ORIGINAL PAPER

ON ZWEIER IDEAL CONVERGENCE SEQUENCES IN INTUITIONISTIC FUZZY n- NORMED SPACES

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Abstract. *In 2007, M. Sengönül defined the notion of zweier sequence spaces* Z *and* Z_0 *consisting of the sequences that are convergent and null convergent respectively. Lately motivated by the above two concepts, Vakeel A. Khan and Nazneen [1] proposed the zweier* $I-$*convergent sequence spaces. In 2013, Nabanita Konwar and Pradip Debnath [2] introduced the* $I_\lambda-$*convergent sequences in intutionstic fuzzy* n*-normed space (IFnNS). In this paper, we introduce a new type of sequence spaces denoted as* $Z^{I_\lambda}_{(\mu,\nu)n}$, $Z^{I_\lambda}_{0(\mu,\nu)n}$, *and* $Z^{I_\lambda}_{\infty(\mu,\nu)n}$, *which are zweier* $I-$*convergent sequences over IFnNS. In addition, we studied the algebraic and topological properties of these spaces and certain inclusion relations.*

Keywords: *Zweier operator, intuitionistic fuzzy* n*-Normed spaces,* $I_\lambda-$*convergence, solid and monotone*

1. INTRODUCTION

In 1999 Kostyrko et. al [3] defined the notion of ideal convergence as the generalisation of statistical convergence [17] that was introduced by Fast [4] and Steinhaus [5] separately in 1951. Lately $I-$convergence for the sequence of functions has been studied by Balcerzak et. al [6], Komisarski [7]. The notion of Intutionstic Fuzzy $n-$normed spaces introdused by Vijayabalaji et. al [8] emerged from the concept of fuzzy sets which was defined by Zadeh [8] and later on studied by Atanossov [9,10]. It has a wide range of applications in the field of science and engineering, e.g., application of fuzzy topology in quantum particle physics that arises in string, chaos control, computer programming etc. In 2013, Nabanita Konwar and Pradip Debnath [2] introduced the notion of $I_\lambda-$convergent sequences in intutionstic fuzzy $n-$normed space (IFnNS). On other hand a new type of sequence spaces using matrix domain was constructed by Altay and Basar [11]. In 2007, M. Sengönül [12] defined the notion of zweier sequence spaces Z and Z_0 consisting of the sequences that are convergent and null convergent respectively.

In the present article, we have connected the construct of $I_\lambda-$convergence of sequence in intuitionistic fuzzy $n-$normed space and zweier operator to construct new kind of sequence spaces which consists of sequences that are zweier $I_\lambda-$ convergent sequence

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space with respect to intuitionistic fuzzy $n-$normed spaces. We denote by $\mathbb{N}$, $\mathbb{C}$, $\mathbb{R}$ and $\mathbb{R}_0^+$ as the set of natural numbers, complex numbers, real numbers and positive real numbers respectively.

2. PRELIMINERIES

Definition 2. 1. [8] Let $*$ be an $t-$norm , $\diamond$ be a $t-$conorm and X be a linear space over the field $K(\mathbb{R}\,\text{or}\,\mathbb{C})$ of dimension $d \geq n.$ An intuitionistic fuzzy $n-$norm (IFnN) on $X,$ denoted by $(\mu,\nu)_n$ is an
object of the form

$$A = \{(x_1, x_2, ..., x_n, t), \mu(x_1, x_2, ..., x_n, t), \nu(x_1, x_2, ..., x_n, t) : (x_1, x_2, ..., x_n, t) \in X^n \times \mathbb{R}\}$$

where μ and ν are fuzzy sets on $X^n \times \mathbb{R},$ μ denotes the degree of membership and ν denotes the degree of non-membership of $(x_1, x_2, ..., x_n, t) \in X^n \times \mathbb{R},$ satisfying the following conditions:
(IFN1) $\forall t \in \mathbb{R}$ with $t \leq 0, \mu(x_1, x_2, ..., x_n, t) = 0.$
(IFN2) $\forall t \in \mathbb{R}$ with $t > 0, \mu(x_1, x_2, ..., x_n, t) = 1$ if and only if $x_1, x_2, ..., x_n$ are linearly dependent.
(IFN3) $\mu(x_1, x_2, ..., x_n, t)$ is invariant under any permutation of $x_1, x_2, ..., x_n$.
(IFN4) $\mu(x_1, x_2, ..., cx_n, t) = \mu(x_1, x_2, ..., x_n, \frac{t}{|c|})$ if $c \neq 0, c \in K.$
(IFN5) $\mu(x_1, x_2, ..., x_n + x'_n, s+t) \geq \mu(x_1, x_2, ..., x_n, s) * \mu(x_1, x_2, ..., x'_n, t) \forall s, t \in \mathbb{R}.$
(IFN6) $\lim_{t\to\infty} \mu(x_1, x_2, ..., x_n, t) = 1.$
(IFN7) $\forall t \in \mathbb{R}$ with $t \leq 0, \nu(x_1, x_2, ..., x_n, t) = 1.$
(IFN8) $\forall t \in \mathbb{R}$ with $t > 0, \nu(x_1, x_2, ..., x_n, t) = 0$ if and only if $x_1, x_2, ..., x_n$ are linearly dependent.
(IFN9) $\nu(x_1, x_2, ..., x_n, t)$ is invariant under any permutation of $x_1, x_2, ..., x_n$
(IFN10) $\nu(x_1, x_2, ..., cx_n, t) = \nu(x_1, x_2, ..., x_n, \frac{t}{|c|})$ if $c \neq 0, c \in K.$
(IFN11) $\nu(x_1, x_2, ..., x_n, s) \diamond \nu(x_1, x_2, ..., x'_n, t) \leq \nu(x_1, x_2, ..., x_n + x'_n, s+t) \forall s, t \in \mathbb{R}.$
(IFN12) $\lim_{t\to\infty} \nu(x_1, x_2, ..., x_n, t) = 0.$
Then (X, A) is called an intuitionistic fuzzy $n-$normed linear space.

