ORIGINAL PAPER

BOUNDARY LAYER IN FLOW OF CASSON FLUID DUE TO STRETCHING SHEET WITH EXPONENTIALLY MOVING FREE STREAM

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Abstract. *The study on boundary layer in flow of non-Newtonian Casson fluid over an exponentially stretching surface in presence of suction effect is carried out. The leading partial differential equations are converted in to ordinary differential equations using suitable similarity transformations. The resulting equations are solved with boundary conditions by fourth order Runge –Kutta method using Matlab software. The values of the velocity ratio parameter are assorted to study the effect of this on the thickness of velocity and thermal boundary respectively in presence of suction with variation in different parameters like Prandtle number, Magnetic parameter ,Casson parameter. Temperature gradient Profiles are plotted by varying the values of Parameter (λ), Prandtle number (Pr) and discussed.*

Keywords: *Casson, MHD, Exponential.*

1. INTRODUCTION

The velocity and thermal boundary layer formation due to the flow of fluid over a stretching surface has numerous applications in industrial process like making of glass fiber, metal spinning, polymer processing, paper production etc. In present days researchers are more inquisitive towards dynamics of non-Newtonian fluids due to its wide applications in industries. But due to its undulate behavior there is no single constitute equation to narrate all the existing physical properties of these non-Newtonian fluids. There are many non-Newtonian fluid models to explain all such behaviors. Casson fluid model is one such non-Newtonian fluid model. Examples of Casson fluids are starch, toothpaste, ketchup, human blood, honey etc. Casson fluid behaves like an elastic solid, and its yield stress is widely used for modeling blood flow in narrow arteries. Casson fluid model for mathematical modeling of blood flow in narrow arteries at low shear rates was studied by Blair [1] and Copley [2]. Many researchers have conducted investigation on the boundary layer flow and heat transfer of Casson fluid [3-9] in presence of different parameters. The exact solution of Casson fluid flow over a permeable stretching and shrinking sheet was discussed by Bhattacharyya et.al [10] .Later the formation of boundary layer with surface heat flux due to the flow of Casson fluid past a symmetric porous wedge was studied by Swathi Mukhopadhyay et .al. [11].

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The study on steady flow in the neighborhood of a stagnation point was first conducted by Hiemenz [12]. Later Mahapathra et.al [13] investigated the stagnation point flow of stretching sheet in presence of different stretching/straining velocity rates. There are many research papers have been published on the boundary layer stagnation point [14-16] due linear variation in the velocity of the stretching sheet. But the boundary layer stagnation point flow of Casson fluid due to exponentially stretching sheet in presence of slip effect was not studied much so motivated by the investigations carried out by the above mentioned researchers on the steady/unsteady flow of Casson fluid due to a stretching sheet and its extensive implementations in many industries, in the present paper, an investigation on the velocity and thermal boundary layer in flow of Casson fluid over an exponentially stretching sheet in presence of suction with moving free stream is carried out. The numerical results are plotted and the effects of different parameters on the boundary layer thickness have been discussed.

2. PROBLEM FORMULATION

Consider the boundary layer flow and heat transfer of a steady, viscous, incompressible due to an exponentially stretching sheet with velocity U and U_∞ as the velocity of exponentially moving free stream .The following are the governing equations of the fluid flow and heat transfer,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}+U_\infty\frac{dU_\infty}{dx}-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho C_p}\left(k\frac{\partial^2 T}{\partial y^2}\right) \tag{3}$$

where U_∞ is the free stream velocity, u and v are velocities in x and y directions ,υ is the kinematic viscosity, ρ is the density, k is the fluid thermal conductivity ,T is the temperature, B_0 is the magnetic field.

In order to reduce the above Partial Differential equations to Ordinary Differential equation, we introduce the following similarity conditions

$$\eta=\sqrt{\frac{c}{2l\upsilon}}\; e^{\left(x/2l\right)}y \tag{4}$$

$$T=T_\infty+\frac{T_0x^2(1-\alpha t)^{-3/2}}{2\upsilon}\;\theta(\eta) \tag{5}$$

$$\psi=\sqrt{2\upsilon lc}\; e^{\left(x/2l\right)}\mathrm{f}(\eta) \tag{6}$$

2.1 BOUNDARY CONDITIONS

The boundary conditions corresponding to the above problem are:

At $y = 0$, u=U$_w$, v= V(x),

u$\rightarrow$U$_\infty$ as y$\rightarrow$ 0

And $T = T_\infty + T_0 e^{\left(\lambda x/2l\right)}$ at y=0

T $\rightarrow T_\infty$ as y $\rightarrow$ ∞.

