ORIGINAL PAPER

NEW VERSION OF FERMI-WALKER DERIVATIVES ACCORDING TO THE TYPE-2 BISHOP FRAME WITH ENERGY

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Abstract. *In this paper, we obtain the Fermi-Walker derivatives of* ξ_1, ξ_2, $\mathbf{B}$ *magnetic curves according to the type-2 Bishop frame in the space. Moreover, we obtain the energy of the Fermi-Walker derivative of magnetic curves according to the type-2 Bishop frame in space. Finally, we have energy relations of some vector fields associated with type-2 Bishop frame in the space.*

Keywords: *magnetic curve; Lorentz force; type-2 Bishop frame; Fermi-Walker derivative; energy.*

1. INTRODUCTION

The closed 2-form $\mathbf{F}$ on Riemannian manifold (M,g) is called a magnetic field. Magnetic curves on a Riemannian manifold (M,g) is a trajectory characterized by a charged particle moving in under the influence of a magnetic field $\mathbf{F}$. If these charged particles enter the magnetic field they are exposed to a force called a Lorentz force. The Lorentz force is an (1, 1)-type tensor field Φ on Riemannian manifold (M,g) and it satisfies that $g(\Phi(X),Y)=\mathbf{F}(X,Y), \quad \forall X,Y \in \chi(M)$. Lorentz force equation is expressed by $\Phi(X)=\mathbf{V}\times X$. Morever the magnetic trajectories of the magnetic field $\mathbf{F}$ is given by

$$\nabla_{\mathbf{T}}\mathbf{T}=\Phi(\mathbf{T})=\mathbf{V}\times\mathbf{T}.$$

Generalized Lorentz equation obtained from the geodesics of M is given by $\nabla_{\mathbf{T}}\mathbf{T}=0$ [1-3].

A charged particle moves along a curve in the magnetic vector field then it is exposed to the magnetic field. The researchers have examined the trajectories of charged particles moving in an area modeled by the homogeneous space $S^2\times\mathbb{R}$ [4]. The notions of $\mathbf{T}$-magnetic, $\mathbf{N}_1$-magnetic and $\mathbf{N}_2$-magnetic curves and some characterizations for them in the semi-Riemannian manifolds have been determined by some researchers [5-9].

The local theory of the curves has been investigated by some researchers by considering Serret-Frenet laws. Bishop frame, which is also called an alternative or a parallel frame of the curves by means of parallel vector fields. The Serret-Frenet and Bishop frames have one thing in common i.e. their tangent vector field. Recently, many studies have been done on the Bishop frames in the Euclidean space [10-12].

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On the other hand, several methods in the research of magnetic curves for a given magnetic field on the constant energy level have been invesitgated by Muntenau in [1]. Also, many researchers have identified energy related studies using different methods [13-21].

In this study, we obtain the Fermi-Walker derivatives of ξ_1, ξ_2, $\mathbf{B}$ magnetic curves according to the type-2 Bishop frame in the space. Moreover, we obtain the energy of the Fermi-Walker derivative of magnetic curves according to the type-2 Bishop frame in space. Finally, we have energy relations of some vector fields associated with type-2 Bishop frame in the space.

2. MATERIALS AND METHODS

At this stage, some basic concepts about curves in space are given.

The Euclidean 3-space supplied with the standard straight metric given by

$$\langle , \rangle = dx_1^2 + dx_2^2 + dx_3^2.$$

Here (x_1, x_2, x_3) is a coordinate system of the Euclidean 3-space.

Considering that $\{\mathbf{T}, \mathbf{N}, \mathbf{B}\}$ is the Serret-Frenet frame of α that the following Frenet-Serret equations can be given.

$$\begin{pmatrix} \mathbf{T}' \\ \mathbf{N}' \\ \mathbf{B}' \end{pmatrix} = \begin{pmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{pmatrix} \begin{pmatrix} \mathbf{T} \\ \mathbf{N} \\ \mathbf{B} \end{pmatrix}, \tag{2.1}$$

where κ and τ are the curvature function and torsion of α, respectively and

$$\begin{aligned} \langle \mathbf{T}, \mathbf{T} \rangle &= \langle \mathbf{N}, \mathbf{N} \rangle = \langle \mathbf{B}, \mathbf{B} \rangle = 1, \\ \langle \mathbf{T}, \mathbf{N} \rangle &= \langle \mathbf{T}, \mathbf{B} \rangle = \langle \mathbf{N}, \mathbf{B} \rangle = 0. \end{aligned}$$

The Bishop frame, which is referred to as the alternative or parallel frame of the curves depending to parallel vector fields, was introduced by L.R. Bishop in 1975. It is a well-defined alternative approach even in the absence of the second derivative of the curve [22]. The Bishop frame is explained as

$$\begin{pmatrix} \mathbf{T}' \\ \mathbf{N}_1' \\ \mathbf{N}_2' \end{pmatrix} = \begin{pmatrix} 0 & k_1 & k_2 \\ -k_1 & 0 & 0 \\ -k_2 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{T} \\ \mathbf{N}_1 \\ \mathbf{N}_2 \end{pmatrix} \tag{2.2}$$

from here

$$\begin{aligned} \langle \mathbf{T}, \mathbf{T} \rangle &= \langle \mathbf{N}_1, \mathbf{N}_1 \rangle = \langle \mathbf{N}_2, \mathbf{N}_2 \rangle = 1, \\ \langle \mathbf{T}, \mathbf{N}_1 \rangle &= \langle \mathbf{T}, \mathbf{N}_2 \rangle = \langle \mathbf{N}_1, \mathbf{N}_2 \rangle = 0. \end{aligned}$$

