ORIGINAL PAPER

EXACT SOLUTIONS OF SRLW EQUATION BY A NEW ANALYTICAL TECHNIQUE

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Abstract. *Evolution equation is among one of the important nonlinear partial differential equations and its exact solutions are of great interest for the researchers around the globe. In this article, a new analytical technique is used to search for exact solutions of nonlinear evolution equation with the aid of symbolic computation. To check the validity of the method developed, we choose the SRLW equation and many new and more general exact solutions have been obtained for the equation, which are of great importance. The efficiency of the developed scheme is confirmed since it provides more exact solutions than other techniques used now a day.*

Keywords: *New (G'/G)-expansion method, Symmetric regularized long wave (SRLW) equation, Homogeneous balance, Solitary wave solutions, Exact solutions.*

1. INTRODUCTION

Nonlinear differential equations are the core of research for many scientists and researchers over the past few decades especially for the fundamental understanding of nature.

Nonlinear evolution equations are one of the important nonlinear partial differential equations that arise from important physical phenomena's. Exact solutions of these physical models are of great significance due to the fact that it not only helps to check the accuracy of computational dynamics but also helps us in understanding the behaviour of the model at different time values.

Phenomena's such as diffusion, dissipation, dispersion, convection and reaction are very important in models representing wave equations. Search for exact solutions gained more attention from researchers after the development of software's like MAPLE, MATLAB, MATHEMATICA, etc which can perform symbolic computations easily with in no time. Methods such as Tanh-function method [1], Hirota method [2], Exp-function method [3], F-expansion method [4], Extended tanh-method [5] and other such methods were developed to find the exact solutions of nonlinear problems.

Wang et al. [6] introduced (G'/G)-expansion method when he obtained solutions of nonlinear models. Exact solutions of nonlinear P.D.Es with variable coefficients were introduced by Zhang et al. [7]. Later Zhang et al. [8] developed improved (G'/G)-expansion method for nonlinear evolution equations. Zayed [9, 10] introduced two new modifications in (G'/G)-expansion method. The transformed rational function method introduced by W. X. Ma [11, 12] uses the same concept as that of (G'/G)-expansion method. For N-soliton and N-wave solutions of the partial differential equation, linear superposition principle [13] and multiple Exp-function method [14] were applied to Hirota bilinear equations and others.

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In this article, we have introduced a new (G'/G)-expansion method [15] to solve the nonlinear partial differential equations. To confirm the performance, accuracy and efficiency of the developed method we have applied it on SRLW equation.

2. THE DEVELOPED METHOD

Assume a nonlinear PDE

$$P(u, u_t, u_x, u_{tt}, u_{tx}, u_{xx}, \cdots) = 0, \tag{1}$$

here u is a function of x and t.

Step 1. Firstly, we introduce a suitable transformation in the form of ξ,

$$u(x,t) = u(\xi),\ \xi = x \pm V\,t, \tag{2}$$

where V is the speed of wave. Equation (2) converts equation (1) into an ordinary differential equation for $u = u(\xi)$:

$$Q(u, u', u'', u''', \cdots) = 0, \tag{3}$$

where Q is a function of $u(\xi)$.

Step 2. Solution of Eq. (3) can be expressed in the form:

$$u(\xi) = \sum_{i=-m}^{m} \alpha_i \left(k + \Phi(\xi)\right)^i, \tag{4}$$

where $\Phi(\xi) = \dfrac{G'(\xi)}{G(\xi)}$. (5)

In equation (4), α_{-m} or α_m do not vanish all together. α_i $(i = 0, \pm 1, \pm 2, \cdots, \pm m)$ and k is a constant and $G = G(\xi)$ is a solution of following equation:

$$G\,G'' = A\,G\,G' + B\,G^2 + C\,(G')^2. \tag{6}$$

Equation (6) is reduced into Riccati equation by using Cole-Hopf transformation $\Phi(\xi) = \ln\left(G(\xi)\right)_\xi = G'(\xi)/G(\xi)$. Thus we have

$$\Phi'(\xi) = B + A\,\Phi(\xi) + (C-1)\,\Phi^2(\xi). \tag{7}$$

The solutions of equation (7) are given in [16].

