ORIGINAL PAPER

EFFECT OF SUCTION/BLOWING ON UNSTEADY MHD SLIP FLOW OF CASSON FLUID OVER A STRETCHING SHEET WITH THERMAL RADIATION

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Abstract. *Analysis of Unsteady Magneto hydro dynamic slip flow of Casson fluid past a vertically stretching sheet in presence of Suction and blowing with heat absorption and generation is carried out numerically. The resulting Partial differential equation is converted in to Ordinary differential equation using similarity conditions and these equations with boundary conditions are solved using fourth order Runge-Kutta method by applying MAT-LAB software. The resulting numerical values are plotted to study velocity and temperature profiles in order to make out changes in the boundary layer thickness by using different parameters. The digression in the velocity and temperature profiles are studied by varying the values of different governing parameters like Slip parameter (μ), Magnetic parameter (M), Unsteadiness parameter (A), Permeability parameter (λ), Casson parameter (β), Prandtle number (Pr), Radiation parameter (Nr), Heat generation and Absorption parameter (δ), and Suction / Blowing parameter (S) .It is observed that variation in slip parameter (μ) has a profound effect on velocity boundary layer thickness and on also on heat transfer rate in presence of suction / blowing.*

Keywords: *MHD, Slip flow, Casson fluid, mixed convection, Heat transfer, Suction and blowing.*

1. INTRODUCTION

The interrogation of Non-Newtonian fluid flows have been carried out in a very wide range by many researchers, the analysis of these are desirable due to their rigorous applications in modern technology and industries like in the manufacturing of plastic films, food stuffs, paper production industries etc.There are many fluids which show non-Newtonian behavior and have got lot of industrial applications therefore, the behavioral study of these industrial non-Newtonian fluids with respect to different boundary conditions has got lot of scope in research. It is impossible to describe the behavior of these fluids by a single constitutive equation due to its precarious nature. There are many important models in the literature like power law and grade two or three. The power law model describes only the shear thinning and shear thickening nature of the fluid but it does not properly predict the normal stress differences that are observed in phenomena like die-swell and rod climbing.

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The Casson fluid model is one type of non-Newtonian fluid model which fits rheological data compare to any general viscoelastic models. Jelly, honey, tomato ketchup, soap, human blood etc. are examples of Casson fluid. Casson fluid is a shear thinning liquid having infinite viscosity at zero rates of shear and a yield stress below which no flow occurs. The analysis of steady flow of a Casson fluid in a tube was first carried out by Fredrickson [1].Further a detailed study on flow of a viscoelastic fluid over a stretching sheet was performed by K. R.Rajagopal et al. [2]. Boyd et.al [3] investigated the non-Newtonian Boundary layer flow of Casson fluid by considering the steady and oscillatory blood flow. Sreenadh et al [4] analyzed the Casson fluid passing in to an inclined tube of non-uniform cross section in presence of multiple stenoses. The magneto hydrodynamic (MHD) boundary layer flow of an electrically conducting Casson fluid over an exponentially permeable shrinking sheet was investigated by Nadeem et al. [5]. Pradeep Porwal and V.H. Badshah [6] analyzed the influence of gravitational force on steady blood flow with Casson fluid along an inclined plane. The effect of suction and blowing on steady boundary layer flow of Casson fluid over an exponentially stretching sheet in the presence of thermal radiation with suction and blowing effect was explored by Pramanik [7]. A numerical study on Steady Mixed Convection Magneto hydrodynamic boundary layer Casson fluid flow and heat transfer in presence of suction and blowing was carried out by B.Lakshmi et.al [8].

