ORIGINAL PAPER

TRANSFORMATION EXPONENTIAL FUNCTION TO LINEAR FUZZY REGRESSION: A FUNCTION GROWTH OF *STREPTOCOCCUS SOBRINUS*

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Manuscript received: 15.10.2018; Accepted paper: 22.12.2018;
Published online: 30.03.2019.

Abstract. *The exponential growth is often used to model population growth as such cell growth while the exponential decay is often to a model population with declining or decreases in size. Fuzzy regression has demonstrated its ability to model bacteria growth process in which the processes have fuzziness and the number of experimental data sets for modeling them is limited. In this manuscript, we try to expand the presentation of exponential growth by adding some improvement method which adding bootstrap and the fuzzy technique. The aim of the study is to estimate the fuzzy and bootstrap parameters of the models The gathered data were compared by measuring the average width of the predicted interval using the least squares method and fuzzy method. The result shows that the average width of the predicted interval using method least square was 0.5116 while fuzzy method was 0.0234. The fuzzy linear regression model was applied mathematical programming had better result compared to method of Least Square (LS).*

Keywords: *Exponential Growth, Bootstrap, Fuzzy Regression.*

1. INTRODUCTION

Streptococcus sobrinus are known to be linked with dental caries in humans. Mutans Streptococci (*S. sobrinus*) is powerfully linked with the growth of dental caries in human. These bacteria are the most common pathogens isolated from human dental plaque, and their prevalence has been reported in epidemiological studies [1-3]. Several methods have been employed for detecting and identifying mutans streptococci, including culturing, direct enzyme tests, enzyme-linked immunosorbent assays, and DNA probes [4]. In previous cross-sectional and longitudinal studies, it was reported that preschool children with primary dentition harboring both *S. mutants* and *S. sobrinus* had a significantly higher incidence of dental caries than those with s. mutans alone. The mutans group of oral streptococci consist of seven species: *S. cricetus*, *S. rattus*, *S. mutans*, *S. sobrinus*, *S. downei*, *S. macacae* and *S. ferus* [5]. Among the group, *S. mutans* and *S. sobrinus* are most frequently isolated from human dental plaque and closely associated with human dental caries [6]. Prevalence of *S. sobrinus* and *S. mutans* in the human oral cavity have been reported by epidemiological studies in which the isolation frequency of *S. mutans* from dental plaque is much higher than that of *S. sobrinus* [1].

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1.1 EXPONENTIAL GROWTH MODEL

When a population grows exponentially, it grows at a rate that is proportional to its size at any time *t*. Suppose the variable *P(t)* (sometimes we use just *P*) represents the population at any time *t*. In addition, let P_0 be the initial population at time $t=0$, that is $P(0)=P_0$. Then if the population grows exponentially, in mathematical terms, this can be written as $\frac{dP}{dt} = kP$, solving for k gives $k = \frac{1dP}{pdt}$. The value *k* is known as the relative growth rate and is a constant. The equation $p(t) = Ae^{kt}$ represents the general solution of the differential equation [7]. When modelling a population with an exponential growth model, if the relative growth rate k is unknown, it should be determined. This is usually done using the known population at two particular times. However, exponential growth models are good predictors for small populations in large populations with abundant resources, usually for relatively short time periods [7].

2. METHODOLOGY

Various kinds of fuzzy regression models are introduced in the literature and many different methods are proposed to estimate the fuzzy parameters of the models. The fuzzy regression analysis is a powerful tool for investigating and predicting data sets by measuring a vague concept that contains a degree of ambiguity, uncertainty or fuzziness [8]. Using regression techniques on the experimental observations has allowed the study of many phenomena in various fields of science as; Agriculture, Chemistry, Medicine, Environment, Psychology, Biology and Economics. This has led to a breakthrough, not only achieved by mathematical developments but also for its application in real situations. These techniques required a set of observations, or they contain some kind of imperfection as a result of inaccuracy or vagueness of the data. In any case, models derived from real data (sufficient or not, imperfect or not) should provide predictive and descriptive capabilities [9]. In this paperwork, an exponential model was applied after transforming the data to a linear model with a mathematical programming by assuming that the dependent variables were crisp while the independent variables were a symmetric fuzzy number. This paper provides an algorithm for the exponential growth model using bacteria growth dataset. We transformed this nonlinear equation in order to get a better result and better significant inferences.