Step 3. In positive integer m in equation (4) is calculated by homogenous balancing principle.

Step 4. Using equations (4)-(6) into equation (3), we obtain polynomials in $\left(k+G'(\xi)/G(\xi)\right)^{i}$ and $\left(k+G'(\xi)/G(\xi)\right)^{-i}$. System of algebraic equations formed can be solved to obtained values of constants α_i $(i=0,\pm 1,\pm 2,\cdots,\pm m)$, k and V.

Step 5. After finding values of constants, using equation (6) we get abundant analytical solutions of the considered problem (1).

3. IMPLEMENTATION OF DEVELOPED METHOD TO SRLW EQUATION

Consider the nonlinear SRLW equation

$$u_{tt}+u_{xx}+uu_{xt}+u_x u_t+u_{xxtt}=0, \tag{8}$$

which arises frequently in many nonlinear physical phenomena's. Exp function method is applied on SRLW equation in [17] to obtain periodic wave solutions. (G'/G)-expansion method is also used to obtain travelling wave solutions of SRLW equation in [18].

Now, using the traveling wave variable $u=u(\xi)$, $\xi=x-Vt$ into Eq. (8) and integrating twice, we obtain

$$(1+V^2)u-\frac{1}{2}Vu^2+V^2u''+C_1=0, \tag{9}$$

where C_1 is an integration constant.

Considering the homogeneous balance between u'' and u^2 in Eq. (9), we obtain $m=2$. Therefore, the trial solution becomes

$$u(\xi)=\alpha_{-2}\left(k+\Phi(\xi)\right)^{-2}+\alpha_{-1}\left(k+\Phi(\xi)\right)^{-1}+\alpha_0+\alpha_1\left(k+\Phi(\xi)\right)+\alpha_2\left(k+\Phi(\xi)\right)^{2}. \tag{10}$$

Using Eq. (10) into Eq. (9), left hand side transforms into polynomials in $\left(k+G'(\xi)/G(\xi)\right)^{i}$ and $\left(k+G'(\xi)/G(\xi)\right)^{-i}$, $(i=0,1,\ 2,\cdots,m)$. Equating the coefficients of same power of the resulted polynomials to zero, we obtain a set of algebraic equations (which are omitted for simplicity) for $\alpha_0,\alpha_1,\alpha_2,\alpha_{-1}$, α_{-2}, k, C_1 and V. Solving by the help of Maple, we have

Set 1. $\alpha_2=12V\left(C-1\right)^2$, $\alpha_1=12V\left(-2C^2k+4Ck+AC-A-2k\right)$,

$$\begin{aligned}
&\alpha_0=V\left(12C^2k^2-12kAC+12k^2+12kA-24Ck^2+8BC-8B+A^2+1\right)+\frac{1}{V},\\
&V=V,\ k=k,\ \alpha_{-1}=0,\ \alpha_{-2}=0,\\
&C_1=\frac{V^3}{2}\left(16B^2C^2+16B^2+8A^2B-8A^2BC+A^4-32B^2C-1\right)-V-\frac{1}{2V},
\end{aligned} \tag{11}$$

Set 2. $\alpha_0=V\left(12C^2k^2+12k^2+12kA-12kAC-24Ck^2+8BC-8B+1+A^2\right)+\dfrac{1}{V}$,

$$\alpha_{-1}=12V\left(2Bk-2C^2k^3+BA-2k^3-3Ak^2-A^2k+3ACk^2+4Ck^3-2BCk\right),$$

$$\alpha_{-2}=12V\left(C^2k^4-2Bk^2+B^2+k^4+2Ak^3+A^2k^2-2ACk^3-2Ck^4-2ABk+2BCk^2\right),$$
$$V=V\,,k=k,\ \alpha_2=0,\ \alpha_1=0\,, \tag{12}$$
$$C_1=\frac{V^3}{2}\left(16B^2C^2+16B^2+8A^2B-8A^2BC+A^4-32B^2C-1\right)-V-\frac{1}{2V},$$

