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AN EFFICIENT CLASS OF PRODUCT ESTIMATORS USING MEASURES OF DISPERSIONS

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Abstract. *The study suggests a class of product estimators for estimating the population mean of variable under investigation in simple random sampling without replacement (SRSWOR) scheme when secondary information on standard deviation, mean deviation, and quartile deviation is available. The expression for Bias and Mean Square Error (MSE) has been derived. A comparison is made both theoretically and numerically with other existing product estimators. It is concluded that compared to other product type estimators, suggested class of estimators estimate the population mean more efficiently.*

Keywords: *standard deviation, mean deviation, quartile deviation, bias and MSE.*

1. INTRODUCTION

In Survey sampling, auxiliary information plays an important role to improve the efficiency of an estimator. A lot of new methods are suggested by researchers to estimate the population mean efficiently through the use of secondary information by taking the benefit of correlation between the study variable and variable under investigation. The following notation would be considered in the research study:

N – Population Size,
n – Sample Size,
Y – Study variable
X – Variable under interest,
$\bar{X}, \bar{Y}$ - Population means,
$\bar{x}, \bar{y}$ - Sample means
S_x S_y -Standard deviation,
C_x C_y - Coefficient of variation,
ρ - Correlation coefficient between X and Y,
$\beta_{2(x)}$ - Kurtosis of auxiliary variable
M_d - Mean deviation,
Q_d– Quartile deviation.

The concept of an auxiliary variable was suggested by Cochran [1] for the first time to improve the efficiency of variable under investigation. Robson [2] proposed the classical product estimator given as,

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$$\bar{y}_{p1} = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)$$

Mathematical expression for MSE and Bias was derived as

$$Bias(\bar{y}_{p1}) = \bar{Y}\lambda\rho C_x C_y$$

$$MSE(\bar{y}_{p1}) = \bar{Y}^2\lambda\left(C_x^2 + C_y^2 + 2\rho C_x C_y\right) \tag{1}$$

where, $\lambda = \left(\frac{1-f}{n}\right)$.

Pandey and Dubey [3] present a new product estimator. This estimator improves the result of classical estimator. The estimator is define by

$$\bar{y}_{p2} = \bar{y}\left(\frac{\bar{x}+C_x}{\bar{X}+C_x}\right)$$

Mathematical expression for MSE and Bias was derived as

$$Bias(\bar{y}_{p2}) = \bar{Y}\theta\lambda\rho C_x C_y$$

$$MSE(\bar{y}_{p2}) = \bar{Y}^2\lambda\left(C_y^2 + \theta^2 C_x^2 + 2\theta\rho C_x C_y\right) \tag{2}$$

where, $\theta = \frac{\bar{X}}{\bar{X}+C_x}$.

Singh and Tailor [4] present a new product estimator given by:

$$\bar{y}_{p3} = \bar{y}\left(\frac{\bar{x}+\rho}{\bar{X}+\rho}\right)$$

Mathematical expression for MSE and Bias was derived as

$$Bias(\bar{y}_{p3}) = \bar{Y}\theta\lambda\rho C_x C_y$$

$$MSE(\bar{y}_{p3}) = \bar{Y}^2\lambda\left(C_y^2 + \theta^2 C_x^2 + 2\theta\rho C_x C_y\right) \tag{3}$$

where, $\theta = \frac{\bar{X}}{\bar{X}+\rho}$.

Singh [5] presented a product estimator utilizing standard deviation to increase its performance. The estimator is follow

$$\bar{y}_{p4} = \bar{y}\left(\frac{\bar{x}+S_x}{\bar{X}+S_x}\right)$$

Mathematical expression for Bias and MSE were obtained as

$$Bias(\bar{y}_{p4}) = \bar{Y}\theta_4\lambda\rho C_x C_y$$

$$MSE(\bar{y}_{p4}) = \bar{Y}^2\lambda\left(C_y^2 + \theta_4^2 C_x^2 + 2\theta_4\rho C_x C_y\right) \tag{4}$$

where, $\theta_4 = \dfrac{\bar{X}}{\bar{X}+S_x}$.

