ORIGINAL PAPER

IMPROVE POINT SPREAD FUNCTION BY USING GAUSSIAN FILTER WITH SINGLE HEXAGONAL APERTURE

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Abstract. *We study the effect of Gaussian filter on the point spread function (psf) for a single hexagonal aperture for diffraction limit system and by inserting a coma aberration (w31=0.25 and 0.5). Also we derived the point spread function for a single hexagonal aperture and when there is a Gaussian filter with different values in width used. the different values of coma aberration was study ,we notice a decrease in intensity and secondary peaks of psf . The pulse to noise ratio increases with the width of the Gaussian filter increases.*
Keywords: *aberrations; geometrical optics; single aperture.*

1. INTRODUCTION

According to the theory of light in the optical system, image of point source is not point, where the diffraction and the scattering make a point object image extends to a spot similar to the Gaussian distribution [1]. The Gaussian filter keeps better presentation of the object [2].

A point spread function can describe the spatial characteristics of remote sensing optical imaging instruments [3, 4], we evaluates the optical system by knowing and measure the point spread function which associated with phase at exited pupil aperture through Fourier transforms [5].

The pattern of diffraction is influenced by many factors, one of these factors is the amount and types of the aberration that play an important role in the distribution of intensity on the image plane [6-9], where when the light entering different points of the spherical lens is not focused to the same point on the optical axis, spherical aberration occurs [10], and when the lens corrects spherical aberration, the point image is formed for a point object located on the axis. But, if point object located on the main axis, the lens even when corrected spherical aberration, we see the formation of a comet-like picture rather than point image this defect is called coma aberration [11, 12], we can elimination from coma aberration by choose spheres radius of lens surface to give a pair of image and object points, while spherical aberration cannot completely corrected. Furthermore, comet corrected lens cannot be free of spherical aberration; also the spherical aberration corrected lens cannot be free of coma aberration [13, 14].

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2. MATERIALS AND METHODS

We can express the complex amplitude in the point (u,v) in the image plane by using Fourier transform to pupil function

$$f(x,y) = \tau(x,y)e^{ikw(x,y)} \tag{1}$$

where $\tau(x,y)$ represents the real amplitude distribution in exit pupil and which is called *pupil transparency* or *transmission function* and often chooses equal one unit, where aperture has uniform power distribution. In this research we used Gaussian filter instead of making it equal unity, to study the effect of the Gaussian filter on point spread function and equal

(2)

where $r^2 = x^2 + y^2$ and A is width of the Gaussian filter and $e^{ikw(x,y)}$ represents the wave front of aberration function where

$$w(x,y) = w_{31}(x^2 + y^2)y + w_{51}(x^2 + y^2)^2 y + \cdots\cdots\cdots \tag{3}$$

$W(x,y)$ is coma aberration factor and (x,y) are exit pupil coordinates. The only condition placed on the function is $\tau(x,y) \leq 1$ to be passive and the rays are passed while $\tau(x,y) \geq 1$ the rays do not pass. The accounts that will be performed include the pupil of the output and through the pupil of the output, the severity in the image plane can be predicted, therefore, the point spread function can be expressed in the following integral formula

$$F(u,v) = N\int\int f(x,y)e^{2\pi i(ux+vy)}\, dx\, dy \tag{4}$$

where (N) represents Normalizing factor and $F(u, v)$ is complex amplitude, and now Let $m = 2\pi v$, $z = 2\pi u$ where *m* and *z* represents the image axies

$$F(z,m) = N\int\int f(x,y)e^{i(zx+my)}\, dx\, dy \tag{5}$$

Since we are dealing with a hexagonal aperture and upon entering the aberrations factor into equation (5) we get

$$F(z,m) = N\int\int \tau(x,y)e^{ikw(x,y)}\, e^{i(zx+my)}\, dx\, dy \qquad 6)$$

When expressing a wave number by the amount $\frac{2\pi}{\lambda}$ and the aberration factor is taken in terms of wavelength , the equation is as follows

