ORIGINAL PAPER

SUPPLY COST MINIMIZATION USING MATHEMATICAL MODELS AND METHODS OF OPTIMIZATION

CONSTANTIN AURELIAN IONESCU[1], MIHAELA DENISA COMAN[2*], GABRIEL CUCUI[3], SORINA GEANINA STANESCU[2]

Manuscript received: 22.02.2018; Accepted paper: 30.04.2018; Published online: 30.06.2018.

Abstract. *Cost minimization represents a priority for the cost accounting of the economic entities. To optimize the supply cost, the supply budget management requires the completion of several stages, namely: the study of the supply policies, the elaboration of the supply budget based on the optimal policies determined by means of the optimization templates and of the supply costs. A supply budget optimization is needed periodically, relying on two policies: the budget policy for the economic period of re-supply and the budget policy for the economic lot to be supplied. In this study we intend to highlight the importance of mathematical models used for supply cost optimization in supply budget organization and management, using determining policies.*

Keywords: *mathematical models, supply cost minimization, supply policies, supply budget.*

1. INTRODUCTION

Mathematical optimization may be described as a science of determining the best solution to mathematically defined problems [21], including ones with economic importance, namely minimizing supply cost. In the majority of economic entities, there appear problems regarding supply, stock management, transport, production distribution and, more recently, cost minimization in supply budget organization and management. Supply represents [12] the process by which the material resources necessary for the company to carry out its activity are provided, under optimum conditions, which process allows the achievement of the company goals through the prism of the assurance of minimum costs. Cost determination represents a decisive factor for the adoption of decisions, as well as for the planning of the future activities [7]. Thus, we may notice a permanent concern of the managers for the determination and the awareness, as accurately as possible, of costs, and for the cost minimization methods, and, within our research, for the minimization of the supply costs, as well [8]. Depending on the complexity of the economic process, stock management develops a large scope, as it includes both issues regarding the management, the optimization of the distribution of stocks within the territory, the distribution and accounting per owners, and issues regarding the reception, storage, maintenance, monitoring, control, re-distribution and use [1]. The role of the stock is

[1]Hyperion University of Bucharest, Faculty of Economics, Bucharest, Romania.
E-mail: ionescuaurelian89@gmail.com.
[2]Valahia University of Targoviste, Institute of Multidisciplinary Research for Science and Technology, 130004 Targoviste, Romania. E-mail: cmndenisa@yahoo.com ([*]Corresponding author); geaninastanescu@yahoo.com.
[3]Valahia University of Targoviste, Faculty of Economic Sciences, 130004 Targoviste, Romania.
E-mail: gabicucui@yahoo.com.

relevant through the fact that it provides certainty and guarantee of a continuous supply of the manufacturing process and a rhythmic sale of the outputs [9]. The reasons leading to the creation of stocks are numerous, namely: the capital invested for stock building is easy to highlight; assurance of the manufacture process continuity; anticipation of a price increase and of the situations when the orders honored by suppliers from other localities cannot be directly introduced in the manufacture process. To be able to achieve a good management of the supply budget, an optimization program for the supply modalities through the integration by the supply department of the key parameters is also necessary [14], namely: stock evolution in time, consumption rhythm and supply time, safety stock and zero stock. Of course, the integration of these four key parameters shall lead to the optimization of the supply modality and it would implicitly contribute to the good management of the supply budget, yet, technologically, these parameters present a quantifiable variability, which is actually extremely necessary as information, to reach the goal proposed by this paper: supply cost optimization in supply budget organization and management.

The literature review regarding supply optimization is plenty of studies regarding aspects of logistic support, supply costs optimization under different constrains [2-4, 18, 23]. Within the scientific research for the supply cost minimization in the supply budget organization and management, for the beginning, we shall consider the theoretical context [5] existing in the literature, related to the supply modality optimization. It refers to the optimization of inputs depending on outputs, avoiding on the one hand the overstock generating additional storage costs, and on the other hand, stock rupture, which can disturb the rest of the activity. Between two successive deliveries, the stock level decreases depending on the output rhythm. The specialty practice [10] recommends the use of the classical saw-like chart. Considering the somewhat constant rhythm of the outputs, the resulting curve-shaped chart, difficult to represent, can be approximated by a straight line (Fig. 1).

