ORIGINAL PAPER

VARIATIONAL ITERATION METHOD FOR THE SIMULATION OF THE EFFECT OF TRANSMISSION COEFFICIENT ON THE SUSCEPTIBLE-EXPOSED-INFECTED-RECOVERED-SUSCEPTIBLE (SEIRS) EPIDEMIC MODEL WITH SATURATED INCIDENCE RATE AND DISEASE-INDUCED DEATH

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Abstract. *In this paper, the variational iteration method (VIM) is implemented to give a numerical solution of a system of linear differential equations that described the effect of transmission coefficient on the model of a susceptible-exposed-infected-recovered-susceptible epidemic with saturated incidence rate and disease-induced death.*

Keywords: *VIM, saturated incidence rate, epidemic model, Lagrange multiplier, disease- induced death.*

1. INTRODUCTION

Numerical simulations of mathematical models have been widely used to understand the spread and control of epidemic diseases. The first mathematical model on smallpox was formulated and solved by Bernoulli [1] in 1760 while the second Nobel Prize was won by Ross [2] in his differential equations model of malaria in 1911.

In this paper, the variational iteration method proposed by He [3-4] is used for the simulation of SEIRS model to be able to gain insight into the effect of the saturation coefficient in the model. This model classifies individual as a susceptible, exposed, infected and recovered. Individual are born into the susceptible S(t) class. This class have never come into contact with the disease though, prone to it. Member of this class that are not able to spread the disease are in exposed class E(t). Infected class I(t) are individuals that are in exposed class E(t) and therefore infected. Finally, after some time, the immune infected individuals moved to recovered R(t).

Authors in [5-7] have studied different epidemiological models while in [5, 7-10], numerical methods, including the variational iteration method has been used for the simulation of some models. Recently in [11-12], the variational iteration method was used to solve some nonlinear differential equations that play significant roles in modeling and simulation.

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2. MATHEMATICAL FORMULATION OF SEIRS EPIDEMIC MODEL

The system of coupled ordinary differential equations that described the model as a modification of [5] is presented below:

$$\left.\begin{aligned}
\frac{dS}{dt} &= N - \frac{\beta SI}{1+m_1 S+m_2 I} - \mu S + \delta R\\
\frac{dE}{dt} &= \frac{\beta SI}{1+m_1 S+m_2 I} - (\mu+\varepsilon)E\\
\frac{dI}{dt} &= \varepsilon E - (\mu+\gamma+d)I\\
\frac{dR}{dt} &= \gamma I - (\mu+\delta)R
\end{aligned}\right\} \qquad (1)$$

where:
d = disease induced death
S(t) = susceptible individual
E(t) = exposed individual
I(t) = infected individual
R(t) = recovered individual
N = birth rate
$\beta =$ Disease transmission coefficient
$\mu =$ Mortality or death rate
$\xi =$ Recovery rate
$\gamma =$ Rate of losing immunity
$m_1 =$ Saturation term for susceptible individual
$m_2 =$ Saturation term for infected individual
$\frac{1}{1+m_1 S+m_2 I} =$ Incidence rate inclusive the saturation terms m_1 and m_2

3. SIMULATION OF THE EFFECT OF TRANSMISSION COEFFICIENT ON THE SUSCEPTIBLE-EXPOSED-INFECTED-RECOVERED-SUSCEPTIBLE (SEIRS) EPIDEMIC MODEL WITH SATURATED INCIDENCE RATE AND DISEASE-INDUCED DEATH BY VIM

Here, the VIM will be used to study the effect of transmission coefficient in the susceptible individual in SEIRS epidemic model with saturated incidence and disease-induced death. According to the Variational Iteration Method, we consider the differential equation.

$$L(u)+N(u)=g(\tau). \qquad (2)$$

where L is a linear operator, N is a non-linear operator, and $g(\tau)$ is an inhomogeneous term. A correction functional to (1) can be constructed as:

$$U_{n+1}(s)=U_n(s)+\int_0^s \lambda\left[LU_n(\tau)+N\breve{u}_n(\tau)-g(\tau)\right]d\tau \tag{3}$$

The general Lagrange multiplier λ can be identified optimally by variational calculus where $\tilde{u}_n(\tau)$ is known as the restricted variation i.e. $\delta\tilde{u}_n(\tau)=0$.