Remark 2.1. [13] The non-decreasing property of $\mu(x_1, x_2, ..., x_n, s)$ follows from (IFN2), and (IFN5) and non-increasing property of $\nu(x_1, x_2, ..., x_n, s)$ follows from (IFN8) and (IFN11).

Definition 2. 2. [11] Let $(X, \mu, \nu, *, \diamond)$ be an IFnNS. For $t > 0$ we define an open ball $B(x, r, t)$ with center $x \in X$ and radius $0 < r < 1$ and $y_1, y_2, ..., y_{n-1} \in X$ as

$$B(x, r, t) = \left\{\mu\left(y_1, y_2, ..., y_{n-1}, y - x.t\right) > 1 - r \text{ and } \nu\left(y_1, y_2, ..., y_{n-1}, y - x.t\right) < r\right\}.$$

Definition 2.3. [14] Let $\lambda = (\lambda_n)$ be a non-decreasing sequence of positive numbers tending to infinity such that $\lambda_{n+1} \leq \lambda_n + 1, \lambda_1 = 1.$ The generalized de la Vallee-Poussin mean is defined by

$$t_n(x) = \frac{1}{\lambda_n} \sum_{k \in J_n} (x_k) \qquad (1.2)$$

where $J_n = [n - \lambda_{n+1}, n]$. A sequence $x = (x_k)$ is said to be $(V, \lambda)-$summability to a number L if $t_n(x) \to L$ as $n \to \infty$ if $\lambda_n = n,$ then $(V, \lambda)-$summability reduces to $(C,1)-$summability.

Definition 2.4. [15] A sequence $x = (x_k)$ is said to be $\lambda-$statistically convergent to the number L if for every $\varepsilon > 0,$

$$\lim_{n\to\infty} \frac{1}{\lambda_n} \left| \left\{ k \in J_n : |x_k - L| \le \varepsilon \right\} \right| = 0. \qquad (1.3)$$

Let S_λ denote the set of all $\lambda-$statistically convergent sequences. If $\lambda_n = n,$ then S_λ is the same as $S.$

Definition 2.5. [16, 19] Let X is non-empty set. Then a family of sets $I \subseteq 2^X$ is said to be an ideal in X if it satisfies the following conditions:
(i) $\varnothing \in I,$
(ii) I is additive. i.e., for each $A, B \in I$ we have $A \cup B \in I,$
(iii) I is hereditary. i.e., for each $A \in I$ and each $B \subseteq A$ then, $B \in I.$

- An ideal $I \subseteq 2^X$ is said to be non-trivial if $X \neq I$ and $X \notin I.$
- An ideal $I \subseteq 2^X$ is said to be an admissible ideal in X if $\{\{x\} : x \in X\} \subset I$.

Definition 2. 6. [16] Let X be a non-empty set. Then a non-empty family of sets $F \subseteq 2^X$ is said to be a filter on X if and only if:
(i) $\varnothing \notin F,$
(ii) for each $A, B \in F$ we have $A \cap B \in F,$
(iii) for each $A \in F$ with $A \subseteq B$, we have $B \in F.$

Remark 2. 2. For each ideal $I,$ there is a filter $F(I)$ corresponding to $I,$ (filter associate with ideal I), that is

$$F(I) = \{K \subseteq X : K^c \in I\}, \text{ where } K^c = X \setminus K.$$

Definition2.7. [25] Let $I \subseteq 2^X$ be a non-trivial ideal. A sequence $x = (x_k)$ is said to be $I-[V, \lambda]-$summability to a number L if, for every $\varepsilon > 0$

$$\left\{ n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k \in J_n} |x_k - L| \ge \varepsilon \right\} \in I.$$

Definition2.8. [2] Let $I \subseteq 2^X$ and let $(X, \mu, \nu, *, \diamond)$ be an IFnNS. A sequence $x = (x_k)$ in X is said to be $I_\lambda-$convergent to $L \in X$ with respect to the intuitionistic fuzzy $n-$norm $(\mu, \nu)_n$ if for every $\varepsilon > 0, t > 0$ and $y_1, y_2, ..., y_{n-1} \in X,$

$$\left\{\begin{array}{l} n\in\mathbb{N}:\frac{1}{\lambda_n}\sum_{k\in J_n}\mu\left(y_1,y_2,...,y_{n-1},x_k-L,t\right)\leq 1-\varepsilon \\ \qquad\qquad\qquad \text{or } \frac{1}{\lambda_n}\sum_{k\in J_n}\nu\left(y_1,y_2,...,y_{n-1},x_k-L,t\right)\geq\varepsilon \end{array}\right\}\in I.$$

In this case L is called the $I_{\lambda}^{(\mu,\nu)_n}$ –limit of the sequence $x=(x_k)$ and we write $I_{\lambda}^{(\mu,\nu)_n}-\lim x=L.$

Definition.2.9. [8] A sequence space E is said to be solid or normal, if $(\alpha_n x_n)\in E$ whenever $(x_n)\in E$ and for any sequence of scalars $(\alpha_n)\in\omega$ with $|\alpha_n|<1$, for every $n\in\mathbb{N}$.