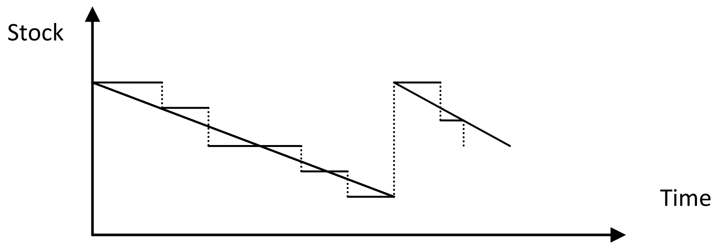

Figure 1. Approximate evolution of stock .

Considering: T = time period when consumption is constant; C = stainless steel consumption; N = number of orders. If there is made the decision that two orders be achieved during the time period T, it results that each time $^{C}/_{2}$ should be ordered, and the owned average stock is to be $^{C}/_{4}$ (Fig. 2). To conclude, the obtained result for achieving the two orders may be synthesized by the following formula:

$$\text{AVERAGE STOCK} = \frac{C}{2N} \quad (1)$$

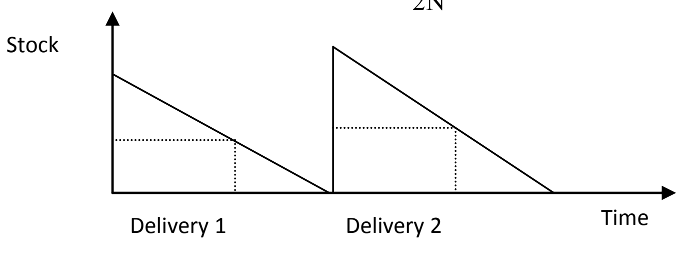

Figure 2. Average stock depending on the number of orders.

To avoid overstock or stock rupture, a good management of the consumption rhythm and of the supply period must be realized, usually through the determination of the following indicators: maximum stock and safety stock. Maximum stock represents the level that must not be exceeded to avoid supplementary storage costs [10]. Safety stock represents the critical stock up to which delivery may be made reaching the zero stock, after which any delay in delivery may lead to a stock rupture. Also, within the stock structure we may distinguish, on the one hand, the *active stock* which is moving between maximum and minimum, and on the other hand the safety stock which, depending on the anticipated consumption rhythm, avoids the stock rupture (Fig. 3) [5].

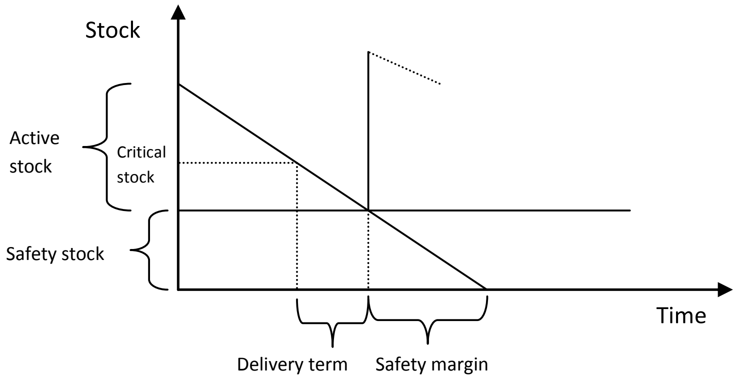

Figure 3. Active stock and safety stock.