Using (3) in (1), the following system of correctional functional can be obtained:

$$S_{n+1}(t)=S_n(t)+\int_0^t \lambda_1(\tau)\left[\frac{dS_n(\tau)}{dt}-N+\frac{\beta \bar{S}_n(\tau)\bar{I}_n(\tau)}{1+m_1\bar{S}_n(\tau)+m_2 I_n(\tau)}-\mu\bar{S}_n(\tau)+\partial\bar{R}_n(\tau)\right]d\tau$$

$$E_{n+1}(t)=E_n(t)+\int_0^t \lambda_2(\tau)\left[\frac{dE_n(\tau)}{dt}-\frac{\beta \bar{S}_n(\tau)\bar{I}_n(\tau)}{1+m_1\bar{S}_n(\tau)+m_2\bar{I}_n(\tau)}+(\mu+\xi)\bar{E}_n(\tau)\right]d\tau$$

$$I_{n+1}(t)=I_n(t)+\int_0^t \lambda_3(\tau)\left[\frac{dI_n(\tau)}{dt}-\xi\bar{E}_n(\tau)+(\mu+\gamma+d)\bar{I}_n(\tau)\right]d\tau \tag{4}$$

$$R_{n+1}(t)=R_n(t)+\int_0^t \lambda_4(\tau)\left[\frac{dR_n(\tau)}{dt}-\gamma\bar{I}_n(\tau)+(\mu+\partial)\bar{R}_n(\tau)\right]d\tau$$

where $\lambda_1, \lambda_2, \lambda_3, and\ \lambda_4$ are general Lagrange Multiplier, $\tilde{S}_{n,}\tilde{E}_{n,}\tilde{I}_{n,}\ and\ \ \tilde{R}_n$ denote restricted variation i.e. $\delta\tilde{S}_n=\delta\tilde{E}_n=\delta\tilde{I}_n=\delta\tilde{R}_n=0$.

The stationary values that corresponds to (4) are:

$$\partial S_{n+1}(t)=\partial S_n(t)+\partial\int_0^t \lambda_1(\tau)\left[\frac{dS_n(\tau)}{dt}-N+\frac{\beta \bar{S}_n(\tau)\bar{I}_n(\tau)}{1+m_1\bar{S}_n(\tau)+m_2\bar{I}_n(\tau)}-\mu\bar{S}_n(\tau)+\partial\bar{R}_n(\tau)\right]d\tau$$

$$\partial E_{n+1}(t)=\partial E_n(t)+\partial\int_0^t \lambda_2(\tau)\left[\frac{dE_n(\tau)}{dt}-\frac{\beta \bar{S}_n(\tau)\bar{I}_n(\tau)}{1+m_1\bar{S}_n(\tau)+m_2\bar{I}_n(\tau)}+(\mu+\xi)\bar{E}_n(\tau)\right]d\tau \tag{5}$$

$$\partial I_{n+1}(t)=\partial I_n(t)+\partial\int_0^t \lambda_3(\tau)\left[\frac{dI_n(\tau)}{dt}-\xi\bar{E}_n(\tau)+(\mu+\gamma+d)\bar{I}_n(\tau)\right]d\tau$$

$$\partial R_{n+1}(t)=\partial R_n(t)+\partial\int_0^t \lambda_4(\tau)\left[\frac{dR_n(\tau)}{dt}-\gamma\bar{I}_n(\tau)+(\mu+\partial)\bar{R}_n(\tau)\right]d\tau$$

Equation (5) gives $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = -1$.