Definition.2.10. [8] A sequence space E is said to be sequence algebra, if $(x_n)*(z_n)=(x_n\cdot z_n)\in E$ whenever $(x_n),(z_n)\in E$.

Definition.2.11: [20] Let $K=\{n_i\in\mathbb{N}:n_1<n_2<...\}\subseteq\mathbb{N}$ and E be a sequence space. A K–step space of E is a sequence space

$$\lambda_K^E=\left\{\left(x_{n_i}\right)\in\omega:(x_n)\in E\right\}.$$

A canonical preimage of a step space of a sequence $\left(x_{n_i}\right)\in\lambda_K^E$ is the sequence $(y_n)\in\omega$ defined as

$$y_n=\begin{cases}x_n, & \text{if } n\in K\\ 0, & \text{otherwise.}\end{cases}$$

A canonical preimage of a step space λ_K^E is a set of canonical preimages of all elements in λ_K^E. i.e., y is a canonical preimage of λ_K^E iff y is canonical preimage of some element $x\in\lambda_K^E$.

Definition.2.12: [8] A sequence space E is said to be monotone, if it is contains all the canonical preimages of it is step space. (i.e., if for all infinite $K\subseteq\mathbb{N}$ and $(x_n)\in E$ the sequence $(\alpha_n x_n)\in E$, where $\alpha_n=1$ for $n\in K$ and $\alpha_n=0$ otherwise, belongs to E).

Lemma.2.1: [18] Every solid space is monotone.

3. ZWEIER SEQUENCE SPACES

Let λ and μ be two sequence spaces and $A=\left(a_{nk}\right)$ be an infinite matrix of real or complex numbers a_{nk}. Then A defines a matrix mapping $A:\lambda\to\mu,$ if for each sequence $x=(x_k)\in\lambda$

The sequence $A_x=\left\{\left(Ax\right)_n\right\}\in\mu,$ where

$$\left(Ax\right)_n = \sum_k a_{nk} x_k .$$

Let $\lambda_A = \left\{ x = \left(x_k\right) : A_X \in \lambda \right\}$, where λ a sequence space. Then λ_A is called matrix domain of an infinite matrix A.

Altay, Basar, Mursaleen [20], Malkowsky [21], Ng, Lee [22] and Wang [23] used the matrix domain to introduce the sequence spaces $\left(\ell_\infty\right)_{R_t, cR_t}$ and $\left(c_0\right)_{R_t}$ and others.

Motivated by Basar and Altay [11], Mehmet Sengönül [12] defined the sequence spaces Z_0 and Z using the sequence $y = \left(y_i\right)$ defined as

$$y_i = px_i + \left(1 - p\right) x_{i-1}$$
$$x_{-1} = 0, p \neq 1, 1 < p < \infty.$$

The sequence y is called the Z^p transform of the sequence $x = (x_i)$, where Z^p denotes the matrix $Z^p = (z_{ik})$ defined by

$$z_{ik} = \begin{cases} p & \text{if } i = k \\ 1 - p & \text{if } i - 1 = k \\ 0 & \text{otherwise.} \end{cases}$$

Mehmet Sengönül [12] introduced the sequence spaces Z_0 and Z as

$$Z_0 = \left\{ x = \left(x_k\right) \in \omega : Z^p\left(x\right) \in c_0 \right\}$$
$$Z = \left\{ x = \left(x_k\right) \in \omega : Z^p\left(x\right) \in c \right\}.$$

Recently Khan, Ebadullah and Yasmeeen [24] introduced the following classes of sequences

$$Z^I = \left\{ \left(x_k\right) \in \omega : \exists\, L \in \mathbb{C} \text{ such that for a given } \varepsilon > 0 \left\{ k \in \mathbb{N} : \left| x_k^{'} - L \right| \geq \varepsilon \right\} \in I \right\},$$

$$Z_0^I = \left\{ \left(x_k\right) \in \omega : \text{for a given } \varepsilon > 0 \left\{ k \in \mathbb{N} : \left| x_k^{'} \right| \geq \varepsilon \right\} \in I \right\}, \text{ where } (x_k^{'}) = (Z^p x_k).$$

4. RESULTS

Using the concepts of $I-$convergence in IF n-NS and zweier sequence spaces, we define the sequence spaces which relate the two concepts in the following manner:

$$Z_{(\mu,\nu)_n}^{I_\lambda} = \left\{ x = (x_k) \in \ell_\infty : \left\{ \begin{array}{l} n \in \mathbb{N} : \dfrac{1}{\lambda_n} \displaystyle\sum_{k \in J_n} \mu\left(y_1, y_2, ..., y_{n-1}, x_k^{'} - L, t\right) \leq 1 - \varepsilon \\ \quad \text{or} \dfrac{1}{\lambda_n} \displaystyle\sum_{k \in J_n} \nu\left(y_1, y_2, ..., y_{n-1}, x_k^{'} - L, t\right) \geq \varepsilon \text{ forsame } \; L \in \mathbb{C} \end{array} \right\} \in I \right\},$$

