ORIGINAL PAPER

INTEGRAL TRANSFORM OF K$_4$-FUNCTION

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Abstract. *In this present work we derived integral transforms such as Euler transform, Laplace transform, and Whittaker transform of K$_4$-function. The results are given in generalized Wright function. Some special cases of the main result are also presented here with new and interesting results. We further extended integral transforms derived here in terms of Gauss Hypergeometric function.*

Keywords: *K$_4$-function; integral transform; Wright function; Gauss Hypergeometric function.*

Subject Classification: *Primary 26A33, 33E20; Secondary 44Axx, 33C20.*

1. INTRODUCTION

The K$_4$-function is defined in [1] by the power series

$$K_4^{(\propto,\beta,\gamma),(c,d):(p,q)}\left(a_1,\dots,a_p;b_1,\dots.b_q;x\right)=\sum_{n=0}^{\infty}\frac{(a_1)_n\dots(a_p)_n(\gamma)_n c^n(x-d)^{(n+\gamma)\alpha-\beta-1}}{(b_1)_n\dots(b_q)_n n!\,\Gamma\left((n+\gamma)\alpha-\beta\right)} \tag{1}$$

where $\propto,\beta,\gamma,x\in C, R(\alpha\gamma-\beta)>0;(a_i)_n(i=1,2,3\dots,p)$ and $\left(b_j\right)_n(j=1,2,3\dots,q)$ are the classical Pochhammer symbols defined as

$$(\gamma)_n=\frac{\Gamma(\gamma+n)}{\Gamma(\gamma)}=\begin{cases}1, & (n=0,\gamma\neq 0)\\ \gamma(\gamma+1)\dots(\gamma+n-1), & (n\in N,\gamma\in C)\end{cases} \tag{2}$$

If any numerator parameter a_i is a zero or negative, then the series in (1.1) terminates to a polynomial in x. The series is convergent for all x if $p>q+1$.

If we set $d=0$ in (1.1), then we obtain the following series

$$K_4^{(\propto,\beta,\gamma),(c,0):(p,q)}\left(a_1,\dots,a_p;b_1,\dots.b_q;x\right)=\sum_{n=0}^{\infty}\frac{(a_1)_n\dots(a_p)_n(\gamma)_n c^n(x)^{(n+\gamma)\alpha-\beta-1}}{(b_1)_n\dots(b_q)_n n!\,\Gamma\left((n+\gamma)\alpha-\beta\right)} \tag{3}$$

Generalized Wright Function

Let $\alpha_i,\beta_j\epsilon\mathbb{R}\backslash\{0\}$and $a_i,b_j\epsilon\ \mathbb{C},\ i=1,2,\dots p\ ;j=1,2,\dots q$,the generalized Wright function introduced by Wright [2] as following :

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$$ {}_{\mathrm{p}}\Psi_q(z) = \quad {}_{\mathrm{p}}\Psi_q\left[\begin{matrix}(a_{i,}\alpha_i)_{1,p}\\(b_{j,}\beta_j)_{1,q}\end{matrix}\middle| z\right] = \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p}\Gamma(a_i+n\alpha_i)}{\prod_{j=1}^{q}\Gamma(b_j+n\beta_j)} \frac{z^n}{n!}, \qquad z\epsilon\mathbb{C}, \tag{4} $$

where $\Gamma(z)$denotes the Euler gamma function [3]. The condition for existence of (11) with its representation in form of Mellin – Barnes integral and of H – function was obtained by Kilbas et al. [4].

Now we state the relation of K_4-Function with various special functions

Mittag- Leffler function [5-6]

If we set$\beta = \alpha - 1, \gamma = 1, c = -c, p = 1, q = 1$, $a_i = 1$ and $b_{j=1}$ in (1.3), we get

$$ K_4^{(\propto,\alpha-1,1),(-c,0):(1,1)}(1;1;x) = E_\alpha(-cx^\alpha) = \sum_{n=0}^{\infty} \frac{(-c)^n x^{n\alpha}}{\Gamma(n\alpha+1)} \tag{5} $$