Furthermore the fluid flow can also be considered as Unsteady because of the impetuous motion of the stretching sheet .So that the flow field, heat and mass transfer would vary with respect to time. The similarity analysis for the investigation of unsteady boundary layer flow and heat transfer over a linear stretching was studied by S.Sharidan [9].Using quasi linearization technique numerical investigation on unsteady boundary layer flow and heat transfer of fluid due to a stretching sheet was carried out by Wubshet Ibrahim et al [10]. Using optimal homotopy analysis method Abolbashari et al [11] investigated the entropy generation for the steady laminar non-Newtonian Nano fluid flow, its heat and mass transfer inclined due to stretching sheet in the presence of velocity slip and convective surface boundary conditions. C.S.K. Raju, N. et al. [12] studied the Magneto Hydrodynamic Casson Fluid flow, its heat and mass transfer behavior due to stretching over an Exponentially Permeable Surface.

The Suction and blowing effect plays a very important role in many engineering activities. Suction is applied to chemical processes to remove reactants. Blowing is used to add reactant, cool the surface, in the prevention of corrosion. The study on steady laminar flow, heat transfer characteristics of a continuously heated vertical plate with suction or injection was conducted by S. Aal-Sanea [13]. Later Anuar Ishak[14] studied the effect of boundary layer flow and heat transfer due to a stretching cylinder in presence of uniform suction and blowing .The numerical study on the unsteady MHD boundary layer flow over an exponentially stretching surface with suction in presence of thermal radiation, internal heat generation/absorption was performed by Elbashbeshy [15]. The boundary layer stagnation point flow heat transfer towards a shrinking sheet with thermal radiation in presence of suction and blowing was studied by Bhattacharya and Layek [16]. However, researchers have also paid attention on the study of formation of boundary layer flow and heat transfer on an exponentially shrinking sheet as it plays a vital role in many engineering processes.

The slip effect on boundary layer flow was studied by many researchers, the slip effect on the Magneto hydro dynamic boundary layer flow over an exponentially sheet in presence of thermal radiation with suction and blowing was studied by Mukhopadhyay and Swathi [17]. A.M. Megahed [18] investigated the MHD Viscous Casson fluid flow and heat transfer with second order velocity and thermal slip over a permeable stretching sheet in the presence of internal heat generation/absorption and thermal radiation. Oyelakin et al [19] presented an unsteady electrically conducting flow of Casson Nano fluid in the presence of slip and convective boundary conditions. Later A.Mahdy [20] conducted a numerical computation on the Unsteady MHD slip flow of a non-Newtonian Casson fluid over a vertical stretching sheet with suction/blowing effect.Recently B.Lakshmi et.al[21] studied Unsteady Slip Flow of Casson Fluid in Presence of Thermal Radiation and Heat Generation/Absorption with Suction/Blowing Effect.

The present study is aimed at Numerical and graphical analysis of the MHD slip flow and heat transfer of Casson fluid on a vertically stretching surface in presence of thermal radiation with suction and blowing effect, the Variation of velocity and temperature with respect to the change in the values of different parameters like slip parameter (μ), Unsteadiness parameter (A), Casson parameter (β), and Permeability parameter (λ) are considered and it is noticed that the variation in the values of these parameters has a profound effect on the velocity and thermal boundary layer thicknesses, and also on heat transfer rate.

2. PROBLEM FORMULATION

Consider an unsteady boundary layer slip flow and heat transfer of a Non-Newtonian fluid along a vertical stretching sheet. The surface is stretched along x-axis and y-axis is considered to perpendicular to it .The velocity of the stretching surface is $U_w = \left(\frac{U_O x}{1-\alpha t} \right)$ and the temperature distribution is $T(x,t) = T_\infty + \frac{T_O}{2\nu} x^2 (1-\alpha t)^{-3/2}$.where reference velocity is noted as U_O, α is a positive constant with dimension reciprocal time, t is the time, T_∞ is the ambient fluid temperature, T_O is the fluid temperature adjacent to the stretching surface. Along y-axis a uniform magnetic field B_O is applied which produces magnetic effect in the x-axis. The induced magnetic field is very small that it can be neglected since the magnetic Reynolds number is assumed to be small.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho} + g\beta_t (T - T_\infty) \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho C_p}\left(k\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} + Q(T - T_\infty)\right) \tag{3}$$

where u and v are the fluid velocity components with respect to x and y axes, υ is the kinematic viscosity, g is the gravitational field, β_t coefficient of thermal expansion, T is the fluid temperature, σ is the electrical conductivity, ρ is the density of the fluid, k is the thermal conductivity, Cp is the specific heat at constant pressure, q_r is the radiation heat flux.