$$Z_{0(\mu,\nu)_n}^{I_\lambda} = \left\{ x = (x_k) \in \ell_\infty : \left\{ \begin{array}{l} n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k \in J_n} \mu\left(y_1, y_2, ..., y_{n-1}, x_k^{'}, t\right) \le 1 - \varepsilon \\ \text{or} \frac{1}{\lambda_n} \sum_{k \in J_n} \nu\left(y_1, y_2, ..., y_{n-1}, x_k^{'}, t\right) \ge \varepsilon \end{array} \right\} \in I \right\},$$

$$Z_{\infty(\mu,\nu)_n}^{I_\lambda} = \left\{ x = (x_k) \in \ell_\infty : \left\{ \begin{array}{l} n \in \mathbb{N} : \exists K > 0 \text{ s.t} \frac{1}{\lambda_n} \sum_{k \in J_n} \mu\left(y_1, y_2, ..., y_{n-1}, x_k^{'}, t\right) \le 1 - K \\ \text{or} \frac{1}{\lambda_n} \sum_{k \in J_n} \nu\left(y_1, y_2, ..., y_{n-1}, x_k^{'}, t\right) \ge K \end{array} \right\} \in I \right\}.$$

We also define an open ball with center x and radius r with respect to t as follows:

$$B(x^{'}, r, t) = \left\{ z = (z_k) \in \ell_\infty : \left\{ \begin{array}{l} n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k \in J_n} \mu\left(y_1, y_2, ..., y_{n-1}, x_k^{'} - z_k^{'}, t\right) > 1 - r \\ \text{and} \frac{1}{\lambda_n} \sum_{k \in J_n} \nu\left(y_1, y_2, ..., y_{n-1}, x_k^{'} - z_k^{'}, t\right) < r \end{array} \right\} \right\}.$$

We will prove the following results:

Theorem 4.1. The sequence spaces $Z_{(\mu,\nu)_n}^{I_\lambda}$ and $Z_{0(\mu,\nu)_n}^{I_\lambda}$ are liner over $\mathbb{R}$.

Proof: We will prove the result for $Z_{(\mu,\nu)_n}^{I_\lambda}$. The proof for the other space will follow similarly. Let $x = (x_k)$, $z = (z_k)$ be two arbitrary elements of the space $Z_{(\mu,\nu)_n}^{I_\lambda}$ and let α, β be scalars. Then for a given $\varepsilon > 0$, there exist $L_1, L_2 \in \mathbb{R}$ such that

$$\left\{ \begin{array}{l} n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k \in J_n} \mu\left(y_1, y_2 ..., y_{n-1}, x_k^{'} - L_1, t\right) \le 1 - \varepsilon \\ \text{or} \frac{1}{\lambda_n} \sum_{k \in J_n} \nu\left(y_1, y_2 ..., y_{n-1}, x_k^{'} - L_1, t\right) \ge \varepsilon \end{array} \right\} \in I$$

and

$$\left\{ \begin{array}{l} n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k \in J_n} \mu\left(y_1, y_2 ..., y_{n-1}, z_k^{'} - L_2, t\right) \le 1 - \varepsilon \\ \text{or} \frac{1}{\lambda_n} \sum_{k \in J_n} \nu\left(y_1, y_2 ..., y_{n-1}, z_k^{'} - L_2, t\right) \ge \varepsilon \end{array} \right\} \in I.$$

Now, let

$$A_1 = \left\{ \begin{array}{l} n \in \mathbb{N} : \frac{1}{\lambda_n} \sum_{k \in J_n} \mu\left(y_1, y_2 ..., y_{n-1}, x_k^{'} - L_1, \frac{t}{2|\alpha|}\right) > 1 - \varepsilon \\ \text{or} \frac{1}{\lambda_n} \sum_{k \in J_n} \nu\left(y_1, y_2 ..., y_{n-1}, x_k^{'} - L_1, \frac{t}{2|\alpha|}\right) < \varepsilon \end{array} \right\} \in F(I),$$

$$A_2=\left\{\begin{array}{l} n\in\mathbb{N}:\frac{1}{\lambda_n}\sum_{k\in J_n}\mu\left(y_1,y_2...,y_{n-1},z_k^{'}-L_2,\frac{t}{2|\beta|}\right)>1-\varepsilon \\ \text{or}\ \frac{1}{\lambda_n}\sum_{k\in J_n}\nu\left(y_1,y_2...,y_{n-1},z_k^{'}-L_2,\frac{t}{2|\beta|}\right)<\varepsilon \end{array}\right\}\in F(I)$$

be such that $A_1^c, A_2^c \in I$. Therefore, the set

$$A_3=\left\{\begin{array}{l} n\in\mathbb{N}:\frac{1}{\lambda_n}\sum_{k\in J_n}\mu\left(y_1,y_2...,y_{n-1},\alpha x_k^{'}+\beta z_k^{'}-(\alpha L_1+\beta L_2),t\right)>1-\varepsilon \\ \text{or}\ \frac{1}{\lambda_n}\sum_{k\in J_n}\nu\left(y_1,y_2...,y_{n-1},\alpha x_k^{'}+\beta z_k^{'}-(\alpha L_1+\beta L_2),t\right)<\varepsilon \end{array}\right\}\supseteq\{A_1\cap A_2\}. \quad (2.1)$$

Thus, the sets on right hand side of equation (2.1) belongs to $F(I)$. By definition of filter associate with ideal, the complement of the set on left hand side of (2.1) belongs to I. This implies that $(\alpha x+\beta z)\in Z_{(\mu,\nu)_n}^{I_\lambda}$. Hence $Z_{(\mu,\nu)_n}^{I_\lambda}$ is linear space.