Agarwal's function [7]

$$ K_4^{(\propto,\alpha-\beta,1),(1,0):(1,1)}(1;1;x) = E_{\alpha,\beta}(x^\alpha) = \sum_{n=0}^{\infty} \frac{x^{n\alpha+\beta-1}}{\Gamma(n\alpha+\beta)} \tag{6} $$

Erdelyi's function [8]

$$ K_4^{(\propto,\alpha-\beta,1),(1,0):(1,1)}(1;1;x) = x^{\beta-1}E_{\alpha,\beta}(x^\alpha) = x^{\beta-1}\sum_{n=0}^{\infty} \frac{x^{n\alpha}}{\Gamma(n\alpha+\beta)}, \alpha > 0, \beta > 0 \tag{7} $$

Miller and Ross's function

$$ K_4^{(1-\beta,1),(c,0):(1,1)}(1;1;x) = E_x(\beta, c) = \sum_{n=0}^{\infty} \frac{c^n x^{n+\beta}}{\Gamma(n+\beta+1)} \tag{8} $$

Wright function [9]

$$ K_4^{(\propto,\beta,\gamma),(c,0):(1,1)}(1;1;x) = \frac{x^{\alpha\gamma-\beta-1}}{\Gamma(\gamma)}\ {}_1\psi_1\left[\begin{matrix}(\gamma,1)\\(\alpha\gamma-\beta,\alpha)\end{matrix}; cx^\alpha\right] \tag{9} $$

where ${}_1\psi_1(x)$ is special case of the generalized Wright's hyper-geometric function ${}_p\psi_q(x)$ defined as [2] see also [10-11]

$$ {}_{\mathrm{p}}\Psi_q(z) = \quad {}_{\mathrm{p}}\Psi_q\left[\begin{matrix}(a_{i,}\alpha_i)_{1,p}\\(b_{j,}\beta_j)_{1,q}\end{matrix}\middle| z\right] = \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p}\Gamma(a_i+n\alpha_i)}{\prod_{j=1}^{q}\Gamma(b_j+n\beta_j)} \frac{z^n}{n!}, \qquad z\epsilon C \tag{10} $$

H-Function [9]

$$ K_4^{(\propto,\beta,\gamma),(c,0):(1,1)}(1;1;x) = \frac{x^{\alpha\gamma-\beta-1}}{\Gamma(\gamma)} H_{1,2}^{1,1}\left[-cx^\alpha \middle| \begin{matrix}(1-\gamma,1)\\(0,1),(1-\alpha\gamma+\beta,\alpha)\end{matrix}\right] \tag{11} $$

If we take $\beta = 0$ and $\gamma = 1$ in (1.10), we get

$$ K_4^{(\propto,0,1),(c,0):(1,1)}(1;1;x) = F_\alpha(c,x) = \sum_{n=0}^{\infty} \frac{c^n (x)^{(n+1)\alpha-1}}{\Gamma((n+1)\alpha)} \tag{12} $$

where $F_\alpha(c,x)$ is the F-function defined by Robotov and Hartley, for example see [12-13].

Euler Transform [14]

The Euler transform of a function $f(x)$ is defined as

$$B(f(x);l,m)=\int_0^1 x^{l-1}(1-x)^{m-1}f(x)dx \quad ;l,m\in C, R(l)>0, R(m)>0 \tag{13}$$

Laplace Transform [14]

The Laplace transform of a function f(t), denoted by F(s), is defined by the function

$$F(s)=L\{f(t);s\}=\int_0^\infty e^{-st}f(t)dt,\ R(s)>0 \tag{14}$$

Provided that the integral (1.13) is convergent for $t>0$ and of exponential order as $t\to\infty$. Also,

$$\int_0^\infty e^{-st}\,t^{p-1}dt=\frac{\Gamma(p)}{s^p},\ R(p)>1, R(s)>1 \tag{15}$$