$$F(z,m) = N\int\int \tau(x,y)e^{i2\pi[w(x,y)+(zx+my)]}\, dx\, dy \tag{7}$$

By substituting the integral boundary in order to include the area of compound aperture as shown in Fig. 1 note the illustration

$$\begin{aligned} F(z,m) &= N\Bigg[\int_{-s}^{s}\int_{-a}^{a}\tau(x,y)e^{i2\pi w(x,y)}\, e^{i2\pi(zx+my)}dydx \\ &+ \int_{s}^{2s}\int_{0}^{\frac{-a}{s}x+2a}\tau(x,y)\, e^{i2\pi w(x,y)}\, e^{i2\pi(zx+my)}\, dydx \\ &+ \int_{-2s}^{-s}\int_{0}^{\frac{a}{s}x+2a}\tau(x,y)e^{i2\pi w(x,y)}\, e^{i2\pi(zx+my)}\, dydx \\ &+ \int_{-2s}^{-s}\int_{0}^{\frac{-a}{s}x-2a}\tau(x,y)e^{i2\pi w(x,y)}\, e^{i2\pi(zx+my)}\, dydx \\ &+ \int_{s}^{2s}\int_{0}^{\frac{a}{s}x-2a}\tau(x,y)e^{i2\pi w(x,y)}\, e^{i2\pi(zx+my)}\, dydx\Bigg] \end{aligned} \quad (8)$$

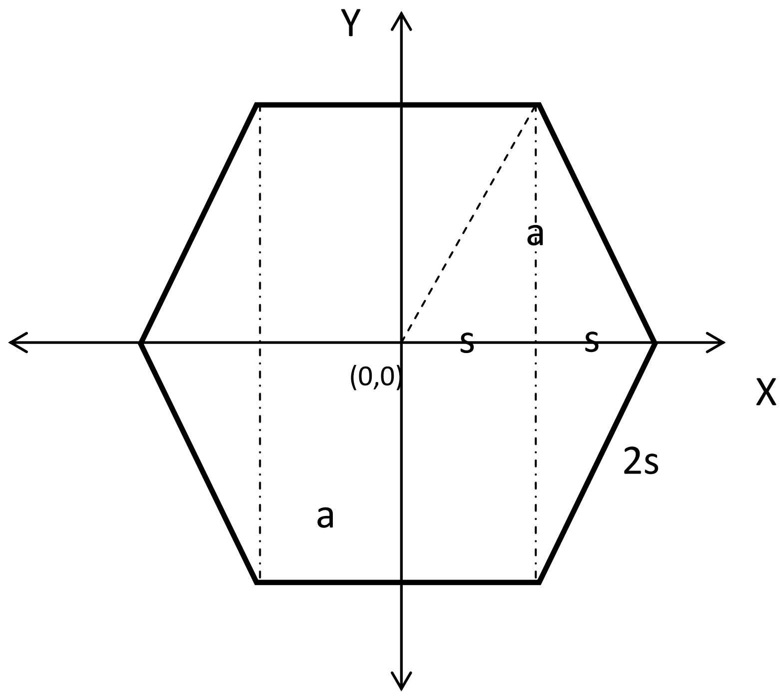

Figure 1. The integral boundary for single hexagonal aperture.

