ORIGINAL PAPER

SMARANDACHE $\Pi_1 B$ CURVES OF BIHARMONIC NEW TYPE CONSTANT Π_2 –SLOPE CURVES ACCORDING TO TYPE-2 BISHOP FRAME IN THE SOL SPACE

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Abstract. *In this paper, we study Smarandache* $\mathbf{\Pi}_1\mathbf{B}$ *curves of biharmonic new type constant* $\mathbf{\Pi}_2-$ *slope curves according to type-2 Bishop frame in the Sol space. Type-2 Bishop equations of Smarandache* $\mathbf{\Pi}_1\mathbf{B}$ *curves are obtained in terms of base curve's type-2 Bishop invariants. Subsequently, we express some interesting relations.*

Keywords: *Type-2 Bishop frame; Sol space; Smarandache* $\mathbf{\Pi}_1\mathbf{B}$ *curve.*

1. INTRODUCTION

The Frenet formulas express physical properties of a particle in modeling geometry. Then, the applied geometric properties of curve itself are irrespective of some motions. Also, the differential geometry of curves and surfaces are reviewed in some aspects [1-14].

Bishop frame is defined an alternative or parallel frame of curves [15-27]. Also, Bishop frame has fascinated attention of researchers from time-to-time and it has been utilized in diverse papers [27-36]. A new version of Bishop frame with the help of a common field using binormal vector field of a regular curve is declared and this new frame is called as " Type-2 Bishop frame" . Hence, spherical images of this frame are studied [17].

In this paper, we characterize Smarandache $\mathbf{\Pi}_1\mathbf{B}$ curves of biharmonic new type constant $\mathbf{\Pi}_2-$ slope curves according to type-2 Bishop frame in the SOL^3. Finally, we express some interesting relations with type-2 Bishop invariants.

2. MATERIALS AND METHODS

Let γ be a unit speed regular curve in SOL^3 and $\{\mathbf{T},\mathbf{N},\mathbf{B}\}$ be its Frenet--Serret frame. Let us express a relatively parallel adapted frame:

$$\begin{aligned}\nabla_{\mathbf{T}}\mathbf{\Pi}_1 &= -\varepsilon_1\mathbf{B},\\ \nabla_{\mathbf{T}}\mathbf{\Pi}_2 &= -\varepsilon_2\mathbf{B},\\ \nabla_{\mathbf{T}}\mathbf{B} &= \varepsilon_1\mathbf{\Pi}_1+\varepsilon_2\mathbf{\Pi}_2,\end{aligned}$$

where

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$$g_{SOL^3}(\mathbf{B},\mathbf{B}) = 1, g_{SOL^3}(\mathbf{\Pi}_1,\mathbf{\Pi}_1)=1, g_{SOL^3}(\mathbf{\Pi}_2,\mathbf{\Pi}_2)=1,$$
$$g_{SOL^3}(\mathbf{B},\mathbf{\Pi}_1) = g_{SOL^3}(\mathbf{B},\mathbf{\Pi}_2)= g_{SOL^3}(\mathbf{\Pi}_1,\mathbf{\Pi}_2)=0.$$

We shall call this frame as Type-2 Bishop Frame. In order to investigate this new frame's relation with Frenet--Serret frame, first we write

$$\tau = \sqrt{\varepsilon_1^2+\varepsilon_2^2}.$$

The relation matrix between Frenet--Serret and type-2 Bishop frames can be expressed

$$\mathbf{T} = \sin \mathrm{A}(s)\mathbf{\Pi}_1 - \cos \mathrm{A}(s)\mathbf{\Pi}_2,$$
$$\mathbf{N} = \cos \mathrm{A}(s)\mathbf{\Pi}_1 + \sin \mathrm{A}(s)\mathbf{\Pi}_2,$$
$$\mathbf{B} = \mathbf{B}.$$

So by Frenet--Serret frame, we may express

$$\varepsilon_1 = -\tau \cos \mathrm{A}(s),$$
$$\varepsilon_2 = -\tau \sin \mathrm{A}(s).$$

The frame $\{\mathbf{\Pi}_1,\mathbf{\Pi}_2,\mathbf{B}\}$ is properly oriented, and τ and $\mathrm{A}(s)=\int_0^s \kappa(s)ds$ are polar coordinates for the curve γ. We shall call the set $\{\mathbf{\Pi}_1,\mathbf{\Pi}_2,\mathbf{B},\varepsilon_1,\varepsilon_2\}$ as type-2 Bishop invariants of the curve γ, [17].

