ORIGINAL PAPER

INTEGRALS INVOLVING GENERALIZED BESSEL-MAITLAND FUNCTION

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Abstract. *The present paper is the investigation of some integrals for the generalized Bessel-Maitland functions, which are expressed in the terms of hypergeometric and beta function. Some interesting special cases involving Jacobi, Legendre polynomial are deduced. The integrals established in this paper are of general character.*

Keywords: *Jacobi polynomial, Bessel-Maitland functions, Legendre polynomial, Gauss hypergeometric function, beta function.*

1. INTRODUCTION

The special function of the form defined by the following series representation as

$$J_{\upsilon}^{\mu}(z)=\sum_{k=0}^{\infty}\frac{(-z)^k}{\Gamma(\upsilon+\mu k+1)\,k!},\quad (\mu>0;\ z\in C) \tag{1.1}$$

is known as Bessel-Maitland function. In fact, the application of Bessel –Maitland function are found in the diverese field on mathematical physics, engineering, biological, chemical in the book of Watson [9].

An interesting generalization of the Bessel function is defined by Jain and Aggarwal [1] as follow.

$$J_{\upsilon,\sigma}^{\mu}(z)=\sum_{k=0}^{\infty}\frac{(-1)^k\,(z/2)^{\upsilon+2\sigma+2k}}{\Gamma(\sigma+k+1)\Gamma(\upsilon+\sigma+\mu k+1)\,k!},\quad Z\in C\setminus(-\infty,0];\ \mu>0;\ \upsilon,\sigma\in C. \tag{1.2}$$

Further, the generalized Bessel –Maitland function investigated and studied by Pathak [3] and defined it as:

$$J_{\upsilon,q}^{\mu,\gamma}(z)=\sum_{k=0}^{\infty}\frac{(\gamma)_{qk}\,(-z)^k}{\Gamma(\upsilon+\mu k+1)\,k!}, \tag{1.3}$$

where $\mu,\ \upsilon,\ \gamma\in C,\ \Re e(\mu)\geq 0,\ \Re e(\upsilon)\geq 0,\ \Re e(\gamma)\geq 0,$ *and* $q\in(0,1)\cup N$ and $(\gamma)_0=1,$
$(\gamma)_{qk}=\Gamma(\gamma+qk)/\Gamma(\gamma),$ denotes the pochhammer symbol.

We have also some special cases related with the Mittag -Leffler function of the generalized Bessel-Maitland function, as follow.

(i) If υ is replaced by $\upsilon-1$ and z by $-z$, (1.3) reduces to generalized Mittag-Leffler function, was given by Shukla and Prajapati [7] as

$$J_{\upsilon-1,q}^{\mu,\gamma}(z)\ =\ E_{\mu,\upsilon}^{\gamma,q}(z), \tag{1.4}$$

where $\mu,\ \upsilon,\ \gamma\in C,\ \Re e(\mu)\geq 0,\ \Re e(\upsilon)\geq 0,\ \Re e(\gamma)\geq 0,$ *and* $q\in(0,1)\cup N$

(ii) If υ is replaced by $\upsilon-1$, z by $-z$ and $q=1$ (1.3) reduces to

$$J_{\upsilon-1,1}^{\mu,\gamma}(z)\ =\ E_{\mu,\upsilon}^{\gamma}(z), \tag{1.5}$$

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was introduced by Prabhakar [4].

(iii) If υ is replaced by $\upsilon-1$, z by $-z$ and $\gamma=1$, $q=1$ (1.3) reduces to

$$J_{\upsilon-1,\,1}^{\mu,1}(z) = E_{\mu,\,\upsilon}(z), \tag{1.6}$$

was studied byWiman [10].

(iv) If $\upsilon=0$, $\gamma=1$, $q=1$ and z is replaced by $-z$, (1.3) reduces to

$$J_{0,1}^{\mu,1}(z) = E_{\mu}(z), \tag{1.7}$$

was introduced by Ghosta Mittag-Leffler [2].

Recently Singh and Rawat [8], established certain integrals for the generalized Mittag- Leffler function. In the present paper, we established integrals with Bessel-Maitland function, add one more dimension to this study by introducing certain integral for the generalized Jacobi polynomials. The integral established in this paper are believed to be a new contribution in the theory of fractional calculus.