The radiation heat flux is evaluated by Rosseland. Approximation as

$$q_r = \frac{4\sigma_s}{3K}\frac{\partial T^4}{\partial y} \tag{4}$$

where σ_s is the Stefan-Boltzman constant and K is the absorption coefficient, T^4 can be linearly expanded in a Taylor's series about T_∞ to get

$$T^4 = 4T_\infty^3 T - 3T_\infty^4 . \tag{5}$$

Substituting equations (4) and (5) in (3) we get

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho C_p}\left((k + \frac{16\sigma_s T_\infty^3}{3K})\frac{\partial^2 T}{\partial y^2} + Q_0(T - T_\infty) \right) \tag{6}$$

The following similarity conditions represented by equations (7),(8),(9) are introduced in order to reduce the above partial differential equations to Ordinary differential equations

$$\eta = \sqrt{\frac{U_O}{\upsilon(1-\alpha t)}}\, y \tag{7}$$

$$T = T_\infty + \frac{T_0 x^2 (1-\alpha t)^{-3/2}}{2\upsilon}\,\theta(\eta) \tag{8}$$

$$\psi = \sqrt{\frac{\upsilon U_0}{1-\alpha t}}\, x f(\eta) \tag{9}$$

2.1. BOUNDARY CONDITIONS

The boundary conditions of the governing equations are

$$y = 0,\ u = U(x,t) + L\left(\frac{\partial u}{\partial y}\right) \tag{10}$$

$$v = -V(x),\ T = T_w(x,t)$$

$$y \to \infty;\ u \to 0, T \to T_\infty .$$

The transformed boundary conditions are

$$\eta = 0, f' = 1 + \mu f'',\ f = S,\ \theta = 1 \tag{11}$$

$$\eta \to \infty,\ f' \to 0, \theta \to 0 .$$

$$\left(1 + \frac{1}{\beta}\right) f'''(\eta) - \left[f'(\eta)\right]^2 + f(\eta) f''(\eta) - A\left(\frac{\eta}{2} f''(\eta) + f'(\eta)\right) - Mf'(\eta) + \lambda\theta(\eta) = 0 \tag{12}$$

$$\theta''(\eta)-\frac{P_r}{1+N_r}\left[\frac{A}{2}\left(\eta\theta'+3\theta\right)+2f'(\eta)\theta(\eta)-f(\eta)\theta'(\eta)-2\delta\theta(\eta)\right]=0 \tag{13}$$

where $P_r=\frac{\rho\upsilon C_p}{k}$ is the Prandtl number, $\delta=\frac{Q_o(1-\alpha t)}{U_O C_p \rho}$ is the heat generation if(δ>0), and absorption (if $\delta<0$) parameter. $N_r=\frac{16\sigma_s T^3}{3kK}$ is the thermal radiation parameter, $A=\frac{\alpha}{U_0}$ Unsteadiness parameter, $M=\frac{\sigma B_O{}^2(1-\alpha t)}{U_O\rho}$ is the Magnetic parameter, $\lambda=\frac{g\beta T_O}{U_0^2}$ is the permeability parameter (where *U₀* is the reference velocity), $\mu=L\left(\frac{U_0}{\upsilon(1-\alpha t)}\right)^{-1/2}$ is the dimensionless parameter.

3. RESULTS AND DISCUSSION

Table 1. The value of f''_w for various values of unsteadiness parameter A for β=0 (which represents the case of Newtonian fluid) where M=0, P_r =0.01, S=0, N_r=0,μ=0, δ=0.

A	Sharidan et al. [9]	Swathi Mukhopadhya [18]	Present study
0.8	-1.261512	-1.261479	-1.2610
1.2	-1.378052	-1.377850	-1.3777

Table 1 gives the values of f''_w got by considering the case of Newtonian fluid (β=0) is compared with the previously obtained results of Sharidan et al [9], Swathi Mukhopadhyay [18] and it was found that the present results are in good agreement with the results obtained by[9] and [18].

