ORIGINAL PAPER

SOME NEW TRAVELLING WAVE SOLUTIONS OF EXTENDED MODEL EQUATION FOR SHALLOW WATER WAVES

QAZI MAHMOOD UL HASSAN[1], KAMRAN AYUB[2], M. YAQUB KHAN[2], MUAVIA MANSOOR[1], KHAN GUL[1]

Manuscript received: 05.01.2018; Accepted paper: 12.04.2018; Published online: 30.06.2018.

Abstract. *Nonlinear mathematical problems and their solutions attain much attention. In soliton theory an efficient tool to attain various types of soliton solutions is the Exp-function technique. Under study article is devoted to find soliton wave solutions of extended model equation for Shallow water waves via a reliable mathematical technique. By use of proposed technique we attain soliton wave solution of various types. The regulation of proposed algorithm is demonstrated by consequent numerical results and computational work. It is observed that under discussion technique is user friendly with minimum computational work; also we can extend for physical problems of different nature.*

Keywords: *Exp-function method, nonlinear equations, extended model equation for Shallow water waves, Maple 18.*

1. INTRODUCTION

In the last few years we have observed an extraordinary progress in soliton theory. Solitons have been studied by various mathematician, physicists and engineers for their applications in physical phenomena's. Firstly soliton waves are observed by an engineer John Scott Russell. Wide ranges of phenomena in mathematics and physics are modeled by differential equations. In nonlinear sciences it is of great importance and interest to explain physical models and attain analytical solutions. In the recent past large series of chemical, biological, physical singularities are feint by nonlinear partial differential equations. At present the prominent and valuable progress are made in the field of physical sciences. The great achievement is the development of various technique to hunt for solitary wave solutions of differential equations. In nonlinear physical sciences, an essential contribution is of exact solutions because of this we can study physical behaviors and discus more features of the problem which give direction to more applications.

At the disruption between chaos, mathematical physics and probability, factional calculus and differential equations are rapidly increasing branches of research. For accurate clarification of innumerable real-time models of nonlinear occurrence differential equation (NDEs) of nonlinear structure have accomplished great notice. Because of its recognizable implementation in branch of sciences and engineering it turn out to be a topic of great notice for scientists in workshops and conferences. In large fields such as porous structures dynamical processes in self-similar or solute transport and fluid flow, material viscoelastic

[1] University of Wah, Faculty of Basic Sciences, Wah Cantt., Pakistan. E-mail: qazimahmood@outlook.com.
[2] Riphah International University, Faculty of Engineering and Applied Sciences, Islamabad, Pakistan.
E-mail: kamranayub88@gmail.com.

theory, economics, bio-sciences, control theory of dynamical systems, geology, diffusive transport akin to diffusion, electromagnetic theory, dynamics of earthquakes, statistics, astrophysics, optics, probability, signal processing and chemical physics, and so on implementations of differential equation models [5-8] are beneficially exerted. As a consequence, hypothesis of nonlinear differential equations has shown fast growth [1-8].

In current times, to solve a nonlinear physical problems Wu and He [9] present an efficient technique called Exp-function method. The technique under study has prospective to deal with the complex nonlinearity of the models with the flexibility. It has been used as an effective tool for diversified nonlinear problems arising in mathematical physics. Through the study of literature exhibits that Exp-function method is extremely reliable and has been effective on a huge range of differential equations [10-24].

After He et al. Mohyud-Din enlarged the Exp-function method and used this algorithm to find soliton wave solutions of differential equations; Oziz used same technique for Fisher's equation; Yusufoglu for MBBN equations; for non-linear higher-order boundary value physical problems ; Momani for travelling wave solutions of KdV equation of fractional order; Zhu for discrete mKdV lattice and the Hybrid-Lattice system; Kudryashov for soliton solutions of the generalized evolution equations arising in wave dynamics ;Wu et al. for the expansion of compaction-like solitary and periodic solutions; Zhang for high-dimensional nonlinear differential equations. It is to be noticed that after applying under study technique and its modifications i.e. Exp-function method to any ordinary nonlinear differential equation, Ebaid [25] proved that $s = f$ and $r = e$ are the only relations of the variables involing in trial solution that can be acquired.

