇒ A ∈ NJDO(X)

2. A ∈ NJDO(X)
⇒ A ⊆ Nint*(Ncl(X)) ∪ Ncl*(Nint(X))
⇒ A ⊆ Ncl(Nint*(Ncl(A)))
⇒ A is a Nano semi pre * open set

$\Rightarrow$ A $\in$ NSP*O(X)

$\Rightarrow$ NJDO(X) $\subseteq$ NSP*O(X)

Remark 3.4. The equality of the above theorem need not be true which can be seen from the following example.

Example 3.5. Let U={a,b,c,d} with U/R= {{a}, {d}, {b,c}} and X={a,c}. Then the topology $\tau_R(X)$ = {U, φ, {a}, {b, c}, {a, b, c}},

NP*O(X) ∪ NS*O(X) = {U, φ, {a}, {b}, {c}, {a, b}, {a, c}, {a, d}, {b, c}, {a, b, c}, {a, b, d}, {a, c, d}}

and

NJDO(U,X) = {U, φ, {a}, {b}, {c}, {a, b}, {a, c}, {a, d}, {b, c}, {a, b, c}, {a, b, d}, {a, c, d},{b, c, d}}.

$NSP^*O(X)$= {U, φ, {a}, {b}, {c}, {a, b},{b,c}, {c,d}, {a, c}, {a, d}, {b,d} {b, c}, {a, b, c}, {a, b, d}, {a, c, d},{b, c, d}}.

Therefore NP*O(X) ∪ NS*O(X) $\neq$ NJDO(U, X) and NJDO(X) $\neq$ $NSP^*O(X)$.

Theorem 3.6. For a Subset S of a Nano Topological space $(U, \tau_R(X))$ the following are equivalent

(a). S is a Nano JD-open

(b). S = Np*int(S) ∪ Ns*int(S)

(c). S $\subseteq$ Np*cl(NpintS)

Proof:

(a) $\Rightarrow$ (b)

Let S be a Nano JD-open,Then we have,

S $\subseteq$ Nint*(Ncl(S)) ∪ Ncl*(Nint(S)

= S ∩ (Nint*(Ncl(S)) ∪ Ncl*(Nint(S)))

= [S ∩ Nint*(Ncl(S)] ∪ [S ∩ Ncl*(Nint(S)]

= Np*int(S) ∪ Ns*int(S) [Proposition 2.11(i),(ii)]

(b) $\Rightarrow$ (c)

S = Np*int(S) ∪ Ns*int(S)

= Np*int(S) ∪ (S ∩ Ncl*(Nint(S)) [Proposition 2.11(ii)]

$\subseteq$ Np*int(S) ∪ Ncl*(Nint(S))

= Np*cl(NpintS) [Proposition 2.11(iii)]

(c) $\Rightarrow$ (a)

S $\subseteq$ Np*cl(NpintS)

= Np*int(S) ∪ Ncl*(Nint(S)) [By proposition 2.11(iii)]

$\subseteq$ Nint*(Ncl(S)) ∪ Ncl*(Nint(S)) and so S is Nano JD-open.

Theorem 3.7. Let S be a Nano JD open set such that $NintS = \varphi$. Then S is Nano Pre^*-open.

Proof: Let S be a Nano JD open set such that $Nint(S) = \varphi$. Since S is Nano JD open set we have, $S \subseteq Ncl^*(NintS) \cup Nint^*(Ncl(S))$

$\Rightarrow S \subseteq Ncl^*(\varphi) \cup Nint^*(Ncl(S))$

$\Rightarrow S \subseteq \varphi \cup Nint^*(Ncl(S))$

$\Rightarrow S \subseteq Nint^*(Ncl(S))$

$\Rightarrow$ S is Nano pre^* -open

Theorem 3.8. The union of any family of Nano JD open sets is a Nano JDopen set.

Proof: Let $\{A_\alpha\}_{\alpha\in\Delta}$ be a family of Nano JD open sets in a space $(U,\tau_R(X))$, then

$A_\alpha \subseteq Nint^*(Ncl(A_\alpha)) \cup Ncl^*(Nint(A_\alpha)), \quad \forall\ \alpha \in \Delta$

Now, $\cup_\alpha A_\alpha \subseteq \bigcup_\alpha Ncl^*(Nint(A_\alpha)) \cup Nint^*(Ncl(A_\alpha))$

$= [\cup_\alpha[Ncl^*(Nint(A_\alpha))]] \cup [\cup_\alpha[Nint^*(Ncl(A_\alpha))]]$

$\subseteq [Nint^*(\cup_\alpha Ncl(A_\alpha))] \cup [Ncl^*(\cup_\alpha Nint(A_\alpha))]$

$\subseteq [Nint^*(Ncl(\cup_\alpha A_\alpha))] \cup [Ncl^*(Nint(\cup_\alpha A_\alpha))]$

$\Rightarrow \cup_\alpha A_\alpha$ is a Nano JD Set

Remark 3.9. The intersection of Nano JD-open sets need not be a Nano JD-open set which can be seen from the following example.