Theorem 4.2. Every open ball $B(x^{'},r,t)$ is an open set in $Z_{(\mu,\nu)_n}^{I_\lambda}$.

Proof: Let $B(x^{'},r,t)$ be an open ball with center x and radius r with respect to t *in* $Z_{(\mu,\nu)_n}^{I_\lambda}$. That is

$$B(x^{'},r,t)=\left\{z=(z_k)\in\ell_\infty:\left\{\begin{array}{l} n\in\mathbb{N}:\frac{1}{\lambda_n}\sum_{k\in J_n}\mu\left(y_1,y_2...,y_{n-1},x_k^{'}-z_k^{'},t\right)>1-r \\ \text{and}\ \frac{1}{\lambda_n}\sum_{k\in J_n}\nu\left(y_1,y_2...,y_{n-1},x_k^{'}-z_k^{'},t\right)<r \end{array}\right\}\right\}.$$

Let $z\in B(x^{'},r,t)$. Then

$$\frac{1}{\lambda_n}\sum_{k\in J_n}\mu\left(y_1,y_2...,y_{n-1},x_k^{'}-z_k^{'},t\right)>1-r \ \text{and}\ \frac{1}{\lambda_n}\sum_{k\in J_n}\nu\left(y_1,y_2...,y_{n-1},x_k^{'}-z_k^{'},t\right)<r.$$

Since

$\frac{1}{\lambda_n}\sum_{k\in J_n}\mu\left(y_1,y_2...,y_{n-1},x_k^{'}-z_k^{'},t\right)>1-r$, there exists $t_0\in(0,t)$ such that

$$\frac{1}{\lambda_n}\sum_{k\in J_n}\mu\left(y_1,y_2...,y_{n-1},x_k^{'}-z_k^{'},t_0\right)>1-r \ \text{and}\ \frac{1}{\lambda_n}\sum_{k\in J_n}\nu\left(y_1,y_2...,y_{n-1},x_k^{'}-z_k^{'},t_0\right)<r.$$

Putting $r_0=\frac{1}{\lambda_n}\sum_{k\in J_n}\mu\left(y_1,y_2...,y_{n-1},x_k^{'}-z_k^{'},t_0\right)$, we have $r_0>1-r$, there exists $s\in(0,1)$ such that $r_0>1-s>1-r$. For $r_0>1-s$, we have $r_1,r_2\in(0,1)$ such that $r_0*r_1>1-s$ and $(1-r_0)\diamond(1-r_2)\le s$.

Putting $r_3=\max\{r_1,r_2\}$. Consider the ball $B\left(z^{'},1-r_3,t-t_0\right)$. We prove that $B\left(z^{'},1-r_3,t-t_0\right)\subset B\left(x^{'},r,t\right)$. Let $q=(q_k)\in B\left(z^{'},1-r_3,t-t_0\right)$, then

$$\frac{1}{\lambda_n}\sum_{k\in J_n}\mu\left(y_1,y_2...,y_{n-1},z_k^{'}-q_k^{'},t-t_0\right)>r_3 \text{ and } \frac{1}{\lambda_n}\sum_{k\in J_n}\nu\left(y_1,y_2...,y_{n-1},z_k^{'}-q_k^{'},t-t_0\right)<1-r_3.$$

Therefore

$$\begin{aligned}\frac{1}{\lambda_n}\sum_{k\in J_n}\mu\left(y_1,y_2...,y_{n-1},x_k^{'}-q_k^{'},t\right)&\geq\frac{1}{\lambda_n}\sum_{k\in J_n}\mu\left(y_1,y_2...,y_{n-1},x_k^{'}-z_k^{'},t_0\right)\\&*\frac{1}{\lambda_n}\sum_{k\in J_n}\mu\left(y_1,y_2...,y_{n-1},z_k^{'}-q_k^{'},t-t_0\right)\\&\geq(r_0*r_3)\geq(r_0*r_1)\geq(1-s)>(1-r)\end{aligned}$$

and

$$\begin{aligned}\frac{1}{\lambda_n}\sum_{k\in J_n}\nu\left(y_1,y_2...,y_{n-1},x_k^{'}-q_k^{'},t\right)&\leq\frac{1}{\lambda_n}\sum_{k\in J_n}\nu\left(y_1,y_2...,y_{n-1},x_k^{'}-z_k^{'},t_0\right)\\&\diamond\frac{1}{\lambda_n}\sum_{k\in J_n}\nu\left(y_1,y_2...,y_{n-1},z_k^{'}-q_k^{'},t-t_0\right)\\&\leq(1-r_0)\diamond(1-r_3)\leq(1-r_0)\diamond(1-r_2)\leq s<r.\end{aligned}$$

Thus $q\in B\left(x^{'},r,t\right)$ and hence $B\left(z^{'},1-r_3,t-t_0\right)\subset B\left(x^{'},r,t\right)$.

Remark 4.1. The sequence space $Z^{I_\lambda}_{(\mu,\nu)_n}$ is an Intuitionistic Fuzzy n – Normed spaces.