Here, $\{\mathbf{T}, \mathbf{N}_1, \mathbf{N}_2\}$ is called Bishop trihedra and k_1 and k_2 are called Bishop curvatures of the curve and the connection between Frenet and the Bishop frame is expressed as follows

$$\begin{pmatrix} \mathbf{T} \\ \mathbf{N} \\ \mathbf{B} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta(s) & \sin\theta(s) \\ 0 & -\sin\theta(s) & \cos\theta(s) \end{pmatrix} \begin{pmatrix} \mathbf{T} \\ \mathbf{N}_1 \\ \mathbf{N}_2 \end{pmatrix},$$

where $\theta(s) = \arctan\frac{k_2}{k_1}$, $\tau(s) = \theta^{'}(s)$ and $\kappa(s) = \sqrt{(k_1)^2 + (k_2)^2}$. Bishop curvatures are defined by $k_1 = \kappa\cos\theta(s)$ and $k_2 = \kappa\sin\theta(s)$ [12].

The Bishop frame or parallel transport frame is an alternative approach to defining a moving frame that is well-defined even when the curve has vanishing second derivative and it is defined by

$$\begin{pmatrix} \xi_1^{'} \\ \xi_2^{'} \\ \mathbf{B}^{'} \end{pmatrix} = \begin{pmatrix} 0 & 0 & -\varepsilon_1 \\ 0 & 0 & -\varepsilon_2 \\ \varepsilon_1 & \varepsilon_2 & 0 \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \\ \mathbf{B} \end{pmatrix}. \tag{2.3}$$

The connection between Frenet and the type-2 Bishop frame is expressed as follows,

$$\begin{pmatrix} \mathbf{T} \\ \mathbf{N} \\ \mathbf{B} \end{pmatrix} = \begin{pmatrix} \sin\theta(s) & -\cos\theta(s) & 0 \\ \cos\theta(s) & \sin\theta(s) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \\ \mathbf{B} \end{pmatrix},$$

where $\theta(s) = \arctan\frac{\varepsilon_2}{\varepsilon_1}$, $\kappa(s) = \theta^{'}(s)$ and $\tau(s) = \sqrt{(\varepsilon_1)^2 + (\varepsilon_2)^2}$. Here type-2 Bishop curvatures are defined by, $\varepsilon_1 = -\tau\cos\theta(s)$ and $\varepsilon_2 = -\tau\sin\theta(s)$ [12].

3. FERMI-WALKER DERIVATIVE

In this section, we give the definitions of ξ_1, ξ_2, $\mathbf{B}$ magnetic curves and Fermi-Walker derivative [23-25].

Definition 3.1. *Let $\alpha : I \subset \mathbb{R} \to \mathbb{R}^3$ be a curve with type-2 Bishop frame in Euclidean 3-space and $\mathbf{F}_{\mathrm{V}}$ be a magnetic field in $\mathbb{R}^3$. If the vector field ξ_1 of the type-2 Bishop frame satisfies the Lorentz force equation $\nabla_{\alpha^{'}}\xi_1 = \Phi(\xi_1) = \mathbf{V}\times\xi_1$, then the curve α is called a ξ_1-magnetic curve according to type-2 Bishop frame [23].*

Proposition 3.2. *Let α be a unit speed ξ_1-magnetic curve according to type-2 Bishop frame in Euclidean 3-space. Then, the Lorentz force according to the type-2 Bishop frame is obtained as*

$$\begin{pmatrix}\Phi(\xi_1)\\ \Phi(\xi_2)\\ \Phi(\mathbf{B})\end{pmatrix}=\begin{pmatrix}0 & 0 & -\varepsilon_1\\ 0 & 0 & \rho_2\\ \varepsilon_1 & -\rho_2 & 0\end{pmatrix}\begin{pmatrix}\xi_1\\ \xi_2\\ \mathbf{B}\end{pmatrix}, \tag{3.1}$$

where ρ_2 *is a certain function defined by* $\rho_2 = g(\Phi\xi_2,\mathbf{B})$, *[23].*

Definition 3.3. *Let* $\alpha : I \subset \mathbb{R} \to \mathbb{R}^3$ *be a curve with type-2 Bishop frame in Euclidean 3-space and* $\mathbf{F_V}$ *be a magnetic field in* $\mathbb{R}^3$. *If the vector field* ξ_2 *of the type-2 Bishop frame satisfies the Lorentz force equation* $\nabla_{\alpha'}\xi_2 = \Phi(\xi_2) = \mathbf{V}\times\xi_2$ *, then the curve* α *is called a* ξ_2*-magnetic curve according to type-2 Bishop frame [23].*

Proposition 3.4. *Let* α *be a unit speed* ξ_2*-magnetic curve according to type-2 Bishop frame in Euclidean 3-space. Then, the Lorentz force according to the type-2 Bishop frame is obtained as*

$$\begin{pmatrix}\Phi(\xi_1)\\ \Phi(\xi_2)\\ \Phi(\mathbf{B})\end{pmatrix}=\begin{pmatrix}0 & 0 & \eta_2\\ 0 & 0 & -\varepsilon_2\\ -\eta_2 & \varepsilon_2 & 0\end{pmatrix}\begin{pmatrix}\xi_1\\ \xi_2\\ \mathbf{B}\end{pmatrix}, \tag{3.2}$$

where η_2 *is a certain function defined by* $\eta_2 = g(\Phi\xi_1,\mathbf{B})$ *[23].*