The Whittaker Transform [15]

$$\int_0^\infty e^{-\frac{t}{2}}\,t^{\zeta-1}W_{\alpha,\beta}(t)dt=\frac{\Gamma\left(\frac{1}{2}+\beta+\zeta\right)\Gamma\left(\frac{1}{2}-\beta+\zeta\right)}{\Gamma\left(\frac{1}{2}-\alpha+\zeta\right)} \tag{16}$$

where $R(\beta\pm\zeta)>-{}^1/_2$and $W_{\alpha,\beta}(t)$is the Whittaker confluent Hypergeometric function

$$W_{\beta,\zeta}(z)=\frac{\Gamma(-2\beta)}{\Gamma(1/2-\alpha-\beta)}M_{\alpha,\beta}(z)+\frac{\Gamma(2\beta)}{\Gamma(1/2+\alpha+\beta)}M_{\alpha,-\beta}(z) \tag{17}$$

where $M_{\alpha,\beta}(z)$ is given by

$$M_{\alpha,\beta}(z)=z^{1/2+\beta}e^{-1/2z}\ {}_1F_1\left(\frac{1}{2}+\beta-\alpha;2\beta+1;z\right) \tag{18}$$

2. INTEGRAL TRANSFORMS OF K$_4$-FUNCTION

Theorem 2.1: (Euler Transform) Let $\alpha,\beta,\gamma,x,l,m\in C, R(\alpha\gamma-\beta)>0\ R(l)>0, R(m)>0$, then there holds the following formula:

$$B\left\{K_4^{(\propto,\beta,\gamma),(c,0):(p,q)}(a_1,\dots,a_p;b_1,\dots.b_q;x);l,m\right\}=\frac{\prod_{j=1}^q\Gamma(b_j)\Gamma(m)}{\prod_{i=1}^p\Gamma(a_i)\,\Gamma(\gamma)}$$
$$\times\ {}_{p+2}\psi_{q+2}\left[\begin{matrix}(a_i,1)_1^p,(\gamma,1),(l+\gamma\alpha-\beta-1,\alpha)\\(b_j,1)_1^q,(\gamma\alpha-\beta,\alpha),(l+\gamma\alpha+m-\beta-1,\alpha)\end{matrix};\ c\right]$$

Proof: Let the left-hand size of (2.1) be denoted by J. By applying (1.3) and using (1.13) to the left-hand side of (2.1), then we have

$$J = B\left\{\sum_{n=0}^{\infty} \frac{(a_1)_n \dots (a_p)_n (\gamma)_n c^n (x)^{(n+\gamma)\alpha-\beta-1}}{(b_1)_n \dots (b_q)_n n!\,\Gamma((n+\gamma)\alpha-\beta)}; l, m\right\}$$

$$= \int_0^1 x^{l-1}(1-x)^{m-1} \sum_{n=0}^{\infty} \frac{(a_1)_n \dots (a_p)_n (\gamma)_n c^n (x)^{(n+\gamma)\alpha-\beta-1}}{(b_1)_n \dots (b_q)_n n!\,\Gamma((n+\gamma)\alpha-\beta)} dx$$

Now changing the order of integration and summation, we get

$$= \sum_{n=0}^{\infty} \frac{(a_1)_n \dots (a_p)_n (\gamma)_n c^n}{(b_1)_n \dots (b_q)_n n!\,\Gamma((n+\gamma)\alpha-\beta)} \int_0^1 x^{l+n\alpha+\gamma\alpha-\beta-1-1}(1-x)^{m-1} dx$$

$$= \sum_{n=0}^{\infty} \frac{(a_1)_n \dots (a_p)_n (\gamma)_n c^n}{(b_1)_n \dots (b_q)_n n!\,\Gamma((n+\gamma)\alpha-\beta)} B(l+n\alpha+\gamma\alpha-\beta-1, m)$$

$$= \sum_{n=0}^{\infty} \frac{(a_1)_n \dots (a_p)_n (\gamma)_n c^n}{(b_1)_n \dots (b_q)_n n!\,\Gamma((n+\gamma)\alpha-\beta)} \frac{\Gamma(l+n\alpha+\gamma\alpha-\beta-1)\Gamma(m)}{\Gamma(l+n\alpha+\gamma\alpha-\beta-1+m)}$$