The equation (8) can be written by using the relationship $e^{i\theta} = \cos\theta + i\sin\theta$, thus we get the following equation

$$\begin{aligned} F(z) = \\ & N\,[\int_{-s}^{s}\int_{-a}^{a}\tau(x,y)\,\big[cos2\pi[w(x,y)+(zx+my)] \\ &\qquad + i\,sin\,2\pi\,[\,w(x,y)+(zx+my)]\big]dydx\ + \end{aligned} \quad (9)$$

$$\int_{s}^{2s}\int_{0}^{\frac{-a}{s}x+2a}\tau(x,y)\left[cos2\pi[w(x,y)+(zx+my)]\right.$$
$$\left.+\, i\, sin\, 2\pi\,[\,w(x,y)+(zx+my)]\right]dydx +$$
$$\int_{-2s}^{-s}\int_{0}^{\frac{a}{s}x+2a}\tau(x,y)\left[cos2\pi[w(x,y)+(zx+my)]\right.$$
$$\left.+\; i\, sin\, 2\pi\,[\,w(x,y)+(zx+my)]\right]dyd +$$
$$\int_{-2s}^{-s}\int_{0}^{\frac{-a}{s}x-2a}\tau(x,y)\left[cos2\pi[w(x,y)+(zx+my)]\right.$$
$$\left.+\, i\, sin\, 2\pi\,[\,w(x,y)+(zx+my)]\right]dydx +$$
$$\int_{s}^{2s}\int_{0}^{\frac{a}{s}x-2a}\tau(x,y)\left[cos2\pi\,[w(x,y)+(zx+my)]\right.$$
$$\left.+\; i\, sin\, 2\pi\,[\,w(x,y)+(zx+my)]\right]dydx\,]$$

because of the similarity in the intensity distribution of the two axes $z\ and\ \ m$ therefore, we keep one of the two axes, so that $m = 0$

$$\begin{aligned}F(z) = N\,[\int_{-s}^{s}\int_{-a}^{a}&\tau(x,y)\left[cos2\pi\,[w(x,y)+(zx)]\right.\\
&\left.+\, i\sin 2\pi\,[\,w(x,y)+(zx)]\right]dydx\\
&+\int_{s}^{2s}\int_{0}^{\frac{-a}{s}x+2a}\tau(x,y)\left[cos2\pi\,[w(x,y)+(zx)]\right.\\
&\left.+\, i\sin 2\pi\,[\,w(x,y)+(zx)]\right]dydx\\
&+\int_{-2s}^{-s}\int_{0}^{\frac{a}{s}x+2a}\tau(x,y)\left[cos2\pi\,[w(x,y)+(zx)]\right.\\
&\left.+\, i\sin 2\pi\,[\,w(x,y)+(zx)]\right]dydx\\
&+\int_{-2s}^{-s}\int_{0}^{\frac{-a}{s}x-2a}\tau(x,y)\left[cos2\pi\,[w(x,y)+(zx)]\right.\\
&\left.+\, i\sin 2\pi\,[\,w(x,y)+(zx)]\right]dydx\\
&+\int_{s}^{2s}\int_{0}^{\frac{a}{s}x-2a}\tau(x,y)\left[cos2\pi\,[w(x,y)+(zx)]\right.\\
&\left.+\, i\sin 2\pi\,[\,w(x,y)+(zx)]\right]dydx\,]\end{aligned} \quad (10)$$

if the intensity of the incident light is incoherent when that the point spread function (PSF) is given by multiplying the function $F(z)$ complex facilities

$$G(z) = |F(z)|^2 \quad (11)$$

$G(z)$ represent the point spread function (PSF) for incoherent light, by substituting the normalizing factor $N = \frac{36}{17\pi^2}$, and from equation (3) by substituting the first term from $w(x, y)$, so the intensity is given according to trigonometric functions by relationship