Gandge et al. [6] proposed modified product estimators utilizing coefficient of variation and correlation coefficiient to increase its efficiency. The estimators is given as

$$\bar{y}_{p5} = k\bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)$$

$$\bar{y}_{p6} = k\bar{y}\left(\frac{\bar{x}+C_x}{\bar{X}+C_x}\right)$$

$$\bar{y}_{p7} = k\bar{y}\left(\frac{\bar{x}+\rho}{\bar{X}+\rho}\right)$$

where k is a constant, suggested as to decrease the MSE of estimators. Mathematical expression for MSE were obtaine as

$$MSE(\bar{y}_{pj}) = \bar{Y}^2\left(1-\frac{A_j^2}{B_j}\right), \quad j=5,6,7 \tag{5}$$

where,

$$k = \frac{1+\lambda C_{yx}}{1+\lambda C_y^2 + \lambda C_x^2 + 4\lambda C_{yx}} = \frac{A_5}{B_5}$$

$$k = \frac{\theta\lambda C_{yx}+1}{\lambda C_y^2 + \theta^2\lambda C_x^2 + 4\theta\lambda C_{xy}+1} = \frac{A_6}{B_6} \qquad \theta = \frac{\bar{X}}{\bar{X}+C_x}$$

$$k = \frac{\theta\lambda C_{yx}+1}{\lambda C_y^2 + \theta^2\lambda C_x^2 + 4\theta\lambda C_{xy}+1} = \frac{A_7}{B_7} \qquad \theta = \frac{\bar{X}}{\bar{X}+\rho}$$

The present study is conducted to develop a modified form of product estimator which estimate the population mean more efficiently by using auxiliary information. Auxiliary information is the information that is priory known to the researcher.

2. ESTIMATORS AND COMPARISIONS

2.1. SUGGESTED ESTIMATORS

The current paper suggested a new modified class of product type estimators by using the auxiliary information on some specific variables. Following is the proposed class of estimator

$$\bar{y}_{bpi} = \bar{y}\left(\frac{\bar{x} - S_x G(x)}{\bar{X} - S_x G(x)}\right)$$

where, G(x) = Specific auxiliary parameter=1, M_d, and Q_d, respectively.

$$\bar{y}_{bp1} = \bar{y}\left(\frac{\bar{x} + S_x}{\bar{X} + S_x}\right)$$

$$\bar{y}_{bp2} = \bar{y}\left(\frac{\bar{x} + S_x M_d}{\bar{X} + S_x M_d}\right)$$

$$\bar{y}_{bp3} = \bar{y}\left(\frac{\bar{x} + S_x Q_d}{\bar{X} + S_x Q_d}\right)$$

Mathematical expression for Bias and MSE of the suggested estimators are derived as

$$Bias(\bar{y}_{bpi}) = \bar{Y}\theta_i \lambda \rho C_x C_y$$

$$MSE(\bar{y}_{bpi}) = \bar{Y}^2 \lambda \left(C_y^2 + \theta_i^2 C_x^2 + 2\theta_i \rho C_x C_y\right) \tag{6}$$

where, i = 1, 2, 3 and $\lambda = \left(\frac{1-f}{n}\right)$

$$\theta_1 = \frac{\bar{X}}{\bar{X} - S_x} \qquad \theta_2 = \frac{\bar{X}}{\bar{X} - S_x M_d} \qquad \theta_3 = \frac{\bar{X}}{\bar{X} + S_x Q_d}$$

2.2. EFFICIENCY COMPARISONS

In In this segment, theoretical compression of efficiency of suggested class of product estimator is made with some existing estimator in the literature. The efficiency is measured in term of MSE. The MSE of suggested class of product estimators will be compare with Robson [2], PandEy and DuBEy [3], Singh and Tailor [4], Singh [5], Gandge et al. [6]

i. Comparison with usual product estimator

The proposed estimator work efficiently than [2] if,

$$MSE(\bar{y}_{bpi}) < \mathrm{MSE}(\bar{\mathrm{y}}_p)$$

$$\bar{Y}^2\lambda\left(C_y^2+\theta_{bi}^2C_x^2+2\theta_{bi}\rho C_xC_y\right)<\bar{Y}^2\lambda\left(C_x^2+C_y^2+2\rho C_xC_y\right)$$