Fuzzy linear regression analysis was first introduced by Tanaka and Watada (1988) in which two factors namely the degree of fitness and the fuzziness of data sets, are considered. A fuzzy linear regression model is commonly presented as follows [9]:

$$\tilde{y} = \tilde{f}_{1r}(x) = \tilde{A}_0 + \tilde{A}_1 x_1 = \tilde{A}_X \qquad (1)$$

where $x = [1, x_1]^T$ is a crisp vector of independent variables and $\tilde{y}$ is the estimated fuzzy output. $\tilde{A} = [\tilde{A}_0, \tilde{A}_1]$ is a vector of fuzzy parameters of the fuzzy linear regression model. $\tilde{A}_j$ is presented in the form of symmetric triangular fuzzy numbers denoted by $\tilde{A}_J = (\alpha_j, c_j), j = 0,1,2,...N$, where its membership function is shown as below:

$$\mu_{A_J}\left(a_j\right)=\left\{1-\frac{a_j-a_j}{c_j}\right\}, a_j-c_j \leq a_j \leq a_j+c_j \tag{2}$$

otherwise, where a_j is the central value of the fuzzy number and c_j is the spread. Therefore the fuzzy linear regression model can be rewritten as shown below.

$$\tilde{y}=\left(\alpha_0, c_0\right)+\left(\alpha_1, c_1\right)x_1 \tag{3}$$

However, the interaction between variables and higher order terms are not included in the fuzzy linear regression defined in (1). In fact, the interaction between variables and higher order terms often exist in physical systems. A simple procedure is commonly used to solve the linear programming problem [10].

2.1. TRANSFORMING FOR EXPONENTIAL GROWTH TO A LINEAR

Exponential growth formula and exponential decay formula is given by $Y=Ae^{bx}$ and $Y=Ae^{-bx}$ and below is the procedure to transform the growth and decay formula into a linear form.

Exponential growth

$$\ln Y=\ln(Ae^{bX}) = \ln(A)+\ln(e^{bX}) \quad (4)$$
$$=\ln(\mathrm{A})+bx$$

2.2. PART 1: CALCULATION OF AN EXPONENTIAL CELL GROWTH USING SAS ALGORITHM

```
Data Cell_streptococcus;
input x y lny;
datalines;

1.00     82.00     4.41
2.00     84.00     4.43
3.00     86.00     4.45
5.00     86.00     4.45
9.00     87.00     4.47
11.00    87.00     4.47
13.00    87.00     4.47
15.00    90.00     4.50
20.00    92.00     4.52
23.00    94.00     4.54
25.00    93.00     4.53
29.00    95.00     4.55
33.00    99.00     4.60
```

```
                                                  ;
run;

/*ADDING BOOTSTRAPPING ALGORITHM TO THE METHOD */
%MACRO bootstrap(data=_last_, booted=booted, boots=2, seed=1234);
   DATA &booted;
   pickobs = INT(RANUNI(&seed)*n)+1;
   SET &data POINT = pickobs NOBS = n;
   REPLICATE=int(i/n)+1;
     i+1;
   IF i > n*&boots THEN STOP;
   RUN;
%MEND bootstrap;
ods rtf file='abc.rtf' style=journal;
%bootstrap(data= Cell_streptococcus,boots=2);
run;
proc print data=booted;
run;

Title "Exponential Equation";
ods graphic/imagename="Exponential Equation";
proc nlin data=booted plots=fit;
parameters A=1 b=0;
model y=A*exp(b*x);
ods output EstSummary=summExp;
run;
Proc reg data=booted;
model lny=x;
run;
ods rtf
close;
```

