ORIGINAL PAPER

COMPARISON BETWEEN METHODS TO GENERATE DATA OF BIVARIATE EXPONENTIAL DISTRIBUTION

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Abstract. *In this paper, we are studying three simulation methods to generate observation for bivariate exponential distribution, and these methods are: method 1, method 2(conditional method) and method 3, and we write simulation programs for each method by Matlab 2015a software, and comparison between these methods by depend on many criterions as MSE, AIC, skw, kur. As well as the run speed criterion for each method to get the best method.*

Keywords: *bivariate exponential, MSE, AIC, skw, kur.*

1. INTRODUCTION

Marshall and Olkin (1967) MOBVE, proposed this distribution with prosperities like marginal exponential, if X, Y are two random vector, then the joint density function is given by the formula: [2,4,6]

$$f(x,y) = \begin{cases} \lambda_1(\lambda_2+\lambda_3)\overline{F}(x,y); y > x > 0\,, x,y > 0 \\ \lambda_2(\lambda_1+\lambda_3)\overline{F}(x,y) \quad ; x > y > 0\,, x,y > 0 \\ \lambda_3\overline{F}(x,x) \qquad\qquad\qquad ; x = y > 0 \end{cases} \tag{1}$$

where

$$\overline{F}(x,y) = \exp\left(-\lambda_1 x - \lambda_2 y - \lambda_3 \max(x,y)\right);\ x,y > 0 \tag{2}$$

The marginal distribution of X, Y are exponential with failure rates $(\lambda_1+\lambda_3)$ and $(\lambda_2+\lambda_3)$ is given by:

$$f_X(x) = (\lambda_1+\lambda_3)e^{-((\lambda_1+\lambda_3))(x-y)} \tag{3}$$

$$f_Y(y) = (\lambda_2+\lambda_3)e^{-((\lambda_2+\lambda_3))(y-x)} \tag{4}$$

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2. THE CONCEPT OF SIMULATION

As a result of appearance several problems and statistical theories which are difficult find a logical analysis by mathematical proof, so it has been translated and transformation these theories to real societies, then they have chosen a number of independent random samples, To get the ideal solution for these problems, so practically these samples which are difficult find at the area because they Requires High cost, Time and effort hence some researchers have gone in the beginning of Twentieth century to apply technique the sampling experiment that which is known today simulation .The simulation process is a digital style to complete the experiments on the electronic calculator, which include types of logical and mathematical operations necessary to describe the behavior and structure of complex real system through a given time period.

3. COMPARATIVE CRITERIA

We will comparison between methods of these distributions by use the criterion Mean squared error (MSE), Akaike information criteria AIC, Mardia's test statistic for Skewness and Kurtosis and etc. as follows:

3.1. MEAN SQUARED ERROR (MSE)

If T is (statistic) estimate for the parameter θ then we called that $E[(T-\theta)^2]$ is MSE:

$$MSE = E[(T-\theta)^2] = V(T) + [\theta - E(T)]^2 \tag{5}$$

Now when the estimate T be unbiased estimator then$\theta = E(T)$, which mean that $[\theta - E(T)]^2$ is equal to zero and $MSE = V(T)$. There is another formula for these estimators specially (for joint estimator) of it as

$$MSE_{model} = det\,\frac{1}{Rep}\sum_{i=1}^{Rep}((T-\theta)\,(T-\theta)') \tag{6}$$

where Rep: Replication of experiment

3.2 AKAIKE INFORMATION CRITERIA AIC

The form of this criterion is either

$$AIC = -2\log(MLE) + 2n \text{ , or } \quad AIC = Nlog(MSE) + 2n \tag{7}$$

where n: number of fitted parameters, N: sample size.