$$C_y^2+\theta_{bi}^2C_x^2+2\theta_{bi}\rho C_xC_y<C_x^2+C_y^2+2\rho C_xC_y$$

$$\theta_{bi}^2C_x+2\theta_{bi}\rho C_y<C_x+2\rho C_y$$

$$\theta_{bi}^2C_x-C_x<2\rho C_y-2\theta_{bi}\rho C_y$$

$$\theta_{bi}^2C_x-C_x<-2\theta_{bi}\rho C_y+2\rho C_y$$

$$C_x\left(\theta_{bi}^2-1\right)<-2\rho C_y\left(\theta_{bi}-1\right)$$

$$-2\rho C_y\left(\theta_{bi}-1\right)>C_x\left(\theta_{bi}^2-1\right)$$

$$2\rho C_y\left(\theta_{bi}-1\right)<-C_x\left(\theta_{bi}+1\right)\left(\theta_{bi}-1\right)$$

$$2\rho C_y<-C_x\left(\theta_{bi}+1\right)$$

$$\rho<\frac{-C_x\left(\theta_{bi}+1\right)}{2C_y} \qquad i=1,2,3 \tag{7}$$

where, $\theta_{b1}=\dfrac{\bar{X}}{\bar{X}-S_x}$ $\theta_{b2}=\dfrac{\bar{X}}{\bar{X}-S_xMD}$ $\theta_{b3}=\dfrac{\bar{X}}{\bar{X}-S_xQD}$.

ii. Comparison with Other Existing Estimator

The suggested class of product estimators will be more efficient then [3-5] if

$$MSE(\bar{y}_{bpi})_j < \mathrm{MSE}(\bar{y}_{pj})$$

$$\bar{Y}^2\lambda\left(C_y^2+\theta_{bi}^2C_x^2+2\theta_{bi}\rho C_xC_y\right)<\bar{Y}^2\lambda\left(C_y^2+\theta_j^2C_x^2+2\theta_j\rho C_xC_y\right)$$

$$\theta_{bi}^2 C_x^2 + 2\theta_{bi}\rho C_x C_y < \theta_j^2 C_x^2 + 2\theta_j \rho C_x C_y$$

$$2\theta_{bi}\rho C_y - 2\theta_j \rho C_y < \theta_j^2 C_x - \theta_{bi}^2 C_x$$

$$2\rho C C_y(\theta_{bi} - \theta_j) < C_x(\theta_j^2 - \theta_{bi}^2)$$

$$2\rho C_y(\theta_{bi} - \theta_j) < -C_x(\theta_{bi}^2 - \theta_j^2)$$

$$2\rho C_y(\theta_{bi} - \theta_j) < -C_x(\theta_{bi} + \theta_j)(\theta_{bi} - \theta_j)$$

$$2\rho C_y < -C_x(\theta_{bi} + \theta_j)$$

$$\rho < \frac{C_x}{2C_y}(\theta_{bi} + \theta_j) \qquad \mathrm{i} = 1,2,3 \; and \; j = 2,3,4 \tag{8}$$

where, $\theta_{b1} = \dfrac{\bar{X}}{\bar{X} - S_x} \quad \theta_{b2} = \dfrac{\bar{X}}{\bar{X} - S_x MD} \quad \theta_{b3} = \dfrac{\bar{X}}{\bar{X} - S_x QD}$.

iii. Comparison with Gandge et al. [6] estimator

Our Proposed estimator will better than [6] if

$$MSE(\bar{y}_{bpi}) < \mathrm{MSE}(\bar{y}_{pj})$$

$$\bar{Y}^2 \lambda\left(C_y^2 + \theta_i^2 C_x^2 + 2\theta_i \rho C_x C_y\right) < \bar{Y}^2\left[1 - \frac{A_j^2}{B_j}\right]$$

$$\lambda\left(C_y^2 + \theta_{bpi}^2 C_x^2 + 2\theta_{bpi}\rho C_x C_y\right) < \left[1 - \frac{A_j^2}{B_j}\right], \quad \text{i=1,2,3 and j=5,6,7} \tag{9}$$

3. RESULTS AND DISCUSSION

For the purpose of numerical illustration, data set of Maddala [6, 7] will be used. The proposed estimator and other existing product estimators will be applied to the data and computed their MSE.