Proof: Define $\tau^{I_\lambda}_{(\mu,\nu)_n}=\left\{A\subset Z^{I_\lambda}_{(\mu,\nu)_n}:\text{for each } x\in A \text{ there exsists } t>0 \text{ and } r\in(0,1) \text{ such that } B\left(x^{'},r,t\right)\subset A\right\}$. Then $\tau^{I_\lambda}_{(\mu,\nu)_n}$ is a topology on $Z^{I_\lambda}_{(\mu,\nu)_n}$.

Theorem 4.3. The topology $\tau^{I_\lambda}_{(\mu,\nu)_n}$ on $Z^{I_\lambda}_{0(\mu,\nu)_n}$ is first countable.

Proof: $\left\{B\left(x^{'},\frac{1}{n},\frac{1}{n}\right):n=1,2,3,\ldots\right\}$ is a local base at x, the topology $\tau^{I_\lambda}_{(\mu,\nu)_n}$ on $Z^{I_\lambda}_{0(\mu,\nu)_n}$ is first countable.

Theorem 4.4. A sequence spaces $Z^{I_\lambda}_{(\mu,\nu)_n}$ and $Z^{I_\lambda}_{0(\mu,\nu)_n}$ are Hausdorff spaces.

Proof: We prove the result for $Z^{I_\lambda}_{(\mu,\nu)_n}$. Similarly the proof follows for $Z^{I_\lambda}_{0(\mu,\nu)_n}$. Let $x,z\in Z^{I_\lambda}_{(\mu,\nu)_n}$ such that $x\neq z$. Then

$$0<\frac{1}{\lambda_n}\sum_{k\in J_n}\mu\left(y_1,y_2...,y_{n-1},x_k^{'}-z_k^{'},t\right)<1 \quad\text{and}\quad 0<\frac{1}{\lambda_n}\sum_{k\in J_n}\nu\left(y_1,y_2...,y_{n-1},x_k^{'}-z_k^{'},t\right)<1.$$

Putting

$$r_1 = \frac{1}{\lambda_n}\sum_{k\in J_n}\mu\left(y_1, y_2 ..., y_{n-1}, x_k^{'} - z_k^{'}, t\right) \text{ and } r_2 = \frac{1}{\lambda_n}\sum_{k\in J_n}\nu\left(y_1, y_2 ..., y_{n-1}, x_k^{'} - z_k^{'}, t\right)$$

and $r = \max\{r_1, 1-r_2\}$. For each $r_0 \in (r,1)$, there exists r_3 and r_4 such that $r_3 * r_3 \geq r_0$ and $(1-r_4)\diamond(1-r_4) \leq (1-r_0)$. Putting $r_5 = \max\{r_3, r_4\}$ and consider the open balls $B\left(x^{'}, 1-r_5, \frac{t}{2}\right)$ and $B\left(z^{'}, 1-r_5, \frac{t}{2}\right)$. Then clearly $B\left(x^{'}, 1-r_5, \frac{t}{2}\right) \cap B\left(z^{'}, 1-r_5, \frac{t}{2}\right) = \phi$. If there exists $q \in B\left(x^{'}, 1-r_5, \frac{t}{2}\right) \cap B\left(z^{'}, 1-r_5, \frac{t}{2}\right)$, then

$$\begin{aligned} r_1 &= \frac{1}{\lambda_n}\sum_{k\in J_n}\mu\left(y_1, y_2 ..., y_{n-1}, x_k^{'} - z_k^{'}, t\right) \\ &\geq \frac{1}{\lambda_n}\sum_{k\in J_n}\mu\left(y_1, y_2 ..., y_{n-1}, x_k^{'} - q_k^{'}, \frac{t}{2}\right) * \frac{1}{\lambda_n}\sum_{k\in J_n}\mu\left(y_1, y_2 ..., y_{n-1}, q_k^{'} - z_k^{'}, \frac{t}{2}\right) \\ &\geq r_5 * r_5 \geq r_3 * r_3 \geq r_0 > r_1 \end{aligned}$$

and

$$\begin{aligned} r_2 &= \frac{1}{\lambda_n}\sum_{k\in J_n}\nu\left(y_1, y_2 ..., y_{n-1}, x_k^{'} - z_k^{'}, t\right) \\ &\leq \frac{1}{\lambda_n}\sum_{k\in J_n}\nu\left(y_1, y_2 ..., y_{n-1}, x_k^{'} - q_k^{'}, \frac{t}{2}\right) \diamond \frac{1}{\lambda_n}\sum_{k\in J_n}\nu\left(y_1, y_2 ..., y_{n-1}, q_k^{'} - z_k^{'}, \frac{t}{2}\right) \\ &\leq (1-r_5)\diamond(1-r_5) \leq (1-r_4)\diamond(1-r_4) \leq (1-r_0) < r_2 \end{aligned}$$

which is a contradiction. Hence $Z^{I_\lambda}_{(\mu,\nu)_n}$ is Hausdorff.

Theorem 4.5. The space $Z^{I_\lambda}_{0(\mu,\nu)_n}$ is solid and monotone.