3.3 MARDIA'S TEST STATISTIC FOR SKEWNESS AND KURTOSIS

If $X_1, X_2, \dots, X_n$ random sample of independent and identical p-variate vectors with unknown mean μ and unknown covariance matrix ∑. Mardia (1970-1974) defined the measure of multivariate skewness and kurtosis as follows:

$$b_{1,p} = \frac{1}{n^2} \sum_{i,j=1}^{n} {g_{ij}}^3 \tag{8}$$

where

$$g_{ij} = (x_i - \overline{x})' S^{-1} \left(x_j - \overline{x}\right) \tag{9}$$

and

$$b_{2,p} = \frac{1}{n} \sum_{i=1}^{n} \{(x_i - \overline{x})' S^{-1} (x_i - \overline{x})\}^2 \tag{10}$$

Under normality of $X_1, X_2, \dots, X_n$, asymptotically MVN , A=n$b_{1,p}$/6 has a X^2distribution with f=p(p+1)(p+2)/6 degrees of freedom and the statistic $B = b_{2,p} - p(p+2)/\sqrt{8p(p+2)/n}$ has asymptotic standard Normal distribution . Based on the statistic A and B, as test for multivariate normality Jarque and Bera (1987) proposed to use the statistic JB=A+B^2 which has asymptotic chi-square distribution with f+1 degrees of freedom, in addition, the distribution is symmetric (null of skewness) around the curve when the value of skewness is zero (sk=0) and (sk=2) for Exponential distribution, and the value of kurtosis for the Normal distribution in univariate case is 3 and for univariate Exponential distribution is 6 ,while the Mardia's kurtosis is p(p+2) for the multivariate distribution of p- variables, which is ku= 2(2+2)=8 when (p=2), another definition of JB criterion like we denoted it in programs in appendix as

$$JB = \frac{n}{6}\left(sk^2 + \frac{1}{4}\, ku^2\right) \tag{11}$$

and p_JB (probability of JB criterion) has belong on the following hypothesis

H∘: the data is belong to multivariate distribution

If p-value <0.05 the hypothesis H∘ is reject and we conclude that the data not belong to multivariate distribution, otherwise the hypothesis H∘ is not reject.

4. FORMULATION OF SIMULATION MODEL

We are choosing Matlab 2015a as a program for this study to write a simulation model [10] to generate observation for bivariate exponential distribution and selecting default value for the parameters

$\lambda_1 = 3, \lambda_2 = 2, \lambda_3 = 1, \mathrm{R} = 10000$of this distribution, in addition to select sample size
n= 15, 50 ,100 and 200 respectively and choosing the number of replication as (R= 10000).

5. SIMULATION METHODS

Method 1:

To generate observation for bivariate exponential distribution by this method we must show the following steps:

1- Generating m_1 observations for Uniform U1(0,1) and m_2 observations for Uniform U2 (0,1).
2- Generate observations for exponential distribution by using this transformation

$$W1j = - ((1\text{-}rao)/m1) \log(U1j),\ j = 1,2,\ ... \tag{12}$$

3- Generate m observations form exponential distribution by using this transformation

$$W2j = -((1\text{-}rao)/m2) \log(U2j),\ j = 1,2,\ ... \tag{13}$$

4- Obtaining one observation of Bivariate exponential $(x_1, y_2)$by using:

$$X_i = \text{sum(w1j)},\ i=1,2,\ ...,\ m_1 \tag{14}$$

$$Y_i = \text{sum(w2j)},\ i=1,2,\ ...,\ m_2 \tag{15}$$

5- To obtain n observations(x_i, y_i) , i=1,2,...n from bivariate exponential, repeat the previous 4 steps n times.

Method 2(Conditional Method):

To generate (X, Y) observation of bivariate exponential distribution MOBVE, we apply the following steps:

- Generate $X = exprnd(\lambda_1 + \lambda_3)$ to get x. (16)
- Generate $V = exprnd(1)$ to get v.
- Given X=x and v,

$$y = \begin{cases} \frac{v}{\lambda_2} & if\ v \le \lambda_2 x \\ x & if\ \lambda_2 x < v < \lambda_2 x + \log\left(\frac{\lambda_1+\lambda_3}{\lambda_3}\right) \\ \frac{1}{(\lambda_2+\lambda_3)\left(v+\lambda_3 x+\log\left(\frac{\lambda_1+\lambda_3}{\lambda_3}\right)\right)} & otherwise \end{cases} \tag{17}$$

Finally, we get to observation of MOBVE.