Determination of the safety stock level may be done either depending on the law of demand corroborated with the existing offer, or depending on the differences noted between clients demands and anticipated outputs [14]. Considering: d_i = the noted differences; I_r = real outputs; I_p = anticipated outputs. Concomitantly:

$$d_i = I_r - I_p \tag{2}$$

More and more companies are working at present with "zero stock" [7], considering that it represents the best means of supply cost decrease. Such companies have a management goal differently budgeted than the companies working with stocks. While such companies aiming at zero stock have as a main concern the organization of their production, companies working with stocks have as their main concern the storage cost minimization. Supply budget management requires the determination of a supply programme by means of supply policies, having as a goal the supply cost minimization. Numerous issues may arise during this stage connected to the application of the supply policies, which issues often generate an array of variables concerning: the demand of new elements, as stocks (existing or not in storage or to be re-supplied); costs that need to be optimized, and the global cost needing to be minimized. From one case to the next, suitable supply policies are to be used, respectively deterministic or probabilistic policies, depending on their feasibility [5]. Our research is being carried out in a stainless steel manufacture company, for 2 standards of stainless steel products (ST_1, ST_2).

2. MATHEMATICAL MODELS AND METHODS USED FOR SUPPLY MINIMIZATION

Considering the certainty in supply of the product A under the form of a uniform demand and the existence of an annual stock, the stock demand becomes certain. In the present case, we are in the Wilson pattern as a deterministic supply policy, one of the oldest deterministic templates, and also the most used pattern [22]. In order to determine the optimal inventory level, several mathematical models has been developed over time [18], but the most used one is the EOQ model (developed by F.W.Haris in 1913 and thoroughly by R.H. Wilson), also known as the Wilson EOQ model [13, 17].

This scientific research pattern requires that we should know the economic lot to be supplied or to be ordered in a time period of one year or, eventually, the optimum number of orders to be realized in a time period of one year (360 days). In case of the economic entity analyzed we intend, according to the previous hypothesis of certainty in supply of the A product as a uniform demand (which product is actually included in the annual consumption for the achievement of the first standard of product, ST_1), to optimize the optimum supply rhythm and the optimum quantity necessary to be supplied. Abbreviations: CA = total annual order; Q = economic lot to be supplied; M = optimum number of orders; T = optimum time period for supply. It results that:

$$\mathrm{M} = {}^{360}\!/_{T} = {}^{CA}\!/_{Q} \tag{3}$$

Under the conditions of the economic entity analyzed, respectively those of an annual consumption of A products in an amount of 162,000 m.u., respectively 13,500 pieces, at the unit price of 12 m.u., the annual percentage for the stock support is 15%, and the cost for putting into production an order is of 300 m.u. The following are to be determined:

- *Optimum supply rhythm*; Abbreviations: CLC = the cost for putting into production an order; CDS = the cost for stock support; CT = total cost. Concomitantly:

$$CLC = 300\, x\, N \tag{4}$$

where,

$$\text{CDS}=\frac{162.000 \times 0.15}{2N} = \frac{12.150}{N}$$

respectively:

$$\text{CT=CLC+CDS=300N+}\frac{12.150}{N}$$
$$N = 6.36 \approx 6\, orders$$

- *Optimum quantity to supply:*

$$\text{CLC=}\frac{13.500}{\text{Q}}\text{ x 300= }\frac{4.050.000}{\text{Q}}$$

$$\text{CDS=}\frac{\text{Q x 12 x 0.15}}{2}\text{=0.9 Q}$$

respectively:

$$\text{CT=CLC+CDS=}\frac{4.050.000}{Q} + 0{,}9Q$$
$$Q = 2{,}121 \approx 2{,}250\, pieces/order$$

Considering the necessary mathematical rounding for the achievement of an integer number of orders, it results that the total cost minimization for the stock re-supply, with A-type products in a quantity of 13,500 pieces is realized by 6 orders of 2,250 pieces per order.