$$\therefore G(z) = N\Bigg[\Bigg|\int_{-s}^{s}\int_{-a}^{a}\tau(x,y)[\cos 2\pi[w_{31}(x^2+y^2)y+(zx)] + i\sin 2\pi[w_{31}(x^2+y^2)y+(zx)]]\,dy\,dx\Bigg|^2$$
$$+\Bigg|\int_{s}^{2s}\int_{0}^{\frac{-a}{s}x+2a}\tau(x,y)[\cos 2\pi[w_{31}(x^2+y^2)y+(zx)] + i\sin 2\pi[w_{31}(x^2+y^2)y+(zx)]]\,dy\,dx\Bigg|^2$$
$$+\Bigg|\int_{-2s}^{-s}\int_{0}^{\frac{a}{s}x+2a}\tau(x,y)[\cos 2\pi[w_{31}(x^2+y^2)y+(zx)] + i\sin 2\pi[w_{31}(x^2+y^2)y+(zx)]]\,dy\,dx\Bigg|^2 \tag{12}$$
$$+\Bigg|\int_{-2s}^{-s}\int_{0}^{\frac{-a}{s}x-2a}\tau(x,y)[\cos 2\pi[w_{31}(x^2+y^2)y+(zx)] + i\sin 2\pi[w_{31}(x^2+y^2)y+(zx)]]\,dy\,dx\Bigg|^2$$
$$+\Bigg|\int_{s}^{2s}\int_{0}^{\frac{a}{s}x-2a}\tau(x,y)[\cos 2\pi[w_{31}(x^2+y^2)y+(zx)] + i\sin 2\pi[w_{31}(x^2+y^2)y+(zx)]]\,dy\,dx\Bigg|^2\Bigg]$$

The equation (12) has been programmed by Mathcad and curves graphically drawn for the point spread function (PSF) (Figs. 2-5).

3. RESULTS AND DISCUSSION

Fig. 2 illustrates the point spread function for single hexagonal aperture when there is no Gaussian filter for diffraction limit system , where we note the function is normalization, and maximum value for intensity is (1), but when we enter coma aberration as much (w_{31}=0.25λ), the intensity value decreases to (0.715) and when coma aberration (w_{31}=0.5λ), the intensity decreases to (0.302).

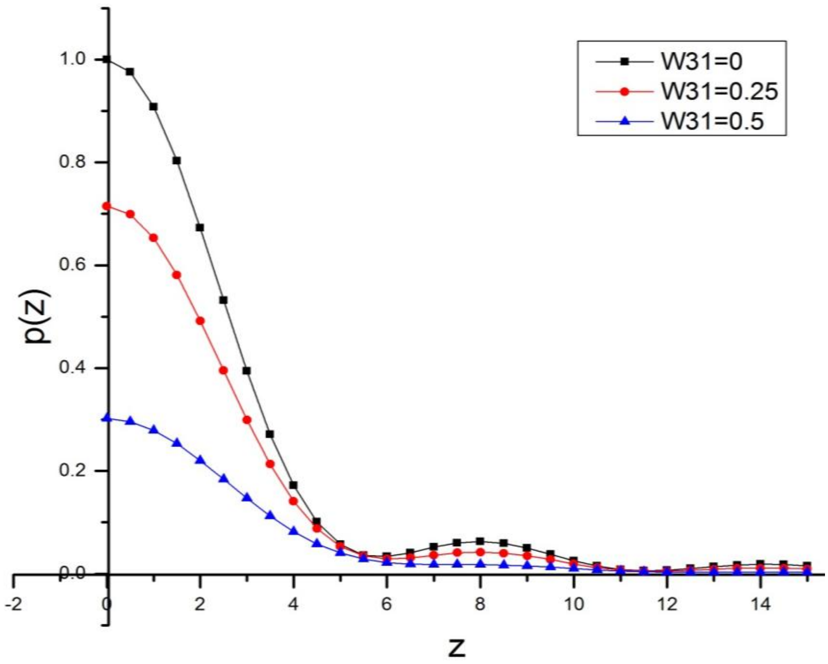

Figure 2. The point spread function to single hexagonal aperture when there is no Gaussian filter for diffraction limit system and coma aberration (0.25λ) and (0.5λ) .

Fig. 3 represents the intensity distribution curves for point spread function for value of Coma aberration (w_{31}=0) and at width values for Gaussian filter (A=0.5, 0.8 and 1), the intensity decreases from (1) to (0.254) when (A=0.5) and at width Gaussian filter (A=0.8), the intensity is increase to value (0.397). Then increase to value (0.469) at width Gaussian filter (A=1).