2. INTEGRALS WITH GENERALIZED JACOBI POLYNOMIALS

The $p_n^{(\alpha,\beta,c,d)}$ is the generalized Jacobi function, which is defined and studied by Kalla et al. [6].

$$P_n^{(\alpha,\beta,c,d)}(x) = \frac{(\alpha+1)_n}{\Gamma(n+1)}\, {}_3F_2\left(\begin{matrix} -n,\ n+\alpha+\beta+1,\ c\,; \\ \alpha+1,\ d\,; \end{matrix}\ \frac{1-x}{2}\right) \tag{2.1}$$

where

$$d \in C-Z^{-}\cup\{0\}:\ \alpha,\ n \in C-Z^{-}\ ;\ \beta \in C\ ;\ \mathrm{Re}(d-\beta-c) > 0\ . \tag{2.2}$$

In dealing with Jacobi function, It is natural to make much use of our knowledge of the hypergeometric function by Rainville ([5], p.45)

$$I_1 \equiv \int_{-1}^{+1}(1-x)^{\lambda}(1+x)^{\delta} P_n^{(\alpha,\,\beta,\,c,\,d)}(x)\ J_{\upsilon,\,q}^{\mu,\,\gamma}[z(1+x)^h]dx$$

$$= \int_{-1}^{+1}(1-x)^{\lambda}(1+x)^{\delta} P_n^{(\alpha,\,\beta,\,c,\,d)}(x) \sum_{k=0}^{\infty}\frac{(\gamma)_{qk}\ [-z(1+x)^h]^k}{\Gamma(\upsilon+\mu k+1)\ k!}\ dx$$

Interchanging the order of integration and summation which is permissible under the condition, then the above expression becomes

$$= \sum_{k=0}^{\infty}\frac{(\gamma)_{qk}\ (-z)^k}{\Gamma(\upsilon+\mu k+1)\ k!}\int_{-1}^{+1}(1-x)^{\lambda}(1+x)^{\delta+hk} P_n^{(\alpha,\,\beta,\,c,\,d)}(x)\ dx \tag{2.3}$$

Now using (2.3) and formula from Kalla et al. [6, p. 372], we get

$$= 2^{\lambda+\delta+1}\,\frac{(\alpha+1)_n\,\Gamma(\lambda+1)}{\Gamma(n+1)}\sum_{k=0}^{\infty}\frac{\Gamma(\delta+hk+1)}{\Gamma(\lambda+\delta+hk+2)}\,J_{\upsilon,\,q}^{\mu,\,\gamma}\left(2^h z\right)\ {}_4F_3\left(\begin{matrix} -n,\ n+\alpha+\beta+1,\ c,\ \lambda+1; \\ \alpha+1,\ d,\ \lambda+\delta+hk+2; \end{matrix}\ \frac{1-x}{2}\right) \tag{2.4}$$

Provided

(i) $\mu,\ \upsilon,\ \gamma\in C,\ \Re e(\mu)\geq 0,\ \Re e(\upsilon)\geq 0,\ \Re e(\gamma)\geq 0,\ and\ \ q\in(0,1)\cup N$

(ii) $\alpha > -1$ *and* $\beta > -1$.

$$I_2 \equiv \int_{-1}^{+1} (1-x)^{\lambda} (1+x)^{\delta} P_n^{(\alpha,\beta,c,d)}(x)\, P_m^{(\rho,\sigma,e,f)}(x)\; J_{\upsilon,q}^{\mu,\gamma}[z(1-x)^h]dx$$

$$= \sum_{k=0}^{\infty} \frac{(\gamma)_{qk}\,(-z)^k}{\Gamma(\upsilon+\mu k+1)\; k!} \int_{-1}^{+1} (1-x)^{\lambda+hk} (1+x)^{\delta} P_n^{(\alpha,\beta,c,d)}(x)\; P_m^{(\rho,\sigma,e,f)}(x)\; dx \qquad (2.5)$$

Now using (2.1) in the above expression, we get

$$= \sum_{k=0}^{\infty} \frac{(\gamma)_{qk}\,(-z)^k}{\Gamma(\upsilon+\mu k+1)\; k!} \frac{(\rho+1)_m}{\Gamma(m+1)} \sum_{k=0}^{\infty} \frac{(-m)_k\,(m+\rho+\sigma+1)_k\,(c)_k}{(\rho+1)_k\,(d)_k\; 2^k k!} \int_{-1}^{+1} (1-x)^{\lambda+hk+k} (1+x)^{\delta} P_n^{(\alpha,\beta,c,d)}(x)\; dx \qquad (2.6)$$