Set 3. $\alpha_2=6V(C-1)^2,\ \alpha_0=V\left(8BC-2A^2-8B+1\right)+\frac{1}{V},$

$$\alpha_{-2}=\frac{3}{4(C-1)^2}\left(16B^2C^2-8BCA^2-32B^2C+16B^2+8A^2B+A^4\right),$$
$$V=V\,,k=\frac{A}{2(C-1)},\alpha_1=0,\ \alpha_{-1}=0, \tag{13}$$
$$C_1=V^3(128B^2C^2+128B^2+8A^4-256B^2C-56A^2BC+56A^2B)-V-\frac{V^3}{2}-\frac{1}{2V},$$

Substituting Eqs. (11)-(13) into Eq. (10), we obtain

$$u_1(\xi)=V\left(12C^2k^2-12kAC+12k^2+12kA-24Ck^2+8BC-8B+A^2+1\right)+\frac{1}{V}$$
$$+12V\left(-2C^2k+4Ck+AC-A-2k\right)\times\left(k+(G'/G)\right)+12V(C-1)^2\times\left(k+(G'/G)\right)^2. \tag{14}$$

$$u_2(\xi)=V\left(12C^2k^2+12k^2+12kA-12kAC-24Ck^2+8BC-8B+1+A^2\right)+\frac{1}{V}$$
$$+12V\left(2Bk-2C^2k^3+BA-2k^3-3Ak^2-A^2k+3ACk^2+4Ck^3-2BCk\right) \tag{15}$$
$$\times\left(k+(G'/G)\right)^{-1}+12V(C^2k^4-2Bk^2+B^2+k^4+2Ak^3+A^2k^2-2ACk^3$$
$$-2Ck^4-2ABk+2BCk^2)\times\left(k+(G'/G)\right)^{-2}.$$

$$u_3(\xi)=\left(V(8BC-2A^2-8B+1)+\frac{1}{V}\right)+6V(C-1)^2\times\left(\frac{A}{2(C-1)}+(G'/G)\right)^2+\frac{3}{4(C-1)^2}$$
$$\times\left(16B^2C^2-8BCA^2-32B^2C+16B^2+8A^2B+A^4\right)\times\left(\frac{A}{2(C-1)}+(G'/G)\right)^{-2}. \tag{16}$$

where $\xi=x-Vt$.

Substituting value of equation (6) into equation (14), we acquire:

When $\Delta=A^2-4BC+4B>0$ and $A(C-1)\neq 0$ (or $B\,(C-1)\neq 0$),

$$u_1^1(\xi)=V\left(12C^2k^2-12kAC+12k^2+12kA-24Ck^2+8BC-8B+A^2+1\right)+\frac{1}{V}$$
$$+12V\left(-2C^2k+4Ck+AC-A-2k\right)\times\left\{k-\frac{1}{2(C-1)}\left(A+\sqrt{\Delta}\tanh(\sqrt{\Delta}\xi/2)\right)\right\} \tag{17}$$
$$+12V(C-1)^2\times\left\{k-\frac{1}{2(C-1)}\left(A+\sqrt{\Delta}\tanh(\sqrt{\Delta}\xi/2)\right)\right\}^2.$$