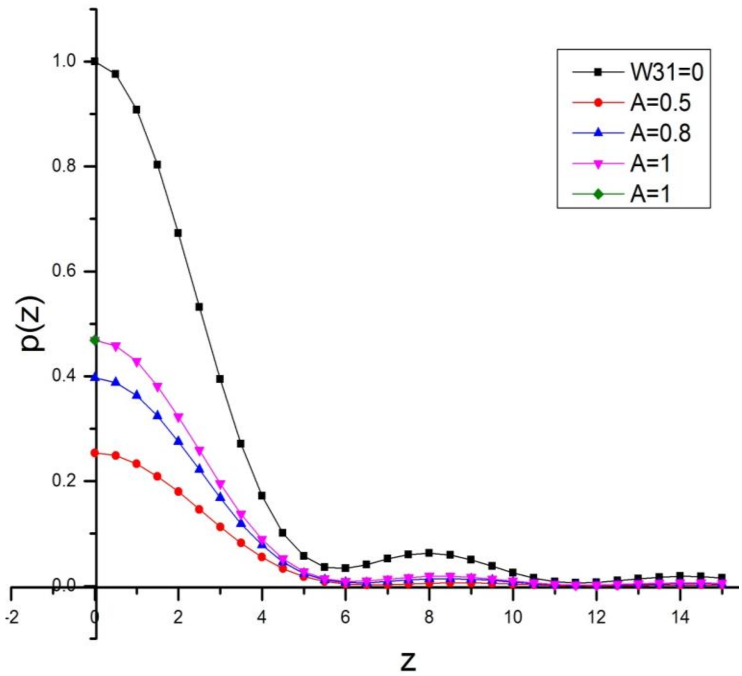

Figure 3. The point spread function to single hexagonal aperture when there is no Gaussian filter and when there is gaussian filter has different width $(A = 0.5)$, $(A = 0.8)$ and $(A = 1)$.

Fig. 4 represents the intensity distribution curves for point spread function for value of Coma aberration (w_{31}=0.25λ) and at width values for Gaussian filter (A=0.5, 0.8 and 1), the intensity decreases from (0.715) to (0.221) when (A=0.5), and at width Gaussian filter

(A=0.8), the intensity is increase to value (0.326). Then increase to value (0.376)at width Gaussian filter (A=1).this lead to an increase in the clarity of the image and an increase in the focal depth of the optical system. We also notice the decay of the secondary peaks or their reduction, and this contributes to the clarity of the image.