Definition 3.5. *Let* $\alpha : I \subset \mathbb{R} \to \mathbb{R}^3$ *be a curve with type-2 Bishop frame in Euclidean 3-space and* $\mathbf{F_V}$ *be a magnetic field in* $\mathbb{R}^3$. *If the vector field* $\mathbf{B}$ *of the type-2 Bishop frame satisfies the Lorentz force equation* $\nabla_{\alpha'}\mathbf{B} = \Phi(\mathbf{B}) = \mathbf{V}\times\mathbf{B}$, *then the curve* α *is called a* $\mathbf{B}$*-magnetic curve according to type-2 Bishop frame [23].*

Proposition 3.6. *Let* α *be a unit speed* $\mathbf{B}$*-magnetic curve according to Bishop frame in Euclidean 3-space. Then, the Lorentz force according to the type-2 Bishop frame is obtained as*

$$\begin{pmatrix}\Phi(\xi_1)\\ \Phi(\xi_2)\\ \Phi(\mathbf{B})\end{pmatrix}=\begin{pmatrix}0 & \gamma_2 & -\varepsilon_1\\ -\gamma_2 & 0 & -\varepsilon_2\\ \varepsilon_1 & \varepsilon_2 & 0\end{pmatrix}\begin{pmatrix}\xi_1\\ \xi_2\\ \mathbf{B}\end{pmatrix}, \tag{3.3}$$

where γ_2 *is a certain function defined by* $\gamma_2 = g(\Phi\xi_1,\xi_2)$ *[23].*

Definition 3.7. *Fermi-Walker derivative of vector field* $\mathbf{X}$ *is defined [26]*

$$\tilde{\nabla}_{\mathbf{T}}\mathbf{X} = \nabla_{\mathbf{T}}\mathbf{X} - \langle \mathbf{T},\mathbf{X}\rangle \nabla_{\mathbf{T}}\mathbf{T} + \langle \nabla_{\mathbf{T}}\mathbf{T},\mathbf{X}\rangle \mathbf{T}. \tag{3.4}$$

Definition 3.8. X *is any vector field along the* $\alpha(s)$ *space curve. If the Fermi--Walker derivative of the vector field* X

$$\tilde{\nabla}_{\mathbf{T}}\mathbf{X} = 0, \tag{3.5}$$

the vector field X *along the curve, parallel to the Fermi--Walker terms, is called [26].*

Theorem 3.9. *Let* α *be a* ξ_1*-magnetic curve with type-2 Bishop frame. Then, Fermi Walker derivatives of Lorentz forces* $\Phi(\xi_1)$, $\Phi(\xi_2)$, $\Phi(\mathbf{B})$ *are given by*

$$\begin{aligned}
\tilde{\nabla}_{\mathbf{T}}\Phi(\xi_1) &= -\varepsilon_1\varepsilon_2\xi_2 - \varepsilon_1'\mathbf{B},\\
\tilde{\nabla}_{\mathbf{T}}\Phi(\xi_2) &= \rho_2\varepsilon_2\xi_2 + \rho_2'\mathbf{B},\\
\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{B}) &= \varepsilon_1'\xi_1 - \rho_2'\xi_2 + \rho_2\varepsilon_2\mathbf{B},
\end{aligned}$$

where $\rho_2 = g(\Phi\xi_2,\mathbf{B})$.

Corollary 3.10. *Lorentz forces* $\Phi(\xi_1)$, $\Phi(\xi_2)$, $\Phi(\mathbf{B})$ *are parellel according to Fermi Walker if*

$$\begin{aligned}
\varepsilon_1 &= constant,\\
\varepsilon_2 &= 0,\\
\rho_2 &= constant.
\end{aligned}$$

Proof. Assume that α is a ξ_1-magnetic curve with type-2 Bishop frame. By using Fermi Walker derivatives, we have

$$\begin{aligned}
\tilde{\nabla}_{\mathbf{T}}\Phi(\xi_1) &= -\varepsilon_1\varepsilon_2\xi_2 - \varepsilon_1'\mathbf{B},\\
\tilde{\nabla}_{\mathbf{T}}\Phi(\xi_2) &= \rho_2\varepsilon_2\xi_2 + \rho_2'\mathbf{B},\\
\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{B}) &= \varepsilon_1'\xi_1 - \rho_2'\xi_2 + \rho_2\varepsilon_2\mathbf{B}.
\end{aligned}$$

Also, *Lorentz forces* $\Phi(\xi_1), \Phi(\xi_2)$, $\Phi(\mathbf{B})$ are parallel to the Fermi--Walker terms, then

$$\varepsilon_1\varepsilon_2 = 0,\ \varepsilon_1' = 0,\ \rho_2\varepsilon_2 = 0 \text{ and } \rho_2' = 0.$$

Therefore, $\varepsilon_1 = constant$, $\varepsilon_2 = 0$ and $\rho_2 = constant$ is obtained.

Theorem 3.11. *Let* α *be a* ξ_2*-magnetic curve with type-2 Bishop frame. Then, Fermi Walker derivatives of Lorentz forces* $\Phi(\xi_1)$, $\Phi(\xi_2)$, $\Phi(\mathbf{B})$ *are given by*

$$\begin{aligned}
\tilde{\nabla}_{\mathbf{T}}\Phi(\xi_1) &= \eta_2\varepsilon_1\xi_1 + \eta_2'\mathbf{B},\\
\tilde{\nabla}_{\mathbf{T}}\Phi(\xi_2) &= -\varepsilon_1\varepsilon_2\xi_1 - \varepsilon_2'\mathbf{B},\\
\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{B}) &= -\eta_2'\xi_1 + \varepsilon_2'\xi_2 + \eta_2\varepsilon_1\mathbf{B},
\end{aligned}$$

where $\eta_2 = g(\Phi\xi_1,\mathbf{B})$.