Example 3.10. U={a,b,c,d} with U/R ={{a},{c},{b,d}} and X={a,b}.Then the Nano topology is defined as $\tau_R(X)$ = {U, φ, {a}, {b, d}, {a, b, d}}. Here the sets {a,c}, {b,c,d} are Nano JD-open but their intersection (i.e) $A \cap B$ which equals to {c} is not a Nano JD open set.

Theorem 3.11. The intersection of a Nano open set and a Nano JD-open set is a Nano JD-open set.

Proof: Let A be a Nano open and B be a Nano JD-open. Consider,
$S = A \cap B$

$= NintA \cap NJDint(B) \subseteq NJDint(A) \cap NJDint(B)$
$= NJDint(A \cap B)$
$= NJDint(S)$

(i.e)., $S \subseteq NJDint(S)$,

But, $NJDint(S) \subseteq S$
Hence, $S = NJDint(S)$ (i.e) $S = A \cap B$ is a Nano JD open set.

Theorem 3.12. If $(U, \tau_R(X))$ is a Nano topological space, then
(a). The intersection of a Nano α^* -open set and a Nano JD-open set is a Nano JD- open set
(b). τ and τ_α^* have the same class of Nano JD open sets.

Proof:
(a). Let A be a Nano α^* -open set and B be a Nano JD-open set
Now, $S = A \cap B = Nint_\alpha^*(A) \cap NJDint(B)$

$\subseteq$ NJDint(A) $\cap$ NJDint(B)
=NJDint($A \cap B$) = NJDint(S).

That is, S $\subseteq$ NJDint(S)

But, NJDint(S) $\subseteq$ S
Hence, S = NJDint(S)
(i.e) S = $A \cap B$ is a nano JD open set.

(b). Let S be an arbitrary Nano JD-open sets with respect to $(U, \tau_R(X))$.
Then S $\subseteq$ Ncl*(Nint(S)) $\cup$ Nint*(Ncl(S))
$= Ncl_\alpha^*(Nint_\alpha(S)) \cup Nint_\alpha^*(Ncl_\alpha(S))$
Which implies S is a Nano JD -open set with respect to τ_α^*
Thus, S $\in$ NJDO(U, τ) $\Leftrightarrow$ S $\in$ NJDO(U, τ_α).
Hence the Theorem.

Theorem 3.13. Let A be a subset of a Nano Topological space $(U, \tau_R(X))$ then
(1). NJDcl(A) = Ns*cl(A) $\cap$ Np*cl(A)
(2). NJDint(A) = Ns*int(A) $\cup$ Np*int(A)

Proof:
(1). Let A be a subset of a Nano Topological space $(U, \tau_R(X))$
Since NJDcl(A) is a Nano JD closed set,
We have,
Ncl*(Nint(NJDcl(A)) $\cap$ Nint*(Ncl(NJDcl(A))) $\subseteq$ NJDcl(A)
Again, Ncl*(Nint(A)) $\cap$ Nint*(Ncl(A))
$\subseteq$ Ncl*(Nint(NJDcl(A)) $\cap$ Nint*(Ncl(NJDcl(A)))
Hence, Nint*(Ncl(A)) $\cap$ Ncl*(Nint(A)) $\subseteq$ NJDcl(A)
Also, A $\cup$ [(Nint*(Ncl(A)) $\cap$ Ncl*(Nint(A))] = Ns*cl(A) $\cap$ Np*cl(A) $\subseteq$ NJDcl(A)
But,NJDcl(A) $\subseteq$ Ns*cl(A) $\cap$ Np*cl(A).
Therefore, NJDcl(A) = Ns*cl(A) $\cap$ Np*cl(A).
Part(2) can be proved in a similar way.

Theorem 3.14. Arbitrary intersection of Nano JD-closed sets is Nano JD-closed set.

Proof: Let $\{B_\alpha\}_{\alpha\in\Delta}$ be a family of Nano JD-closed sets in a Nano topological space $(U, \tau_R(X))$, then $(Nint^*(Ncl(B_\alpha)) \cap Ncl^*(Nint(B_\alpha)) \subseteq B_\alpha, \forall \alpha \in \Delta$
Since, $\{{B_\alpha}^c\}_{\alpha\in\Delta}$ is an arbitrary family of Nano JD- open sets, by theorem 3.8,

$\cup_{\alpha\epsilon\,\Delta} B_\alpha^c$ is a Nano JD- open set. But, $\cup_{\alpha\epsilon\Delta} B_\alpha^c = [\cap\alpha\in\Delta B\alpha]^C$ which implies $\cap_{\alpha\epsilon\Delta} B_\alpha$ is a Nano JD-closed set.

