ORIGINAL PAPER

ON FUZZY INTEGRO DIFFERENTIAL EQUATIONS BY USING MODIFIED VARIATIONAL ITERATION METHOD

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Abstract.*The purpose of this paper is to show use of modified variational iteration technique for understanding fuzzy integro-differential equations. Illuminating integro differential equations with correct parameter inside numerous displaying physical issues isn't exactly simple or better to state unthinkable in genuine issues. To defeat this trouble fuzzy idea is utilized. Generally it's hard to comprehend nonlinear fuzzy IDEs numerically.*

Keywords: *Modified Variational iteration Method, Fuzzy integro differential equation, crisp number.*

1. INTRODUCTION

Integro differential equations assume a critical part inside different orders of science and mathematics. These happen in an assortment of utilizations that is gotten from a differential equation. Numerous uses of science and engineering are characterized by integral equations, for example, Volterra's population development models, organic species, and warmth exchange. These likewise frequently show up in electro measurements, low recurrence electromagnetic issues and flexible waves. Essential thought of fuzzy arithmetic was 1st introduced by Lotfi Asker Zadeh's in 1965 after his production on fuzzy set hypothesis [1, 2]. Therefore in 1978 Dubois and Prade acquainted the idea of number-crunching activities on fuzzy numbers or can state they displayed the fuzzy analytics [3, 4], at that point and in addition time pass a wide range of fields of science utilize this idea of fuzzy set hypothesis and presented fuzzy functions, relations, gatherings, subgroups etc. The fundamental introductory theory of fuzzy DEs was first encompassed by Kaleva and Seikkala. Kaleva had point by point FDE to the extent the (H-auxiliary). Buckley and Feuring have decided an average meaning of a first request fuzzy starting quality issue. Recently twenty years prior in Japan a man name M. Sugeno presented the idea of fuzzy integrals [5, 6], at that point it's turning into an exploration situated subject [7-9]. The fields of fuzzy IEs have been growing fast in current couple of years [10-16].

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2. ANALYSIS OF MVIM TO FUZZY INTEGRO DIFFERENTIAL EQUATION

Considering the general n^{th} order nonlinear FIDE-2

$$x^{(n)}(t,\alpha)=f(t,\alpha)+\lambda\int_{a(t)}^{b(t)}k(t,s)N(x(s,\alpha))ds \tag{1}$$

with

$$x(0,\alpha)=(a_0,b_0),x'(0,\alpha)=(a_1,b_1),x''(0,\alpha)=(a_2,b_2),...,x^{(n-1)}(0,\alpha)=(a_{n-1},b_{n-1})$$

where $x^{(n)}(t,\alpha)$ is nth order derivative of fuzzy function $x(t,\alpha)$ also $f(t,\alpha)$ is fuzzy function given in advance, λ is constant parameter, $k(t,s)$ is kernel, N(x(s, α)) is the nonlinear term of FIDE-2.

The parametric representation of Eq. (1) is as follow

$$\begin{cases}\underline{x}^{(n)}(t,\alpha)=\underline{f}(t,\alpha)+\lambda\int_{a(t)}^{b(t)}k(t,s)N(\underline{x}(s,\alpha))ds\\ \overline{x}^{(n)}(t,\alpha)=\overline{f}(t,\alpha)+\lambda\int_{a(t)}^{b(t)}k(t,s)N(\overline{x}(s,\alpha))ds\end{cases}\qquad 0\le\alpha\le 1 \tag{2}$$

where $x^{(n)}(t,\alpha)=(\underline{x}^{(n)}(t,\alpha),\overline{x}^{(n)}(t,\alpha))$, $f(t,\alpha)=(\underline{f}(t,\alpha),\overline{f}(t,\alpha))$.