The use of deterministic policies, particularly of the Wilson pattern supposes the occurrence of peculiarities that should be considered, such as [10]: occurrence, along the circuit, of some trade discounts such as: discount, reduction or a decreasing rate, which may lead to the modification of the optimum supply rhythm and of the optimum quantity to supply. Under these circumstances, the optimum decision is made depending on the economy achieved, as a result of the trade discounts applied, compared to the increase of the storage costs for the additional quantities supplied; the authorized stock rupture may cause different consequences, and the quite complex calculation templates lead, in their great majority, to a calculation of the shortage cost; the spontaneous exits from stock may be, partly or totally covered, either by means of the calculation methods for the safety stock or by scheduling inputs into the stock, using policies of determination of the supply optimum [16,19], particularly Wilson pattern.

3. THE APPLICATION OF PROBABILISTIC POLICIES

In case of application of probabilistic policies, the supply occurs under conditions of uncertainty, since the demand is non-uniform, namely randomly and continuous [14]. As policies are multiple and different, for the economic entity analyzed, we shall study a pattern of minimization of the average cost for stock management, in a situation in which the demand is randomly. We shall start from the following hypotheses: demand is a discrete randomly variable, assuming different values with multiple probabilities; the time periods between two successive deliveries are constantly determined; the supply demand is equally distributed throughout the time period; the storage and shortage costs per product, per time period and per non-delivered product are known; the delivery term is considered a sure indicator.

According to the previously mentioned hypotheses - the determination of the optimum supply of product B should be calculated, both for the determination of the quantity to be ordered, and for the minimization of the average management cost of the stock; this product is included in the annual consumption for the achievement of the second standard of the finished product - ST_2. We are considering two distinctive cases, respectively:

A. Case of a demand under the stock at the beginning of the time period under analysis; Within the analyzed case of the economic entity analyzed, the products named in the supply programme, respectively A and B, included in the material consumption for the manufacture of the finished products ST_1 and ST_2 do not record a demand under the stock at the beginning of the time period under analysis. The theoretical approach in this situation may be useful only for certain materials included in the consumption of the sectors and services providing auxiliary activities, since they are not part of the list of materials named for the directly productive consumption. Therefore, we shall not develop a case analysis here.

B. Case of a demand over the stock at the beginning of the time period under analysis; Abbreviations: C_{ad} = discrete randomly demand; Q = stock at the beginning of the time period under analysis; p_n = probability for the time period under analysis; T = time period of storage; S_m = average stock; C_s = total cost of stock; C_{sp} = unit cost of stock T_1 ; C_{pp} = unit cost of stock T_2; C_p = shortage cost per unit without delivery, and per time period under analysis.

$$C_{ad} > Q \tag{5}$$

Within the limits of the approached hypotheses [5], we may note that we are dealing with a stock cost borne by the time period T_1 and a shortage cost for the time period T_2 (Fig. 4).

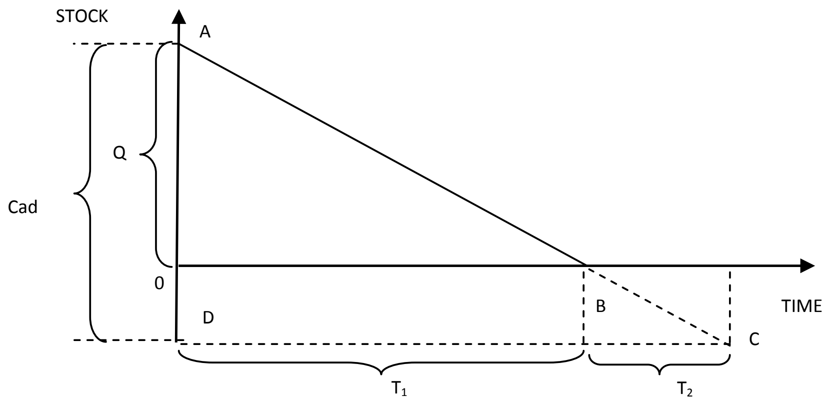

Figure 4. Demand over the stock at the beginning of the time period under analysis.