2.3. PART 2: CALCULATION OF FUZZY LEAST SQUARES (FLS) FOR EXPONENTIAL GROWTH

```
ods rtf file='abc.rtf' style=journal;
Proc nlp;
min Y;
decvar a0c a0w a1c a1w;
bounds a0w>=0, a1w>=0;
lincon a0c+5*a1c-a0w-5*a1w<=4.45;
lincon a0c+2*a1c-a0w-2*a1w<=4.43;
lincon a0c+9*a1c-a0w-9*a1w<=4.47;
lincon a0c+2*a1c-a0w-2*a1w<=4.43;
lincon a0c+5*a1c-a0w-5*a1w<=4.45;
lincon a0c+2*a1c-a0w-2*a1w<=4.43;
lincon a0c+1*a1c-a0w-1*a1w<=4.41;
lincon a0c+2*a1c-a0w-2*a1w<=4.43;
```

```
lincon a0c+11*a1c-a0w-11*a1w<=4.47;
lincon a0c+2*a1c-a0w-2*a1w<=4.43;
lincon a0c+1*a1c-a0w-1*a1w<=4.41;
lincon a0c+11*a1c-a0w-11*a1w<=4.47;
lincon a0c+2*a1c-a0w-2*a1w<=4.43;
lincon a0c+29*a1c-a0w-29*a1w<=4.55;
lincon a0c+33*a1c-a0w-33*a1w<=4.6;
lincon a0c+23*a1c-a0w-23*a1w<=4.54;
lincon a0c+11*a1c-a0w-11*a1w<=4.47;
lincon a0c+9*a1c-a0w-9*a1w<=4.47;
lincon a0c+9*a1c-a0w-9*a1w<=4.47;
lincon a0c+13*a1c-a0w-13*a1w<=4.47;
lincon a0c+5*a1c-a0w-5*a1w<=4.45;
lincon a0c+9*a1c-a0w-9*a1w<=4.47;
lincon a0c+5*a1c-a0w-5*a1w<=4.45;
lincon a0c+11*a1c-a0w-11*a1w<=4.47;
lincon a0c+2*a1c-a0w-2*a1w<=4.43;
lincon a0c+23*a1c-a0w-23*a1w<=4.54;
lincon a0c+5*a1c+a0w+5*a1w>=4.45;
lincon a0c+2*a1c+a0w+2*a1w>=4.43;
lincon a0c+9*a1c+a0w+9*a1w>=4.47;
lincon a0c+2*a1c+a0w+2*a1w>=4.43;
lincon a0c+5*a1c+a0w+5*a1w>=4.45;
lincon a0c+2*a1c+a0w+2*a1w>=4.43;
lincon a0c+1*a1c+a0w+1*a1w>=4.41;
lincon a0c+2*a1c+a0w+2*a1w>=4.43;
lincon a0c+11*a1c+a0w+11*a1w>=4.47;
lincon a0c+2*a1c+a0w+2*a1w>=4.43;
lincon a0c+1*a1c+a0w+1*a1w>=4.41;
lincon a0c+11*a1c+a0w+11*a1w>=4.47;
lincon a0c+2*a1c+a0w+2*a1w>=4.43;
lincon a0c+29*a1c+a0w+29*a1w>=4.55;
lincon a0c+33*a1c+a0w+33*a1w>=4.6;
lincon a0c+23*a1c+a0w+23*a1w>=4.54;
lincon a0c+11*a1c+a0w+11*a1w>=4.47;
lincon a0c+9*a1c+a0w+9*a1w>=4.47;
lincon a0c+9*a1c+a0w+9*a1w>=4.47;
lincon a0c+13*a1c+a0w+13*a1w>=4.47;
lincon a0c+5*a1c+a0w+5*a1w>=4.45;
lincon a0c+9*a1c+a0w+9*a1w>=4.47;
lincon a0c+5*a1c+a0w+5*a1w>=4.45;
lincon a0c+11*a1c+a0w+11*a1w>=4.47;
lincon a0c+2*a1c+a0w+2*a1w>=4.43;
lincon a0c+23*a1c+a0w+23*a1w>=4.54;
Y=a0w*26+237*a1w;
run;
ods rtf close;
```

3. RESULTS AND DISCUSSION

PART 1: PARAMETER ESTIMATION FOR EXPONENTIAL CELL GROWTH

Table 1. Parameter Estimate of an Exponential Equation.