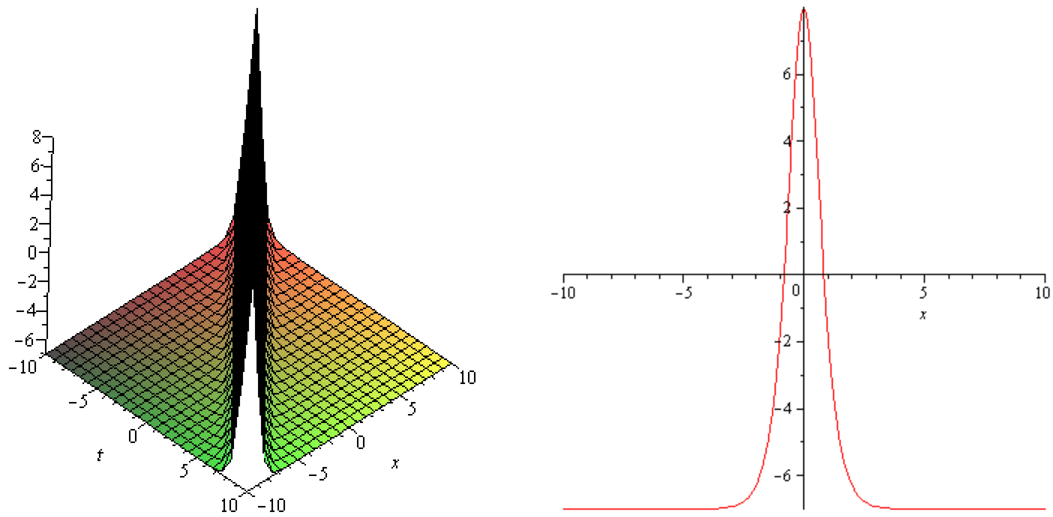

Figure 1. Soliton solution for Eq. (17).

$$
\begin{aligned}
u_1^2(\xi) &= V\left(12C^2k^2 - 12kAC + 12k^2 + 12kA - 24Ck^2 + 8BC - 8B + A^2 + 1\right) + \frac{1}{V} \\
&+ 12V\left(-2C^2k + 4Ck + AC - A - 2k\right) \times \left\{k - \frac{1}{2(C-1)}\left(A + \sqrt{\Delta}\coth(\sqrt{\Delta}\xi/2)\right)\right\} \qquad (18) \\
&+ 12V(C-1)^2 \times \left\{k - \frac{1}{2(C-1)}\left(A + \sqrt{\Delta}\coth(\sqrt{\Delta}\xi/2)\right)\right\}^2.
\end{aligned}
$$

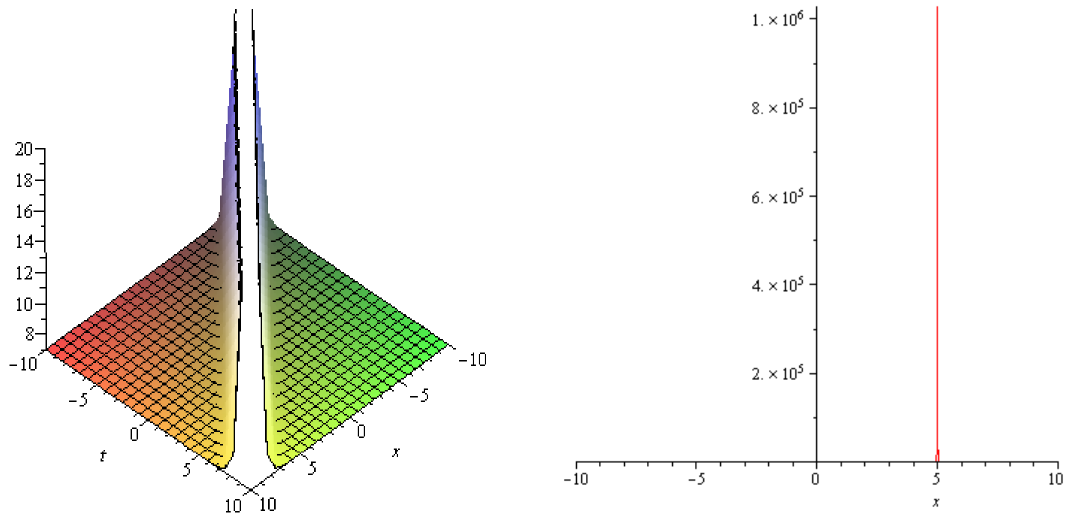

Figure 2. Soliton solution for Eq. (18).

$$
\begin{aligned}
u_1^3(\xi) &= V\left(12C^2k^2 - 12kAC + 12k^2 + 12kA - 24Ck^2 + 8BC - 8B + A^2 + 1\right) + \frac{1}{V} \\
&+ 12V\left(-2C^2k + 4Ck + AC - A - 2k\right) \times \left\{k - \frac{1}{2(C-1)}\left(A + \sqrt{\Delta}\left(\tanh(\sqrt{\Delta}\xi) \pm i\,\mathrm{sech}(\sqrt{\Delta}\xi)\right)\right)\right\} \qquad (19) \\
&+ 12V(C-1)^2 \times \left\{k - \frac{1}{2(C-1)}\left(A + \sqrt{\Delta}\left(\tanh(\sqrt{\Delta}\xi) \pm i\,\mathrm{sech}(\sqrt{\Delta}\xi)\right)\right)\right\}^2.
\end{aligned}
$$