Method 3

This method depends on generate three vectors U, V, W from the form of univariate exponential distribution that we generate it from matlab software form as follows:

$$\mathrm{U} = \mathrm{exprnd}(1/(\lambda_1 - \lambda_3), \mathrm{n}, \mathrm{m}) \tag{18}$$

$$\mathrm{V} = \mathrm{exprnd}(1/(\lambda 2 - \lambda_3), \mathrm{n}, \mathrm{m}) \tag{19}$$

$$\mathrm{W} = \mathrm{exprnd}\ (1/\lambda_3, \mathrm{n}, \mathrm{m}) \tag{20}$$

then we take the minimum value of all two vectors to get two random vectors observation X, Y as follows:

X=min (U, W) and Y=min (V, W)

Then we get $(x, y) \sim BVE$, where$\lambda_1, \lambda_2, \lambda_3$ are the parameters of bivariate exponential distribution, n is sample size and m is dimension of random variable, and we clarify and detail it in Special program in appendix.

6. RESULTS AND DISCUSSION

6.1 SIMULATION RESULTS

After we show the special methods to generate observation for bivariate exponential distribution, we shall offer the following results of these methods in Tables 1-3 and then calculated in general Table 4 to compare between them in addition to summaries Tables 5-7 as follows:

Table 1. Simulation Results for method 1 when λ_1=3, λ_2=2, λ_3=1, R=10000.

Sample size	$\hat{\boldsymbol{\lambda}}_1$	$\hat{\boldsymbol{\lambda}}_2$	$\hat{\boldsymbol{\lambda}}_3$	$\hat{\boldsymbol{\rho}}$	MSE	skw Sig. of skw	kur Sig. of kur	P_JB	time	AIC
15	2.7462	1.5399	0.8572	-0.0036	0.2965	2.5382 0.1748	8.2443 0.4529	0.9779	22.5109	-12.2345
50	2.7441	1.5427	0.8574	0.0006	0.2950	0.7018 0.2107	7.8445 0.4453	0.0840	83.4761	-55.0423
100	2.7433	1.5428	0.8572	-0.0020	0.2953	0.2255 0.4397	7.9223 0.4613	0.5878	112.8511	-115.9860
200	2.7429	1.5428	0.8571	-0.0015	0.2956	0.1573 0.2632	7.1314 0.0623	0.2735	276.3090	-237.7747

Table 2. Simulation Results for method 2 when λ1=3, λ2=2, λ3=1, R=10000.

Sample size	$\hat{\boldsymbol{\lambda}}_1$	$\hat{\boldsymbol{\lambda}}_2$	$\hat{\boldsymbol{\lambda}}_3$	$\hat{\boldsymbol{\rho}}$	MSE	skw Sig. of skw	kur Sig. of kur	P_JB	time	AIC
15	3.0356	1.7123	0.9496	0.1133	0.0866	0.3208 0.9382	4.9792 0.0718	0.7165	22.4017	-30.7001
50	3.0450	1.7045	0.9499	0.0867	0.0919	0.1007 0.9332	6.5801 0.1047	0.8972	54.1167	-113.3616
100	3.0498	1.7024	0.9504	0.0829	0.0935	0.1923 0.5241	7.5086 0.2695	0.6514	93.2418	-230.9876
200	3.0449	1.7089	0.9508	0.0824	0.0892	0.1241 0.3876	8.9084 0.0542	0.2768	204.8917	-477.4590

Table 3. Simulation Results for method3 when λ1=3, λ2=2, λ3=1, R=10000.

Sample size	$\hat{\lambda}_1$	$\hat{\lambda}_2$	$\hat{\lambda}_3$	$\hat{\rho}$	MSE	skw Sig. of skw	kur Sig. of kur	P_JB
15	2.2768	1.2857	0.7125	0.2686	1.1160	1.1824 0.5652	7.0419 0.3214	0.5989
50	2.2790	1.2858	0.7130	0.2557	1.1123	0.1857 0.8182	6.3279 0.0697	0.3117
100	2.2858	1.2841	0.7140	0.2540	1.1044	0.5222 0.0689	7.6659 0.3381	0.1908
200	2.2865	1.2837	0.7140	0.2516	1.1039	0.1998 0.1550	8.0779 0.4452	0.6428

Table 4. Comparison Result between Tables 1-3.