The transformed ordinary differential equations (12) and (13) subjected to boundary conditions (12) are solved numerically using MATLAB software and the results are plotted as shown figs [1-16] to study the behavior of velocity and temperature profiles by considering various values for the parameters M, A, λ, β, Pr, Nr, μ ,δ, and S.

Figs. 1-2 gives the velocity and temperature profiles for different values of the slip parameter (μ) in presence of suction/blowing. The slip parameter μ measures the magnitude of slip takes place at the surface. From the fig 1 it is contemplated that, as the value of the slip parameter increases there will be diminution in the velocity distribution of the fluid, because as the value of μ increases this permits more fluid to flow over the surface due to which the flow retards near the stretching surface this declines fluid velocity which leads to the thinning of velocity boundary layer. Further as the value of the slip parameter tends to infinity (i.e. μ $\rightarrow\infty$) the velocity boundary layer will be more suppressed and finally it will disappear this is because if the numerical value slip parameter μ to tends to an undefined number infinity, then the fluid velocity and free stream velocity velocity will concur with each other.

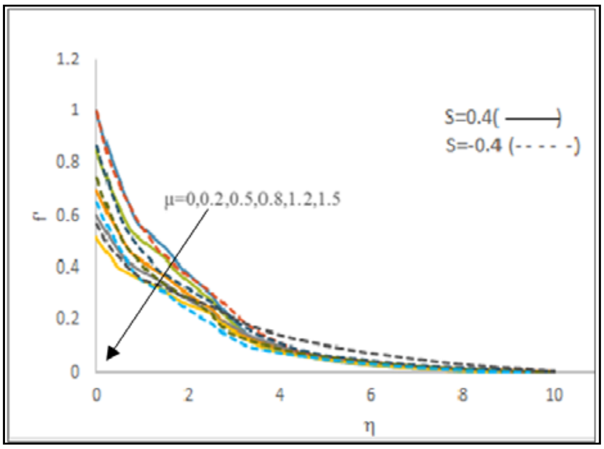


Figure 1. Velocity Profile for the slip parameter (μ) with M=0.6, A=0.1, β=0.5, P_r=6.2, N_r=5.

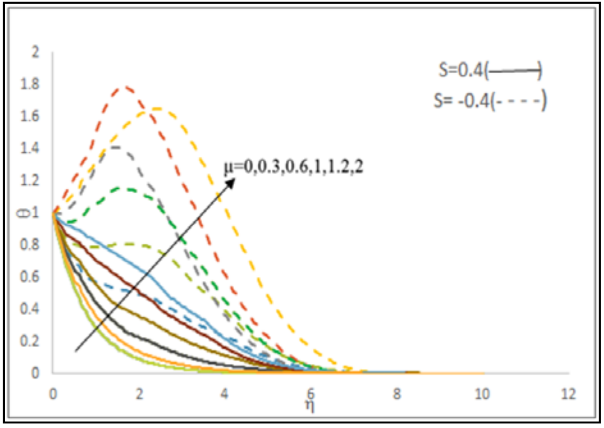
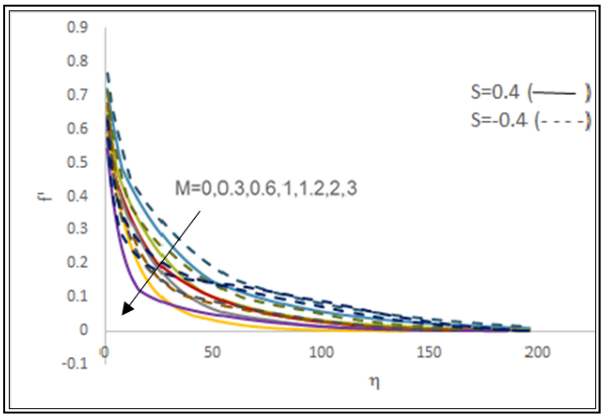


Figure 2. Temperature Profile for slip parameter (μ) with M=0.6, A=0.1, β=0.5, P_r=6.2, N_r =5.

