ORIGINAL PAPER

ON THE DEVELOPMENT OF RATIO TYPE ESTIMATORS USING AUXILIARY INFORMATION

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Abstract. *The paper produces some new modified forms of the ratio estimators using the auxiliary information. The large sample properties, that is, the bias and mean squared error up to the first order of approximation are determined. The comparison is made with other existing estimators by using an applied data. It has been observed that the proposed estimators have a fewer mean squared error and leads to the efficient results as compared to the classical ratio estimator, Sisodia and Dwivedi, Singh and Kakran, Upadhyaya and Singh estimators.*

Keywords: *skewness; kurtosis; quartile deviation; ratio estimator; MSE.*

1. INTRODUCTION

To estimate the unknown population mean it is usual practice to use the sampling techniques. In this context, different estimators have been developed by many researchers using the auxiliary information and still the research is going on. The auxiliary information can improve the efficiency of an estimator. There exist two situations to use the auxiliary information. The study variable (y) may have a positive or negative correlation with the auxiliary information (x). We prefer a ratio type estimators if there is positive correlation between the study and auxiliary veritable. The ratio estimator is modified by many researchers. Such as, Subramani [1], Ekpenyong and Enang [2], Abid et al. [3], Soponviwatkul and Lawson [4], Abbas et al. [5], and Zaman and Dunder [6].

The paper circulates the study on modification to the ratio type estimators by using the supplementary information under simple random sampling without replacement. The following notations are used in the proposed estimators:

$\beta_{1(x)}, \beta_{2(x)}$: Coefficient of Skewness and Kurtosis of the auxiliary variables

$Q.D$: Quartile Deviation

M_d: Median of the auxiliary variable

$\gamma = \frac{Q.D}{S_x}$: The ratio between quartile and Standard deviation of the auxiliary variable.

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2. ESTIMATORS AND COMPARISIONS

2.1. EVALUATION OF EXISTING RATIO-TYPE ESTIMATORS

In order to improve the estimation of the unknown population mean, many researchers have presented their own modified estimators, the description of which as under including with the usual population mean.

The usual estimator of the population mean is given by

$$t_0 = \bar{y}$$

$$MSE(t_0) = \frac{1-f}{n}\bar{Y}^2 C_y^2 \tag{1}$$

The classical ratio estimator with the mean square error is characterized as

$$t_r = \bar{y}\frac{\bar{X}}{\bar{x}}$$

$$MSE(t_r) = \frac{1-f}{n}\bar{Y}^2\left[C_y^2 + C_x^2 - 2\rho_{yx}C_yC_x\right] \tag{2}$$

Sisodia and Dwivedi [7] improved the classical ratio estimator by adding the coefficient of variation (C_x) as the auxiliary information. The suggested estimator is given by

$$t_{SD} = \bar{y}\frac{\bar{X}+C_x}{\bar{x}+C_x}$$

The related mean square error up to the first order of approximation is defined as

$$MSE(t_{SD}) \cong \frac{1-f}{n}\bar{Y}^2\left[C_y^2 + C_x^2\alpha(\alpha - 2K)\right] \tag{3}$$

Singh and Kakran [8] replaced the coefficient of variation by the coefficient of kurtosis as auxiliary variable. The estimator is then take the following form

$$t_{SK} = \bar{y}\frac{\bar{X}+\beta_{2(x)}}{\bar{x}+\beta_{2(x)}}$$

The corresponding mean square error is defined as

$$MSE(t_{SK}) \cong \frac{1-f}{n}\bar{Y}^2\left[C_y^2 + C_x^2\delta(\delta - 2K)\right] \tag{4}$$

Upadhyaya and Singh [9] proposed a combined ratio estimator by using two auxiliary informations, that is, the Coefficient of Kurtosis $\beta_{2(x)}$ and the Coefficient of variation (C_x). The combined ratio estimator is given by

$$t_{US1} = \bar{y}\,\frac{\bar{X}\beta_{2(x)} + C_x}{\bar{x}\beta_{2(x)} + C_x}$$

The mean square error is then expressed as

$$MSE\left(t_{US1}\right) \cong \frac{1-f}{n}\bar{Y}^2\left[C_y^2 + C_x^2\omega_1\left(\omega_1 - 2K\right)\right] \tag{5}$$

In the same paper of Upadhyaya and Singh [9] suggested another estimator by changing the places of the auxiliary information's. The modified estimator is then take the following form

$$t_{US2} = \bar{y}\,\frac{\bar{X}C_x + \beta_{2(x)}}{\bar{x}C_x + \beta_{2(x)}}$$

Up to the first order of approximation, the mean square error is defined as

$$MSE\left(t_{US2}\right) \cong \frac{1-f}{n}\bar{Y}^2\left[C_y^2 + C_x^2\omega_2\left(\omega_2 - 2K\right)\right] \tag{6}$$

2.2. PROPOSED RATIO-TYPE ESTIMATORS

In this section, we suggests six new modified forms of the ratio estimator by using the auxiliary information including coefficient of Skewness, coefficient of Kurtosis, median, quartile deviation, and ratio between the quartile deviation and standard deviation. The estimators are given by

$$t_{p_1} = \bar{y}\left[\frac{\bar{X}\left(\beta_{1(x)} - \beta_{2(x)}\right) + C_x}{\bar{x}\left(\beta_{1(x)} - \beta_{2(x)}\right) + C_x}\right]$$