Sample size	Meth. No	$\hat{\lambda}_1$	$\hat{\lambda}_2$	$\hat{\lambda}_3$	$\hat{\rho}$	MSE	skw Sig. of skw	kur Sig. of kur	P_JB	time	AIC
15	1	2.7462	1.5399	0.8572	-0.0036	0.2965	2.5382 0.1748	8.2443* 0.4529	0.9779*	22.5109	-12.2345
	2	3.0356	1.7123	0.9496	0.1133	0.0866*	0.3208* 0.9382	4.9792 0.0718	0.7165	22.4017	-30.7001*
	3	2.2768	1.2857	0.7125	0.2686*	1.1160	1.1824 0.5652	7.0419 0.3214	0.5989	11.5909*	7.6466
50	1	2.7441	1.5427	0.8574	0.0006	0.2950	0.7018 0.2107	7.8445* 0.4453	0.0840	83.4761	-55.0423
	2	3.0450	1.7045	0.9499	0.0867	0.0919*	0.1007* 0.9332	6.5801 0.1047	0.8972*	54.1167	- 113.3616*
	3	2.2790	1.2858	0.7130	0.2557*	1.1123	0.1857 0.8182	6.3279 0.0697	0.3117	11.7625*	11.3206
100	1	2.7433	1.5428	0.8572	-0.0020	0.2953	0.2255 0.4397	7.9223* 0.4613	0.5878	112.8511	-115.9860
	2	3.0498	1.7024	0.9504	0.0829	0.0935*	0.1923* 0.5241	7.5086 0.2695	0.6514*	93.2418	- 230.9876*
	3	2.2858	1.2841	0.7140	0.2540*	1.1044	0.5222 0.0689	7.6659 0.3381	0.1908	8.2057*	15.9284
200	1	2.7429	1.5428	0.8571	-0.0015	0.2956	0.1573 0.2632	7.1314 0.0623	0.2735	276.3090	-237.7747
	2	3.0449	1.7089	0.9508	0.0824	0.0892*	0.1241* 0.3876	8.9084* 0.0542	0.2768	204.8917	- 477.4590*
	3	2.2865	1.2837	0.7140	0.2516*	1.1039	0.1998 0.1550	8.0779 0.4452	0.6428*	14.7265*	25.7780

Table 5. Number times of excellence for each method and the total ratio .

Sample size	Methods		
	Method1	Method2	Method3
15	2	3	2
50	1	4	2
100	1	4	2
200	0	4	3
Total of each method	4	15	9
Ratio	14%	54%	32%

Table 6. The average of all parameters and criterion of each method for bivariate exponential distribution.

Methods	$\hat{\lambda}_1$	$\hat{\lambda}_2$	$\hat{\lambda}_3$	$\hat{\rho}$	MSE	skw	kur	P_JB	time	AIC
Method 1	2.7441	1.5421	0.8572	-0.0016	0.2956	0.9057	7.7856	0.4808	123.7868	-105.2594
Method 2	3.0438	1.7070	0.9502	0.0913	0.0903	5.8295	6.9941	0.6355	93.6630	-213.1271
Method 3	2.2820	1.2848	0.7134	0.2575	1.1092	0.5225	7.2784	0.4361	11.5714	60.6736

Table 7. Number times of excellence for each method and the total ratio to excellence according to the average of each method for bivariate exponential distribution.