With $\lambda_i, i = 1 \ldots 4$, we obtained the following iterative scheme:.

$$
\begin{aligned}
S_{n+1}(t) &= S_n(t) - \int_0^t \left[\frac{dS_n(\tau)}{dt} - N + \frac{\beta S_n(\tau)\bar{I}_n(\tau)}{1+m_1 S_n(\tau)+m_2 I_n(\tau)} - \mu S_n(\tau) + \partial R_n(\tau) \right] d\tau \\
E_{n+1}(t) &= E_n(t) - \int_0^t \left[\frac{dE_n(\tau)}{dt} - \frac{\beta S_n(\tau) I_n(\tau)}{1+m_1 \bar{S}_n(\tau)+m_2 \bar{I}_n(\tau)} + (\mu+\xi)E_n(\tau) \right] d\tau \\
I_{n+1}(t) &= I_n(t) - \int_0^t \left[\frac{dI_n(\tau)}{dt} - \xi E_n(\tau) + (\mu+\gamma+d) I_n(\tau) \right] d\tau \\
R_{n+1}(t) &= R_n(t) - \int_0^t \left[\frac{dR_n(\tau)}{dt} - \gamma I_n(\tau) + (\mu+\partial) R_n(\tau) \right] d\tau
\end{aligned} \tag{6}
$$

For numerical results, we used the following parameters:

$$
\begin{aligned}
&S_0(t) = 15,\ I_0(t) = 10,\ E_0(t) = 13,\ R_0(t) = 11,\ \varepsilon = 0.25,\ \delta = 0.05, \mu = 0.3, \\
&N = 49, \gamma = 0.1, m_1 = 0.1, m_2 = 0.2, d = 0.1
\end{aligned} \tag{7}
$$

when $n = 4$, the following results can be readily obtained by Maple 18.

$$
\begin{aligned}
S_1(t) &= 15 + 44.35t - 150\beta t \\
E_1(t) &= 13 - 8.85t - 150\beta t \\
I_1(t) &= 10 - 0.750t - 10dt \\
R_1(t) &= 11 - 2.85t
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
&S_2(t) = 15 + 44.35t - 150\beta t + 11.0875\beta t^3 + 147.8333333\beta dt^3 - 37.5\beta^2 t^2 - 500\beta^2 dt^3 \\
&-186.125\beta t^2 + 75\beta dt^2 + 750\beta^2 t^2 - 8.93375t^2 + 0.1dt^2 \\
&E_2(t) = 13 - 8.85t - 150\beta t + 11.0875\beta t^3 + 147.8333333\beta dt^3 - 37.5\beta^2 t^3 - 500\beta^2 dt^3 \\
&-167.375\beta t^2 + 75\beta dt^2 + 0.22375t^2 + 0.1dt^2 \\
&I_2(t) = 10 - 0.750t - 10dt + 11.0875\beta t^3 - 0.95625t^2 - 18.75\beta^2 t^2 + 2.375dt^2 + 5d^2 t^2 \\
&R_2(t) = 11 - 2.85t + 0.46125t^2 - 0.5t^2 d
\end{aligned} \tag{9}
$$