$$u_1^4(\xi)=V\left(12C^2k^2-12kAC+12k^2+12kA-24Ck^2+8BC-8B+A^2+1\right)+\frac{1}{V}$$
$$+12V\left(-2C^2k+4Ck+AC-A-2k\right)\times\left\{k-\frac{1}{2(C-1)}\left(A+\sqrt{\Delta}\left(\coth(\sqrt{\Delta}\xi)\pm\operatorname{csc}h(\sqrt{\Delta}\xi)\right)\right)\right\} \tag{20}$$
$$+12V(C-1)^2\times\left\{k-\frac{1}{2(C-1)}\left(A+\sqrt{\Delta}\left(\coth(\sqrt{\Delta}\xi)\pm\operatorname{csc}h(\sqrt{\Delta}\xi)\right)\right)\right\}^2.$$

$$u_1^5(\xi)=V\left(12C^2k^2-12kAC+12k^2+12kA-24Ck^2+8BC-8B+A^2+1\right)+\frac{1}{V}$$
$$+12V\left(-2C^2k+4Ck+AC-A-2k\right)$$
$$\times\left\{k-\frac{1}{4(C-1)}\left(2A+\sqrt{\Delta}\left(\tanh(\sqrt{\Delta}\xi/4)+\coth(\sqrt{\Delta}\xi/4)\right)\right)\right\} \tag{21}$$
$$+12V(C-1)^2\times\left\{k-\frac{1}{4(C-1)}\left(2A+\sqrt{\Delta}\left(\tanh(\sqrt{\Delta}\xi/4)+\coth(\sqrt{\Delta}\xi/4)\right)\right)\right\}^2.$$

$$u_1^6(\xi)=V\left(12C^2k^2-12kAC+12k^2+12kA-24Ck^2+8BC-8B+A^2+1\right)+\frac{1}{V}$$
$$+12V\left(-2C^2k+4Ck+AC-A-2k\right)$$
$$\times\left[k+\frac{1}{2(C-1)}\left\{-A+\frac{\pm\sqrt{\Delta(F^2+H^2)}-F\sqrt{\Delta}\cosh(\sqrt{\Delta}\xi)}{F\sinh(\sqrt{\Delta}\xi)+B}\right\}\right] \tag{22}$$
$$+12V(C-1)^2\times\left[k+\frac{1}{2(C-1)}\left\{-A+\frac{\pm\sqrt{\Delta(F^2+H^2)}-F\sqrt{\Delta}\cosh(\sqrt{\Delta}\xi)}{F\sinh(\sqrt{\Delta}\xi)+B}\right\}\right]^2.$$

$$u_1^7(\xi)=V\left(12C^2k^2-12kAC+12k^2+12kA-24Ck^2+8BC-8B+A^2+1\right)+\frac{1}{V}$$
$$+12V\left(-2C^2k+4Ck+AC-A-2k\right)$$
$$\times\left[k+\frac{1}{2(C-1)}\left\{-A+\frac{\pm\sqrt{\Delta(F^2+H^2)}+F\sqrt{\Delta}\cosh(\sqrt{\Delta}\xi)}{F\sinh(\sqrt{\Delta}\xi)+B}\right\}\right] \tag{23}$$
$$+12V(C-1)^2\times\left[k+\frac{1}{2(C-1)}\left\{-A+\frac{\pm\sqrt{\Delta(F^2+H^2)}+F\sqrt{\Delta}\cosh(\sqrt{\Delta}\xi)}{F\sinh(\sqrt{\Delta}\xi)+B}\right\}\right]^2.$$

here $F, H \in \mathbb{R}$

$$u_1^8(\xi)=V\left(12C^2k^2-12kAC+12k^2+12kA-24Ck^2+8BC-8B+A^2+1\right)+\frac{1}{V}$$
$$+12V\left(-2C^2k+4Ck+AC-A-2k\right)\times\left\{k+\frac{2B\cosh(\sqrt{\Delta}\xi/2)}{\sqrt{\Delta}\sinh(\sqrt{\Delta}\xi/2)-A\cosh(\sqrt{\Delta}\xi/2)}\right\} \quad (24)$$
$$+12V\left(C-1\right)^2\times\left\{k+\frac{2B\cosh(\sqrt{\Delta}\xi/2)}{\sqrt{\Delta}\sinh(\sqrt{\Delta}\xi/2)-A\cosh(\sqrt{\Delta}\xi/2)}\right\}^2.$$