Again using (2.1) and (2.6), we obtain

$$= \sum_{k=0}^{\infty} \frac{(\gamma)_{qk}\,(-z)^k}{\Gamma(\upsilon+\mu k+1)\; k!} \frac{\Gamma(\rho+m+1)}{\Gamma(m+1)} \frac{(\alpha+1)_m}{\Gamma(n+1)} \sum_{k=0}^{\infty} \frac{(-m)_k\,(-n)_k\,(m+\rho+\sigma+1)_k\,(n+\alpha+\beta+1)_k\,(c)_k\,(e)_k}{\Gamma(\rho+k+1)\;(\alpha+1)_k\,(d)_k\;(f)_k\;2^{2k}\,(k!)^2}$$

$$\times \int_{-1}^{+1} (1-x)^{\lambda+hk+2k} (1+x)^{\delta}\; dx \qquad (2.7)$$

Using the known result by Rainville ([5], p.261)

$$\int_{-1}^{+1} (1-x)^{\alpha+n} (1+x)^{\beta+n}\; dx = 2^{2n+\alpha+\beta+1}\; B(\alpha+n+1,\; \beta+n+1) \qquad (2.8)$$

From equation (2.7)

$$= 2^{\lambda+\delta+1} \frac{\Gamma(\rho+m+1)}{\Gamma(m+1)} \frac{\Gamma(\alpha+n+1)}{\Gamma(n+1)} \sum_{k=0}^{\infty} \frac{(-m)_k\,(-n)_k\,(m+\rho+\sigma+1)_k\,(n+\alpha+\beta+1)_k\,(c)_k\,(e)_k}{\Gamma(\rho+k+1)\;\Gamma(\alpha+k+1)\;(d)_k\;(f)_k\;2^{2k}\;(k!)^2}$$

$$\times\; J_{\upsilon,q}^{\mu,\gamma}\left(2^h z\right) B(\lambda+hk+2k+1,\; \delta+1) \qquad (2.9)$$

Provided

(i) $\Re e(\mu) \ge 0,\ \Re e(\upsilon) \ge 0,\ \Re e(\gamma) \ge 0,$ *and* $q \in (0,1) \cup N$

(ii) $\Re(\beta) > -1$, λ and h are positive numbers.

$$I_3 \equiv \int_{-1}^{+1} (1-x)^{\lambda} (1+x)^{\delta} P_n^{(\alpha,\beta,c,d)}(x)\; J_{\upsilon,q}^{\mu,\gamma}[z(1-x)^h (1+x)^t]dx$$

$$= \sum_{k=0}^{\infty} \frac{(\gamma)_{qk}\,(-z)^k}{\Gamma(\upsilon+\mu k+1)\; k!} \int_{-1}^{+1} (1-x)^{\lambda+hk} (1+x)^{\delta+tk} P_n^{(\alpha,\beta,c,d)}(x)\; dx$$

(2.10)

Using (2.1) in (2.10), we obtain

$$= \sum_{k=0}^{\infty} \frac{(\gamma)_{qk}\,(-z)^k}{\Gamma(\upsilon+\mu k+1)\; k!} \frac{\Gamma(\alpha+1)_n}{\Gamma(n+1)} \sum_{k=0}^{\infty} \frac{(-n)_k\,(n+\alpha+\beta+1)_k\,(c)_k}{\Gamma(\alpha+1)_k\;(d)_k\;2^k (k!)} \int_{-1}^{+1} (1-x)^{n+\lambda+hk+k-n} (1+x)^{n+\delta+tk-n}\, dx \qquad (2.11)$$

Now using (2.8) and (2.11), we get

$$= 2^{\lambda+\delta+1} \frac{\Gamma(\alpha+1)_n}{\Gamma(n+1)} \sum_{k=0}^{\infty} \frac{(-n)_k\,(n+\alpha+\beta+1)_k\;(c)_k}{\Gamma(\alpha+1)_k\;(d)_k\;(k!)} J_{\upsilon,q}^{\mu,\gamma}\left(2^{h+t} z\right) B(\lambda+hk+k+1,\; \delta+tk+1) \qquad (2.12)$$