The mean square error of the above estimator is given by

$$MSE\left(t_{p_1}\right) = \frac{1-f}{n}\bar{Y}^2\left[C_y^2 + \theta_1^2 C_x^2 - 2\theta_1\rho_{yx}C_yC_x\right] \tag{7}$$

$$t_{p_2} = \bar{y}\left[\frac{\bar{X}\left(Q.D\right) + \left(M_d - Q.D\right)}{\bar{x}\left(Q.D\right) + \left(M_d - Q.D\right)}\right]$$

The mean square error of the above estimator is given by

$$MSE\left(t_{p_2}\right)=\frac{1-f}{n}\bar{Y}^2\left[C_y^2+\theta_2^2C_x^2-2\theta_2\rho_{yx}C_yC_x\right] \tag{8}$$

$$t_{p_3}=\bar{y}\left[\frac{\bar{X}\left(Q.D\right)+\left(\delta-M_d\right)}{\bar{x}\left(Q.D\right)+\left(\delta-M_d\right)}\right]$$

The mean square error of the above estimator is given by

$$MSE\left(t_{p_3}\right)=\frac{1-f}{n}\bar{Y}^2\left[C_y^2+\theta_3^2C_x^2-2\theta_3\rho_{yx}C_yC_x\right] \tag{9}$$

$$t_{p_4}=\bar{y}\left[\frac{\bar{X}\left(Q.D\right)+\left(\delta-Q.D\right)}{\bar{x}\left(Q.D\right)+\left(\delta-Q.D\right)}\right]$$

The mean square error of the above estimator is given by

$$MSE\left(t_{p_4}\right)=\frac{1-f}{n}\bar{Y}^2\left[C_y^2+\theta_4^2C_x^2-2\theta_4\rho_{yx}C_yC_x\right] \tag{10}$$

$$t_{p_5}=\bar{y}\left[\frac{\bar{X}C_x+\left(\beta_{1(x)}-\beta_{2(x)}\right)}{\bar{x}C_x+\left(\beta_{1(x)}-\beta_{2(x)}\right)}\right]$$

The mean square error of the above estimator is given by

$$MSE\left(t_{p_5}\right)=\frac{1-f}{n}\bar{Y}^2\left[C_y^2+\theta_5^2C_x^2-2\theta_5\rho_{yx}C_yC_x\right] \tag{11}$$

The mean square error of the above estimator is given by

$$t_{p_6}=\bar{y}\left[\frac{\bar{X}\left(\beta_{1(x)}-\beta_{2(x)}\right)+\left(Q.D\right)}{\bar{x}\left(\beta_{1(x)}-\beta_{2(x)}\right)+\left(Q.D\right)}\right]$$

$$MSE\left(t_{p_6}\right)=\frac{1-f}{n}\bar{Y}^2\left[C_y^2+\theta_6^2C_x^2-2\theta_6\rho_{yx}C_yC_x\right] \tag{12}$$

The mean squared error of the suggested estimators are summarized as

$$MSE\left(t_{p_i}\right)=\frac{1-f}{n}\bar{Y}^2\left[C_y^2+\theta_i^2C_x^2-2\theta_i\rho_{yx}C_yC_x\right], \tag{13}$$

where, $i=1,2,3,4,5,6$ and

$$\theta_1 = \frac{\bar{X}\left(\beta_{1(x)} - \beta_{2(x)}\right)}{\left(\beta_{1(x)} - \beta_{2(x)}\right) + C_x},\ \theta_2 = \frac{\bar{X}\left(Q.D\right)}{\bar{X}\left(Q.D\right) + \left(M_d - Q.D\right)},\ \theta_3 = \frac{\bar{X}\left(Q.D\right)}{\bar{X}\left(Q.D\right) + \left(\gamma - M_d\right)},$$

$$\theta_4 = \frac{\bar{X}\left(Q.D\right)}{\bar{X}\left(Q.D\right) + \left(\gamma - Q.D\right)},\ \theta_5 = \frac{\bar{X}C_x}{\bar{X}C_x + \left(\beta_{1(x)} - \beta_{2(x)}\right)},\ \theta_6 = \frac{\bar{X}\left(\beta_{1(x)} - \beta_{2(x)}\right)}{\left(\beta_{1(x)} - \beta_{2(x)}\right) + Q.D}$$

$$\gamma = \frac{Q.D}{S_x}.$$

2.3. COMPARISION OF EFFICIENCIES

This section presents the theoretical as well as the numerical illustration for the comparison of the proposed estimators with other existing estimators under the scheme of simple random sampling without replacement. The numerical values are given in Table 2.

