ORIGINAL PAPER

NUMERICAL ACCURACY OF ERRORS IN DIFFERENT FUNCTIONS USING INTERPOLATION TECHNIQUE

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Abstract. *In this paper, we introduce the comparative study of errors in different functions using error formula. Some examples are also discussed. This work presents a numerical analysis of error formula. Also to check the performance of the considered method an error associated with Lagrange Interpolation has considered. Errors are analyzed by comparing the actual sampled values with the values obtained by Lagrange's Interpolation formula.*

Keywords: *Interpolation, Exponential functions, Trigonometric functions, Logarithmic functions.*

1. INTRODUCTION

Numerical analysis is the area of mathematics and computer sciences that creates analyzes and impliments algorithms for solving numerically the problems of continuous mathematics.[1] Such problem originate generally from real-world applications of Algebra, geometry and calculas and they involve variables which vary continuously: these problem occur throughout the natural sciences, social sciences, engineering, medicine and business. From very ancient time Interpolation is being used for various purposes [2].

In this paper Lagrange's Interpolation Error formula is used.

$$R_n = \frac{1}{(n+1)!}(x - x_0)^{n+1} f^{(n+1)}(\in) \qquad x_0 < \in < x$$

For higher order Interpolation:

$$E_n\,(f; x) = \frac{w(x)}{(n+1)!} f^{(n+1)}(\in)$$

where $w(x) = (x - x_0)(x - x_1)$………..$(x - x_n)$. [3]

Theorem: If g(x) is a continuous function on same interval [a, b] and differentiable on (a, b) and if g(a) = 0, g(b) = 0, then there is atleast one point ∈ inside (a, b) for which g'(∈) = 0.

We notice that if $x=x_0$ or $x=x_1$ then $E_1(f; x) = 0$. If $x \in (x_0, x_1)$, then for this x we define a function g(t)

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$$g(t) = f(t) - p(t) - \left[f(x) - p(x) \right] \frac{(t - x_0)(t - x_1)}{(x - x_0)(x - x_1)}$$

It is easy to varify that g(t) = 0 at the three distinct points $t = x_0$, $t = x_1$ and $t = x$. The function g(t) satisfies the conditions of the Rolle's theorem.

Applying the Rolle's theorem on the intervals (x_0, t) and (t, x_1) separately, we get

$$g'(\in_1) = 0,\ x_0 < \in_1 < t \text{ and } g'(\in_2) = 0,\ t < \in_2 < x_1.$$

Now

$$g''(t) = f''(t) - \frac{2\left[f(x) - p(x) \right]}{(x - x_0)(x - x_1)}$$

Setting $g''(\in) = 0$ and solving above equation for f(x) we get

$$f(x) = p(x) + \frac{1}{2}(x - x_0)(x - x_1) f''(\in)$$

Therefore, the truncation error in linear interpolation is given by

$$E_1(f; x) = \frac{1}{2}(x - x_0)(x - x_1) f''(\in).$$

Analysis and implementation
Truncation Error Bounds -
Linear Interpolation-

$$E_1(f; x) = \frac{1}{2}(x - x_0)(x - x_1) f''(\in).$$

Quadratic Interpolation

$$E_2(f; x) = f(x) - p_2(x)$$

$$E_2(f; x) = \frac{1}{3!}(x - x_0)(x - x_1)(x - x_2) f'''(\in)$$

Higher order Interpolation

$$E_2(f; x) = \frac{1}{3!}(x - x_0)(x - x_1)(x - x_2) f'''(\in).$$

where $w(x) = (x - x_0)(x - x_1)\ldots(x - x_n)$ [4].

Numerical examples

Example 1. [5]

x	y=log$x$
2	0.30103
2.5	0.39794
3	0.47712

$$E_n\left(f;x\right)=\frac{w\left(x\right)}{\left(n+1\right)!}f^{\left(n+1\right)}\left(\in\right)=0.00175$$

Example 2.

x	y=log$x$
2	0.30103
2.5	0.39794
3	0.47712
3.5	0.54407
4	0.60206

$$E_n\left(f;x\right)=0.000273$$

Example 3.

x	y=log$x$
2	0.30103
2.5	0.39794
3	0.47712
3.5	0.54407
4	0.60206
4.5	0.6532125
5	0.69897

$$E_n\left(f;x\right)=0.000208334839$$

Example 4. [6]

x	y=sin$x$
0	0
π/4	0.70711
π/2	1.0

$$E_n\left(f;x\right) = 0.02392$$

Example 5.

x	y=sin$x$
0	0
π/8	0.3827
π/6	0.5
π/4	0.70711
π/2	1.0

$$E_n\left(f;x\right) = 0.000005927$$

Example 6.

x	y=sin$x$
0	0
π/12	0.25882
π/10	0.30902
π/8	0.3827
π/6	0.5
π/4	0.70711
π/2	1.0

$$E_n\left(f;x\right) = 0.000000003553058352$$

Example 7.

x	$y=e^{2x}$
0	1
0.5	2.718
1	7.389

$$E_n\left(f;x\right) = 0.413784$$

Example 8.

x	$y = e^{2x}$
0	1
0.5	2.718
1	7.389
1.2	11.0232
2	54.5982

$$E_n(f;x) = 0.397474896$$

Example 9.

x	$y = e^{2x}$
0	1
0.5	2.718
1	7.389
1.2	11.0232
2	54.5982
2.5	148.4132
3	403.4288

$$E_n(f;x) = 1.158002602$$

Example 10.

x	y=sin$x$
0.1	0.09983
0.2	0.19867

$$E_n(f;x) = 0.00025$$

COMPARISION OF RESULTS

From above examples we see that when we increase the functions then error decreases for all type of functions i.e. logarithmic functions, exponential functions, trigonometric functions.

CONCLUSION

In this paper, accordingly to the analysis the performance of errors on different types of function is presented. Experimental results show that when functions will increase then error will decrease for all type of functions i.e. logarithmic functions, exponential functions, trigonometric function

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