With respect to the orthonormal basis $\{\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3\}$, we can write

$$\mathbf{\Pi}_1 = \pi_1^1\mathbf{e}_1+\pi_1^2\mathbf{e}_2+\pi_1^3\mathbf{e}_3,$$
$$\mathbf{\Pi}_2 = \pi_2^1\mathbf{e}_1+\pi_2^2\mathbf{e}_2+\pi_2^3\mathbf{e}_3.$$
$$\mathbf{B} = B^1\mathbf{e}_1+B^2\mathbf{e}_2+B^3\mathbf{e}_3,$$

Theorem 2.1. Let $\gamma : I \to \mathrm{SOL}^3$ be a unit speed non-geodesic biharmonic new type constant $\mathbf{\Pi}_2-$slope curves according to type-2 Bishop frame in the SOL^3. Then, the parametric equations of γ are

$$\mathbf{x}(s) = e^{-\frac{1}{\kappa}\cos[\kappa s]\cos \mathrm{E}+\frac{1}{\kappa}\sin[\kappa s]\sin \mathrm{E}-\mathrm{R}_3}[\sin[\kappa s]\cos \mathrm{E}\cos[\mathrm{R}_1 s+\mathrm{R}_2]$$
$$-\cos[\kappa s]\sin \mathrm{E}\cos[\mathrm{R}_1 s+\mathrm{R}_2]]ds,$$
$$\mathbf{y}(s) = \int e^{\frac{1}{\kappa}\cos[\kappa s]\cos \mathrm{E}-\frac{1}{\kappa}\sin[\kappa s]\sin \mathrm{E}+\mathrm{R}_3}[\sin[\kappa s]\cos \mathrm{E}\sin[\mathrm{R}_1 s+\mathrm{R}_2]$$
$$-\cos[\kappa s]\sin \mathrm{E}\sin[\mathrm{R}_1 s+\mathrm{R}_2]]ds,$$
$$\mathbf{z}(s) = \frac{1}{\kappa}\cos[\kappa s]\cos \mathrm{E}-\frac{1}{\kappa}\sin[\kappa s]\sin \mathrm{E}+\mathrm{R}_3,$$

where $\mathrm{R}_1,\mathrm{R}_2,\mathrm{R}_3$ are constants of integration.

This study is organised as follows: Firstly, we study Smarandache $\mathbf{\Pi}_1\mathbf{B}$ curves of biharmonic new type constant $\mathbf{\Pi}_2-$ slope curves according to type-2 Bishop frame in the SOL^3. Secondly, type-2 Bishop equations of Smarandache $\mathbf{\Pi}_1\mathbf{B}$ curves are obtained in terms of base curve's type-2 Bishop invariants. Finally, we express some interesting relations and illustrate some examples of our main results.

3. SMARANDACHE $\mathbf{\Pi}_1\mathbf{B}$ CURVES

Definition 3.1. Let $\gamma : I \to \mathrm{SOL}^3$ be a unit speed curve in the Sol Space SOL^3 and $\{\mathbf{\Pi}_1, \mathbf{\Pi}_2, \mathbf{B}\}$ be its moving type-2 Bishop frame. Smarandache $\mathbf{\Pi}_1\mathbf{B}$ curves are defined by

$$\bar{\gamma} = \frac{1}{\sqrt{2\varepsilon_1^2 + \varepsilon_2^2}} (\mathbf{\Pi}_1 + \mathbf{B}). \tag{3.1}$$

Then, we have the following theorem.