Starting from the average stock (S_m) determined for that time period T_1, respectively:

$$S_m = \frac{Q}{2} \times C_{sp} \times \frac{T_1}{T_2} \tag{6}$$

We can determine the total cost for storage (C_s):

$$C_s = C_{sp} \times \frac{Q^2}{2C_{ad}} \tag{7}$$

For the time period T_2, the total shortage cost is determined according to the formula:

$$C_p = C_{pp} \times \frac{(C_{ad} - Q)}{2} \times \frac{T_2}{T} \tag{8}$$

or by successive replacements:

$$C_p = C_{pp} \times \frac{(C_{ad} - Q)^2}{2\, C_{ad}} \tag{9}$$

Both cases of storage and of shortage must be balanced by the probability index (p_i), depending on the degree of accomplishment of the demand (C_{ad}). In the case of the economic entity analyzed, we have the following elements and information for the case study analysis: probability (pi) of the chain of randomly demands (Table 1.); storage cost of 20 m.u. per product and time period under analysis (2 days); shortage cost of 30 m.u. per non-delivered product and per time period under analysis (2 days).

Table no. 1. Probability of randomly demands

C_{ad}	1	2	3	4	Comments
Probability (p_i)	0.1	0.2	0.4	0.6	x

There arises the issue of determining the quantities to be supplied at the beginning of each time period under analysis under the conditions of minimization of the stock management cost. Abbreviations: C_{pm} = average total shortage cost; C_{pm1}= storage cost with no shortage; C_{pm2}= storage cost including shortage; C_{pm3}= shortage cost with shortage; where:

$$C_{pm1} = 20 \sum (Q - \frac{C_{ad}}{2}) \times p_i \tag{10}$$

$$C_{pm2} = 10Q^2 \sum \frac{1}{C_{ad}} \times p_i \qquad (11)$$

$$C_{pm3} = 15 \sum \left[\left(\frac{C_{ad} - Q}{C_{ad}} \right)^2 \right] \times p_i \qquad (12)$$

$$C_{pm} = C_{pm1} + C_{pm2} + C_{pm3} \qquad (13)$$

After the calculations made, the following table has been filled in (Table 2.).

Table 2. Calculation of the average total shortage cost.

Q	C_{pm1}	C_{pm2}	C_{pm3}	TOTAL
1	1	1 (1)	0	2.00
2	8	4	0	12.00
3	12	11.70	0	23.70
4	24	24	0	48.00

The quantity to be supplied at the beginning of each time period of 2 days is Q = 1 for recording the minimization of the management cost C_{mp} = 2 m.u.

The use of templates for the optimization of the supply costs has allowed us to select supply policies. Two significant elements for the correction of the supply budget have been determined: the economic time period for re-supply; volume of the economic lot to be supplied. By applying these policies, the minimal supply budget needs to consider - on the one hand - the chart of orders to be achieved, and - on the other hand - the exact quantities that need to be supplied. Also, one must realize the connection of the information lying at the basis of the drafting of the minimal supply budget with the policies regarding the sale, including stock refilling and re-supply [17]. The minimal supply budget structure shall contain the following: orders, consumptions, sales and stocks at the end of the time periods. For a better communication and compatibility, we have preferred the exploitation of the minimal supply budget in monetary units. A performance can be repeated only if the rhythm is maintained or accelerated cyclically, by planning, controlling the processes and estimating the results [6]. For the reflection of all the necessary information, we have preferred the use of two methods in the drafting of the minimal supply budget: the budget method based on the economic period needed for re-supply and the budget method based on the economic lot to be supplied.