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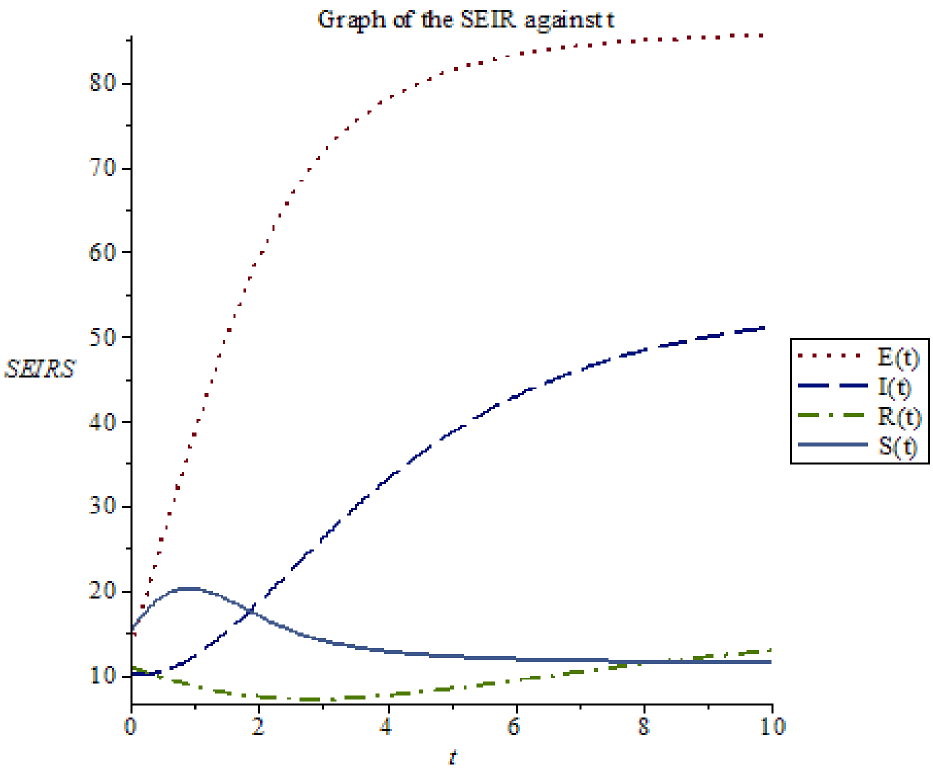