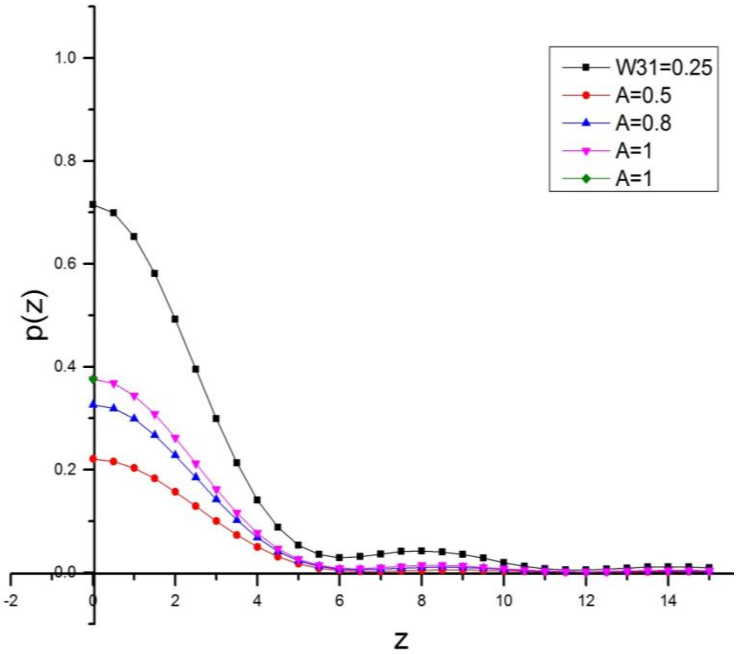

Figure 4. The point spread function for single hexagonal aperture at coma aberration (0.25λ) when there is no Gaussian filter and when gaussian filter has different width $(A = 0.5)$, $(A = 0.8)$ and $(A = 1)$.

The effect is greater in Fig. 5, which represents the intensity distribution curves for point spread function for value of Coma aberration (w_{31}=0.5λ) and at width values for Gaussian filter (A=0.5, 0.8 and 1), the intensity decreases from (0.302) to (0.157) when (A=0.5), and at width Gaussian filter (A=0.8) , the intensity is increase to value (0.201), then increase to value (0.218)at width Gaussian filter (A=1).

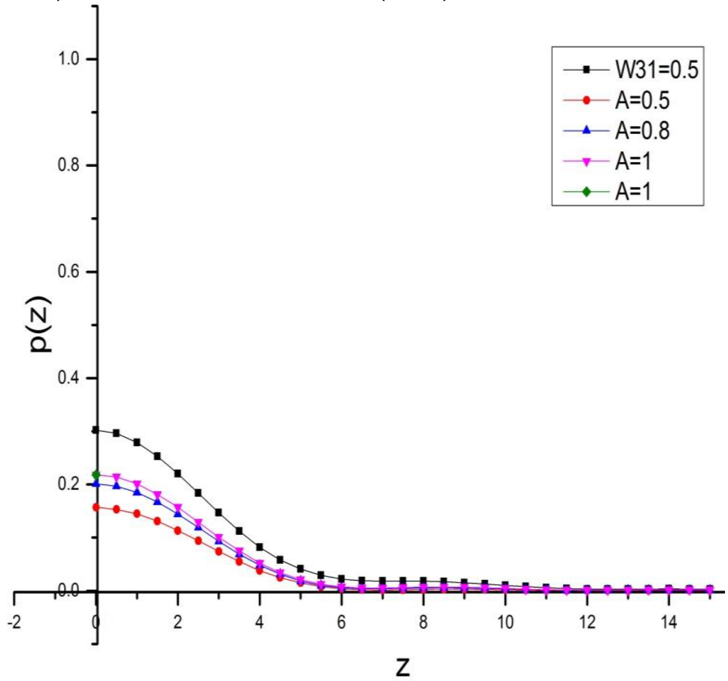

Figure 5. The point spread function for single hexagonal aperture at coma aberration(0.5λ) when there is no Gaussian filter and when gaussian filter has different width $(A = 0.5)$, $(A = 0.8)$ and $(A = 1)$.

From the above it is clear that the Gaussian filter works to reduce the secondary peaks, this case is called Apodization and this is a positive result that increase the ratio of light to noise in the image plan and increase clarity of the image. Also, the width of the curve decreases as the width of the Gaussian filter decreases, and this positively affects the resolving power of optical systems.

4. CONCLUSION

In this research, we study the effect of using the Gaussian filter on a point spread function for single hexagonal aperture and when there is coma aberration. Therefore, conclusions may be limited to the following points: the optical intensity can be increased (signal to noise ratio increased) by increasing the width of the Gaussian filter; the Gaussian filter reduces secondary peaks (secondary peaks are often undesirable in optical system) this case is called Apodization; the apodization method can be used to improve the image when the intensity is ($I \geq 0.8$) because the intensity distribution maintains the shape and specifications of the image, and otherwise apodization would be impractical because the intensity distribution loses all its specification. Gaussian filter causes a decrease in the resolving power of optical system.

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