Theorem 3.2. Let $\gamma : I \to \mathrm{SOL}^3$ be a unit speed non-geodesic biharmonic constant $\mathbf{\Pi}_2-$slope curves according to type-2 Bishop frame in the SOL^3. Then, the equation of Smarandache $\mathbf{\Pi}_1\mathbf{B}$ curves of biharmonic constant $\mathbf{\Pi}_2-$slope curves is given by

$$\begin{aligned} \bar{\gamma}(s) = {} & \frac{1}{\sqrt{2\varepsilon_1^2 + \varepsilon_2^2}} [-\sin[\mathrm{R}_1 s + \mathrm{R}_2] + \cos \mathrm{E} \cos[\mathrm{R}_1 s + \mathrm{R}_2]]\mathbf{e}_1 \\ & + \frac{1}{\sqrt{2\varepsilon_1^2 + \varepsilon_2^2}} [\cos[\mathrm{R}_1 s + \mathrm{R}_2] + \cos \mathrm{E} \sin[\mathrm{R}_1 s + \mathrm{R}_2]]\mathbf{e}_2 \\ & + \frac{1}{\sqrt{2\varepsilon_1^2 + \varepsilon_2^2}} [-\sin \mathrm{E}]\mathbf{e}_3, \end{aligned} \tag{3.2}$$

where $\mathrm{R}_1, \mathrm{R}_2$ are constants of integration.

We have the following corollary of Theorem 3.2.

Corollary 3.3. Let $\gamma : I \to \mathrm{SOL}^3$ be a unit speed non-geodesic biharmonic constant $\mathbf{\Pi}_2-$slope curve according to type-2 Bishop frame in the SOL^3. Then, the parametric equations of Smarandache $\mathbf{\Pi}_1\mathbf{B}$ curve of biharmonic constant $\mathbf{\Pi}_2-$slope curve are given by

$$x_{\bar{\gamma}}(s) = e^{\frac{\sin \mathrm{E}}{\sqrt{2\varepsilon_1^2+\varepsilon_2^2}}} \frac{1}{\sqrt{2\varepsilon_1^2+\varepsilon_2^2}}[-\sin[\mathrm{R}_1 s+\mathrm{R}_2]+\cos\mathrm{E}\cos[\mathrm{R}_1 s+\mathrm{R}_2]],$$

$$y_{\bar{\gamma}}(s) = e^{\frac{-\sin \mathrm{E}}{\sqrt{2\varepsilon_1^2+\varepsilon_2^2}}} \frac{1}{\sqrt{2\varepsilon_1^2+\varepsilon_2^2}}[\cos[\mathrm{R}_1 s+\mathrm{R}_2]+\cos\mathrm{E}\sin[\mathrm{R}_1 s+\mathrm{R}_2]], \qquad (3.3)$$

$$z_{\bar{\gamma}}(s) = \frac{1}{\sqrt{2\varepsilon_1^2+\varepsilon_2^2}}[-\sin\mathrm{E}],2.3$$

where $\mathrm{R}_1,\mathrm{R}_2$ are constants of integration.

Proof: According to Theorem 3.2, we have (3.3). The conclusion holds. This ends the proof.

We can now state the main result of the paper.

In this section, we shall call the set $\{\overline{\mathbf{\Pi}}_1,\overline{\mathbf{\Pi}}_2,\overline{\mathbf{B}}\}$ as type-2 Bishop frame, $\bar{\varepsilon}_1$ and $\bar{\varepsilon}_2$ as Bishop curvatures of Smarandache $\mathbf{\Pi}_1\mathbf{B}$ curve.