This article is keen to the soliton like solutions of extended model equation for Shallow water waves by applying a novel technique, the applications of under study nonlinear equation are very vast in different areas of physical sciences and engineering. Additionally, such type of equation found in different physical phenomenon related to fluid mechanic, astrophysics, solid state physics, chemical kinematics, ion acoustic waves in plasma, control and optimization theory, nonlinear optics etc.

2. ANALYSIS OF THE METHOD

We consider the general nonlinear partial differential equation of the type

$$P(\Psi, \Psi_t, \Psi_x, \Psi_y, \Psi_{xx}, \Psi_{yy} .....) = 0. \tag{1}$$

Invoking a transformation

$$\xi = kx + my + nz + \Omega t. \tag{2}$$

Here k, m, n, Ω are all constants with $k, \Omega. \neq 0$. Equation (1) can be converted into ordinary differential equation of the form

$$Q(\Psi, \Psi', \Psi'', \Psi''', ...) = 0. \tag{3}$$

Where prime signify differentiation w.r.t ξ.

Permitting to Exp-function technique, the solitary wave solutions can be articulated in the subsequent procedure

$$\Psi(\xi) = \frac{\sum_{i=-e}^{f} a_i \exp[i\xi]}{\sum_{j=-r}^{s} b_j \exp[j\xi]}. \tag{4}$$

In last equation r, e, s and f are the positive integers and need to be calculated, a_i and b_j are constants. Equation (4) can be expressed in the subsequent corresponding way

$$\Psi(\xi) = \frac{a_{-e} \exp(-e\xi) + ... + a_f \exp(f\xi)}{b_{-r} \exp(-r\xi) + ... + b_s \exp(s\xi)}. \tag{5}$$

The outcome of equivalent formulation is an imperative and vital analytic solutions of the governing differential equation. Calculating value of varibles involving in the trial solution by using [25], finally results in

$$r = e, s = f. \tag{6}$$

3. SOLUTION PROCEDURE

Ablowitz et al, Hirota and Satsuma established two model equation for shallow water waves. In this work we extend the model to other two completely integrable models for shallow water waves. Ablowitz et al introduced the equation:

$$v_t - v_{xxt} - 4vv_t - 2v_x \int v_t dx + v_x = 0.$$

Consider the following Shallow water equation

$$v_t - v_{xxt} - 4vv_t - 2v_x \int v_t dx + v_x + v_{xxx} + 6vv_x = 0. \tag{7}$$

Using the potential $v = \Psi_x$ we have

$$\Psi_{xt} - \Psi_{xxxt} - 4\Psi_x \Psi_{xt} - 2\Psi_{xx}\Psi_t + \Psi_{xx} + \Psi_{xxxx} + 6\Psi_x \Psi_{xx} = 0. \tag{8}$$

Using wave transformation (2), equation (8) can be converted to an ordinary differential equation

$$\Omega k\Psi'' + k^3\Omega\Psi^{(iv)} - 6k^2\Omega\Psi'\Psi'' + k^2\Psi'' + k^4\Psi^{(iv)} + 6k^3\Psi'' = 0. \tag{9}$$

Where the prime denotes the differentiation with respect to ξ . The solution of the equation (8) can be expressed in the form, equation (5). To determine the value of e, f and r, s by using [25], we have

$$r = e, s = f.$$

We choose the values of e, f and r, s, but we will illustrate that the final solution does not strongly depend upon the choice of values of e, f and r, s. For simplicity, we set $r = e = 1$ and $s = f = 1$

$$\Psi(\xi) = \frac{a_{-1}\exp[-\xi] + ... + a_1 \exp[\xi]}{b_{-1}\exp[-\xi] + ... + b_1 \exp[\xi]}. \tag{10}$$

Substituting equation (10) into equation (8), we have

$$\frac{1}{A}\begin{bmatrix} e_5 \exp(5\xi) + e_4 \exp(4\xi) + e_3 \exp(3\xi) + e_2 \exp(2\xi) + e_1 \exp(\xi) + e_0 \\ + e_{-1}\exp(-\xi) + e_{-2}\exp(-2\xi) + e_{-3}\exp(-3\xi) + e_{-4}\exp(-4\xi) + e_{-5}\exp(-5\xi) \end{bmatrix} = 0. \tag{11}$$