Fig 2 shows that fluid temperature increases moderately with rise in value of slip parameter in presence of suction, but temperature increases abruptly in presence of blowing with increase in the value of the slip parameter μ this results in diminution of heat transfer rate.



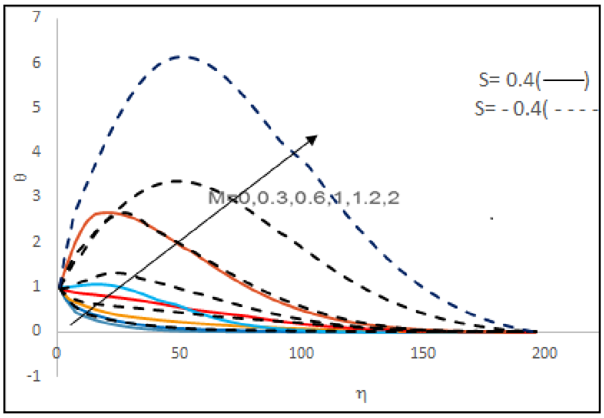


Figure 3. Velocity profile for the magnetic parameter (M) with δ=0.4, A=λ=0.1, β=0.5, P_r=6.2, N_r =5, μ=0.7.

Figure 4. Temperature Profile for the Magnetic parameter (M) with δ=0.4, A=λ=0.1, β=0.5, P_r=6.2, N_r =5, μ=0.7.

Figs. 3-4 illustrates the effect of Magnetic parameter (M) on velocity and temperature profiles respectively in presence of Suction (S>0) and blowing(S<0). It is clear from fig.3 that the increase in the Magnetic parameter decreases velocity of fluid flow and the denseness of the velocity boundary layer respectively. This is by the virtue of the fact that the application of Magnetic field develops a drag-like force called Lorentz force in the direction opposite to the fluid flow, which resists the fluid flow and results in the declination of velocity boundary layer thickness. Furthermore it is clearly depicted from Fig. 4 that the increase in the magnetic parameter has a tendency to enhance the thickness of the thermal boundary layer this impact is more dominant in case of S<0.

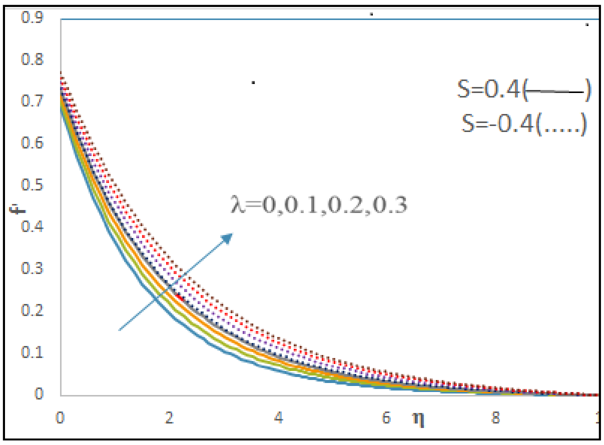


Figure 5.Velocity profile for permeability parameter (λ) with δ= -0.5, A=M=0.1, β=0.5, P_r=0.72, N_r=5, μ=0.7.

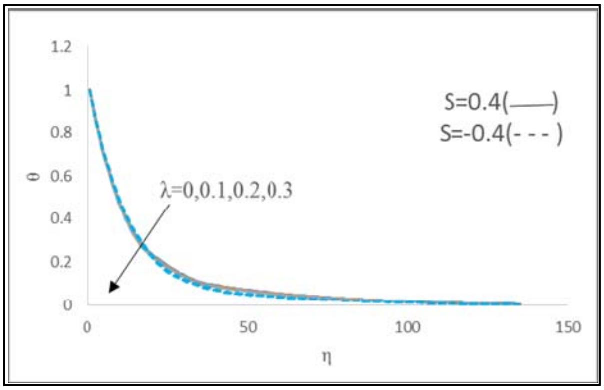


Figure 6.Temperature Profile for permeability parameter (λ) with δ= -0.5, A=M=0.1, β=0.5, P_r=0.72, N_r=5, μ=0.7.