Next we use (1.2), then we obtain

$$= \sum_{n=0}^{\infty} \frac{\Gamma(a_1+n) \dots \Gamma(a_p+n)\Gamma(b_1) \dots \Gamma(b_q)\Gamma(\gamma+n)c^n}{\Gamma(b_1+n) \dots \Gamma(b_q+n)\Gamma(a_1) \dots \Gamma(a_p)\Gamma(\gamma) n!\,\Gamma((n+\gamma)\alpha-\beta)} \times \frac{\Gamma(l+n\alpha+\gamma\alpha-\beta-1)\Gamma(m)}{\Gamma(l+n\alpha+\gamma\alpha-\beta-1+m)}$$

$$= \frac{\prod_{j=1}^{q}\Gamma(b_j)\Gamma(m)}{\prod_{i=1}^{p}\Gamma(a_i)\,\Gamma(\gamma)} \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p}\Gamma(a_i+n)\,\Gamma(\gamma+n)\Gamma(l+n\alpha+\gamma\alpha-\beta-1)\Gamma(m)}{\prod_{j=1}^{q}\Gamma(b_j+n)\, n!\,\Gamma((n+\gamma)\alpha-\beta)\Gamma(l+n\alpha+\gamma\alpha-\beta-1+m)} c^n$$

Whose last summation in view of (1.4), is easily seen to arrive at the results of (2.1).This completes the proof.

Setting $p = 1, q = 1, a_1 \dots a_p = 1, b_1 \dots b_q = 1$in (2.1) then we obtain the following result:

Corollary 2.2 Let $\alpha, \beta, \gamma, x, l, m \in C, R(\alpha\gamma - \beta) > 0\ R(l) > 0, R(m) > 0$, then there holds the following formula:

$$B\left\{K_4^{(\alpha,\beta,\gamma),(c,0):(1,1)}(1;1;x); l, m\right\} =$$

$$\frac{\Gamma(m)}{\Gamma(\gamma)}\ {}_2\psi_2\left[\begin{matrix}(\gamma,1),(l+\gamma\alpha-\beta-1,\alpha)\\(\gamma\alpha-\beta,\alpha),(l+\gamma\alpha+m-\beta-1,\alpha)\end{matrix}; c\right]$$

$$= \frac{\Gamma(m)}{\Gamma(\gamma)} H_{2,3}^{1,2}\left[c\left|\begin{matrix}(1-\gamma,1),(2-l-\gamma\alpha+\beta,\alpha)\\(1-\gamma\alpha+\beta,\alpha),(2-l-\gamma\alpha+\beta-m,\alpha)\end{matrix}\right.\right]$$

Now on setting $\alpha = 1, \beta = -\beta, \gamma = 1$in (2.2) then we obtain the following result:

Corollary 2.3. Let$\beta, x, l, m \in C, R(1-\beta) > 0\ R(l) > 0, R(m) > 0$, then there holds the following formula:

$$B\left\{K_4^{(1,-\beta,1),(c,0):(1,1)}(1;1;x);l,m\right\} = \mathfrak{C}\ \ {}_2F_2\left[\begin{matrix}1,(l+\beta)\\(1+\beta),(l+\beta+m)\end{matrix};c\right]$$

where $\mathfrak{C} = \frac{\Gamma(m)\Gamma(l+\beta)}{\Gamma(l+\beta+m)\beta!}$

Theorem 2.4. (Laplace Transform) Let $\alpha,\beta,\gamma,x,l,m \in C, R(\alpha\gamma-\beta) > 0\ R(l) > 0, R(m) > 0$, then there holds the following formula:

$$L\left\{K_4^{(\propto,\beta,\gamma),(c,0):(p,q)}\big(a_1,\ldots.,a_p;b_1,\ldots.b_q;x\big);s\right\} = \frac{\prod_{j=1}^{q}\Gamma\big(b_j\big)\,s^{\beta-\alpha\gamma}}{\prod_{i=1}^{p}\Gamma(a_i)\,\Gamma(\gamma)}$$
$$\times\ {}_{p+1}\psi_q\left[\begin{matrix}(a_i,1)_1^p,(\gamma,1)\\ \big(b_j,1\big)_1^q,\end{matrix};\ cs^{-\alpha}\right]$$