Provided

(i) $\Re e(\mu)\geq 0,\ \Re e(\upsilon)\geq 0,\ \Re e(\gamma)\geq 0,\ and\ \ q\in(0,1)\bigcup N$

(ii) $\Re(\alpha)>-1\ \ and\ \ \Re(\beta)>-1.$

$$I_4\equiv\int_{-1}^{+1}(1-x)^{\lambda}(1+x)^{\delta}P_n^{(\alpha,\,\beta,\,c,\,d)}(x)\ J_{\upsilon,\,q}^{\mu,\,\gamma}[z(1+x)^{-h}]dx$$

$$=\sum_{k=0}^{\infty}\frac{(\gamma)_{qk}\ (-z)^k}{\Gamma(\upsilon+\mu k+1)\ k!}\int_{-1}^{+1}(1-x)^{\lambda}(1+x)^{\delta-hk}P_n^{(\alpha,\,\beta,\,c,\,d)}(x)\ dx \tag{2.13}$$

By using (2.1) in (2.13), we obtain

$$=\sum_{k=0}^{\infty}\frac{(\gamma)_{qk}\ (-z)^k}{\Gamma(\upsilon+\mu k+1)\ k!}\frac{\Gamma(\alpha+1)_n}{\Gamma(n+1)}\sum_{k=0}^{\infty}\frac{(-n)_k\ (n+\alpha+\beta+1)_k\ (c)_k}{\Gamma(\alpha+1)_k\ \ (d)_k\ \ 2^k(k!)}\int_{-1}^{+1}(1-x)^{n+\lambda+k-n}(1+x)^{n+\delta-hk-n}\ dx \tag{2.14}$$

Now using (2.8) in (2.14), we get

$$=2^{\lambda+\delta+1}\frac{\Gamma(\alpha+1)_n}{\Gamma(n+1)}\sum_{k=0}^{\infty}\frac{(-n)_k\ (n+\alpha+\beta+1)_k\ \ (c)_k}{\Gamma(\alpha+1)_k\ \ (d)_k\ \ (k!)}J_{\upsilon,\,q}^{\mu,\,\gamma}\left(2^{-h}z\right)B(\lambda+k+1,\ \ \delta-hk+1) \tag{2.15}$$

Provided

(i) $\Re e(\mu)\geq 0,\ \Re e(\upsilon)\geq 0,\ \Re e(\gamma)\geq 0,\ and\ \ q\in(0,1)\bigcup N$

(ii) $\Re(\alpha)>-1\ \ and\ \ \Re(\beta)>-1.$

$$I_5\equiv\int_{-1}^{+1}(1-x)^{\lambda}(1+x)^{\delta}P_n^{(\alpha,\,\beta,\,c,\,d)}(x)\ J_{\upsilon,\,q}^{\mu,\,\gamma}[z(1-x)^{h}(1+x)^{-t}]dx$$

$$=\sum_{k=0}^{\infty}\frac{(\gamma)_{qk}\ (-z)^k}{\Gamma(\upsilon+\mu k+1)\ k!}\int_{-1}^{+1}(1-x)^{\lambda+hk}(1+x)^{\delta-tk}P_n^{(\alpha,\,\beta,\,c,\,d)}(x)\ dx \tag{2.16}$$

Using (2.1) in (2.16), we obtain

$$=\sum_{k=0}^{\infty}\frac{(\gamma)_{qk}\ (-z)^k}{\Gamma(\upsilon+\mu k+1)\ k!}\frac{\Gamma(\alpha+1)_n}{\Gamma(n+1)}\sum_{k=0}^{\infty}\frac{(-n)_k\ (n+\alpha+\beta+1)_k\ (c)_k}{\Gamma(\alpha+1)_k\ \ (d)_k\ \ 2^k(k!)}\int_{-1}^{+1}(1-x)^{n+\lambda+hk+k-n}(1+x)^{n+\delta-tk-n}\ dx \tag{2.17}$$

Now using (2.8) and (2.17), we get

$$=2^{\lambda+\delta+1}\frac{\Gamma(\alpha+1)_n}{\Gamma(n+1)}\sum_{k=0}^{\infty}\frac{(-n)_k\ (n+\alpha+\beta+1)_k\ \ (c)_k}{\Gamma(\alpha+1)_k\ \ (d)_k\ \ (k!)}J_{\upsilon,\,q}^{\mu,\,\gamma}\left(2^{h-t}z\right)B(\lambda+hk+k+1,\ \delta-tk+1) \tag{2.18}$$