Corollary 3.12. *Lorentz forces* $\Phi(\xi_1)$, $\Phi(\xi_2)$, $\Phi(\mathbf{B})$ *are parellel according to Fermi-Walker if*

$$
\begin{aligned}
\varepsilon_1 &= 0, \\
\varepsilon_2 &= constant, \\
\eta_2 &= constant.
\end{aligned}
$$

Proof. It is clear with Theorem 3.11.

Theorem 3.13. *Let α be a $\mathbf{B}$-magnetic curve with type-2 Bishop frame. Then, Fermi Walker derivatives of Lorentz forces $\Phi(\xi_1)$, $\Phi(\xi_2)$, $\Phi(\mathbf{B})$ are given by*

$$
\begin{aligned}
\tilde{\nabla}_{\mathbf{T}}\Phi(\xi_1) &= \gamma_2'\xi_2 - \varepsilon_1'\mathbf{B}, \\
\tilde{\nabla}_{\mathbf{T}}\Phi(\xi_2) &= -\gamma_2'\xi_1 - \varepsilon_2'\mathbf{B}, \\
\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{B}) &= \varepsilon_1'\xi_1 + \varepsilon_2'\xi_2,
\end{aligned}
$$

where $\gamma_2 = g(\Phi\xi_1, \xi_2)$.

Theorem 3.14. *Lorentz forces $\Phi(\xi_1)$, $\Phi(\xi_2)$, $\Phi(\mathbf{B})$ are parellel according to Fermi Walker if*

$$
\begin{aligned}
\varepsilon_1 &= constant, \\
\varepsilon_2 &= constant, \\
\gamma_2 &= constant.
\end{aligned}
$$

Proof. It is clear with Theorem 3.13.

4. ENERGY OF FERMI-WALKER DERIVATIVE OF MAGNETIC CURVES

In our this part, we define Fermi-Walker derivative and energy with *Sasaki metric* [27].

Definition 4.1. *For two Riemannian manifolds (M,ρ) and (π,H) energy of a differentiable map $f:(M,\rho)\to(\pi,\mathrm{H})$ is defined by*

$$
\varepsilon(f) = \frac{1}{2}\int_M \sum_{a=1}^{\pi} \mathrm{H}(df(\mathbf{b}_a), df(\mathbf{b}_a))v,
$$

where v is the canonical volume form on M [28]. *Sasaki metric defined as*

$$
\rho_S(\varsigma_1,\varsigma_2) = \rho(d\omega(\varsigma_1), d\omega(\varsigma_2)) + \rho(Q(\varsigma_1), Q(\varsigma_2)).
$$

Now, we study relationship between Fermi-Walker derivative and Frenet fields of curves. Fermi transport and derivative have the following theories.

Fermi-Walker transport is defined by

$$\nabla_{\mathbf{T}}^{FW}\mathbf{X}=\nabla_{\mathbf{T}}\mathbf{X}+\mathbf{T}<\mathbf{X},\nabla_{\mathbf{T}}\mathbf{T}>-\nabla_{\mathbf{T}}\mathbf{T}<\mathbf{X},\mathbf{T}>=0.$$

$\nabla_{\mathbf{T}}^{FW}\mathbf{X}$ is called Fermi Walker derivative for $\mathbf{X}$ by $\mathbf{T}$ along the particle [29].

Theorem 4.2. *Let α be a unit speed ξ_1-magnetic curve according to type-2 Bishop frame in Euclidean 3-space. Then, energies of $\Phi(\xi_1)$, $\Phi(\xi_2)$, $\Phi(\mathbf{B})$ with Sasakian metric are given by*

$$\varepsilon(\Phi(\xi_1)) = \frac{1}{2}\int_\alpha (1+(\varepsilon_1)^4+(\varepsilon_1\varepsilon_2)^2+(\varepsilon_1^{'})^2)ds,$$

$$\varepsilon(\Phi(\xi_2)) = \frac{1}{2}\int_\alpha (1+(\rho_2\varepsilon_1)^2+(\rho_2\varepsilon_2)^2+(\rho_2^{'})^2)ds,$$

$$\varepsilon(\Phi(\mathrm{B})) = \frac{1}{2}\int_\alpha (1+(\varepsilon_1^{'})^2+(\rho_2^{'})^2+(\rho_2\varepsilon_2-\varepsilon_1^2)^2)ds.$$

Proof. Let α be a unit speed ξ_1-magnetic curve according to type-2 Bishop frame in Euclidean 3-space. Then, by using $\nabla_{\mathbf{T}}\Phi(\xi_1)=-\varepsilon_1^2\xi_1-\varepsilon_1\varepsilon_2\xi_2-\varepsilon_1^{'}\mathbf{B}$, we have

$$\begin{aligned}\varepsilon(\Phi(\xi_1)) &= \frac{1}{2}\int_\alpha (1+\langle\nabla_{\mathbf{T}}\Phi(\xi_1),\nabla_{\mathbf{T}}\Phi(\xi_1)\rangle)ds\\ &= \frac{1}{2}\int_\alpha (1+\langle-\varepsilon_1^2\xi_1-\varepsilon_1\varepsilon_2\xi_2-\varepsilon_1^{'}\mathbf{B},-\varepsilon_1^2\xi_1\\ &\quad -\varepsilon_1\varepsilon_2\xi_2-\varepsilon_1^{'}\mathbf{B}\rangle)ds\\ &= \frac{1}{2}\int_\alpha (1+(\varepsilon_1)^4+(\varepsilon_1\varepsilon_2)^2+(\varepsilon_1^{'})^2)ds.\end{aligned}$$