Proof: By applying (1.3) and using (1.14) to the left hand side of (2.4), then we have

$$= L\left\{\sum_{n=0}^{\infty}\frac{(a_1)_n\ldots\big(a_p\big)_n(\gamma)_n c^n(x)^{(n+\gamma)\alpha-\beta-1}}{(b_1)_n\ldots\big(b_q\big)_n n!\,\Gamma\big((n+\gamma)\alpha-\beta\big)};s\right\}$$
$$= \int_0^{\infty} e^{-sx}\sum_{n=0}^{\infty}\frac{(a_1)_n\ldots\big(a_p\big)_n(\gamma)_n c^n(x)^{(n+\gamma)\alpha-\beta-1}}{(b_1)_n\ldots\big(b_q\big)_n n!\,\Gamma\big((n+\gamma)\alpha-\beta\big)}dx$$

Now changing the order of integration and summation

$$= \sum_{n=0}^{\infty}\frac{(a_1)_n\ldots\big(a_p\big)_n(\gamma)_n c^n}{(b_1)_n\ldots\big(b_q\big)_n n!\,\Gamma\big((n+\gamma)\alpha-\beta\big)}\int_0^{\infty} e^{-sx}\,(x)^{(n+\gamma)\alpha-\beta-1}dx$$
$$= \sum_{n=0}^{\infty}\frac{(a_1)_n\ldots\big(a_p\big)_n(\gamma)_n c^n}{(b_1)_n\ldots\big(b_q\big)_n n!}\ \ L\left\{\frac{(x)^{(n+\gamma)\alpha-\beta-1}}{\Gamma\big((n+\gamma)\alpha-\beta\big)};s\right\}$$

Now on using (1.2) we get

$$= \sum_{n=0}^{\infty}\frac{\Gamma(a_1+n)\ldots\Gamma\big(a_p+n\big)\Gamma(b_1)\ldots\Gamma\big(b_q\big)\Gamma(\gamma+n)c^n}{\Gamma(b_1+n)\ldots\Gamma\big(b_q+n\big)\Gamma(a_1)\ldots\Gamma\big(a_p\big)\Gamma(\gamma)n!}L\left\{\frac{(x)^{(n+\gamma)\alpha-\beta-1}}{\Gamma((n+\gamma)\alpha-\beta)};s\right\}$$

In accordance with the definition of (1.4), we obtain the result (2.4). This completes the proof of the theorem.

On setting $p=1,q=1,a_1\ldots a_p=1,b_1\ldots b_q=1$in (2.4) then we obtain the following results

Corollary 2.5 (a) Let$\alpha,\beta,\gamma,x,l,m \in C, R(\alpha\gamma-\beta) > 0\ R(l) > 0, R(m) > 0$, then there holds the following formula:

$$\boldsymbol{L\left\{K_4^{(\propto,\beta,\gamma),(c,0):(1,1)}(1;1;x);s\right\} = L\left\{\frac{x^{\alpha\gamma-\beta-1}}{\Gamma(\gamma)}\ {}_1\psi_1\left[\begin{matrix}(\gamma,1)\\(\alpha\gamma-\beta,\alpha)\end{matrix};cx^{\alpha}\right]\right\}}$$
$$= s^{-\alpha\gamma+\beta}\ {}_1F_0[\gamma;cs^{-\alpha}]$$

Corollary 2.5 (b) Let $\alpha,\beta,\gamma,x,l,m \in C, R(\alpha\gamma-\beta) > 0\ R(l) > 0, R(m) > 0$, then there holds the following formula:

$$L\left\{K_4^{(\propto,\beta,\gamma),(c,0):(1,1)}(1;1;x);s\right\} = L\left\{\frac{x^{\alpha\gamma-\beta-1}}{\Gamma(\gamma)} H_{1,2}^{1,1}\left[-cx^{\alpha}\left|\begin{matrix}(1-\gamma,1)\\(0,1)(1-\alpha\gamma+\beta,\alpha)\end{matrix}\right.\right]\right\}$$

$$= \frac{s^{\beta-\alpha\gamma}}{\Gamma(\gamma)}\ {}_1\psi_0[(\gamma,1);cs^{-\alpha}]$$

Special Cases of theorem 2.4

(i) If we take $\beta = \alpha - 1, \gamma = 1, c = -c, p = 1, q = 1, a_1 \dots a_p = 1, b_1 \dots b_q = 1$in (2.4), then we obtain the following Laplace Transform of Mittag-Leffler function

$$L\left\{K_4^{(\propto,\alpha-1,1),(-c,0):(1,1)}(1;1;x);s\right\} = L\{E_{\alpha}(-cx^{\alpha});s\} = \frac{s^{\alpha}}{s(s^{\alpha}+c)}$$

The above equation is a known result given by ([16] equation (2)).