Where $A = (b_1 \exp(\xi) + b_0 + b_{-1}\exp(-\xi))^5$ and e_{i} are the constants acquired by Maple 18. Associating the coefficients of $\exp(i\xi)$ equal to zero, we gain

$$e_i = 0, i = 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5. \tag{12}$$

The equation (7) is satisfied by following solution sets.

1st Solution set:

$$\left\{\Omega = \frac{k(1+k^2)}{k^2-1}, a_{-1} = -\frac{b_{-1}(2kb_0 - a_0)}{b_0}, a_0 = a_0, a_1 = 0, b_{-1} = b_{-1}, b_0 = b_0, b_1 = 0\right\}$$

The above values yields solution given as

$$\Psi(x,t) = \frac{\frac{b_{-1}(2kb_0 - a_0)}{b_0} e^{-kx-\Omega t} + a_0}{b_{-1} e^{-kx-\Omega t} + b_0}.$$

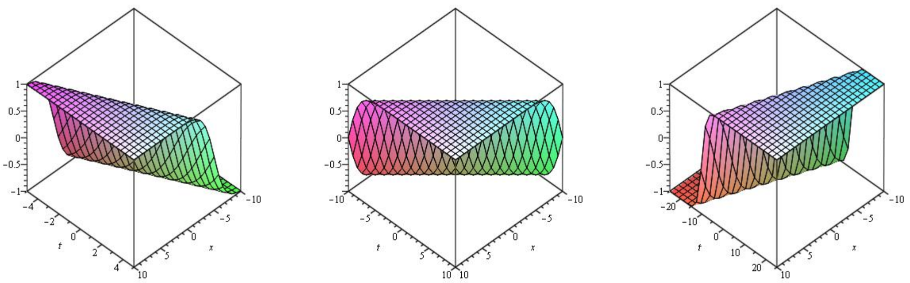

Figure 1. Travelling wave solution for different values of parameters.

2nd Solution set:

$$\left\{\begin{array}{l} \Omega = \frac{k(1+k^2)}{k^2-1}, a_{-1} = \frac{(-6ka_1b_0a_0b_1^2 + 4ka_1^2b_1b_0^2 - a_1^3b_0^2 - b_1^2a_1a_0^2 + 2a_1^2b_1b_0a_0 + 4k^2b_1^3b_0a_0 - 4k^2b_1^2b_0^2a_1 + 2a_0^2b_1^3k)}{4k^2b_1^4}, \\ a_0 = a_0, a_1 = a_1, b_{-1} = -\frac{(2kb_0a_0b_1^2 - 2ka_1b_1b_0^2 + a_1^2b_0^2 + b_1^2a_0^2 - 2a_1b_1b_0a_0)}{4k^2b_1^3}, b_0 = b_0, b_1 = b_1 \end{array}\right\}$$

The above values yields the solution

$$\frac{\left(-6ka_1b_0a_0b_1^2+4ka_1^2b_1b_0^2-a_1^3b_0^2-b_1^2a_1a_0^2+2a_1^2b_1b_0a_0+4k^2b_1^3b_0a_0-4k^2b_1^2b_0^2a_1+2a_0^2b_1^3k\right)}{4k^2b_1^4}e^{-kx-\Omega t}$$

$$\Psi\left(x,t\right)=\frac{+a_0+a_1e^{kx+\Omega t}}{-\dfrac{\left(2kb_0a_0b_1^2-2ka_1b_1b_0^2+a_1^2b_0^2+b_1^2a_0^2-2a_1b_1b_0a_0\right)}{4k^2b_1^3}e^{-kx-\Omega t}+b_0+b_1e^{kx+\Omega t}}.$$