(a) Comparison with usual estimator - The usual estimator of the population mean with the suggested estimators by the relation given below

$$MSE\left(t_{P_i}\right) < Var\left(t_0\right)$$

$$\left(\theta_i C_x - 2\rho C_y\right) < 0$$

(b) Comparison with classical ratio estimator - The suggested estimators are more efficient from the classical ratio estimator if holds the inequality given below

$$MSE\left(t_{P_i}\right) < MSE\left(t_r\right)$$

$$\left\{C_x\left(\theta_i + 1\right) - 2\rho_{yx}C_y\right\} < 0$$

(c) Comparison with Sisodia and Dwivedi [7] - The suggested estimators are perform well as compared to the Sisodia and Dwivedi by the following condition

$$MSE\left(t_{P_i}\right) < MSE\left(t_{SD}\right)$$

$$\left\{C_x\left(\theta_i + \alpha\right) - 2\rho_{yx}C_y\right\} < 0$$

(a) Comparison with Singh and Kakran [8]

The proposed estimators lead to a better result than Singh and Kakran if the following inequality exists

$$MSE\left(t_{P_i}\right) < MSE\left(t_{SK}\right)$$

$$\left\{C_x\left(\theta_i + \delta\right) - 2\rho_{yx}C_y\right\} < 0$$

(b) Comparison with Upadhyaya and Singh (1) [9]

The suggested estimators are provide a best results than Upadhyaya and Singh (1), if

$$MSE\left(t_{P_i}\right) < MSE\left(t_{US1}\right)$$

$$\left\{C_x\left(\theta_i + \omega_1\right) - 2\rho_{yx}C_y\right\} < 0$$

(c) Comparison with Upadhyaya and Singh (2) [9]

The comparison is studied between the proposed estimators and Upadhyaya and Singh (2) by the given relation of mean square error

$$MSE\left(t_{P_i}\right) < MSE\left(t_{US2}\right)$$

$$\left\{C_x\left(\theta_i + \omega_2\right) - 2\rho_{yx}C_y\right\} < 0$$

3. RESULTS AND DISCUSSION

In this section, we have considered a real data set of Kadilar and Cingi [10]. The data set represents the quantity of apple production (as research variable "y") and number of apple trees (as auxiliary variable "x") in 106 Aegean area villages in 1990. The data set values are given in Table 1. We have computed the mean square error of the proposed estimators and compared it with usual estimator, Sisodia and Dwivedi, Singh and Kakran, Upadhyaya and singh estimator. The results are given in Table 3. It has been observed that the proposed estimators provide more efficient results.

In Table 1, we observe the statistics about the population. It would be significant to comment that the sample of size has no effect on the efficiency comparison of the estimators.

Table 1. Data Statistics.

N=106	n=20	$\bar{Y} = 2212.59$
$\bar{X} = 27421.70$	$C_y = 5.22$	$C_x = 2.10$
$S_{yx} = 568176176.59$	$S_y = 11551.53$	$S_x = 57460.61$
$\rho_{yx} = 0.86$	$\beta_{1(x)} = 2.122$	$\beta_{2(x)} = 34.57$

Table 2. Numerical illustration of the Conditions of efficiency comparison.

Estimators	t_0	t_r	t_{SD}	t_{SK}	t_{US1}	t_{US2}
t_{p_1}	-6.878395	-4.778395	-4.778556	-4.781039	-4.778400	-4.779655
t_{p_2}	-6.878369	-4.778369	-4.778530	-4.781013	-4.778374	-4.779629
t_{p_3}	-6.878354	-4.778354	-4.778515	-4.780998	-4.778359	-4.779614
t_{p_4}	-6.878323	-4.778323	-4.778484	-4.780968	-4.778328	-4.779583
t_{p_5}	-6.877216	-4.777216	-4.777377	-4.779860	-4.777221	-4.778476
t_{p_6}	-6.849312	-4.749312	-4.749473	-4.751956	-4.749317	-4.750572

The efficiency conditions were evaluated for the proposed estimators. Table 2 demonstrates the efficiency condition results. Tables 3 indicate the MSE and constant values for the existing and proposed estimators. It is clear that the proposed estimator is superior to existing estimators. That is, proposed estimators have smaller MSE values than the other estimators considered here.

Table 3. Constants, and MSE of the suggested and existing estimators.

Estimators	**Constant**	**Mean square error**
t_0	NA	5411348
t_r	NA	2542740
t_{SD}	0.9999234	2542893
t_{SK}	0.9987409	2545251
t_{US1}	0.9999978	2542745
t_{US2}	0.9994	2543936
t_{p_1}	**1.000002**	**2542736**
t_{p_2}	**1.000015**	**2542711**
t_{p_3}	**1.000022**	**2542697**
t_{p_4}	**1.000036**	**2542668**
t_{p_5}	**1.000564**	**2541617**
t_{p_6}	**1.013851**	**2515305**

4. CONCLUSION

The paper presents a class of ratio estimator of a finite population mean by using the auxiliary information under simple random sampling scheme. The comparison of the proposed estimators with others is carried out theoretically and numerically. On the basis of the results it has been determined that the suggested new ratio -type estimators increase the efficiency. The findings is not surprising because the conditions are satisfied as in Table 2. Furthermore more, among the proposed estimators the estimator t_{p_6} provide an efficient result than $t_{p_1}, t_{p_2}, t_{p_3}, t_{p_4}$ and t_{p_5}.

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