Theorem 2.4. Let $\gamma: I \to \mathrm{SOL}^3$ be a unit speed non-geodesic biharmonic constant $\mathbf{\Pi}_2-$slope curve with constant curvatures according to type-2 Bishop frame in the SOL^3. Then, type-2 Bishop frame of Smarandache $\mathbf{\Pi}_1\mathbf{B}$ curve of biharmonic constant $\mathbf{\Pi}_2-$slope curve are given by

$$\begin{aligned}
\overline{\mathbf{\Pi}}_1 = {} & [[\mathrm{W}\varepsilon_1\sin\mathrm{A}(s)-\frac{\mathrm{W}}{\bar{\kappa}}\varepsilon_1^2\cos\mathrm{A}(s)]\cos\mathrm{E}\cos[\mathrm{R}_1 s+\mathrm{R}_2] \\
& +[\mathrm{W}\varepsilon_2\sin\mathrm{A}(s)-\frac{\mathrm{W}}{\bar{\kappa}}\varepsilon_1\varepsilon_2\cos\mathrm{A}(s)]\sin\mathrm{E}\cos[\mathrm{R}_1 s+\mathrm{R}_2] \\
& -[-\mathrm{W}\varepsilon_1\sin\mathrm{A}(s)-\frac{\mathrm{W}}{\bar{\kappa}}(\varepsilon_1^2+\varepsilon_2^2)\cos\mathrm{A}(s)]\sin[\mathrm{R}_1 s+\mathrm{R}_2]]\mathbf{e}_1 \\
& +[[\mathrm{W}\varepsilon_1\sin\mathrm{A}(s)-\frac{\mathrm{W}}{\bar{\kappa}}\varepsilon_1^2\cos\mathrm{A}(s)]\cos\mathrm{E}\sin[\mathrm{R}_1 s+\mathrm{R}_2] \\
& +[\mathrm{W}\varepsilon_2\sin\mathrm{A}(s)-\frac{\mathrm{W}}{\bar{\kappa}}\varepsilon_1\varepsilon_2\cos\mathrm{A}(s)]\sin\mathrm{E}\sin[\mathrm{R}_1 s+\mathrm{R}_2] \\
& +[-\mathrm{W}\varepsilon_1\sin\mathrm{A}(s)-\frac{\mathrm{W}}{\bar{\kappa}}(\varepsilon_1^2+\varepsilon_2^2)\cos\mathrm{A}(s)]\cos[\mathrm{R}_1 s+\mathrm{R}_2]]\mathbf{e}_2 \\
& +[-[\mathrm{W}\varepsilon_1\sin\mathrm{A}(s)-\frac{\mathrm{W}}{\bar{\kappa}}\varepsilon_1^2\cos\mathrm{A}(s)]\sin\mathrm{E} \\
& +[\mathrm{W}\varepsilon_2\sin\mathrm{A}(s)-\frac{\mathrm{W}}{\bar{\kappa}}\varepsilon_1\varepsilon_2\cos\mathrm{A}(s)]\cos\mathrm{E}]\mathbf{e}_3,
\end{aligned}$$