(ii) If we take $\beta = \alpha - \beta, \gamma = 1, c = 1, p = 1, q = 1, a_1 \dots a_p = 1, b_1 \dots b_q = 1$in (2.4), then we obtain the Laplace Transform of Agarwal’s function given as

$$L\left\{K_4^{(\propto,\alpha-\beta,1),(1,0):(1,1)}(1;1;x);s\right\} = L\{E_{\alpha,\beta}(x^{\alpha});s\} = \frac{s^{\alpha-\beta}}{s^{\alpha}-1}$$

It is a known result given by ([16] equation (4)).

(iii) If we take $\beta = 0, \gamma = 1, p = 1, q = 1, a_1 \dots a_p = 1, b_1 \dots b_q = 1$in (2.4) then we obtain the following Laplace Transform

$$L\left\{K_4^{(\propto,0,1),(c,0):(1,1)}(1;1;x);s\right\} = L\{F_{\alpha}(c,x);s\} = \frac{1}{s^{\alpha}-c}$$

where $F_{\alpha}(c,x)$is the F-function defined by Robotnov and Hartley [8].The above equation is a known result given by ([16] equation (8)).

(iv) If we take $\alpha = 1, \beta = -\beta, \gamma = 1, p = 1, q = 1, a_1 \dots a_p = 1, b_1 \dots b_q = 1$ in (2.4), then the following Laplace Transform of Miller and Ross’s function is obtained as

$$L\left\{K_4^{(1,-\beta,1),(c,0):(1,1)}(1;1;x);s\right\} = L\{E_x(\beta,c);s\} = \frac{s^{-\beta}}{s-c}$$

It is a known result given by ([16] equation (10)).

Theorem 2.6. (Whittaker Transform) Let $\alpha,\beta,\gamma,t,l,m \in C, R(\alpha\gamma-\beta) > 0\ R(l) > 0, R(m) > 0$, then there holds the following formula:

$$\int_0^{\infty} e^{-\frac{t}{2}} t^{\tau-1} W_{\rho,\delta}(t) K_4^{(\propto,\beta,\gamma),(c,0):(p,q)}\left(a_1,\dots,a_p;b_1,\dots.b_q;t\right)dt = \frac{\prod_{j=1}^{q}\Gamma(b_j)}{\prod_{i=1}^{p}\Gamma(a_i)\,\Gamma(\gamma)}$$

$$\times\ {}_{p+3}\psi_{q+2}\left[\begin{matrix}(a_i,1)_1^p,(\gamma,1),\left(\delta+\gamma\alpha+\tau-\beta-\frac{1}{2},\alpha\right),\left(-\delta+\gamma\alpha+\tau-\beta-\frac{1}{2},\alpha\right)\\(b_j,1)_1^q,(\gamma\alpha-\beta,\alpha),\left(\gamma\alpha+\tau-\beta-\rho-\frac{1}{2},\alpha\right)\end{matrix};\quad c\right]$$

Proof: By applying (1.3) to the left hand side of (2.6), then we have

$$\int_0^\infty e^{-\frac{t}{2}} t^{\tau-1} W_{\rho,\delta}(t) \sum_{n=0}^\infty \frac{(a_1)_n \dots (a_p)_n (\gamma)_n c^n (t)^{(n+\gamma)\alpha-\beta-1}}{(b_1)_n \dots (b_q)_n n!\, \Gamma((n+\gamma)\alpha-\beta)} dt$$

Now changing the order of integration and summation, we obtain

$$= \sum_{n=0}^\infty \frac{(a_1)_n \dots (a_p)_n (\gamma)_n c^n}{(b_1)_n \dots (b_q)_n n!\, \Gamma((n+\gamma)\alpha-\beta)} \int_0^\infty e^{-\frac{t}{2}} t^{\tau-1} (t)^{(n+\gamma)\alpha-\beta-1} W_{\rho,\delta}(t)\, dt$$