Proposition 3.15. Let S be a subset of a Nano Topological space (U, τ_R(X)). Then :

1. NJDcl(Nint(S)) = Nint(NJDcl(S)) =Nint*(Ncl(Nint(S))
2. NJDint(Ncl(S)) = Ncl(NJDint(S) = Ncl*(Nint(Ncl(S))
3. NJDcl(Npint(S)) = Np*int(Npcl(S))
4. NJDint(Npcl(S)) =Np*cl(Npint(S))
5. Npint(NJDcl(S)) = Np*cl(S) ∩ Nint*(Ncl(S))
6. Npcl(NJDint(S)) = Np*int(S) ∩ Ncl*(Nint(S))
7. Npint(NJDcl(S)) = NJDcl(Npint(S)) = Np*int(Npcl(S))
8. Npcl(NJDint(S)) = NJDint(Npcl(S)) = Npcl(Npint(S))
9. Nαint(NJDcl(S)) = NJDcl(Nαint(s)) = Nscl(Nαints) ∩ Npcl(S)

Proof: The proof is obvious.

Theorem 3.16. A subset A of a Nano topological space (U, τ_R(X)) is Nano JD open if and only if every Nano g-closed set F containing A, there exist a Nano maximal open set M contained in Ncl(A) and Nano minimal closed set N containing Nint(A) such that A ⊆ M ∪ N ⊆F.

Proof: Let A be a Nano JD-open in a Nano topological space (U, τ_R(X)). Then

$$A \subseteq Nint^*(Ncl(A)) \cup Ncl^*(Nint(A) \quad (1)$$

Let A ⊆ F and F is Nano g-closed, so that Ncl*(A) ⊆ F. Since every Nano g-closed set is a Nano closed set, we also have Ncl(A) ⊆ F .

Let M = Nint*(Ncl(A)) , then M is the Nano maximal open set contained in Ncl(A).

Let N = Ncl*(Nint(A), then N is the Nano minimal closed containing Nint(A). Again,

A ⊆ Ncl(A) ⊆ F & Nint*(Ncl(A)) ⊆ Ncl(A) .

$$\Rightarrow Nint^*(Ncl(A)) \subseteq F \quad (2)$$

Next, Nint(A) ⊆ A and Ncl*(Nint(A) ⊆ cl*(A) & cl*(A) ⊆ F.

$$\Rightarrow Ncl^*(Nint(A) \subseteq F \quad (3)$$

From (2) and (3), we have

$$Nint^*(Ncl(A)) \cup Ncl^*(Nint(A) \subseteq F \quad (4)$$

Combining (1) and (4), we have,

A ⊆ Nint*(Ncl(A)) ∪ Ncl*(Nint(A) ⊆ F (or) A ⊆ M ∪ N ⊆ F

Conversely, assume that the condition holds good (i.e) $A \subseteq M \cup N \subseteq F$ where A is a subset of a topological space. F is closed and M is the maximal open set contained in cl(A). Therefore, M = Nint*(Ncl(A)) & N = Ncl*(Nint(A) .

Thus the above condition reduces to $A \subseteq$ Nint*(Ncl(A)) $\cup$ Ncl*(Nint(A) $\subseteq$ F, which implies A is a Nano JD-open set.

Theorem 3.17. A subset in a topological space (U, $\tau_R(X)$) is Nano JD-open iff there exist Nano pre-open set U in (U, $\tau_R(X)$) such that $U \subseteq A \subseteq$ Np*cl(U).

Proof: Let A be Nano JD-open. Then by Theorem 3.6

$$A \subseteq Np^*cl(Npint(A) \qquad (1)$$

Now, as usual, NpintA $\subseteq$ A and U = pintA , a Nano pre-open set. Hence from (1), it follows that $U \subseteq$ Np*cl(U). Conversely, for a set A there exists a Nano pre-open set U such that

$$U \subseteq A \subseteq Np^*cl(U) \qquad (2)$$

Since Npint(A) is the maximal Nano pre-open set contained in A, hence,

$$U \subseteq Npint(A) \subseteq A \qquad (3)$$

Now, Np*cl(U) $\subseteq$ Np*cl(Npint(A)) (From (3)) (4)

Combining (2) and (3) we get $A \subseteq$ Np*cl(Npint(A) where A is a Nano JD-open set.

Theorem 3.18. Let V be a Nano JD-open set in a Nano topological space (U, $\tau_R(X)$), then V \Nint*(Ncl(Nint(V))) is a Nano pre* open set.

Proof: Let V be a Nano JD-open in a space (U, $\tau_R(X)$), then $V \subseteq$ Ncl* (Nint(V) $\cup$ N int*(Ncl(V)).

Since Nint* (A) $\subseteq$ A for all $A \subseteq X$, hence substituting Ncl(NintV) for A.

We have Nint* (Ncl(Nint(V)) $\subseteq$ Ncl(NintV).

This means that Nint*(Ncl(Nint(V)) is a semi closed set.

And in turn it is Nano JD-closed. Now, S = V – Nint* (Ncl(Nint(V)) is Nano JD-open.

Also, Nint(S)= φ.

Using Theorem 3.7 and the above facts we prove that S is a Nano pre*−open set.

Hence the Theorem.

4. CONCLUSION

In this paper, we have introduced the concept of Nano JD-closed sets and have studied the characterizations of Nano JD closed and Nano JD open sets. Also, have introduced Nano JD-interior, Nano JD closure operators and have recognized a number of remarkable properties. As conclusion, this paper is just a beginning of a new structure and it shall inspire many to contribute to the cultivation of Nano topology in the field of Mathematics.

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