The correctional functional formula is read as

$$\begin{cases}\underline{x}_{n+1}(t,\alpha)=\underline{x}_n(t,\alpha)+\int_0^t\lambda(\zeta)[\underline{x}_n^i(\zeta,\alpha)-\underline{f}(\zeta,\alpha)-\lambda\int_0^\zeta k(\zeta,s)N(\underline{x}(s,\alpha))ds]d\zeta\\ \overline{x}_{n+1}(t,\alpha)=\overline{x}_n(t,\alpha)+\int_0^t\lambda(\zeta)[\overline{x}_n^i(\zeta,\alpha)-\overline{f}(\zeta,\alpha)-\lambda\int_0^\zeta k(\zeta,\alpha)N(\overline{x}(s,\alpha))ds]d\zeta\end{cases} \tag{3}$$

Inserting the Adomian polynomials in Eq. (3), we have

$$\begin{cases}\underline{x}_{n+1}(t,\alpha)=\underline{x}_n(t,\alpha)+\int_0^t\lambda(\zeta)[\underline{x}_n^i(\zeta,\alpha)-\underline{f}(\zeta,\alpha)-\lambda\int_0^\zeta k(\zeta,s)\sum_{n=0}^{\infty}A_n(\underline{x}(s,\alpha))ds]d\zeta\\ \overline{x}_{n+1}(t,\alpha)=\overline{x}_n(t,\alpha)+\int_0^t\lambda(\zeta)[\overline{x}_n^i(\zeta,\alpha)-\overline{f}(\zeta,\alpha)-\lambda\int_0^\zeta k(\zeta,\alpha)\sum_{n=0}^{\infty}A_n(\overline{x}(s,\alpha))ds]d\zeta\end{cases} \tag{4}$$

where A_n can be used for nonlinear term and A_n given by in Eq. (4)

$$A_n=\frac{1}{n}\frac{d^n}{d\lambda^n}\left[F\left(\sum_{i=0}^{n}\lambda^i x_i\right)\right]_{\lambda=0}\qquad n=0,1,2,3,..., \tag{5}$$

A_n called Adomian polynomials

Finally the solution is as follow

$$\begin{cases} \underline{x}(t,\alpha) = \lim\limits_{n\to\infty} \underline{x}_n(t,\alpha) \\ \overline{x}(t,\alpha) = \lim\limits_{n\to\infty} \overline{x}_n(t,\alpha) \end{cases}. \tag{6}$$

3. NUMERICAL APPLICATIONS

In this section, we have talked about the different sorts of non-linear FVIDEs and discover their solutions by utilizing MVIM.

Example 3.1 Consider the following nonlinear FFIDE

$$x'(t) = \left(\alpha(1-\frac{17}{24}t), \frac{1}{\alpha}(1-\frac{17}{24}t)\right) + \frac{1}{2}\int_0^1 stx^2(s)ds, \qquad 0 \le t, s \le 1,\ \ 0 \le \alpha \le 1 \tag{7}$$

with condition

$$x(0,\alpha) = (\alpha, \frac{1}{\alpha}). \tag{8}$$

The parametric equations are

$$\begin{cases} \underline{x}'(t) = \alpha(1-\frac{17}{24}t) + \frac{1}{2}\int_0^1 st\underline{x}^2(s)ds \\ \overline{x}'(t,\alpha) = \frac{1}{\alpha}(1-\frac{17}{24}t) + \frac{1}{2}\int_0^1 st\overline{x}^2(s)ds \end{cases} \tag{9}$$

The correctional functional is given as

$$\begin{cases} \underline{x}_{n+1}(t,\alpha) = \underline{x}_n(t,\alpha) - \int_0^t [\underline{x}'_n(\xi,\alpha) - \alpha + \frac{17}{24}\alpha\zeta - \int_0^1 s\zeta\, \underline{x}_n^2(s,\alpha)ds]d\xi \\ \overline{x}_{n+1}(t,\alpha) = \overline{x}_n(t,\alpha) - \int_0^t [\overline{x'}_n(\xi,\alpha) - \frac{1}{\alpha} + \frac{17}{24}(\frac{1}{\alpha})\zeta - \int_0^1 s\zeta\, \overline{x}_n^2(s,\alpha)ds]d\xi \end{cases} \tag{10}$$