Provided

(i) $\Re e(\mu)\geq 0,\ \Re e(\upsilon)\geq 0,\ \Re e(\gamma)\geq 0,\ and\ \ q\in(0,1)\bigcup N$

(ii) $\Re(\alpha)>-1\ \ and\ \ \Re(\beta)>-1.$

3. SPECIAL CASES

For $c = d$, (2.1) reduces in to Jacobi polynomial $P_n^{(\alpha,\beta)}(x)$ which is defined in Rainville [5]

$$P_n^{(\alpha,\beta)}(x) = \frac{(\alpha+1)_n}{n!}\ {}_3F_2\left(\begin{matrix} -n,\ n+\alpha+\beta+1; \\ \alpha+1; \end{matrix}\ \frac{1-x}{2}\right) \tag{3.1}$$

If we set $\alpha = \beta = 0$, the polynomial in (3.1) becomes the Legendre polynomial ([5]).

From (3.1), If $x = 1$ it also follow that $P_n^{(\alpha,\beta)}(x)$ is a polynomial of degree n and that

$$P_n^{(\alpha,\beta)}(1) = \frac{(\alpha+1)_n}{n!} \tag{3.2}$$

From I_1 integral, we obtain

$$I_6 \equiv \int_{-1}^{+1}(1-x)^{\lambda}(1+x)^{\delta} P_n^{(\alpha,\beta)}(x)\ J_{\upsilon,q}^{\mu,\gamma}[z(1+x)^h]dx \tag{3.3}$$

$$= 2^{\lambda+\delta+1}\frac{(\alpha+1)_n\,\Gamma(\lambda+1)}{\Gamma(n+1)}\sum_{k=0}^{\infty}\frac{\Gamma(\delta+hk+1)}{\Gamma(\lambda+\delta+hk+2)}\,J_{\upsilon,q}^{\mu,\gamma}\left(2^h z\right)\ {}_3F_2\left(\begin{matrix} -n,\ n+\alpha+\beta+1,\ \lambda+1; \\ \alpha+1,\ \lambda+\delta+hk+2; \end{matrix}\ \frac{1-x}{2}\right)$$

Now from I_2 integral, we obtain

$$I_7 \equiv \int_{-1}^{+1}(1-x)^{\lambda}(1+x)^{\delta} P_n^{(\alpha,\beta)}(x)\,P_m^{(\rho,\sigma)}(x)\ J_{\upsilon,q}^{\mu,\gamma}[z(1-x)^h]dx$$

$$= 2^{\lambda+\delta+1}\frac{\Gamma(\rho+m+1)}{\Gamma(m+1)}\frac{\Gamma(\alpha+n+1)}{\Gamma(n+1)}\sum_{k=0}^{\infty}\frac{(-m)_k(-n)_k(m+\rho+\sigma+1)_k(n+\alpha+\beta+1)_k}{\Gamma(\rho+k+1)\,\Gamma(\alpha+k+1)\ 2^{2k}\ (k!)^2}$$

$$\times\ J_{\upsilon,q}^{\mu,\gamma}\left(2^h z\right)B(\lambda+hk+2k+1,\ \delta+1) \tag{3.4}$$

If we replace $\lambda = \lambda - 1$ and $\alpha = \beta = \rho = \sigma = \delta = 0$, then the integral I_7 transforms in to the following integral involving Legendre polynomials.

$$I_8 \equiv \int_{-1}^{+1}(1-x)^{\lambda-1}P_n(x)\ J_{\upsilon,q}^{\mu,\gamma}[z(1-x)^h]dx$$

$$= 2^{\lambda}\sum_{k=0}^{\infty}\frac{(-m)_k(-n)_k(m+1)_k(n+1)_k}{\Gamma(k+1)\,\Gamma(k+1)\ 2^{2k}\ (k!)^2}\ J_{\upsilon,q}^{\mu,\gamma}\left(2^h z\right)B(\lambda+hk+2k,\ 1) \tag{3.5}$$