$$\begin{aligned}
\overline{\mathbf{\Pi}}_2 = & [[-\mathsf{W}\varepsilon_1 \cos \mathsf{A}(s) - \frac{\mathsf{W}}{\overline{\kappa}}\varepsilon_1^2 \sin \mathsf{A}(s)] \cos \mathsf{E} \cos[\mathsf{R}_1 s + \mathsf{R}_2] \\
& + [-\mathsf{W}\varepsilon_2 \cos \mathsf{A}(s) - \frac{\mathsf{W}}{\overline{\kappa}}\varepsilon_1\varepsilon_2 \sin \mathsf{A}(s)] \sin \mathsf{E} \cos[\mathsf{R}_1 s + \mathsf{R}_2] \\
& - [\mathsf{W}\varepsilon_1 \cos \mathsf{A}(s) - \frac{\mathsf{W}}{\overline{\kappa}}(\varepsilon_1^2 + \varepsilon_2^2) \sin \mathsf{A}(s)] \sin[\mathsf{R}_1 s + \mathsf{R}_2]]\mathbf{e}_1 \\
& + [[-\mathsf{W}\varepsilon_1 \cos \mathsf{A}(s) - \frac{\mathsf{W}}{\overline{\kappa}}\varepsilon_1^2 \sin \mathsf{A}(s)] \cos \mathsf{E} \sin[\mathsf{R}_1 s + \mathsf{R}_2] \\
& + [-\mathsf{W}\varepsilon_2 \cos \mathsf{A}(s) - \frac{\mathsf{W}}{\overline{\kappa}}\varepsilon_1\varepsilon_2 \sin \mathsf{A}(s)] \sin \mathsf{E} \sin[\mathsf{R}_1 s + \mathsf{R}_2] \\
& + [\mathsf{W}\varepsilon_1 \cos \mathsf{A}(s) - \frac{\mathsf{W}}{\overline{\kappa}}(\varepsilon_1^2 + \varepsilon_2^2) \sin \mathsf{A}(s)] \cos[\mathsf{R}_1 s + \mathsf{R}_2]]\mathbf{e}_2 \\
& + [-[-\mathsf{W}\varepsilon_1 \cos \mathsf{A}(s) - \frac{\mathsf{W}}{\overline{\kappa}}\varepsilon_1^2 \sin \mathsf{A}(s)] \sin \mathsf{E} \\
& + [-\mathsf{W}\varepsilon_2 \cos \mathsf{A}(s) - \frac{\mathsf{W}}{\overline{\kappa}}\varepsilon_1\varepsilon_2 \sin \mathsf{A}(s)] \cos \mathsf{E}]\mathbf{e}_3,
\end{aligned} \tag{3.4}$$

$$\begin{aligned}
\overline{\mathbf{B}} = & [[-\frac{\mathsf{W}^2}{\overline{\kappa}}(\varepsilon_1^2\varepsilon_2 + \varepsilon_2(\varepsilon_1^2 + \varepsilon_2^2)) \cos \mathsf{E} \cos[\mathsf{R}_1 s + \mathsf{R}_2] + \frac{\mathsf{W}^2}{\overline{\kappa}}(\varepsilon_1^3 \\
& + \varepsilon_1(\varepsilon_1^2 + \varepsilon_2^2)) \sin \mathsf{E} \cos[\mathsf{R}_1 s + \mathsf{R}_2]]]\mathbf{e}_1 \\
& + [[-\frac{\mathsf{W}^2}{\overline{\kappa}}(\varepsilon_1^2\varepsilon_2 + \varepsilon_2(\varepsilon_1^2 + \varepsilon_2^2)) \cos \mathsf{E} \sin[\mathsf{R}_1 s + \mathsf{R}_2] + \frac{\mathsf{W}^2}{\overline{\kappa}}(\varepsilon_1^3 + \varepsilon_1 \\
& (\varepsilon_1^2 + \varepsilon_2^2)) \sin \mathsf{E} \sin[\mathsf{R}_1 s + \mathsf{R}_2]]]\mathbf{e}_2 \\
& + [[\frac{\mathsf{W}^2}{\overline{\kappa}}(\varepsilon_1^2\varepsilon_2 + \varepsilon_2(\varepsilon_1^2 + \varepsilon_2^2)) \sin \mathsf{E} + \frac{\mathsf{W}^2}{\overline{\kappa}}(\varepsilon_1^3 + \varepsilon_1(\varepsilon_1^2 + \varepsilon_2^2)) \cos \mathsf{E}]]\mathbf{e}_3,
\end{aligned}$$

where $\mathsf{R}_1, \mathsf{R}_2$ *are constants of integration and*

$$\mathsf{W} = \frac{1}{\sqrt{2\varepsilon_1^2 + \varepsilon_2^2}}.$$

Proof: Assume that γ is a unit speed non-geodesic biharmonic constant Π_2 –slope curve according to type-2 Bishop frame.

From (1.1) and (3.1) we have above system. This completes the proof.