Using $\nabla_{\mathbf{T}}\Phi(\xi_2)=\rho_2\varepsilon_1\xi_1+\rho_2\varepsilon_2\xi_2+\rho_2^{'}\mathbf{B}$ equation and from energy formula we have

$$\begin{aligned}\varepsilon(\Phi(\xi_2)) &= \frac{1}{2}\int_\alpha (1+\langle\nabla_{\mathbf{T}}\Phi(\xi_2),\nabla_{\mathbf{T}}\Phi(\xi_2)\rangle)ds\\ &= \frac{1}{2}\int_\alpha (1+(\rho_2\varepsilon_1)^2+(\rho_2\varepsilon_2)^2+(\rho_2^{'})^2)ds.\end{aligned}$$

Similarly by using $\nabla_{\mathbf{T}}\Phi(\mathbf{B})=\varepsilon_1^{'}\xi_1-\rho_2^{'}\xi_2+(\rho_2\varepsilon_2-\varepsilon_1^2)\mathbf{B}$ and from energy formula we get

$$\begin{aligned}\varepsilon(\Phi(\mathbf{B})) &= \frac{1}{2}\int_\alpha (1+\langle\nabla_{\mathbf{T}}\Phi(\mathbf{B}),\nabla_{\mathbf{T}}\Phi(\mathbf{B})\rangle)ds\\ &= \frac{1}{2}\int_\alpha (1+(\varepsilon_1^{'})^2+(\rho_2^{'})^2+(\rho_2\varepsilon_2-\varepsilon_1^2)^2)ds.\end{aligned}$$

Theorem 4.3. *Let α be a unit speed ξ_2-magnetic curve according to type-2 Bishop frame in Euclidean 3-space. Then, energy of $\Phi(\xi_1)$, $\Phi(\xi_2)$, $\Phi(\mathbf{B})$ with Sasakian metric are given by*

$$\varepsilon(\Phi(\xi_1)) = \frac{1}{2}\int_\alpha (1+(\eta_2\varepsilon_1)^2+(\eta_2\varepsilon_2)^2+(\eta_2')^2)ds,$$

$$\varepsilon(\Phi(\xi_2)) = \frac{1}{2}\int_\alpha (1+(\varepsilon_1\varepsilon_2)^2+(\varepsilon_2)^4+(\varepsilon_2')^2)ds,$$

$$\varepsilon(\Phi(\mathrm{B})) = \frac{1}{2}\int_\alpha (1+(\eta_2')^2+(\varepsilon_2')^2+(\eta_2\varepsilon_1-\varepsilon_2^2)^2)ds.$$

Proof. Let α be a unit speed ξ_2-magnetic curve according to type-2 Bishop frame in Euclidean 3-space. Then, by using $\nabla_{\mathbf{T}}\Phi(\xi_1)=\eta_2\varepsilon_1\xi_1+\eta_2\varepsilon_2\xi_2+\eta_2'\mathbf{B}$ equation and from energy formula

$$\begin{aligned}\varepsilon(\Phi(\xi_1)) &= \frac{1}{2}\int_\alpha (1+\langle\nabla_{\mathbf{T}}\Phi(\xi_1),\nabla_{\mathbf{T}}\Phi(\xi_1)\rangle)ds\\ &= \frac{1}{2}\int_\alpha (1+(\eta_2\varepsilon_1)^2+(\eta_2\varepsilon_2)^2+(\eta_2')^2)ds\end{aligned}$$

is obtained.

Using $\nabla_{\mathbf{T}}\Phi(\xi_2)=-\varepsilon_1\varepsilon_2\xi_1-\varepsilon_2^2\xi_2-\varepsilon_2'\mathbf{B}$ equation and from energy formula we have

$$\begin{aligned}\varepsilon(\Phi(\xi_2)) &= \frac{1}{2}\int_\alpha (1+\langle\nabla_{\mathbf{T}}\Phi(\xi_2),\nabla_{\mathbf{T}}\Phi(\xi_2)\rangle)ds\\ &= \frac{1}{2}\int_\alpha (1+(\varepsilon_1\varepsilon_2)^2+(\varepsilon_2)^4+(\varepsilon_2')^2)ds.\end{aligned}$$

Similarly by using $\nabla_{\mathbf{T}}\Phi(\mathbf{B})=-\eta_2'\xi_1+\varepsilon_2'\xi_2+(\eta_2\varepsilon_1-\varepsilon_2^2)\mathbf{B}$ and from energy formula we get

$$\begin{aligned}\varepsilon(\Phi(\mathbf{B})) &= \frac{1}{2}\int_\alpha (1+\langle\nabla_{\mathbf{T}}\Phi(\mathbf{B}),\nabla_{\mathbf{T}}\Phi(\mathbf{B})\rangle)ds\\ &= \frac{1}{2}\int_\alpha (1+(\eta_2')^2+(\varepsilon_2')^2+(\eta_2\varepsilon_1-\varepsilon_2^2)^2)ds.\end{aligned}$$