Proof: Let $x = (x_k) \in Z^{I_\lambda}_{0(\mu,\nu)_n}$ and Let $\varepsilon > 0$ be given. Then we have

$$\left\{ n \in \mathbb{N} : \frac{1}{\lambda_n}\sum_{k\in J_n}\mu\left(y_1, y_2 ..., y_{n-1}, x_k^{'}, t\right) \leq 1-\varepsilon \text{ or } \frac{1}{\lambda_n}\sum_{k\in J_n}\nu\left(y_1, y_2 ..., y_{n-1}, x_k^{'}, t\right) \geq \varepsilon \right\} \in I. \quad (2.2)$$

Let $\alpha = (\alpha_k)$ be a sequence of scalars with $|\alpha| \leq 1$ for all $k \in \mathbb{N}$. Therefore,

$$\mu\left(y_1, y_2 ..., y_{n-1}, \alpha x_k^{'}, t\right) \leq \mu\left(y_1, y_2 ..., y_{n-1}, x_k^{'}, t\right) \text{or } \nu\left(y_1, y_2 ..., y_{n-1}, \alpha x_k^{'}, t\right) \leq \nu\left(y_1, y_2 ..., y_{n-1}, x_k^{'}, t\right).$$

Thus, from the above inequality and (2.2), we have

$$\left\{ n \in \mathbb{N} : \frac{1}{\lambda_n}\sum_{k\in J_n}\mu\left(y_1, y_2 ..., y_{n-1}, \alpha x_k^{'}, t\right) \leq 1-\varepsilon \text{ or } \frac{1}{\lambda_n}\sum_{k\in J_n}\nu\left(y_1, y_2 ..., y_{n-1}, \alpha x_k^{'}, t\right) \geq \varepsilon \right\} \in I.$$

Thus, $(\alpha x_k) \in Z_{0(\mu,\nu)_n}^{I_\lambda}$ whenever $(x_k) \in Z_{0(\mu,\nu)_n}^{I_\lambda}$. Hence the space $Z_{0(\mu,\nu)_n}^{I_\lambda}$ is solid and hence by Lemma (1.1), the space $Z_{0(\mu,\nu)_n}^{I_\lambda}$ is monotone.

Theorem 4.6. Let $Z_{(\mu,\nu)_n}^{I_\lambda}$ be an IFnNS and $\tau_{(\mu,\nu)_n}^{I_\lambda}$ is a topology on $Z_{(\mu,\nu)_n}^{I_\lambda}$. Then a sequence $(x_k) \in Z_{(\mu,\nu)_n}^{I_\lambda}$ convergent to x if and only if

$$\frac{1}{\lambda_n}\sum_{k\in J_n}\mu\left(y_1, y_2..., y_{n-1}, x_k^{'} - x^{'}, t\right) \to 1 \text{ and } \frac{1}{\lambda_n}\sum_{k\in J_n}\nu\left(y_1, y_2..., y_{n-1}, x_k^{'} - x^{'}, t\right) \to 0 \text{ as } k \to \infty.$$

Proof: Fix $t > 0$. Suppose $x_k \to x$. Then for $r \in (0,1)$, there exists $n_0 \in \mathbb{N}$ such that $x_k \in B\left(x^{'}, r, t\right)$ for all $k \geq n_0$,

$$B\left(x^{'}, r, t\right) = \left\{ n \in \mathbb{N} : \frac{1}{\lambda_n}\sum_{k\in J_n}\mu\left(y_1, y_2..., y_{n-1}, x_k^{'} - x^{'}, t\right) > 1 - r \text{ and } \frac{1}{\lambda_n}\sum_{k\in J_n}\nu\left(y_1, y_2..., y_{n-1}, x_k^{'} - x^{'}, t\right) < r \right\},$$

Then

$$1 - \frac{1}{\lambda_n}\sum_{k\in J_n}\mu\left(y_1, y_2..., y_{n-1}, x_k^{'} - x^{'}, t\right) < r \text{ and } \frac{1}{\lambda_n}\sum_{k\in J_n}\nu\left(y_1, y_2..., y_{n-1}, x_k^{'} - x^{'}, t\right) < r.$$

Hence

$$\frac{1}{\lambda_n}\sum_{k\in J_n}\mu\left(y_1, y_2..., y_{n-1}, x_k^{'} - x^{'}, t\right) \to 1 \text{ and } \frac{1}{\lambda_n}\sum_{k\in J_n}\nu\left(y_1, y_2..., y_{n-1}, x_k^{'} - x^{'}, t\right) \to 0 \text{ as } k \to \infty.$$

Conversely, if for each $t > 0$,

$$\frac{1}{\lambda_n}\sum_{k\in J_n}\mu\left(y_1, y_2..., y_{n-1}, x_k^{'} - x^{'}, t\right) \to 1 \text{ and } \frac{1}{\lambda_n}\sum_{k\in J_n}\nu\left(y_1, y_2..., y_{n-1}, x_k^{'} - x^{'}, t\right) \to 0 \text{ as } k \to \infty,$$

then for $r \in (0,1)$, there exists $n_0 \in \mathbb{N}$ such that

$$1 - \frac{1}{\lambda_n}\sum_{k\in J_n}\mu\left(y_1, y_2..., y_{n-1}, x_k^{'} - x^{'}, t\right) < r \text{ and } \frac{1}{\lambda_n}\sum_{k\in J_n}\nu\left(y_1, y_2..., y_{n-1}, x_k^{'} - x^{'}, t\right) < r$$

for all $k \geq n_0$. It follows that

$$\frac{1}{\lambda_n}\sum_{k\in J_n}\mu\left(y_1, y_2..., y_{n-1}, x_k^{'} - x^{'}, t\right) > 1 - r \text{ and } \frac{1}{\lambda_n}\sum_{k\in J_n}\nu\left(y_1, y_2..., y_{n-1}, x_k^{'} - x^{'}, t\right) < r$$

for all $k \geq n_0$. Thus $(x_k) \in B\left(x^{'}, r, t\right)$ for all $k \geq n_0$ and hence $x_k \to x$.