Using the derivative formulae of the type-2 Bishop frame, we get

Theorem 3.5. Let $\gamma : I \to \mathsf{SOL}^3$ be a unit speed non-geodesic biharmonic constant $\mathbf{\Pi}_2$ –slope curve with constant curvatures according to type-2 Bishop frame in the SOL^3. Then, type-2 Bishop frame of Smarandache $\mathbf{\Pi}_1\mathbf{B}$ curve of biharmonic constant $\mathbf{\Pi}_2$ –slope curve are given by

$$\begin{aligned}
\nabla_{\bar{\mathbf{T}}}\overline{\mathbf{\Pi}}_1 &= -\bar{\varepsilon}_1[[-\frac{\mathrm{W}^2}{\bar{\kappa}}(\varepsilon_1^2\varepsilon_2+\varepsilon_2(\varepsilon_1^2+\varepsilon_2^2))\cos\mathrm{E}\cos[\mathrm{R}_1 s+\mathrm{R}_2] \\
&+\frac{\mathrm{W}^2}{\bar{\kappa}}(\varepsilon_1^3+\varepsilon_1(\varepsilon_1^2+\varepsilon_2^2))\sin\mathrm{E}\cos[\mathrm{R}_1 s+\mathrm{R}_2]]]\mathbf{e}_1 \\
&-\bar{\varepsilon}_1[[-\frac{\mathrm{W}^2}{\bar{\kappa}}(\varepsilon_1^2\varepsilon_2+\varepsilon_2(\varepsilon_1^2+\varepsilon_2^2))\cos\mathrm{E}\sin[\mathrm{R}_1 s+\mathrm{R}_2] \\
&+\frac{\mathrm{W}^2}{\bar{\kappa}}(\varepsilon_1^3+\varepsilon_1(\varepsilon_1^2+\varepsilon_2^2))\sin\mathrm{E}\sin[\mathrm{R}_1 s+\mathrm{R}_2]]]\mathbf{e}_2 \\
&-\bar{\varepsilon}_1[[\frac{\mathrm{W}^2}{\bar{\kappa}}(\varepsilon_1^2\varepsilon_2+\varepsilon_2(\varepsilon_1^2+\varepsilon_2^2))\sin\mathrm{E}+\frac{\mathrm{W}^2}{\bar{\kappa}}(\varepsilon_1^3+\varepsilon_1(\varepsilon_1^2+\varepsilon_2^2))\cos\mathrm{E}]]\mathbf{e}_3,
\end{aligned}$$

$$\begin{aligned}
\nabla_{\bar{\mathbf{T}}}\overline{\mathbf{\Pi}}_2 &= -\bar{\varepsilon}_2[[-\frac{\mathrm{W}^2}{\bar{\kappa}}(\varepsilon_1^2\varepsilon_2+\varepsilon_2(\varepsilon_1^2+\varepsilon_2^2))\cos\mathrm{E}\cos[\mathrm{R}_1 s+\mathrm{R}_2] \\
&+\frac{\mathrm{W}^2}{\bar{\kappa}}(\varepsilon_1^3+\varepsilon_1(\varepsilon_1^2+\varepsilon_2^2))\sin\mathrm{E}\cos[\mathrm{R}_1 s+\mathrm{R}_2]]]\mathbf{e}_1 \\
&-\bar{\varepsilon}_2[[-\frac{\mathrm{W}^2}{\bar{\kappa}}(\varepsilon_1^2\varepsilon_2+\varepsilon_2(\varepsilon_1^2+\varepsilon_2^2))\cos\mathrm{E}\sin[\mathrm{R}_1 s+\mathrm{R}_2] \\
&+\frac{\mathrm{W}^2}{\bar{\kappa}}(\varepsilon_1^3+\varepsilon_1(\varepsilon_1^2+\varepsilon_2^2))\sin\mathrm{E}\sin[\mathrm{R}_1 s+\mathrm{R}_2]]]\mathbf{e}_2 \\
&-\bar{\varepsilon}_2[[\frac{\mathrm{W}^2}{\bar{\kappa}}(\varepsilon_1^2\varepsilon_2+\varepsilon_2(\varepsilon_1^2+\varepsilon_2^2))\sin\mathrm{E}+\frac{\mathrm{W}^2}{\bar{\kappa}}(\varepsilon_1^3+\varepsilon_1(\varepsilon_1^2+\varepsilon_2^2))\cos\mathrm{E}]]\mathbf{e}_3,
\end{aligned}$$