Methods	Method 1	Method 2	Method 3
Total of each method	1	6	3
Ratio	10%	60%	30%

6.2 DISCUSSION

Form the above tables we can discuss the following results:

- We can see in Tables 1-3 that values of $\hat{\boldsymbol{\lambda}}_1, \hat{\boldsymbol{\lambda}}_2$ ***and*** $\hat{\boldsymbol{\lambda}}_3$ are approach to the default value $\boldsymbol{\lambda_1 = 3, \lambda_2 = 2, \lambda_3 = 1}$, when the sample size are increasing from the smaller (15) to greater (200) in all the methods .
- Note that the criterion AIC in all simulation results in Tables 1-2 above are inversely proportional to the sample size, which mean, when sample size is increasing the criterion AIC are decreasing, but in method 3 the value of AIC is directly proportional to the sample size .and the value of MSE is directly proportional with sample size in Tables1-2, but inversely proportional to the sample size in method 3.
- The values of criterion skewness (skw) are approach to zero when the sample size is increasing in each method, also the values of kurtosis criterion (kur) are nearly to (8) in each method when the sample size start to be increasing, in addition, the values of the joint criterion (p_JB) in every method and in all cases of sample size are greater than (0.05) then we accept the hypothesis $H_\circ$ that we assumed it in the previous section 3.
- Table 4 shown the comparison between the simulation results of these methods, such that the best value to the criterion MSE, AIC when sample size is (15), (50) in method 2.
- In Table 4 of comparison the best value of skewness criterion in all value of sample size in method 2 and the best value of kurtosis criterion is in method 1 when sample size are (15), (50) and (100).
- In Table 5 we make a comparison to know the number of times of excellence and total ratio of excellence($ratio = \frac{sub\ number}{total\ number} * 100$) for each method and we get that the upper ratio to (method 2) is (54%) and we get it is the best method to generate observation of bivariate exponential distribution. while the second method (method 3) have got on (32%) as a ratio.

- In Table 6 we take the average of all the parameter and criterion of each method, this average got from accumulated the four values of parameter and criterion in addition to correlation in all case of sample size after that we divided it by four ,and so on for all other values, in addition to the Table 7 contain the comparison of number of the times of excellence and the ratio to excellence according to the average of each method, finally we deduced that the best method is (method 2) because it get (60%) as a better ratio .

7. CONCLUSIONS

The best method to generate observation of bivariate exponential distribution is "method 2" (conditional method) gave a closely values from the default values, and this method got on 54% as a total ratio of the number times of excellence through the comparison between the other methods, and got 60% as a total ratio of the average of a number times of excellence, and therefore can be relied upon to generate.

All values of the parameters and the criteria of all methods to generate the observations to bivariate distribution in this study got from simulation programs, almost very closely to the default values that assumed it in the beginning of formulation of Simulation Models.