Theorem 4.7. The spaces $Z_{0(\mu,\nu)_n}^{I_\lambda}$ and $Z_{(\mu,\nu)_n}^{I_\lambda}$ are sequence algebra.

Proof: Let $x=(x_k), z=(z_k)\in Z_{0(\mu,\nu)_n}^{I_\lambda}$. Then

$$I_\lambda^{(\mu,\nu)_n}-\lim x=0 \text{ and } I_\lambda^{(\mu,\nu)_n}-\lim z=0. \quad (2.3)$$

Therefore, from (2.3), we have $I_\lambda^{(\mu,\nu)_n}-\lim(x\cdot z)=0$. This implies that

$$\left\{n\in\mathbb{N}:\frac{1}{\lambda_n}\sum_{k\in J_n}\mu\left(y_1,y_2...,y_{n-1},x_k^{'}\cdot z_k^{'},t\right)\le 1-r \text{ or } \frac{1}{\lambda_n}\sum_{k\in J_n}\nu\left(y_1,y_2...,y_{n-1},x_k^{'}\cdot z_k^{'},t\right)\ge r\right\}\in I.$$

Thus, $(x\cdot z)\in Z_{0(\mu,\nu)_n}^{I_\lambda}$. Hence $Z_{0(\mu,\nu)_n}^{I_\lambda}$ is sequence algebra. Similarly, we can prove that $Z_{(\mu,\nu)_n}^{I_\lambda}$, is a sequence algebra.

Theorem 4.8. The inclusion relation $Z_{0(\mu,\nu)_n}^{I_\lambda}\subset Z_{(\mu,\nu)_n}^{I_\lambda}\subset Z_{\infty(\mu,\nu)_n}^{I_\lambda}$ holds.

Proof: The inclusion $Z_{0(\mu,\nu)_n}^{I_\lambda}\subset Z_{(\mu,\nu)_n}^{I_\lambda}$ is obvious. Now, let $x=(x_k)\in Z_{(\mu,\nu)_n}^{I_\lambda}$. Then there exists a number $L\in\mathbb{R}$ such that $I_\lambda^{(\mu,\nu)_n}-\lim x=L$. That is, the set

$$\left\{n\in\mathbb{N}:\frac{1}{\lambda_n}\sum_{k\in J_n}\mu\left(y_1,y_2...,y_{n-1},x_k^{'}-L,t\right)\le 1-r \text{ or } \frac{1}{\lambda_n}\sum_{k\in J_n}\nu\left(y_1,y_2...,y_{n-1},x_k^{'}-L,t\right)\ge r\right\}\in I.$$

We have

$$\left\{n\in\mathbb{N}:\frac{1}{\lambda_n}\sum_{k\in J_n}\mu\left(y_1,y_2...,y_{n-1},x_k^{'}-L+L,t\right)\le 1-r \text{ or } \frac{1}{\lambda_n}\sum_{k\in J_n}\nu\left(y_1,y_2...,y_{n-1},x_k^{'}-L+L,t\right)\ge r\right\}$$
$$\supseteq\left\{\begin{array}{l}\left\{n\in\mathbb{N}:\frac{1}{\lambda_n}\sum_{k\in J_n}\mu\left(y_1,y_2...,y_{n-1},x_k^{'}-L,t\right)\le 1-r \text{ or } \frac{1}{\lambda_n}\sum_{k\in J_n}\nu\left(y_1,y_2...,y_{n-1},x_k^{'}-L,t\right)\ge r\right\}\\ \cup\left\{n\in\mathbb{N}:\frac{1}{\lambda_n}\sum_{k\in J_n}\mu\left(y_1,y_2...,y_{n-1},L,t\right)\le 1-r \text{ or } \frac{1}{\lambda_n}\sum_{k\in J_n}\nu\left(y_1,y_2...,y_{n-1},L,t\right)\ge r\right\}\end{array}\right\}.$$

From this it easily follows that the sequence (x_k) must belongs to $Z_{\infty(\mu,\nu)_n}^{I_\lambda}$.

5. DISCUSSION

Statistical convergence which was given by Fast and Steinhaus independently in the year 1951 was generalized to ideal convergence by Kostryko et al. [3] in 1999. Intuitionistic fuzzy n- normed space was defined by Vijayabalaji et al. in his paper [8]. Later on, in 2016, Konwar et al. gave more generalized definition of $I_\lambda-$*convergent in intuitionistic fuzzy* n-normed space. Using matrix domain some sequence spaces were constructed by Altar, Basar and Mursaleen. M. Sengönül in 2007, introduced zweier sequence spaces denoted by Z_0 and Z. Inspired by these ideas, we in the current article introduce sequence spaces $Z_{(\mu,\nu)_n}^{I_\lambda}, Z_{0(\mu,\nu)_n}^{I_\lambda}$

and $Z^{I_\lambda}_{\infty(\mu,\nu)_n}$ and studied the properties of these space like linearity, inclusion relations, topological and algebraic properties

6. CONCLUSION

In the present article, we have introduced new kind of sequence spaces using the concept of zweier sequence spaces in intuitionistic fuzzy n- normed space. We investigated the elementary properties of these spaces such as linearity, inclusion relations etc. These results will provide new tool to deal with the convergence problems in the field of science and engineering.

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