$$u_1^9(\xi)=V\left(12C^2k^2-12kAC+12k^2+12kA-24Ck^2+8BC-8B+A^2+1\right)+\frac{1}{V}$$
$$+12V\left(-2C^2k+4Ck+AC-A-2k\right)\times\left\{k+\frac{2B\sinh(\sqrt{\Delta}\xi/2)}{\sqrt{\Delta}\cosh(\sqrt{\Delta}\xi/2)-A\sinh(\sqrt{\Delta}\xi/2)}\right\} \quad (25)$$
$$+12V\left(C-1\right)^2\times\left\{k+\frac{2B\sinh(\sqrt{\Delta}\xi/2)}{\sqrt{\Delta}\cosh(\sqrt{\Delta}\xi/2)-A\sinh(\sqrt{\Delta}\xi/2)}\right\}^2.$$

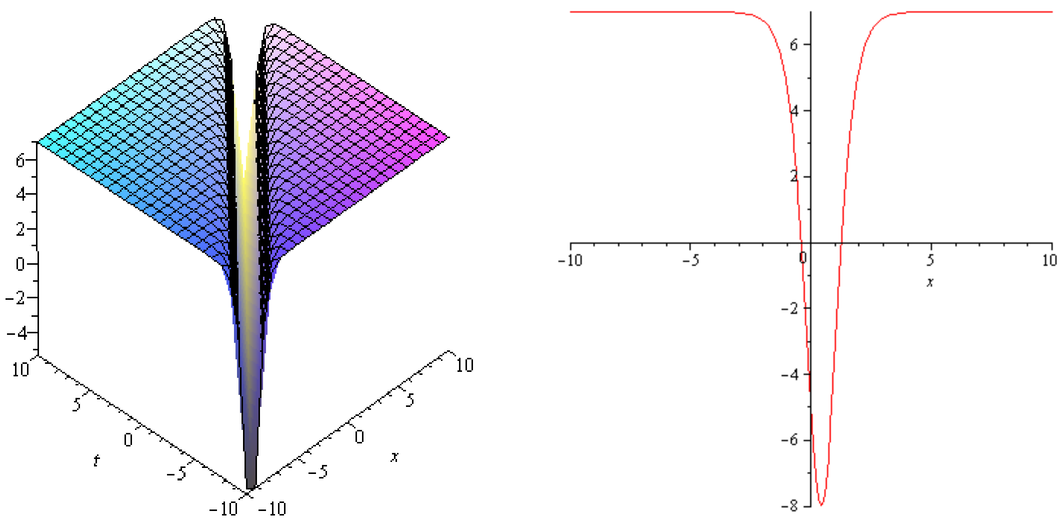

Figure 3. Soliton solution for Eq. (25).

$$u_1^{10}(\xi)=V\left(12C^2k^2-12kAC+12k^2+12kA-24Ck^2+8BC-8B+A^2+1\right)+\frac{1}{V}$$
$$+12V\left(-2C^2k+4Ck+AC-A-2k\right)\times\left\{k+\frac{2B\cosh(\sqrt{\Delta}\xi)}{\sqrt{\Delta}\sinh(\sqrt{\Delta}\xi)-A\cosh(\sqrt{\Delta}\xi)\pm i\sqrt{\Delta}}\right\} \quad (26)$$
$$+12V\left(C-1\right)^2\times\left\{k+\frac{2B\cosh(\sqrt{\Delta}\xi)}{\sqrt{\Delta}\sinh(\sqrt{\Delta}\xi)-A\cosh(\sqrt{\Delta}\xi)\pm i\sqrt{\Delta}}\right\}^2.$$