Consequently,

$$\begin{cases} \underline{x}_0(t,\alpha) = \alpha \\ \overline{x}_0(t,\alpha) = \frac{1}{\alpha}, \end{cases} \tag{11}$$

$$\begin{cases} \underline{x}_1(t,\alpha)=\alpha+\alpha t-\dfrac{17}{48}\alpha t^2+\dfrac{\alpha^2 t^2}{8} \\ \overline{x}_1(t,\alpha)=\dfrac{1}{\alpha}+\dfrac{1}{\alpha}t-\dfrac{17}{48}(\dfrac{1}{\alpha})t^2+\dfrac{(\frac{1}{\alpha})^2 t^2}{8} \end{cases}, \tag{12}$$

$$\begin{cases} \underline{x}_2(t,\alpha)=\alpha+\alpha t-\dfrac{17}{48}\alpha t^2+\dfrac{143\alpha^2 t^2}{384}+\dfrac{\alpha^3 t^2}{64} \\ \overline{x}_2(t,\alpha)=\dfrac{1}{\alpha}+\dfrac{1}{\alpha}t-\dfrac{17}{48}(\dfrac{1}{\alpha})t^2+\dfrac{143(1/\alpha)^2 t^2}{384}+\dfrac{(1/\alpha)^3 t^2}{64} \end{cases}, \tag{13}$$

$\vdots,$

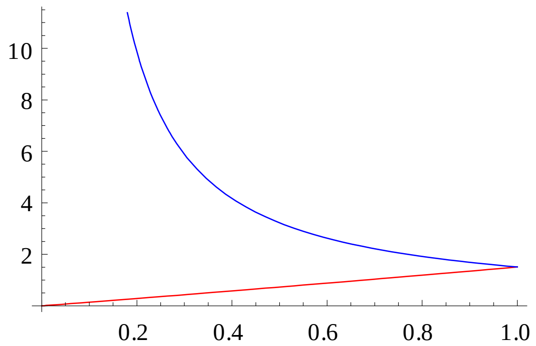

Figure 1 Shows the graphical behavior $(\underline{x}(t,\alpha),\ \overline{x}(t,\alpha))$ **with** $t=0.5$

Example 3.2 Consider the non-linear FVFIDE-2

$$x'(t,\alpha)=((\alpha^2+2\alpha),(3-\alpha))+\int_0^t\int_0^1 \chi^2(s,\alpha)dsdr, \tag{14}$$

with

$$x(0,\alpha)=(0,0). \tag{15}$$

The correctional function is read as

$$\begin{cases} \underline{x}_{n+1}(t,\alpha)=\underline{x}_n(t,\alpha)-\int_0^t[\underline{x}_n'(\zeta,\alpha)-(\alpha^2+2\alpha)-\int_0^\zeta\int_0^1\sum_{n=0}^{\infty}A_n(\underline{x}(s,\alpha))ds]d\zeta \\ \overline{x}_{n+1}(t,\alpha)=\overline{x}_n(t,\alpha)-\int_0^t[\overline{x}_n'(\zeta,\alpha)-(3-\alpha)-\int_0^\zeta\int_0^1\sum_{n=0}^{\infty}A_n(\overline{x}(s,\alpha))ds]d\zeta \end{cases} \tag{16}$$

Consequently,

$$\begin{cases} \underline{x}_1(t,\alpha)=(\alpha^2+2\alpha)t \\ \overline{x}_1(t,\alpha)=(3-\alpha)t \end{cases}, \tag{17}$$

$$\begin{cases} \underline{x}_2(t,\alpha)=(\alpha^2+2\alpha)t \\ \overline{x}_2(t,\alpha)=(3-\alpha)t \end{cases}, \tag{18}$$

$$\begin{cases} \underline{x}_3(t,\alpha)=(\alpha^2+2\alpha)t+(\alpha^2+2\alpha)^2\dfrac{t^2}{6} \\ \overline{x}_3(t,\alpha)=(3-\alpha)t+(3-\alpha)^2\dfrac{t^2}{6} \end{cases}, \tag{19}$$

$$\begin{cases} \underline{x}_4(t,\alpha)=(\alpha^2+2\alpha)t+(\alpha^2+2\alpha)^2\dfrac{t^2}{3} \\ \overline{x}_4(t,\alpha)=(3-\alpha)t+(3-\alpha)^2\dfrac{t^2}{3} \end{cases}, \tag{20}$$

$$\begin{cases} \underline{x}_5(t,\alpha)=(\alpha^2+2\alpha)t+(\alpha^2+2\alpha)^2\dfrac{t^2}{3}+(\alpha^2+2\alpha)^3\dfrac{t^2}{24} \\ \overline{x}_5(t,\alpha)=(3-\alpha)t+(3-\alpha)^2\dfrac{t^2}{3}+(3-\alpha)^3\dfrac{t^2}{24} \end{cases}, \tag{21}$$