The velocity and temperature profile for various values of permeability parameter (λ) is show in Figs. 5-6. It is noted that, with the rise in the value of λ, the velocity profile increases and the temperature profile decreases in presence of suction and blowing. But it can be seen from the above figure that the increase in the fluid velocity is more rapid in presence of blowing.

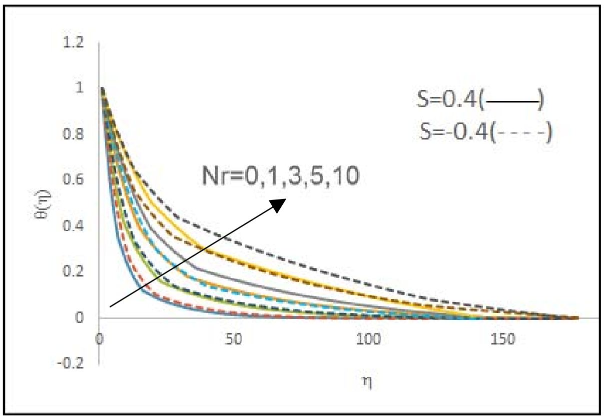


Figure 7. Temperature Profile for the Radiation parameter (Nr) with δ=-0.5, A=M=0.1, β=0.5, P_r=0.72, N_r=5, μ=0.7.

The above Fig. 7 defines the influence of thermal radiation (Nr) temperature profile. It is noted that when the value of Nr is increased the temperature increases. Also the rise in the value of thermal radiation parameter Nr has a tendency to increase the thickness of the thermal boundary layer this is because as ***Nr*** increases, the mean absorption coefficient k decreases this in turn enhances the parting of the radiative heat flux. Since the transformation of the radiative heat to the fluid is more this results in increase of fluid temperature.

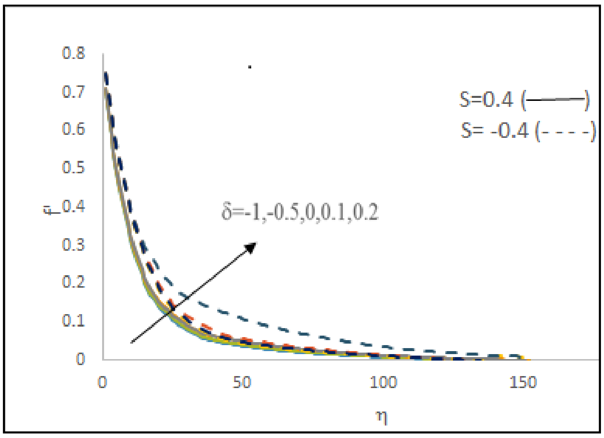


Figure 8. Velocity Profile for heat generation/absorption coefficient (δ) with M=λ=0.1, β=0.5, P_r=0.72, N_r=5, μ=0.7.

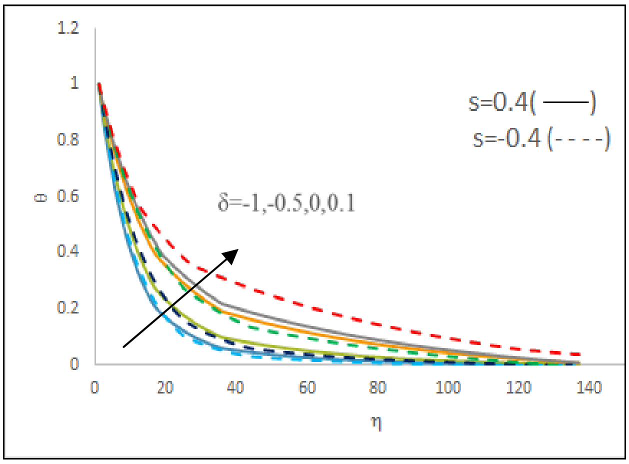


Figure 9. Temperature Profile for heat generation/absorption Coefficient (δ) with M=λ=0.1, β=0.5, P_r=0.72, N_r=5, μ=0.7.