Theorem 4.4. *Let α be a unit speed $\mathbf{B}$-magnetic curve according to type-2 Bishop frame in Euclidean 3-space. Then, energy of $\Phi(\xi_1)$, $\Phi(\xi_2)$, $\Phi(\mathbf{B})$ with Sasakian metric are given by*

$$\varepsilon(\Phi(\xi_1)) = \frac{1}{2}\int_\alpha (1+(\varepsilon_1)^4+(\gamma_2'-\varepsilon_1\varepsilon_2)^2+(\gamma_2\varepsilon_2+\varepsilon_1')^2)ds,$$

$$\varepsilon(\Phi(\xi_2)) = \frac{1}{2}\int_\alpha (1+(\gamma_2'+\varepsilon_1\varepsilon_2)^2+(\varepsilon_2)^4+(\gamma_2\varepsilon_1-\varepsilon_2')^2)ds,$$

$$\varepsilon(\Phi(\mathrm{B})) = \frac{1}{2}\int_\alpha (1+(\varepsilon_1')^2+(\varepsilon_2')^2+(\varepsilon_1^2+\varepsilon_2^2)^2)ds.$$

Proof. Let α be a unit speed $\mathbf{B}$-magnetic curve according to type-2 Bishop frame in Euclidean 3-space. Then, by using $\nabla_\mathbf{T}\Phi(\boldsymbol{\xi}_1) = -\varepsilon_1^2\boldsymbol{\xi}_1 + (\gamma_2' - \varepsilon_1\varepsilon_2)\boldsymbol{\xi}_2 - (\gamma_2\varepsilon_2 + \varepsilon_1')\mathbf{B}$ and from energy formula

$$\begin{aligned}\varepsilon(\Phi(\boldsymbol{\xi}_1)) &= \frac{1}{2}\int_\alpha (1+\langle\nabla_\mathbf{T}\Phi(\boldsymbol{\xi}_1),\nabla_\mathbf{T}\Phi(\boldsymbol{\xi}_1)\rangle)ds\\ &= \frac{1}{2}\int_\alpha (1+(\varepsilon_1)^4+(\gamma_2'-\varepsilon_1\varepsilon_2)^2+(\gamma_2\varepsilon_2+\varepsilon_1')^2)ds\end{aligned}$$

is obtained.

Similarly by using $\nabla_\mathbf{T}\Phi(\boldsymbol{\xi}_2) = -(\gamma_2' + \varepsilon_1\varepsilon_2)\boldsymbol{\xi}_1 - \varepsilon_2^2\boldsymbol{\xi}_2 + (\gamma_2\varepsilon_1 - \varepsilon_2')\mathbf{B}$ and from energy formula we have

$$\begin{aligned}\varepsilon(\Phi(\boldsymbol{\xi}_2)) &= \frac{1}{2}\int_\alpha (1+\langle\nabla_\mathbf{T}\Phi(\boldsymbol{\xi}_2),\nabla_\mathbf{T}\Phi(\boldsymbol{\xi}_2)\rangle)ds\\ &= \frac{1}{2}\int_\alpha (1+(\gamma_2'+\varepsilon_1\varepsilon_2)^2+(\varepsilon_2)^4+(\gamma_2\varepsilon_1-\varepsilon_2')^2)ds.\end{aligned}$$

On the other hand, by using $\nabla_\mathbf{T}\Phi(\mathbf{B}) = \varepsilon_1'\boldsymbol{\xi}_1 + \varepsilon_2'\boldsymbol{\xi}_2 - (\varepsilon_1^2+\varepsilon_2^2)\mathbf{B}$, we obtain

$$\begin{aligned}\varepsilon(\Phi(\mathbf{B})) &= \frac{1}{2}\int_\alpha (1+\langle\nabla_\mathbf{T}\Phi(\mathbf{B}),\nabla_\mathbf{T}\Phi(\mathbf{B})\rangle)ds\\ &= \frac{1}{2}\int_\alpha (1+(\varepsilon_1')^2+(\varepsilon_2')^2+(\varepsilon_1^2+\varepsilon_2^2)^2)ds.\end{aligned}$$

Theorem 4.5. *Energy of* $\tilde{\nabla}_\mathbf{T}\Phi(\boldsymbol{\xi}_1)$, $\tilde{\nabla}_\mathbf{T}\Phi(\boldsymbol{\xi}_2)$, $\tilde{\nabla}_\mathbf{T}\Phi(\mathbf{B})$ *with Sasakian metric are presented*

$$\varepsilon(\tilde{\nabla}_{\xi_1}\Phi(\xi_1)) = \frac{1}{2}\int_\alpha (1+(\varepsilon_1'\varepsilon_1)^2+((\varepsilon_1\varepsilon_2)'+\varepsilon_1'\varepsilon_2)^2+(\varepsilon_1\varepsilon_2^2-\varepsilon_1'')^2)ds,$$

$$\varepsilon(\tilde{\nabla}_{\xi_1}\Phi(\xi_2)) = \frac{1}{2}\int_\alpha (1+(\rho_2'\varepsilon_1)^2+((\rho_2\varepsilon_2)'+\rho_2'\varepsilon_2)^2+(\rho_2''-\rho_2\varepsilon_2^2)^2)ds,$$

$$\varepsilon(\tilde{\nabla}_{\xi_1}\Phi(\mathrm{B})) = \frac{1}{2}\int_\alpha (1+((\varepsilon_1''+\rho_2\varepsilon_2\varepsilon_1)^2+(\rho_2\varepsilon_2^2-\rho_2'')^2+(\rho_2'\varepsilon_2-\varepsilon_1'\varepsilon_1+(\rho_2\varepsilon_2)')^2)ds.$$