$\vdots,$

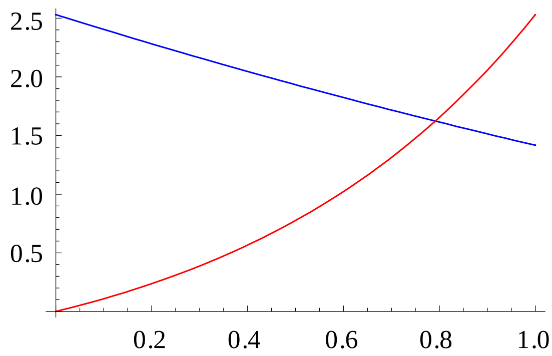

Figure 2 Shows the graphical behavior $(\underline{x}(t,\alpha),\ \overline{x}(t,\alpha))$ **with** $t = 0.5$

Example 3.3 Consider the non-linear FVIDE-2

$$x'(t,\alpha) = g(t,\alpha) + \int_0^t x^2(s,\alpha)ds, \tag{22}$$

where $\lambda = 1,\ \ 0 \le s \le x,\ 0 \le \alpha \le 1,\ k(t,s) = 1\ \ and\ \ g(t,\alpha) = (e^x(\alpha^2+1), e^x(4-\alpha))$ with condition

$$x(0,\alpha) = ((\alpha^2+1),(4-\alpha)). \tag{23}$$

The correction functional formula is read as

$$\begin{cases} \underline{x}_{n+1}(t,\alpha)=\underline{x}_n(t,\alpha)-\int_0^t[\underline{x}_n'(\zeta,\alpha)-e^{\zeta}(\alpha^2+1)-\int_0^{\zeta}\sum_{n=0}^{\infty}A_n(\underline{x}(s,\alpha))ds]d\zeta \\ \overline{x}_{n+1}(t,\alpha)=\overline{x}_n(t,\alpha)-\int_0^t[\overline{x}_n'(\zeta,\alpha)-e^{\zeta}(4-\alpha)-\int_0^{\zeta}\sum_{n=0}^{\infty}A_n(\overline{x}(s,\alpha))ds]d\zeta \end{cases} \quad (24)$$

Consequently,

$$\begin{cases} \underline{x}_1(t,\alpha)=(\alpha^2+1)+(\alpha^2+1)e^t+\frac{(\alpha^2+1)^2t^2}{2} \\ \overline{x}_1(t,\alpha)=(4-\alpha)+(4-\alpha)e^t+\frac{(4-\alpha)^2t^2}{2} \end{cases}, \quad (25)$$

$$\begin{cases} \underline{x}_2(t,\alpha)=(\alpha^2+1)+(\alpha^2+1)e^t+(\alpha^2+1)^2t^2+2(\alpha^2+1)^2e^t+\frac{(\alpha^2+1)^3t^4}{12} \\ \overline{x}_2(t,\alpha)=(4-\alpha)+(4-\alpha)e^t+(4-\alpha)^2t^2+2(4-\alpha)^2e^t+\frac{(4-\alpha)^3t^4}{12} \end{cases}, \quad (26)$$

$\vdots,$

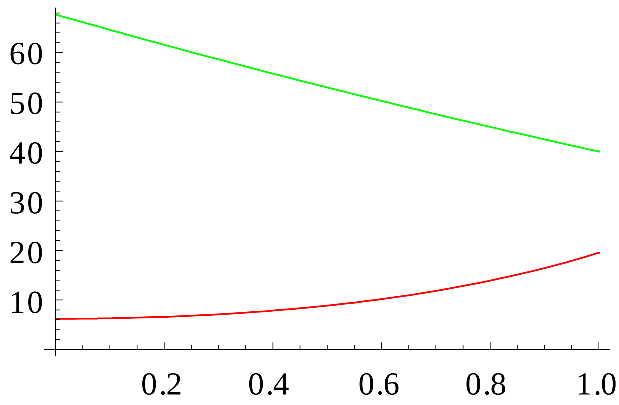

Figure 3 Shows the graphical behavior ($\underline{x}(t,\alpha), \overline{x}(t,\alpha)$ **) with** $t=0.5$