The partial differential equations (2) and (3) are transformed in to the following non-linear ordinary differential equations using the similarity conditions (4), (5) and (6).

$$\left(1+\tfrac{1}{\beta}\right) f'''(\eta) - 2\left[f'(\eta)\right]^2 + f(\eta)f''(\eta) + 2Mf' + 2\left(\frac{a}{c}\right)^2 = 0 \tag{8}$$

$$\theta''(\eta) + P_r\left(\mathrm{f}(\eta)\theta'(\eta)\text{-}\lambda \mathrm{f}'(\eta)\theta(\eta)\right) = 0 \tag{9}$$

The transformed boundary conditions are

$$\eta = 0,\ f' = 1,\ f = S,\ \theta = 1 \tag{10}$$

$$\eta \rightarrow \infty,\ f' \rightarrow a/c, \theta \rightarrow 0\,.$$

3. RESULTS AND DISCUSSIONS

The transformed ordinary differential equations (2) and (3) subjected to boundary conditions (10) are solved numerically using MATLAB software and the results are plotted as shown by the figs. 1-7 to study the behavior of velocity and temperature profiles by considering various values for the parameters M, λ, β, Pr, a/c and S.

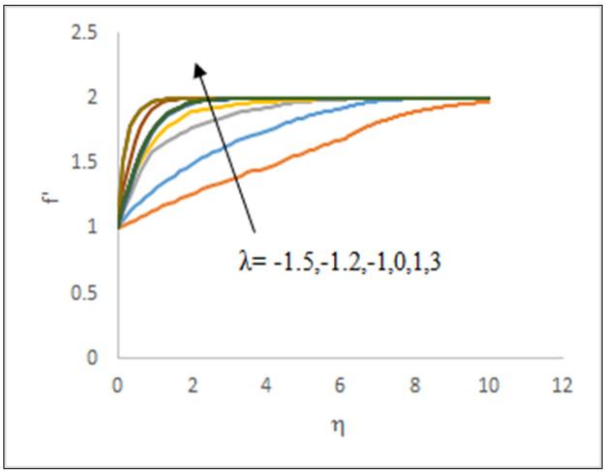

Figure 1. Velocity Profile for the parameter (λ) with M=0.5, a/c=2, β=2, Pr=0.5, S =0.4.

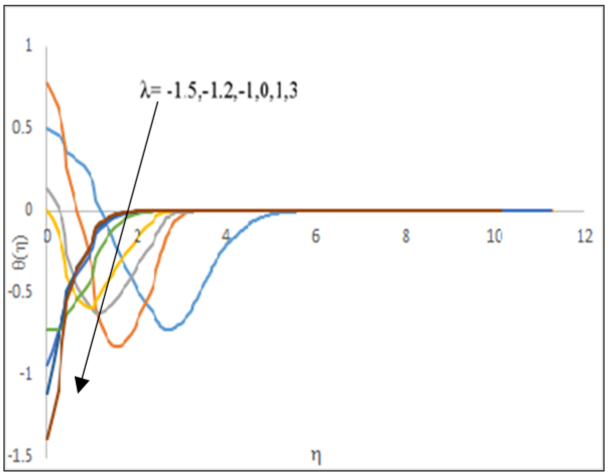

Figure 2. Temperature Profile for the parameter (λ) with M=0.5, a/c=2, β=2, Pr=0.5, S =0.4.

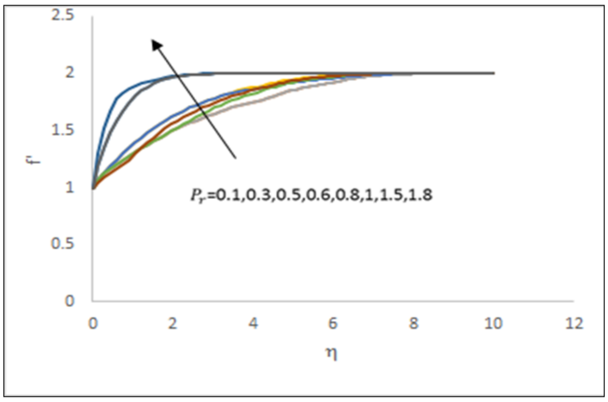

Figure 3. Velocity Profile for the Prandtle number (Pr) with M=0.5, a/c=2, β=2, λ=-1.3, S=0.4.

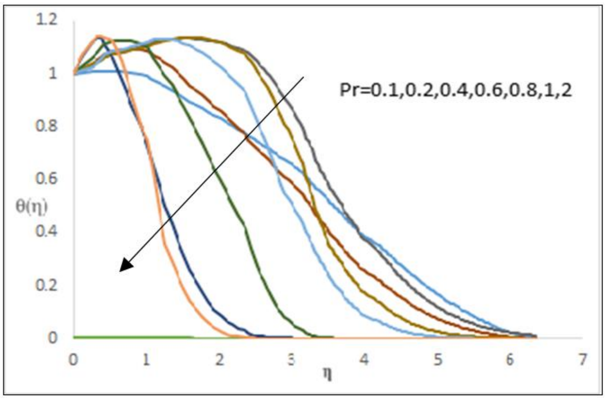

Figure 4. Temperature Profile for the Prandtle number (Pr) with M=0.5, a/c=2, β=2, λ=-1.3, S =0.4.