Parameter	Estimate	Approx Std Error	Approximate 95%	Confidence Limits
a	82.9121	0.2208	82.4563	83.3679
b	0.00512	0.000197	0.00471	0.00553

Table 1 shows that the parameter estimate of an exponential equation. The obtained equation is given by

$$Y = 82.9121e^{0.00512x} \quad (5)$$

Exponential growth formula is given by $Y = Ae^{bx}$ after estimation the parameter, we obtained the equation as (6). Using the equation of exponential growth equation we can estimate the growth of cell at the certain point $\hat{Y} = 82.9121e^{0.00512x}$. Taking an algorithm, we obtained

$$\ln y = \ln(82.9121e^{0.00512x}) = \ln(82.9121) + \ln(e^{0.00512x}) = 4.41778 + 0.00512x$$

$$\ln y = 4.41778 + 0.00512x \quad (6)$$

So the estimation of parameter estimates with the standard error is given by

$$\ln y = 4.41778 + 0.00512x$$

$$\text{Std Errors} = (0.2208)(0.000197) \quad (7)$$

So the upper limits of prediction interval for exponential model is computed using the equation

$$\ln y = 4.41778+0.2208)+(0.00512+0.000197)\, x$$

$$\ln y = (4.63858)+(0.005317)\, x \quad (8)$$

and the lower limits of prediction interval for exponential model is computed using the equation

$$\ln y = (4.41778-0.2208)+(0.00512-0.000197)\, x$$

$$\ln y = (4.19698)+(0.004923)\, x \quad (9)$$

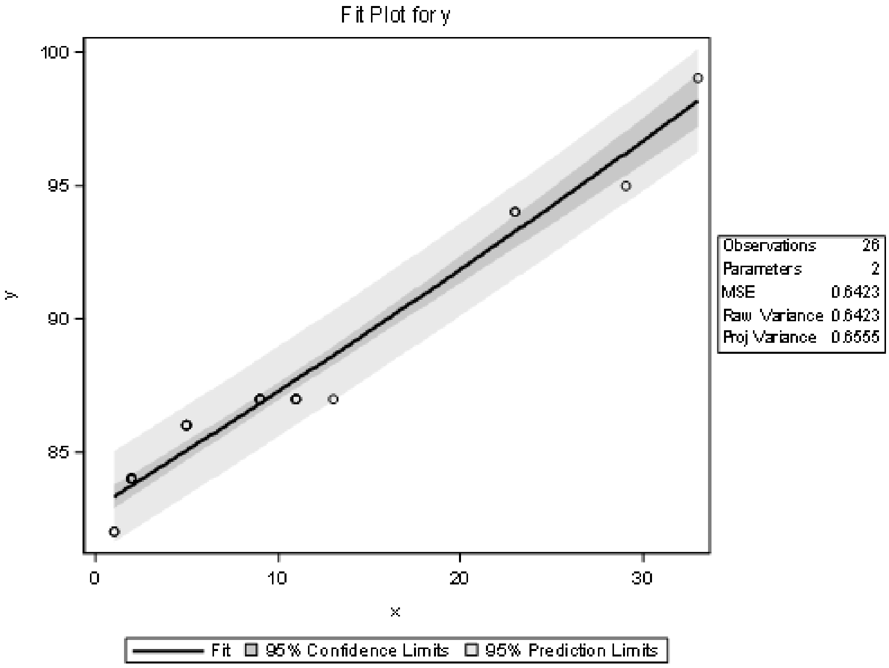

Figure 1. Plot of an Exponential.