Example 3.4 Consider the following nonlinear FFIDE

$$x'(t)=\left(\alpha(-\frac{47}{45}-\frac{193}{90}t),\frac{1}{\alpha}(-\frac{47}{45}-\frac{193}{90}t)\right)+\frac{1}{12}\int_{-1}^{1}(t-s)x^2(s)ds,\ \ 0\le t,s\le 1,\ \ 0\le\alpha\le 1 \quad (27)$$

with condition

$$x(0,\alpha)=(\alpha,\frac{1}{\alpha}). \quad (28)$$

The parametric equations are

$$\begin{cases} \underline{x}'(t) = \alpha(-\frac{47}{45} - \frac{193}{90}t) + \frac{1}{2}\int_{-1}^{1}(t-s)\underline{x}^2(s)ds \\ \overline{x}'(t,\alpha) = \frac{1}{\alpha}(-\frac{47}{45} - \frac{193}{90}t) + \frac{1}{2}\int_{-1}^{1}(t-s)\overline{x}^2(s)ds \end{cases} \qquad (29)$$

The correctional functional formula is given as

$$\begin{cases} \underline{X}_{n+1}(t,\alpha) = \underline{X}_n(t,\alpha) - \int_0^t [\underline{X}'_n(\xi,\alpha) + \alpha\frac{47}{45} + \frac{193}{90}\alpha\zeta - \int_0^1 (\zeta - s)\,\underline{X}_n^2(s,\alpha)ds]d\xi \\ \overline{X}_{n+1}(t,\alpha) = \overline{X}_n(t,\alpha) - \int_0^t [\overline{X'}_n(\xi,\alpha) + -(\frac{1}{\alpha})\frac{47}{45} + \frac{193}{90}(\frac{1}{\alpha})\zeta - \int_0^1 (\zeta - s)\,\overline{X}_n^2(s,\alpha)ds]d\xi \end{cases} \qquad (30)$$

Consequently,

$$\begin{cases} \underline{x}_0(t,\alpha)=\alpha \\ \overline{x}_0(t,\alpha)=\frac{1}{\alpha}, \end{cases} \qquad (31)$$

$$\begin{cases} \underline{x}_1(t,\alpha)=\alpha-\frac{47}{45}\alpha t-\frac{193}{180}\alpha t^2+\frac{\alpha^2 t^2}{12} \\ \overline{x}_1(t,\alpha)=\frac{1}{\alpha}-\frac{47}{45\alpha}t-(\frac{193}{180\alpha})t^2-\frac{(\frac{1}{\alpha})^2 t^2}{12}, \end{cases} \qquad (32)$$

$$\begin{cases} \underline{x}_2(t,\alpha)=\alpha-\frac{47}{45}\alpha t-\frac{193}{180}\alpha t^2-\frac{193\alpha^2 t^2}{3240}+\frac{\alpha^3 t^2}{216}+\frac{94\alpha^2 t}{810} \\ \overline{x}_2(t,\alpha)=\frac{1}{\alpha}-\frac{47}{45\alpha}t-(\frac{193}{180\alpha})t^2-\frac{193(1/\alpha)^2 t^2}{3240}+\frac{(1/\alpha)^3 t^2}{216}+\frac{94(1/\alpha)^2 t}{810}, \end{cases} \qquad (33)$$

$\vdots$,

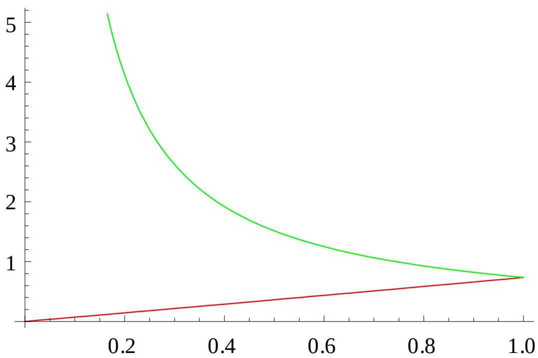

Figure 4 Shows the graphical behavior ($\underline{x}(t,\alpha)$, $\overline{x}(t,\alpha)$) with $t = 0.22225$

4. CONCLUSION

In this paper, utilization of modified VIM has been reached out for fathoming diverse sorts of non-linearfuzzy integro differential equations. For this reason we have demonstrated diverse numerical illustrations and discover their solutions by MVIM. This technique gives quick convergence; consequently by the use of less number of emphases we get estimated and also exactsolution. This procedure demonstrated solid and emotional potential from accomplished outcomes and can be stretched out to other complex nature of issues.

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