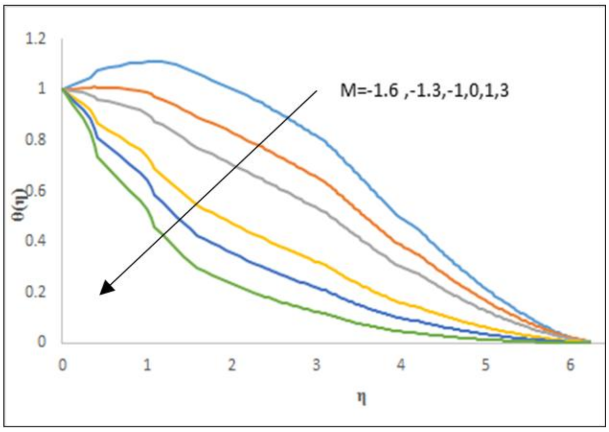

Figure 5.Temperature Profile for the Magnetic Parameter (M) with a/c=2, β=2, λ=-1.

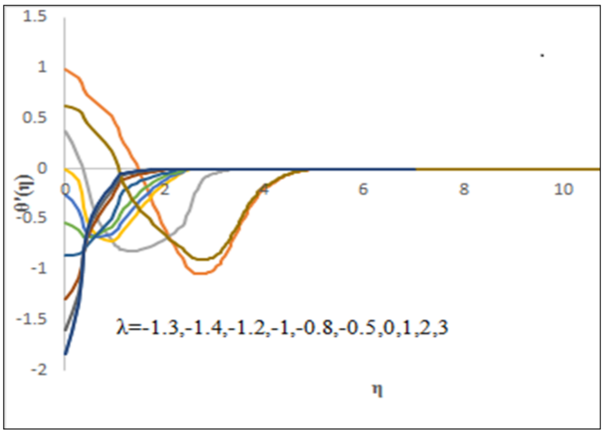

Figure 6. Temperature gradient Profile for the parameter (λ) with M=0.5, a/c=2, β=2, Pr=0.6, S=0.4.

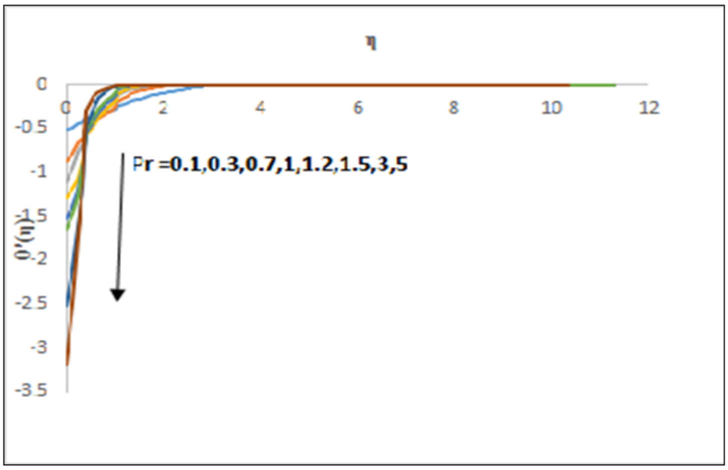

Figure 7. Temperature gradient Profile for the parameter (λ) with M=0.5, a/c=2, β=2,Pr=0.6 S=0.4.

The effect of the parameter λ on the velocity and temperature profiles respectively are presented in Figs. 1-2. It has been noticed that the velocity profile increases with increases in the values of λ. But there will be fall in the temperature profile with rise in λ, that is there will be diminution of thermal boundary layer thickness with increase in the value of λ. Figs. 3-4 shows the velocity and temperature profiles for different values of Prandtle number (P_r). With the increase in the Prandtl number, the velocity boundary layer thickness increases but the thermal boundary thickness decreases. The temperature gradient θ' profiles for different values λ and P_r with a/c=2 are displayed in Figs. 6-7. The temperature gradient at the stretching sheet $\theta'(\eta)$<0 with for rise in the value of Prandtle number. This implies heat absorption at the sheet turns more with increase in Prandtle number. Also the temperature gradient $\theta'(\eta) < 0$ and $\theta'(\eta) > 0$ respectively for different values of λ, this means the absorption of heat and transfer of heat from the stretching sheet takes place based on the value of λ.

4. CONCLUSIONS

The findings of the above work can be briefed as follows:

- For increases in the value of λ with fixed value of velocity ratio, the thermal boundary layer thickness decreases and velocity boundary layer becomes bulky in presence of suction.
- The velocity boundary layer thickness increases but the thermal boundary thickness decreases for different values Pr with fixed values of velocity ratio, suction.
- Absorption of heat at the stretching sheet and transfer of heat from the stretching sheet to the fluid vice versa increases with increase in prandtle number in presence of suction.
- Both heat absorption and heat transfer takes place for depending on the input values given to the parameter λ.

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