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APPENDIX

Program of simulation Bivariate Exponential Distribution

```
clear all;num=1;
while num<4
    num=input('number of program 1 method1. 2 method2. 3 method3.?');
    switch num
        case 1

     %program 1 generate observation of Bivariate Exponential distribution
by method1
            disp('Result of method1')
n=input('sample size?');% n is sample size.
Rep=input('R of Rep.=');% number of replications.
l1=input('l1=');
l2=input('l2=');
l3=input('l3=');
rao=l3/(l1+l2+l3);n3=n*rao;t0=cputime();
rand('seed',n);
sm_x=0;sm_y=0;SR=[0 0;0 0];SP=[0 0;0 0];L=3;
sx=zeros(1,n);sy=zeros(1,n);
for i=1:Rep
    for j=1:n
     u1=unifrnd(0,1,n,1);
     u2=unifrnd(0,1,n,1);
     w1=-((1-rao)/l1)*log(u1);
     w2=-((1-rao)/l2)*log(u2);
     x(j)=sum(w1);
     y(j)=sum(w2);
    end
    sm_x=sm_x+sum(x);
    sm_y=sm_y+sum(y);
    [RHO,PVAL] = corrcoef(x,y);
    SR=SR+RHO;SP=SP+PVAL;
   sx=sx+x;sy=sy+y;
end
s_x=sm_x/Rep/n;s_y=sm_y/Rep/n;
l1_h=[n/s_x-n3/s_y]/[1+n3/n]
l2_h=[n/s_y-n3/s_x]/[1+n3/n]
l3_h=n3*[1/s_x+1/s_y]/[1+n3/n]
RHO=SR/Rep
PVAL=SP/Rep
lo=[l1 l2 l3];lo_h=[l1_h l2_h l3_h];
MS=(lo_h-lo)*(lo_h-lo)';
MSE=abs(det(MS))        % MSE criterion for model
AIC=n*log(MSE)+2*L      % AIC criterion for model
 x=sx/Rep;y=sy/Rep;X=[x;y]';
 [n,p] = size(X); alpha = 0.05;
difT = [];
for     j = 1:p
   difT = [difT,(X(:,j) - mean(X(:,j)))];
end;
S = cov(X);         % Variance-covariance matrix
D = difT * inv(S) * difT';  % Mahalanobis' distances matrix
b1p = (sum(sum(D.^3))) / n^2;  % Multivariate skewness coefficient
b2p = trace(D.^2) / n; % Multivariate kurtosis coefficient
v = (p*(p+1)*(p+2)) / 6;        % Degrees of freedom
g1 = (n*b1p) / 6;               % Skewness test statistic (approximates to
a chi-square distribution)
P1 = 1 - chi2cdf(g1,v);  % Significance value of skewness
```

```
g2 = (b2p-(p*(p+2))) / ...
    (sqrt((8*p*(p+2))/n));      % Kurtosis test statistic (approximates to
a unit-normal distribution)
P2 = 1-normcdf(abs(g2)); % Significance value of kurtosis
sk=b1p
ku=b2p
stats.Ps  = P1
stats.Pk  = P2
ks=skewness(X);ku=kurtosis(X)-3;
kwen=ks*ks';kur=ku*ku';
jb=n/6*(kwen+kur/4)
p_jb=1-chi2cdf(jb,v)
Mean_Ku=p*(p+2)*(p+1+n)/n
time=cputime()-t0
        case 2
   % program 2 generate observation of Bivariate Exponential distribution
by method2
            disp('Result of method2')
n=input('sample size ?');
Rep=input('R of Rep.=');
l1=input('l1=');
l2=input('l2=');
l3=input('l3=');
rao=l3/(l1+l2+l3);n3=n*rao;t0=cputime();
rand('seed',n);
sm_x=0;sm_y=0;SR=[0 0;0 0];SP=[0 0;0 0];L=3;
sx=zeros(1,n);sy=zeros(1,n);
for i=1:Rep
    for j=1:n
   v1=exprnd(1/(l1+l3));x(j)=v1;z=v1;
    v=exprnd(1);
    if v<l2*z
        y(j)=v/l2;
      else
        if v>l2*z & v<(l2*z+log(l1/l3+1))
            y(j)=z;
      else
        y(j)=(1/(l2+l3))*(v+l3*z+log(l1/l3+1));
    end
    end
  end
    sm_x=sm_x+sum(x);
    sm_y=sm_y+sum(y);
    [RHO,PVAL] = corrcoef(x,y);
    SR=SR+RHO;SP=SP+PVAL;sx=sx+x;sy=sy+y;
end
s_x=sm_x/Rep;s_y=sm_y/Rep;
l1_h=[n/s_x-n3/s_y]/[1+n3/n]
l2_h=[n/s_y-n3/s_x]/[1+n3/n]
l3_h=n3*[1/s_x+1/s_y]/[1+n3/n]