The statistics about the population parameters is given below. It is noted that study variable is 68% negatively correlated with auxiliary variable. It is of keen interest that the efficiency of the estimators is independent of a sample size.

N = 16, n = 4, $\bar{X}$ = 75.4313, $\bar{Y}$ = 7.6375, $y_{\max}$ =634, S_x =7.43753, S_y =1.739823,

C_x =0.0986, C_y =0.2278, M_d=3.96672, Q_d=4.9584, and ρ_{xy} = -0.6823.

Table 1: MSE and Constants (θ,k) of the suggested and other estimators.

Estimator	MSE	(θ)	(k)	MSE
$\bar{y}_{p1}$	$\bar{Y}^2\lambda\left(C_x^2+C_y^2+2\rho C_x C_y\right)$	--	--	0.338664236
$\bar{y}_{p2}$	$\bar{Y}^2\lambda\left(C_y^2+\theta^2 C_x^2+2\theta\rho C_x C_y\right)$	0.9987	--	0.338824365
$\bar{y}_{p3}$	$\bar{Y}^2\lambda\left(C_y^2+\theta^2 C_x^2+2\theta\rho C_x C_y\right)$	1.00913	--	0.337552597
$\bar{y}_{p4}$	$\bar{Y}^2\left(1-\frac{A^2}{B}\right)$ $\frac{A}{B}=\frac{1+\lambda C_{yx}}{1+\lambda C_y^2+\lambda C_x^2+4\lambda C_{yx}}=k$	--	0.9971	0.338058556
$\bar{y}_{p5}$	$\bar{Y}^2\left[1-\frac{A^2}{B}\right]$ $\frac{A}{B}=\frac{\theta\lambda C_{yx}+1}{\lambda C_y^2+\theta^2\lambda C_x^2+4\theta\lambda C_{xy}+1}=k$	0.9987	0.9942	0.338216576
$\bar{y}_{p6}$	$\bar{Y}^2\left[1-\frac{A^2}{B}\right]$ $\frac{A}{B}=\frac{\theta\lambda C_{yx}+1}{\lambda C_y^2+\theta^2\lambda C_x^2+4\theta\lambda C_{xy}+1}=k$	1.00913	0.9971	0.336962917
$\bar{y}_{bp1}$	$\bar{Y}^2\lambda\left(C_y^2+\theta_1^2 C_x^2+2\theta_1\rho C_x C_y\right)$	1.10939	--	0.326529579
$\bar{y}_{bp2}$	$\bar{Y}^2\lambda\left(C_y^2+\theta_2^2 C_x^2+2\theta_2\rho C_x C_y\right)$	1.95656	--	0.31876063
$\bar{y}_{bp3}$	$\bar{Y}^2\lambda\left(C_y^2+\theta_3^2 C_x^2+2\theta_3\rho C_x C_y\right)$	1.64236	--	0.303807719

4. CONCLUSIONS

The present study introduce a class of product estimator for estimating finite population which utilizes auxiliary information in the form of measure of dispersion i.e, population standard deviation, quartile deviation and mean deviation of specific known population. The class of product estimators has been proposed under SRSWOR scheme. Mathematical equation for Bias and MSE has been derived for the proposed estimators. Theoretical comparison of the efficiency of suggested class of estimators has been made with others. Theoretical results are supported by numerical illustration by using data set on real population. Table 1 revealed that the MSE and Bias of suggested estimators are smaller than others in the literature. Thus the suggested estimators work perfectly than usual and other already derived product estimators of the population mean. Besides from efficient result proposed estimator also easy to understand and simple to use. Moreover among the other proposed regression estimators in this study, the estimator $\bar{y}_{bp3}$ performs better than other four estimators both in term of MSE and Bias.

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