$$\begin{aligned}
\nabla_{\bar{\mathbf{T}}}\overline{\mathbf{B}} &= \bar{\varepsilon}_1[[\mathrm{W}\varepsilon_1\sin\mathrm{A}(s)-\frac{\mathrm{W}}{\bar{\kappa}}\varepsilon_1^2\cos\mathrm{A}(s)]\cos\mathrm{E}\cos[\mathrm{R}_1 s+\mathrm{R}_2] \\
&+[\mathrm{W}\varepsilon_2\sin\mathrm{A}(s)-\frac{\mathrm{W}}{\bar{\kappa}}\varepsilon_1\varepsilon_2\cos\mathrm{A}(s)]\sin\mathrm{E}\cos[\mathrm{R}_1 s+\mathrm{R}_2] \\
&-[-\mathrm{W}\varepsilon_1\sin\mathrm{A}(s)-\frac{\mathrm{W}}{\bar{\kappa}}(\varepsilon_1^2+\varepsilon_2^2)\cos\mathrm{A}(s)]\sin[\mathrm{R}_1 s+\mathrm{R}_2]]\mathbf{e}_1 \\
&+\bar{\varepsilon}_1[[\mathrm{W}\varepsilon_1\sin\mathrm{A}(s)-\frac{\mathrm{W}}{\bar{\kappa}}\varepsilon_1^2\cos\mathrm{A}(s)]\cos\mathrm{E}\sin[\mathrm{R}_1 s+\mathrm{R}_2] \\
&+[\mathrm{W}\varepsilon_2\sin\mathrm{A}(s)-\frac{\mathrm{W}}{\bar{\kappa}}\varepsilon_1\varepsilon_2\cos\mathrm{A}(s)]\sin\mathrm{E}\sin[\mathrm{R}_1 s+\mathrm{R}_2] \\
&+[-\mathrm{W}\varepsilon_1\sin\mathrm{A}(s)-\frac{\mathrm{W}}{\bar{\kappa}}(\varepsilon_1^2+\varepsilon_2^2)\cos\mathrm{A}(s)]\cos[\mathrm{R}_1 s+\mathrm{R}_2]]\mathbf{e}_2 \\
&+\bar{\varepsilon}_1[-[\mathrm{W}\varepsilon_1\sin\mathrm{A}(s)-\frac{\mathrm{W}}{\bar{\kappa}}\varepsilon_1^2\cos\mathrm{A}(s)]\sin\mathrm{E} \\
&+[\mathrm{W}\varepsilon_2\sin\mathrm{A}(s)-\frac{\mathrm{W}}{\bar{\kappa}}\varepsilon_1\varepsilon_2\cos\mathrm{A}(s)]\cos\mathrm{E}]\mathbf{e}_3
\end{aligned}$$