The velocity and temperature profile for various values of heat generation and absorption coefficient is as shown in the Figs. 8-9. It has been observed that the fluid velocity and temperature increases with increases in the value of δ.

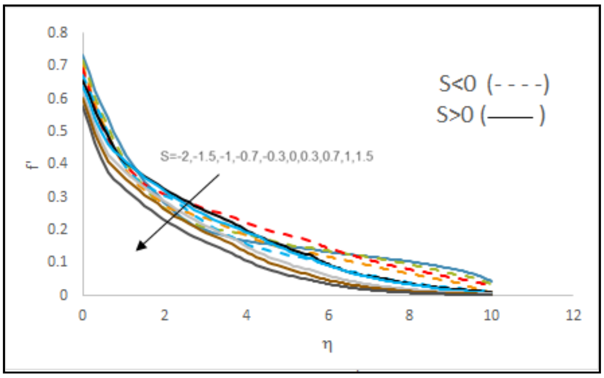


Figure 10. Velocity Profile for suction and blowing with M=λ=A=0.1, β=0.5, Pr=6.2, Nr =5, μ=0.7, δ=0.4.

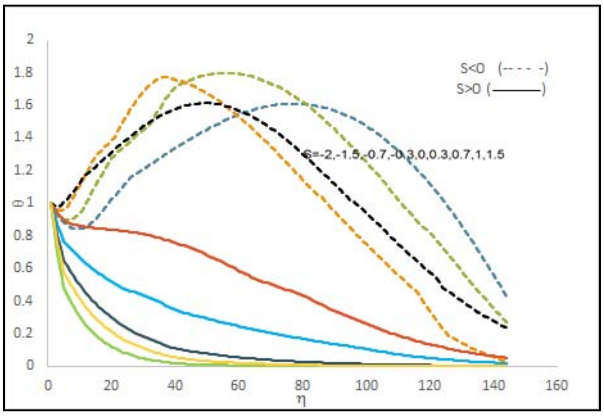


Figure 11.Temperature Profile for suction and blowing with M=λ=A=0.1,β=0.5, Pr=6.2, Nr =5, μ=0.7,δ=0.4.

The effect of suction (S<0) and Blowing (S>0) parameters on the velocity and temperature profiles have been studied numerically and the results are shown in fig 10 and 11.It can be seen that the Suction/Blowing have profound effect on fluid velocity and temperature. The fluid velocity is retarded with the increase in the wall suction S (S>0) because of the influence of viscous effect this in turn decreases the wall shear stress this causes the thinning of the boundary layer as shown in fig10. And fig 11 represents the temperature profile in presence of Suction and blowing .We can see that the increase in **S** (S>0) decreases the temperature of the fluid this suppresses the heat transfer rate. In other words it can be noted that the Suction parameter causes thinning thermal boundary layers. However the temperature increases abruptly in presence of blowing (S<0).This causes thickening of thermal boundary layer.

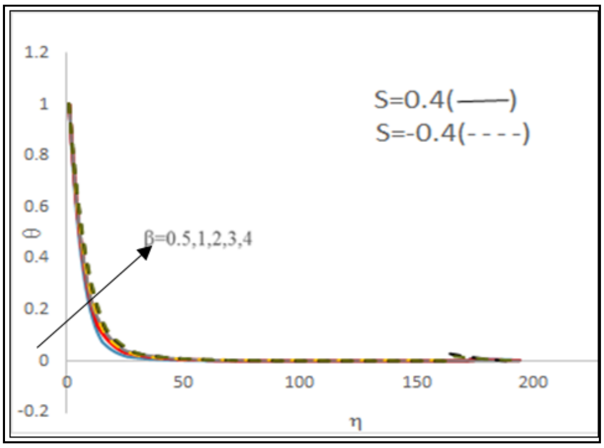


Figure 12. Temperature Profile for Casson parameter (β) with M=0.6, λ=0.A=0.7, β=0.5, P_r=0.72, N_r=0.1, μ=0.7, δ=0.4.