Proof. Let α be a unit speed $\boldsymbol{\xi}_1$-magnetic curve according to type-2 Bishop frame in Euclidean 3-space. Then

$$\begin{aligned}\varepsilon(\tilde{\nabla}_\mathbf{T}\Phi(\boldsymbol{\xi}_1)) &= \frac{1}{2}\int_\alpha (1+\langle\nabla_\mathbf{T}(\tilde{\nabla}_\mathbf{T}\Phi(\boldsymbol{\xi}_1)),\nabla_\mathbf{T}(\tilde{\nabla}_\mathbf{T}\Phi(\boldsymbol{\xi}_1))\rangle)ds\\ &= \frac{1}{2}\int_\alpha (1+((\varepsilon_1'\varepsilon_1)^2+((\varepsilon_1\varepsilon_2)'+\varepsilon_1'\varepsilon_2)^2+(\varepsilon_1\varepsilon_2^2-\varepsilon_1'')^2)ds\end{aligned}$$

is obtained.

Similarly by using $\tilde{\nabla}_{\mathbf{T}}\Phi(\xi_2) = \rho_2\varepsilon_2\xi_2 + \rho_2^{'}\mathbf{B}$ equation and from energy formula we have

$$\begin{aligned}\varepsilon(\tilde{\nabla}_{\mathbf{T}}\Phi(\xi_2)) &= \frac{1}{2}\int_{\alpha}(1+\langle\nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\xi_2)),\nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\xi_2))\rangle)ds \\ &= \frac{1}{2}\int_{\alpha}(1+((\rho_2^{'}\varepsilon_1)^2+((\rho_2\varepsilon_2)^{'}+\rho_2^{'}\varepsilon_2)^2+(\rho_2^{''}-\rho_2\varepsilon_2^2)^2)ds.\end{aligned}$$

Then by using $\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{B}) = \varepsilon_1^{'}\xi_1 - \rho_2^{'}\xi_2 + \rho_2\varepsilon_2\mathbf{B}$ equation and from energy formula, we have

$$\begin{aligned}\varepsilon(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{B})) &= \frac{1}{2}\int_{\alpha}(1+\langle\nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{B})),\nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{B}))\rangle)ds \\ &= \frac{1}{2}\int_{\alpha}(1+((\varepsilon_1^{''}+\rho_2\varepsilon_2\varepsilon_1)^2+(\rho_2\varepsilon_2^2-\rho_2^{''})^2+(\rho_2^{'}\varepsilon_2-\varepsilon_1^{'}\varepsilon_1+(\rho_2\varepsilon_2)^{'})^2)ds.\end{aligned}$$

Theorem 4.6. *Energy of* $\tilde{\nabla}_{\mathbf{T}}\Phi(\xi_1)$, $\tilde{\nabla}_{\mathbf{T}}\Phi(\xi_2)$, $\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{B})$ *with Sasakian metric are presented*

$$\varepsilon(\tilde{\nabla}_{\mathrm{T}}\Phi(\xi_1)) = \frac{1}{2}\int_{\alpha}(1+((\eta_2\varepsilon_1)^{'}+\eta_2^{'}\varepsilon_1)^2+(\eta_2^{'}\varepsilon_2)^2+(\eta_2^{''}-\eta_2\varepsilon_1^2)^2)ds,$$

$$\varepsilon(\tilde{\nabla}_{\mathrm{T}}\Phi(\xi_2)) = \frac{1}{2}\int_{\alpha}(1+((\varepsilon_1\varepsilon_2)^{'}+\varepsilon_1\varepsilon_2^{'})^2+(\varepsilon_2^{'}\varepsilon_2)^2+(\varepsilon_1^2\varepsilon_2-\varepsilon_2^{''})^2)ds,$$

$$\varepsilon(\tilde{\nabla}_{\mathrm{T}}\Phi(\mathrm{B})) = \frac{1}{2}\int_{\alpha}(1+(\eta_2\varepsilon_1^2-\eta_2^{''})^2+(\varepsilon_2^{''}+\eta_2\varepsilon_2)^2+(\eta_2^{'}\varepsilon_1-\varepsilon_2^{'}\varepsilon_2+(\eta_2\varepsilon_1)^{'})^2)ds.$$

Proof: Let α be a unit speed ξ_2-magnetic curve according to type-2 Bishop frame in Euclidean 3-space. Then, we have

$$\begin{aligned}\varepsilon(\tilde{\nabla}_{\mathbf{T}}\Phi(\xi_1)) &= \frac{1}{2}\int_{\alpha}(1+\langle\nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\xi_1)),\nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\xi_1))\rangle)ds \\ &= \frac{1}{2}\int_{\alpha}(1+((\eta_2\varepsilon_1)^{'}+\eta_2^{'}\varepsilon_1)^2+(\eta_2^{'}\varepsilon_2)^2+(\eta_2^{''}-\eta_2\varepsilon_1^2)^2)ds.\end{aligned}$$