$$
\begin{aligned}
&+\bar{\varepsilon}_2[[-\mathsf{W}\varepsilon_1\cos\mathsf{A}(s)-\frac{\mathsf{W}}{\bar{\kappa}}\varepsilon_1^2\sin\mathsf{A}(s)]\cos\mathsf{E}\cos[\mathsf{R}_1 s+\mathsf{R}_2]\\
&+[-\mathsf{W}\varepsilon_2\cos\mathsf{A}(s)-\frac{\mathsf{W}}{\bar{\kappa}}\varepsilon_1\varepsilon_2\sin\mathsf{A}(s)]\sin\mathsf{E}\cos[\mathsf{R}_1 s+\mathsf{R}_2]\\
&-[\mathsf{W}\varepsilon_1\cos\mathsf{A}(s)-\frac{\mathsf{W}}{\bar{\kappa}}\left(\varepsilon_1^2+\varepsilon_2^2\right)\sin\mathsf{A}(s)]\sin[\mathsf{R}_1 s+\mathsf{R}_2]]\mathbf{e}_1\\
&+\bar{\varepsilon}_2[[-\mathsf{W}\varepsilon_1\cos\mathsf{A}(s)-\frac{\mathsf{W}}{\bar{\kappa}}\varepsilon_1^2\sin\mathsf{A}(s)]\cos\mathsf{E}\sin[\mathsf{R}_1 s+\mathsf{R}_2]\\
&+[-\mathsf{W}\varepsilon_2\cos\mathsf{A}(s)-\frac{\mathsf{W}}{\bar{\kappa}}\varepsilon_1\varepsilon_2\sin\mathsf{A}(s)]\sin\mathsf{E}\sin[\mathsf{R}_1 s+\mathsf{R}_2]\\
&+[\mathsf{W}\varepsilon_1\cos\mathsf{A}(s)-\frac{\mathsf{W}}{\bar{\kappa}}\left(\varepsilon_1^2+\varepsilon_2^2\right)\sin\mathsf{A}(s)]\cos[\mathsf{R}_1 s+\mathsf{R}_2]]\mathbf{e}_2\\
&+\bar{\varepsilon}_2[-[-\mathsf{W}\varepsilon_1\cos\mathsf{A}(s)-\frac{\mathsf{W}}{\bar{\kappa}}\varepsilon_1^2\sin\mathsf{A}(s)]\sin\mathsf{E}\\
&+[-\mathsf{W}\varepsilon_2\cos\mathsf{A}(s)-\frac{\mathsf{W}}{\bar{\kappa}}\varepsilon_1\varepsilon_2\sin\mathsf{A}(s)]\cos\mathsf{E}]\mathbf{e}_3,
\end{aligned}
\tag{3.5}
$$

where $\mathsf{R}_1,\mathsf{R}_2$ *are constants of integration and*

$$
\mathsf{W}=\frac{1}{\sqrt{2\varepsilon_1^2+\varepsilon_2^2}}.
$$

Proof: Type-2 Bishop equations of Smarandache $\mathbf{\Pi}_1\mathbf{B}$ curve of biharmonic constant $\mathbf{\Pi}_2-$slope curve are

$$
\begin{aligned}
\nabla_{\bar{\mathbf{T}}}\bar{\mathbf{\Pi}}_1 &= -\bar{\varepsilon}_1\bar{\mathbf{B}},\\
\nabla_{\bar{\mathbf{T}}}\bar{\mathbf{\Pi}}_2 &= -\bar{\varepsilon}_2\bar{\mathbf{B}},\\
\nabla_{\bar{\mathbf{T}}}\bar{\mathbf{B}} &= \bar{\varepsilon}_1\bar{\mathbf{\Pi}}_1+\bar{\varepsilon}_2\bar{\mathbf{\Pi}}_2.
\end{aligned}
\tag{3.6}
$$

Substituting (3.4) in (3.6), we get (3.5). Thus, we obtain the theorem. This completes the proof.

5. CONCLUSIONS

In the differential geometry, curves theory is a ultimate important applications. Special curves and their interpretations have been studied for a long time and are still being studied. The operation of this special curves is seen in nature, computer aided design, mechanic tools and computer graphics etc.

In our paper, we obtain Smarandache $\mathbf{\Pi}_1\mathbf{B}$ curves of biharmonic new type constant $\mathbf{\Pi}_2-$ slope curves according to type-2 Bishop frame in the SOL^3. Type-2 Bishop equations of Smarandache $\mathbf{\Pi}_1\mathbf{B}$ curves are obtained in terms of base curve's type-2 Bishop invariants. Finally, we express some interesting relations and illustrate some examples of our main results.

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