Now from I_3 integral, we obtain

$$I_9 \equiv \int_{-1}^{+1}(1-x)^{\lambda}(1+x)^{\delta} P_n^{(\alpha,\beta)}(x)\ J_{\upsilon,q}^{\mu,\gamma}[z(1-x)^h(1+x)^t]dx$$

$$= 2^{\lambda+\delta+1}\frac{\Gamma(\alpha+1)_n}{\Gamma(n+1)}\sum_{k=0}^{\infty}\frac{(-n)_k(n+\alpha+\beta+1)_k}{\Gamma(\alpha+1)_k\ (k!)}\,J_{\upsilon,q}^{\mu,\gamma}\left(2^{h+t}z\right)B(\lambda+hk+k+1,\ \delta+tk+1) \tag{3.6}$$

If we set $\lambda = \lambda - 1,\ \delta = \delta - 1$ and $\alpha = \beta = 0$, then the integral I_9 transforms in to the following integral involving Legendre polynomials.

$$I_{10} \equiv \int_{-1}^{+1}(1-x)^{\lambda-1}(1+x)^{\delta-1}P_n(x)\ J_{\upsilon,q}^{\mu,\gamma}[z(1-x)^h(1+x)^t]dx$$

$$= 2^{\lambda+\delta-1}\sum_{k=0}^{\infty}\frac{(-n)_k(n+1)_k}{(k!)^2}\,J_{\upsilon,q}^{\mu,\gamma}\left(2^{h+t}z\right)B(\lambda+hk+k,\ \delta+tk) \tag{3.7}$$

Now from I_4 integral, we obtain

$$I_{11} \equiv \int_{-1}^{+1} (1-x)^{\lambda} (1+x)^{\delta} P_n^{(\alpha,\beta)}(x)\ J_{\upsilon,q}^{\mu,\gamma}[z(1+x)^{-h}]dx$$

$$= 2^{\lambda+\delta+1} \frac{\Gamma(\alpha+1)_n}{\Gamma(n+1)} \sum_{k=0}^{\infty} \frac{(-n)_k\ (n+\alpha+\beta+1)_k}{\Gamma(\alpha+1)_k\ (k!)} J_{\upsilon,q}^{\mu,\gamma}\left(2^{-h} z\right) B(\lambda+k+1,\ \ \delta-hk+1) \qquad (3.8)$$

Now from I_5 integral, we obtain

$$I_{12} \equiv \int_{-1}^{+1} (1-x)^{\lambda} (1+x)^{\delta} P_n^{(\alpha,\beta)}(x)\ J_{\upsilon,q}^{\mu,\gamma}[z(1-x)^{h}(1+x)^{-t}]dx$$

$$= 2^{\lambda+\delta+1} \frac{\Gamma(\alpha+1)_n}{\Gamma(n+1)} \sum_{k=0}^{\infty} \frac{(-n)_k\ (n+\alpha+\beta+1)_k}{\Gamma(\alpha+1)_k\ (k!)} J_{\upsilon,q}^{\mu,\gamma}\left(2^{h-t} z\right) B(\lambda+hk+k+1,\ \delta-tk+1) \qquad (3.9)$$

If we replace $\lambda = \lambda - 1,\ \delta = \delta - 1$ and $\alpha = \beta = 0$, then the integral I_{12} takes the following integral involving Legendre polynomials.

$$I_{13} \equiv \int_{-1}^{+1} (1-x)^{\lambda-1} (1+x)^{\delta-1} P_n(x)\ J_{\upsilon,q}^{\mu,\gamma}[z(1-x)^{h}(1+x)^{-t}]dx$$

$$= 2^{\lambda+\delta+1} \sum_{k=0}^{\infty} \frac{(-n)_k\ (n+1)_k}{(k!)^2} J_{\upsilon,q}^{\mu,\gamma}\left(2^{h-t} z\right) B(\lambda+hk+k,\ \ \delta-tk) \qquad (3.10)$$

4. CONCLUDING REMARKS

In the present paper, we investigate new integrals involving the generalized Bessel-Maitland function, in terms of the hypergeometric function and beta function. Some special cases of integrals invovling the generalized Mittag -Leffler function have been investigated in the literature by a many authors with different arguments.

It is interesting to observe that the results given by Singh and Rawat [8, eq.16, 19, 22, 25, 26, 27 and 28] follow from the special cases results derived in this paper, if we use (1.4) and some suitable parametric replacements.

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