Now on using (1.16), we have

$$= \sum_{n=0}^\infty \frac{(a_1)_n \dots (a_p)_n (\gamma)_n c^n}{(b_1)_n \dots (b_q)_n n!\, \Gamma((n+\gamma)\alpha-\beta)}$$
$$\times \frac{\Gamma\left(\frac{1}{2}+\delta+n\alpha+\gamma\alpha+\tau-\beta-1\right)\Gamma\left(\frac{1}{2}-\delta+n\alpha+\gamma\alpha+\tau-\beta-1\right)}{\Gamma\left(n\alpha+\gamma\alpha+\tau-\beta-1-\rho+\frac{1}{2}\right)}$$
$$= \sum_{n=0}^\infty \frac{(a_1)_n \dots (a_p)_n (\gamma)_n c^n}{(b_1)_n \dots (b_q)_n n!\, \Gamma((n+\gamma)\alpha-\beta)}$$
$$\times \frac{\Gamma\left(\delta+\gamma\alpha+\tau-\beta-\frac{1}{2}+n\alpha\right)\Gamma\left(-\delta+\gamma\alpha+\tau-\beta-\frac{1}{2}+n\alpha\right)}{\Gamma\left(\gamma\alpha+\tau-\beta-\rho-\frac{1}{2}+n\alpha\right)}$$

In view of (1.2) and (1.4), It completes the proof.

Corollary 2.7 On setting $\alpha = 1, \beta = -\beta, \gamma = 1, p = 1, q = 1, a_1 \dots a_p = 1, b_1 \dots b_q = 1$ in (2.6), then the following formula holds true for the Whittaker transform of Miller and Ross's function

$$\int_0^\infty e^{-\frac{t}{2}} t^{\tau-1} W_{\rho,\delta}(t) K_4^{(1,-\beta,1),(c,0):(1,1)}(1;1;t) dt = \int_0^\infty e^{-\frac{t}{2}} t^{\tau-1} W_{\rho,\delta}(t) E_t(\beta,c) dt$$
$$= \mathcal{C}\ {}_3F_2\left(\begin{matrix} 1, \frac{1}{2}+\delta+\beta+\tau, \frac{1}{2}-\delta+\beta+\tau \\ 1+\beta, \frac{1}{2}-\rho+\beta+\tau \end{matrix}; c\right)$$

where $\mathcal{C} = \frac{\Gamma\left(\frac{1}{2}+\delta+\beta+\tau\right)\Gamma\left(\frac{1}{2}-\delta+\beta+\tau\right)}{\Gamma(1+\beta)\Gamma\left(\frac{1}{2}-\rho+\beta+\tau\right)}$. Here equation (2.7) is a new result.

Corollary 2.8: On setting $p = 1, q = 1, a_1 \dots a_p = 1, b_1 \dots b_q = 1$ in (2.6), then the following formula holds true for the Whittaker transform of Fox Wright function

$$\int_0^\infty e^{-\frac{t}{2}} t^{\tau-1} W_{\rho,\delta}(t) K_4^{(\alpha,\beta,\gamma),(c,0):(1,1)}(1;1;t)dt$$

$$= \int_0^\infty e^{-\frac{t}{2}} t^{\tau-1} W_{\rho,\delta}(t) \frac{t^{\gamma\alpha-\beta-1}}{\Gamma(\gamma)} \ {}_1\psi_1 \left[\begin{matrix} (\gamma,1) \\ (\gamma\alpha-\beta,\alpha) \end{matrix}; ct^\alpha \right] dt$$

$$= \frac{1}{\Gamma(\gamma)} \ {}_3\psi_2 \left[\begin{matrix} (\gamma,1), \left(\delta+\tau+\gamma\alpha-\beta-\frac{1}{2},\alpha\right), \left(-\delta+\tau+\gamma\alpha-\beta-\frac{1}{2},\alpha\right) \\ (\gamma\alpha-\beta), \left(\tau+\gamma\alpha-\beta-\rho-\frac{1}{2},\alpha\right) \end{matrix}; c \right]$$

This is a new result.

3. CONCLUSION

The integral transforms presented in this paper are new and can be further extended in more useful results due to their generalized nature, which can be applied in various fields of applied mathematics, engineering etc. Some extentions of the main results are also considered as special cases which are well known results of eminent researchers.

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