5. CONCLUSIONS

Cost determination is a decisive factor in making decisions and planning the future activities. For a better management of the supply budget, an optimization programme for the supply modalities and for the integration of the key parameters is needed. To reach the intended goal, namely cost minimization in the supply budget organization and management, we have carried out a research by means of case studies, confirming the supply policies on the one hand, under conditions of certainty in supply, and on the other hand, under conditions of uncertainty in supply. In this respect, we have determined, for the first variant, the economic time period, the economic lot to be supplied, the storage cost for the time period under analysis, the cost for putting the order into production for the time period under analysis, the supply price for the time period under analysis and the total supply cost, and for the second variant, we have determined the shortage cost and the total cost. After the determination of the minimal economic results, we continued by drafting the supply budget, based on the optimal supply policies, respecting the structure of these budget categories, and using, at the same time, the corrections required by the economic time period needed for re-supply and the volume of the economic lot to be supplied. To conclude, the mathematical models and methods used by entities management must be carefully chosen, in order to optimize the priorities of the investment and strategic future plans.

REFERENCES

[1] Alazard, C., Sėpari S., *Contrôle de gestion*, Editura Dunod, Paris, 2001.
[2] Balakrishnan, A. et al., *Coordinating the Distribution Chain: New Models for New Challenges*. In: Geunes J. et al. (Eds) *Supply Chain Management: Models, Applications, and Research Directions. Applied Optimization*, Vol. 62, Springer, Boston, 2002.
[3] Brookson, St., *Gérer un budget,* Edition Pratique, Italy, 2001.
[4] Croitoru, G. et al. *Journal of Science and Arts*, **1**(42), 139, 2018
[5] Coucoureux, M. et al, *Contrôle de gestion*, Edition Nathan, Paris, 2007.
[6] Coman, M.D., Coman, M.., *Valahian Journal of Economic Studies*, **3**, 93, 2012.
[7] Cucui, I., Man, M., *Costs and Cost Control*, Ed. Economica, Bucharest, 2004.
[8] Man, M. et al., Costs occurred by implementing the logistical study into the company, *Proceedings of 2nd World Multiconference on Applied Economics, Business and Development*, 104-109, 2010.
[9] Cucui, I. et al., *Cost Control*, Ed. Niculescu, Bucharest, 2003.
[10] Cuyaubere, T., Muller, J., *Contrôle de gestion*, 5e Edition, Groupe Revue fiducieaire, Paris, 2000.
[11] Danila, A. et al., *Journal of Science and Arts*, **4**(41), 761, 2017
[12] Davasse, H, Langlois, G., *Comptabilité de gestion. Gestion previsionnelle*, Ed. Foucher, Paris, 2001.
[13] Erlenkotter, D., *Operations Research*, **38**, 937, 1990.
[14] Horngren, Ch. et al., *Contrôle de gestion et gestion budgétaire***,** 3e Edition, Pearson Education France, Paris, 2006.
[15] Kouvelis, P., Rosenblatt, M.J., *Supply Chain Management: Models, Applications, and Research Directions*, **62**, 271, 2005.
[16] Kumar, R., *Global Journal of finance and economic management*, **5**, 1, 2016
[17] Lwiki, T. et al., *International Journal of Business, Humanities and Technology*, **3**(5), 75, 2013.
[18] Mula, J. et al., *European Journal of Operational Research*, **204**(3), 377, 2010.
[19] Sarbjit, S., *International Journal of Computer Applications*, **33**, 9, 2011
[20] Selmer, C.**,** *Construire et défendre son budget***,** Editura Dunod, Paris, 2003.
[21] Snyman, J., *Practical mathematical optimization: an introduction to basic optimization theory and classical and new gradient-based algorithms*, Springer Science & Business Media, New York, USA, 2005.
[22] Vallin, Ph., *Rairo Operations Research*, **33**(1), 47, 1999.
[23] Zeng, A.Z., *An Optimization Framework for Evaluating Logistics Costs in a Global Supply Chain: an Application to the Commercial Aviation Industry*. In: Geunes, J. et al., (eds) *Supply Chain Management: Models, Applications, and Research Directions. Applied Optimization*, Vol 62. Springer, Boston, 2002.