RHO=SR/Rep
PVAL=SP/Rep
lo=[l1 l2 l3];lo_h=[l1_h l2_h l3_h];
MS=(lo_h-lo)*(lo_h-lo)';
MSE=abs(det(MS))        % MSE criterion for model
AIC=n*log(MSE)+2*L      % AIC criterion for model
x=sx/Rep;y=sy/Rep;X=[x;y]';
 [n,p] = size(X); alpha = 0.05;
difT = [];
for      j = 1:p
   difT = [difT,(X(:,j) - mean(X(:,j)))];
```

```
end;
S = cov(X);          % Variance-covariance matrix
D = difT * inv(S) * difT';  % Mahalanobis' distances matrix
b1p = (sum(sum(D.^3))) / n^2;  % Multivariate skewness coefficient
b2p = trace(D.^2) / n; % Multivariate kurtosis coefficient
v = (p*(p+1)*(p+2)) / 6;        % Degrees of freedom
g1 = (n*b1p) / 6;               % Skewness test statistic (approximates to
a chi-square distribution)
P1 = 1 - chi2cdf(g1,v);  % Significance value of skewness
g2 = (b2p-(p*(p+2))) / ...
    (sqrt((8*p*(p+2))/n));      % Kurtosis test statistic (approximates to
a unit-normal distribution)
P2 = 1-normcdf(abs(g2)); % Significance value of kurtosis
sk=b1p
ku=b2p
stats.Ps  = P1
stats.Pk  = P2
ks=skewness(X);ku=kurtosis(X)-3;
kwen=ks*ks';kur=ku*ku';
jb=n/6*(kwen+kur/4)
p_jb=1-chi2cdf(jb,v)
Mean_Ku=p*(p+2)*(p+1+n)/n
time=cputime()-t0
        case 3
   % program 3 generate observation of Bivariate Exponential distribution
by method3
   disp('Result of method3')
 n=input('sample size ?');
Rep=input('R of Rep.=');t0=cputime();
rand('seed',n);
L1=input('L1?');
L2=input('L2?');
L3=min(L1,L2)-1;
rao=L3/(L1+L2+L3);n3=n*rao;
sm_x=0;sm_y=0;SR=[0 0;0 0];SP=[0 0;0 0];L=3;
sx=zeros(1,n);sy=zeros(1,n);
for i=1:Rep
   U=exprnd(1/(L1-L3),1,n);
   V=exprnd(1/(L2-L3),1,n);
   W=exprnd(1/L3,1,n);
   x=min(U,W);
   y=min(V,W);
  sm_x=sm_x+sum(x);
  sm_y=sm_y+sum(y);
  [RHO,PVAL] = corrcoef(x,y);
    SR=SR+RHO;SP=SP+PVAL;
    sx=sx+x;sy=sy+y;
end
s_x=sm_x/Rep;s_y=sm_y/Rep;
L1_h=[n/s_x-n3/s_y]/[1+n3/n]
L2_h=[n/s_y-n3/s_x]/[1+n3/n]
L3_h=n3*[1/s_x+1/s_y]/[1+n3/n]
RHO=SR/Rep
PVAL=SP/Rep
Lo=[L1 L2 L3];Lo_h=[L1_h L2_h L3_h];
MS=(Lo_h-Lo)*(Lo_h-Lo)';
MSE=abs(det(MS))        % MSE criterion for model
AIC=n*log(MSE)+2*L      % AIC criterion for model
x=sx/Rep;y=sy/Rep;X=[x;y]';
 [n,p] = size(X); alpha = 0.05;
difT = [];
```

```
for     j = 1:p
   difT = [difT,(X(:,j) - mean(X(:,j)))];
end;
S = cov(X);         % Variance-covariance matrix
D = difT * inv(S) * difT';  % Mahalanobis' distances matrix
b1p = (sum(sum(D.^3))) / n^2;  % Multivariate skewness coefficient
b2p = trace(D.^2) / n; % Multivariate kurtosis coefficient
v = (p*(p+1)*(p+2)) / 6;        % Degrees of freedom
g1 = (n*b1p) / 6;               % Skewness test statistic (approximates to
a chi-square distribution)
P1 = 1 - chi2cdf(g1,v);  % Significance value of skewness
g2 = (b2p-(p*(p+2))) / ...
    (sqrt((8*p*(p+2))/n));      % Kurtosis test statistic (approximates to
a unit-normal distribution)
P2 = 1-normcdf(abs(g2)); % Significance value of kurtosis
sk=b1p
ku=b2p
stats.Ps  = P1
stats.Pk  = P2
ks=skewness(X);ku=kurtosis(X)-3;
kwen=ks*ks';kur=ku*ku';
jb=n/6*(kwen+kur/4)
p_jb=1-chi2cdf(jb,v)
Mean_Ku=p*(p+2)*(p+1+n)/n
time=cputime()-t0
        otherwise
            disp('End of select')
            break
    end
end
```