Similarly by using $\tilde{\nabla}_{\mathbf{T}}\Phi(\xi_2) = -\varepsilon_1\varepsilon_2\xi_1 - \varepsilon_2^{'}\mathbf{B}$ equation and from energy formula we have

$$\begin{aligned}\varepsilon(\tilde{\nabla}_{\mathbf{T}}\Phi(\xi_2)) &= \frac{1}{2}\int_{\alpha}(1+\langle\nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\xi_2)),\nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\xi_2))\rangle)ds \\ &= \frac{1}{2}\int_{\alpha}(1+((\varepsilon_1\varepsilon_2)^{'}+\varepsilon_1\varepsilon_2^{'})^2+(\varepsilon_2^{'}\varepsilon_2)^2+(\varepsilon_1^2\varepsilon_2-\varepsilon_2^{''})^2)ds.\end{aligned}$$

Then by using $\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{B}) = -\eta_2^{'}\xi_1 + \varepsilon_2^{'}\xi_2 + \eta_2\varepsilon_1\mathbf{B}$ equation and from energy formula we have

$$\varepsilon(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{B})) = \frac{1}{2}\int_{\alpha}(1+\langle\nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{B})),\nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{B}))\rangle)ds$$
$$= \frac{1}{2}\int_{\alpha}(1+(\eta_2\varepsilon_1^2-\eta_2^{''})^2+(\varepsilon_2^{''}+\eta_2\varepsilon_2)^2+(\eta_2^{'}\varepsilon_1-\varepsilon_2^{'}\varepsilon_2+(\eta_2\varepsilon_1)^{'})^2)ds.$$

Theorem 4.7. *Energy of* $\tilde{\nabla}_{\mathbf{T}}\Phi(\xi_1)$, $\tilde{\nabla}_{\mathbf{T}}\Phi(\xi_2)$, $\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{B})$ *with Sasakian metric are presented*

$$\varepsilon(\tilde{\nabla}_{\mathrm{T}}\Phi(\xi_1)) = \frac{1}{2}\int_{\alpha}(1+(\varepsilon_1^{'}\varepsilon_1)^2+(\gamma_2^{''}-\varepsilon_1^{'}\varepsilon_2)^2+(\varepsilon_1^{''}+\gamma_2^{'}\varepsilon_2)^2)ds,$$

$$\varepsilon(\tilde{\nabla}_{\mathrm{T}}\Phi(\xi_2)) = \frac{1}{2}\int_{\alpha}(1+(\gamma_2^{''}+\varepsilon_2^{'}\varepsilon_1)^2+(\varepsilon_2^{'}\varepsilon_2)^2+(\gamma_2^{'}\varepsilon_1-\varepsilon_2^{''})^2)ds,$$

$$\varepsilon(\tilde{\nabla}_{\mathrm{T}}\Phi(\mathrm{B})) = \frac{1}{2}\int_{\alpha}(1+(\varepsilon_1^{''})^2+(\varepsilon_2^{''})^2+(\varepsilon_1^{'}\varepsilon_1+\varepsilon_2^{'}\varepsilon_2)^2)ds.$$

Proof. Let α be a unit speed $\mathbf{B}$-magnetic curve according to type-2 Bishop frame in Euclidean 3-space. Then, we get

$$\varepsilon(\tilde{\nabla}_{\mathbf{T}}\Phi(\xi_1)) = \frac{1}{2}\int_{\alpha}(1+\langle\nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\xi_1)),\nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\xi_1))\rangle)ds$$
$$= \frac{1}{2}\int_{\alpha}(1+(\varepsilon_1^{'}\varepsilon_1)^2+(\gamma_2^{''}-\varepsilon_1^{'}\varepsilon_2)^2+(\varepsilon_1^{''}+\gamma_2^{'}\varepsilon_2)^2)ds.$$

Similarly by using $\tilde{\nabla}_{\mathbf{T}}\Phi(\xi_2)=-\gamma_2^{'}\xi_1-\varepsilon_2^{'}\mathbf{B}$ equation and from energy formula we have

$$\varepsilon(\tilde{\nabla}_{\mathbf{T}}\Phi(\xi_2)) = \frac{1}{2}\int_{\alpha}(1+\langle\nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\xi_2)),\nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\xi_2))\rangle)ds$$
$$= \frac{1}{2}\int_{\alpha}(1+(\gamma_2^{''}+\varepsilon_2^{'}\varepsilon_1)^2+(\varepsilon_2^{'}\varepsilon_2)^2+(\gamma_2^{'}\varepsilon_1-\varepsilon_2^{''})^2)ds.$$

Then by using $\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{B})=\varepsilon_1^{'}\xi_1+\varepsilon_2^{'}\xi_2$ equation and from energy formula we have

$$\varepsilon(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{B})) = \frac{1}{2}\int_{\alpha}(1+\langle\nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{B})),\nabla_{\mathbf{T}}(\tilde{\nabla}_{\mathbf{T}}\Phi(\mathbf{B}))\rangle)ds$$
$$= \frac{1}{2}\int_{\alpha}(1+(\varepsilon_1^{''})^2+(\varepsilon_2^{''})^2+(\varepsilon_1^{'}\varepsilon_1+\varepsilon_2^{'}\varepsilon_2)^2)ds.$$

5. CONCLUSIONS

Fermi-Walker transport and inextensible flows play an important role in geometric design and theorical physics. A recent problem in the field of classical differential geometry consists in the study of Fermi-Walker transport and energy relations.

In this paper, we have studied Fermi-Walker derivative and Fermi-Walker parallelism in space. In this paper, we obtain the Fermi-Walker derivatives of some magnetic curves according to the Bishop frame in the space. Moreover, we gey the energy of the Fermi-Walker derivative of magnetic curves according to the type-2 Bishop frame in space. Finally, we have energy relations of some vector fields associated with type-2 Bishop frame in the space.

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