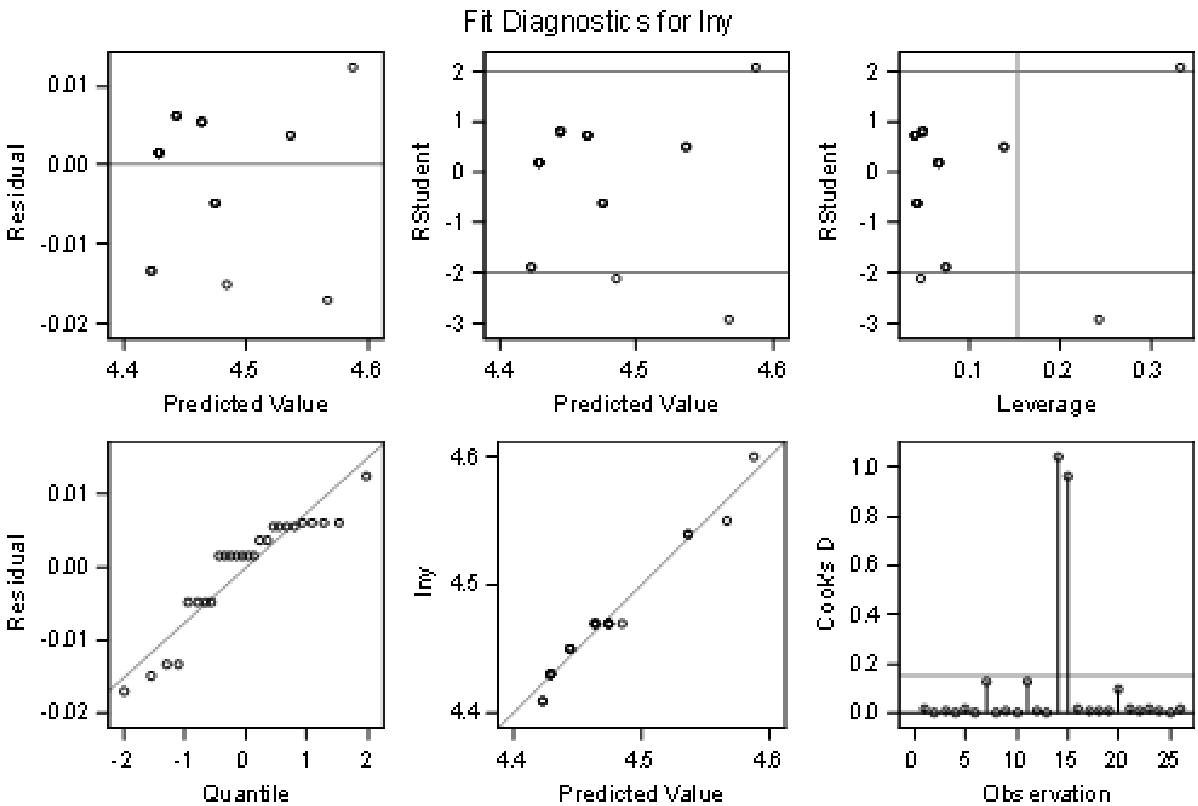

Figure 2. Fit Diagnostics for ln y.

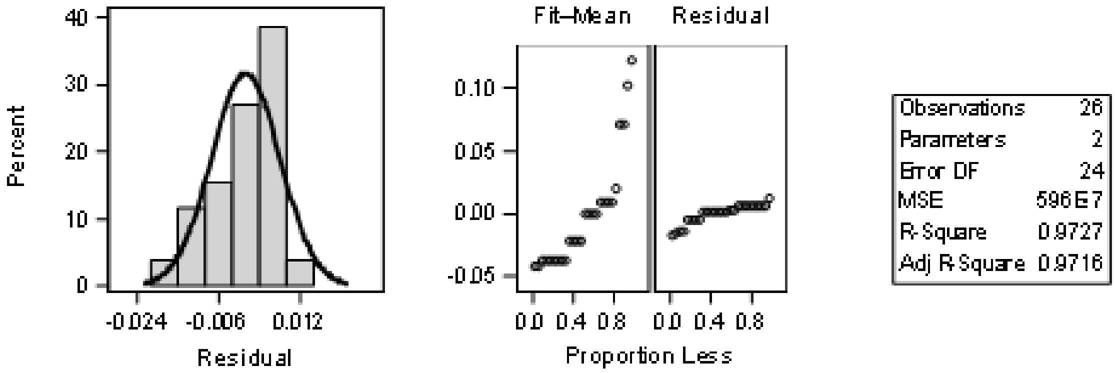

Figure 2. Fit Diagnostics for ln y (*continued*).