The above Fig. 12 shows the temperature profiles for numerical values of Casson parameter β. The temperature profile for β shows that the temperature of the fluid increases with increase in the value of the Casson parameter β in presence of suction and blowing this leads to the enhancement in the thickness in the thermal boundary layer.

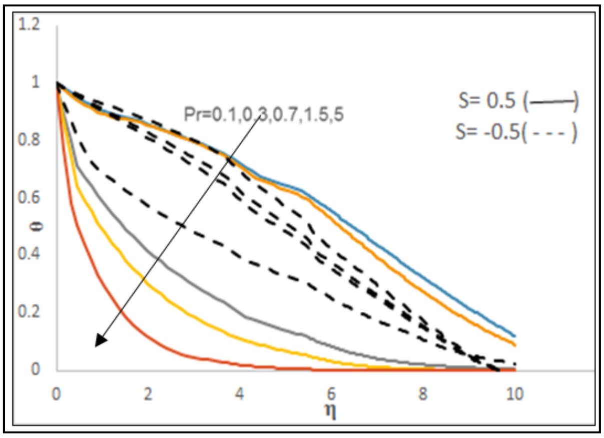


Figure 13. Temperature Profile for Prandtle number (P_r) with δ=0.4, A=λ=0.1, β=0.5, N_r=5, μ=0.7, M=0.6.

From the present analysis it is clear that the temperature fiel shows declaration for larger value of Prandtle number. That is the thermal boundary layer thickness decreases and it becomes thinner than the velocity boundary layer thickness this is because when the Prandtl number is high, viscous diffusion plays a vital role. The temperature field is more suppressed in presence of suction. This causes enhancement in the rate of heat transfer since rate of heat transfer varies with thermal boundary layer thickness.

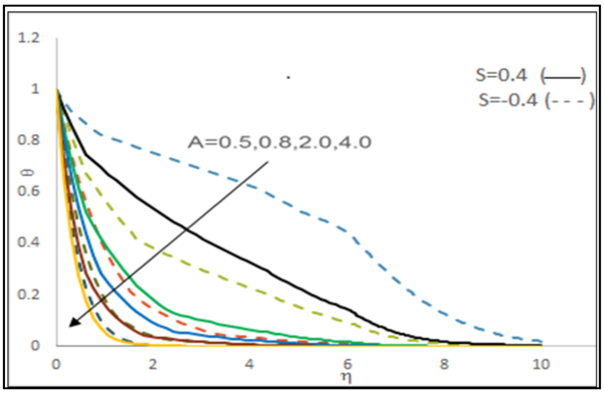


Figure 14. Temperature Profile for the unsteadiness parameter (A) with $\delta=0.4$, $A=\lambda=0.1$, $\beta=0.5$, $N_r=5$, $\mu=0.7$, $M=0.6$.

Fig. 14 shows that as the unsteadiness parameter A increases, the temperature profiles decrease. This effect is more significant in presence of Suction (S=0.4).The physical explanation for this is, when unsteadiness increases, heat transfer rate decreases or less heat will be transferred from the stretching sheet to the fluid. Hence the temperature profiles drops.

4. CONCLUSIONS

The conclusions of the above work can be outlined as follows:

- In presence of Suction and blowing, the velocity boundary layer thickness decreases with rise in the value of slip parameter.
- With increase in the value of slip the temperature increases and suppresses the heat transfer rate, this effect is more rapid in presence of blowing.
- With increase in the permeability parameter the fluid velocity increases and temperature decreases in presence of suction and blowing.
- The temperature boundary layer thicknesses increase as the Radiation parameter increases; heat transfer rate decreases with increasing Unsteadiness parameter (A).
- With increase in wall suction the velocity and thermal boundary layer becomes thin. However in presence of blowing the temperature increases rapidly by increasing the thickness of temperature boundary layer.

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