Figure 1. Graph of SEIRS against t when $\beta = 1$**.**

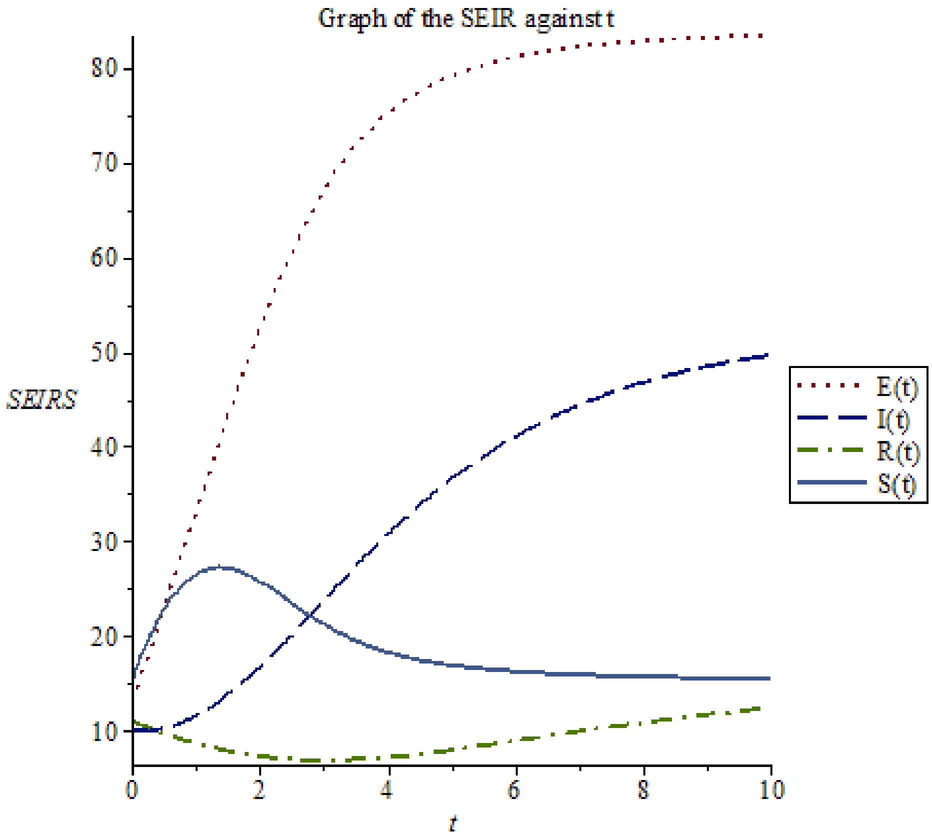

Figure 2. Graph of SEIRS against t when $\beta = 0.75$**.**

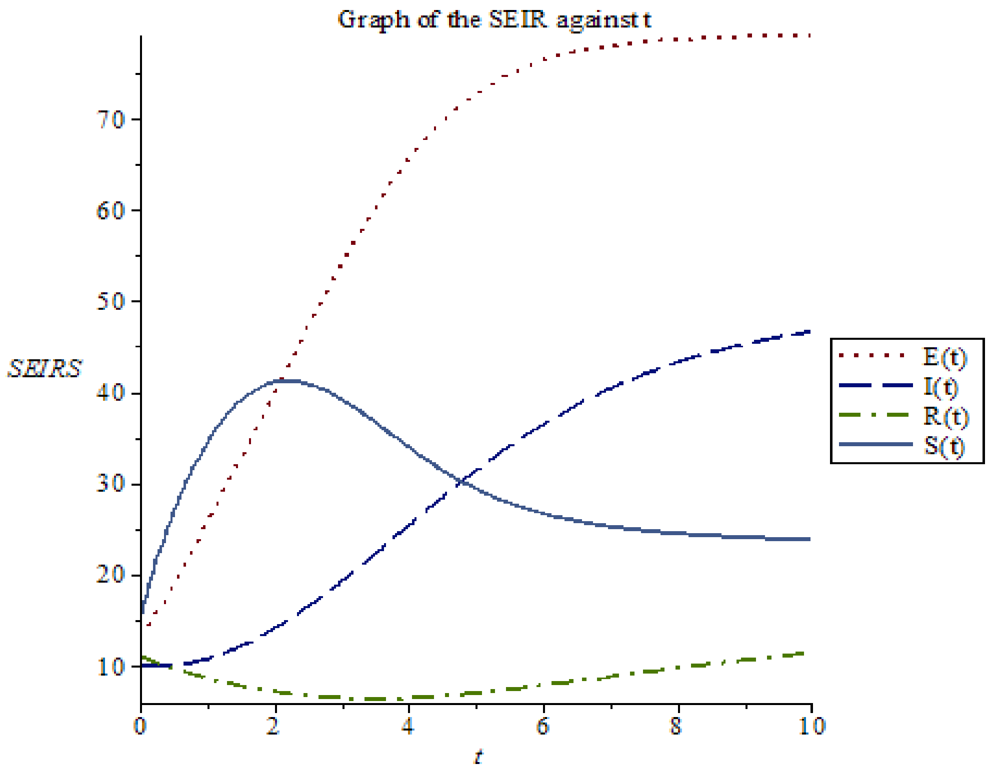

Figure 3. Graph of SEIRS against t when $\beta = 0.5$ **.**

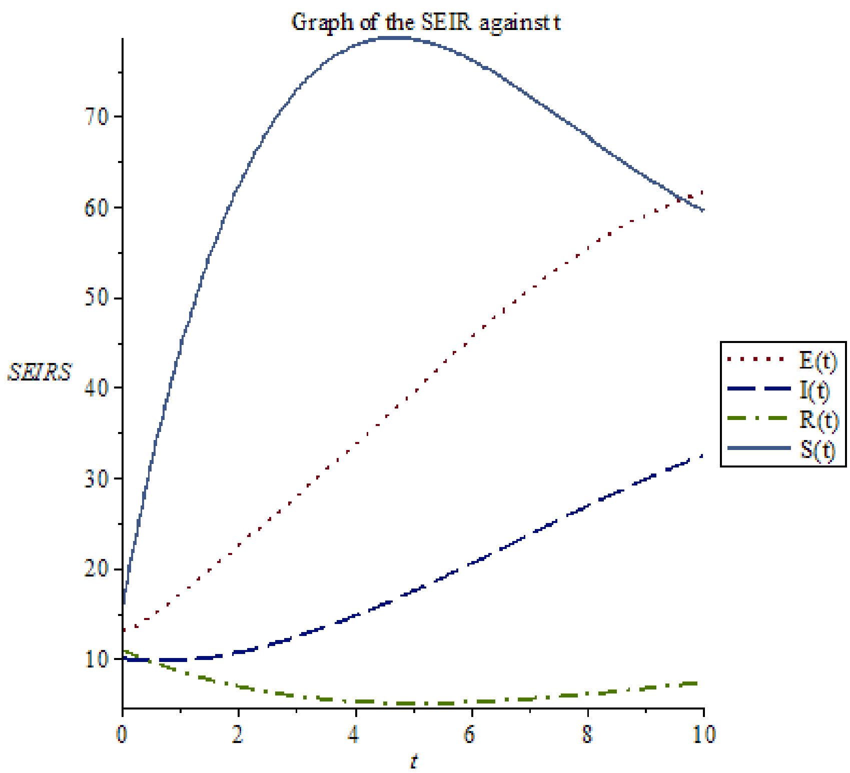

Figure 4. Graph of SEIRS against t when $\beta = 0.25$ **.**

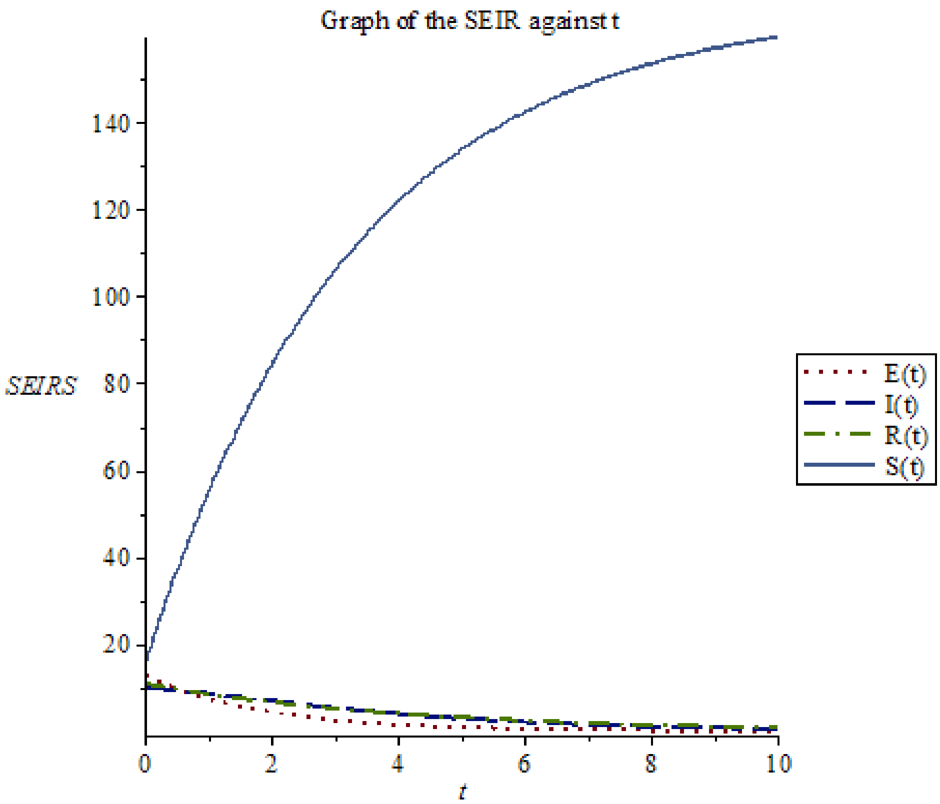

Figure 5. Graph of SEIRS against t when $\beta = 0.0005$.

4. RESULTS AND DISCUSSION

From Figures I-V, the simulation reveals the effect of disease transmission coefficient in its eradication. As the coefficient decreases, more people move to the susceptible compartment while the exposed and the infected classes reduce drastically. Numerically, since the disease induced-death is insignificant, it is therefore, recommended that other measures such as disease transmission coefficient be used for a better stability.

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