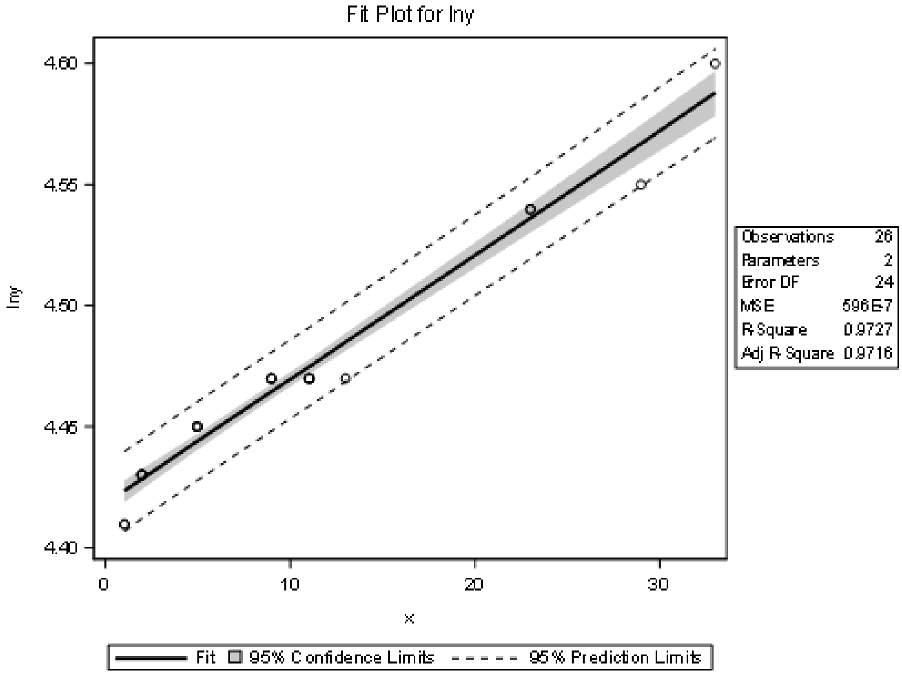

Figure 3. Plot ln *y* vs *x*.

PART 2: PARAMETER ESTIMATION FOR FUZZY LEAST SQUARE EXPONENTIAL CELL GROWTH

Table 2. Optimization Results for the Parameter Estimates.

N	Parameter	Estimate	Gradient Objective Function
1	a0c	4.414107	0
2	a0w	0.009107	26.000000
3	a1c	0.005179	0
4	a1w	0.000179	237.000000

Value of Objective Function = 0.2791071429

Parameter estimates is given: a0c=4.414107, a0w=0.041176, a1c=0.005179 and a1w=0.000179. For the fuzzy upper limit of prediction for exponential model is computed using the equation as follows:

$$\ln y = (4.414107+0.009107) + (0.005179+0.000179)\, x \quad (10)$$

$$= 4.423214 + 0.005358x$$

and the lower limits of prediction interval for the exponential model is computed using the equation

$$\ln y = (4.414107 - 0.009107) + (0.005179 - 0.000179)\, x \quad (11)$$

$$= 4.405 + 0.005x$$

Table 3. Average Width for Fitted Regression Models.

Method of Least Squares		**Method of Fuzzy Regression**	
(upper of width prediction)-(lower of width prediction)*i* *i*= 1,2,…,13	$\sum_{i=1}^{13}$ (Width / 13) = 0.5116	(upper of width prediction)-(lower of width prediction) *i*= 1,2,…,13	$\sum_{i=1}^{13}$ (Width / 13) = 0.0234

Table 3 clearly shows that fuzzy regression methodology is capable of handling situation in which predictor variables are highly correlated. From this table, average width for former was found to be 0.5116, while that for latter was only 0.0234, indicating thereby the superiority of fuzzy regression methodology. In classical regression, it is assumed that each error in the predicted variable is the same, whereas, in fuzzy regression, the individual membership function describes the unique uncertainty of the variable.

4. DISCUSSION AND CONCLUSION

This paperwork gives the explanation of exponential growth by adding some improvement method which adding bootstrap and the fuzzy technique using SAS software. The different result can see in table 3. The advantage of using fuzzy regression is that some of the strict assumptions of the statistical model can be relaxed. Another benefit of fuzzy regression is that it can provide a better generalization of data trends or patterns compared to crisp data. From the analysis, it concludes that fuzzy linear regression mode was applied mathematical programming had better result compared to Method of Least Square (LS).

***Acknowledgement**: The authors would like to express their gratitude to Universiti Sains Malaysia for providing the research funding (Grant no.1001/PPSG/8012278, School of Dental Sciences, University Sains Malaysia).*

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