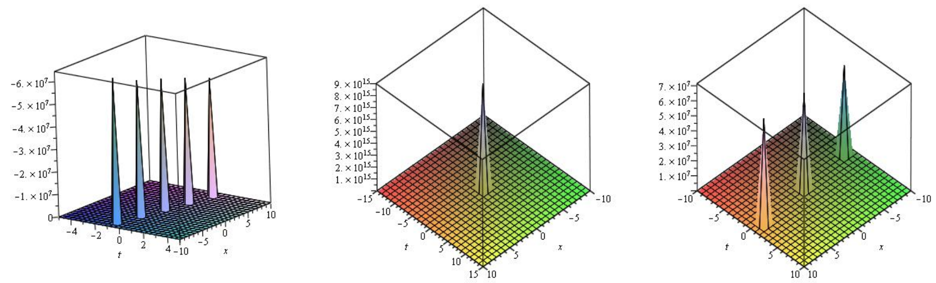

Figure 2. Travelling wave solution for different values of parameters.

3rd Solution set:

$$\left\{\Omega=\frac{k\left(1+4k^2\right)}{4k^2-1},a_{-1}=-\frac{b_{-1}\left(4kb_1-a_1\right)}{b_1},a_1=a_1,a_0=0,b_{-1}=b_{-1},b_1=b_1,b_0=0\right\}$$

The above values yields solution of the form

$$\Psi(x,t)=-\frac{4b_{-1}e^{-kx-\Omega t}kb_1-b_{-1}e^{-kx-\Omega t}a_1-a_1e^{kx+\Omega t}b_1}{b_1\left(b_{-1}e^{-kx-\Omega t}+b_1e^{kx+\Omega t}\right)}.$$

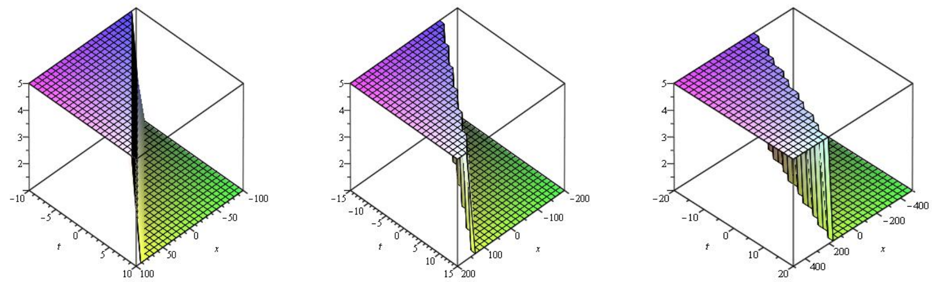

Figure 3. Travelling wave solution for different values of parameters.

4. RESULTS AND DISCUSSION

From the above figures (Figs. 1-3) we note that soliton is a wave which preserves its shape after it collides with another wave of the same kind. By solving extended model equation for Shallow water waves, we attain desired solitary wave solutions for different value of random parameters. The solitary wave moves toward right if the velocity is positive or left directions if the velocity is negative and the amplitudes and velocities are controlled by various parameters. Solitary waves show more complicated behaviors which are controlled by various parameters. Figures signify graphical representation for different values of parameters. In both cases, for various values of parameters, we attain identical solitary wave solutions which obviously comprehend that final solution does not effectively based upon these parameters. So we can choose arbitrary values of such parameters. Since the solutions does not strongly depends on additional free parameters, we choose different parameters as input to our simulations.

5. CONCLUSION

This article is devoted to attain, test and analyze the novel soliton wave solutions and physical properties of nonlinear partial differential equation. For this, extended model equation for Shallow water waves is considered and we apply Exp function method. We attain desired soliton solutions of various types for different values of parameters. It is guaranteed the accuracy of the attain results by backward substitution into the original equation with Maple 18. The scheming procedure of this method is simplest, straightforward and productive. We conclude that the under study technique is more reliable and have minimum computational task, so widely applicable. In precise we can say this method is quite competent and much operative for evaluating exact solution of NLEEs. The validity of given algorithm is totally hold up with the help of the computational work, the graphical representations and successive numerical results. Results obtained by this method are very encouraging and reliable for solving any other type of NLEEs. The graphical representations clearly indicate the solitary solutions.

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