$$u_1^{11}(\xi) = V\left(12C^2k^2 - 12kAC + 12k^2 + 12kA - 24Ck^2 + 8BC - 8B + A^2 + 1\right) + \frac{1}{V}$$
$$+12V\left(-2C^2k + 4Ck + AC - A - 2k\right) \times \left\{k + \frac{2B\sinh(\sqrt{\Delta}\xi)}{\sqrt{\Delta}\cosh(\sqrt{\Delta}\xi) - A\sinh(\sqrt{\Delta}\xi) \pm \sqrt{\Delta}}\right\} \quad (27)$$
$$+12V(C-1)^2 \times \left\{k + \frac{2B\sinh(\sqrt{\Delta}\xi)}{\sqrt{\Delta}\cosh(\sqrt{\Delta}\xi) - A\sinh(\sqrt{\Delta}\xi) \pm \sqrt{\Delta}}\right\}^2.$$

When $\Delta = A^2 - 4BC + 4B < 0$ and $A(C-1) \neq 0$ (or $B(C-1) \neq 0$),

$$u_1^{12}(\xi) = V\left(12C^2k^2 - 12kAC + 12k^2 + 12kA - 24Ck^2 + 8BC - 8B + A^2 + 1\right) + \frac{1}{V}$$
$$+12V\left(-2C^2k + 4Ck + AC - A - 2k\right) \times \left\{k + \frac{1}{2(C-1)}\left(-A + \sqrt{-\Delta}\tan(\sqrt{-\Delta}\xi/2)\right)\right\} \quad (28)$$
$$+12V(C-1)^2 \times \left\{k + \frac{1}{2(C-1)}\left(-A + \sqrt{-\Delta}\tan(\sqrt{-\Delta}\xi/2)\right)\right\}^2.$$

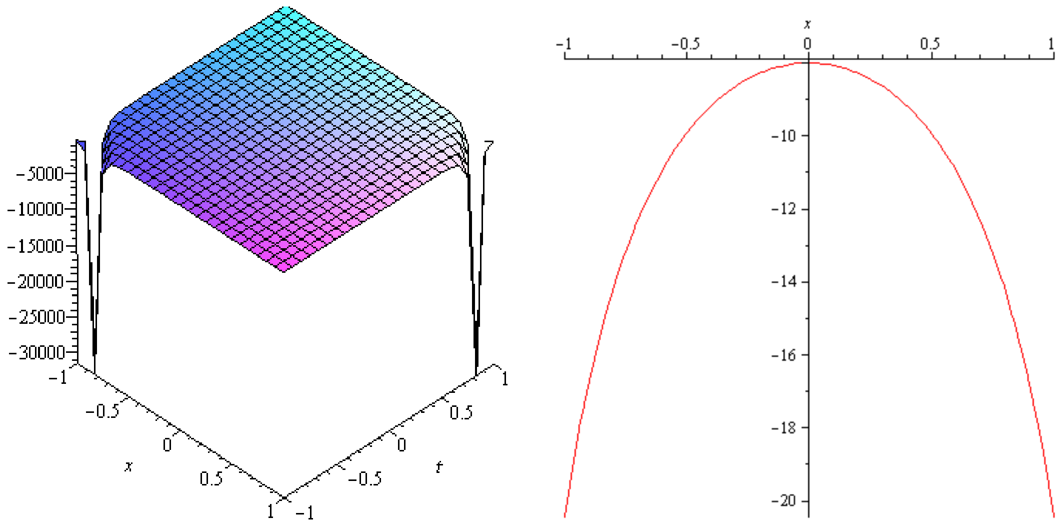

Figure 4. Soliton solution for Eq. (28).

$$u_1^{13}(\xi) = V\left(12C^2k^2 - 12kAC + 12k^2 + 12kA - 24Ck^2 + 8BC - 8B + A^2 + 1\right) + \frac{1}{V}$$
$$+12V\left(-2C^2k + 4Ck + AC - A - 2k\right) \times \left\{k - \frac{1}{2(C-1)}\left(A + \sqrt{-\Delta}\cot(\sqrt{-\Delta}\xi/2)\right)\right\} \quad (29)$$
$$+12V(C-1)^2 \times \left\{k - \frac{1}{2(C-1)}\left(A + \sqrt{-\Delta}\cot(\sqrt{-\Delta}\xi/2)\right)\right\}^2.$$

$$
\begin{aligned}
u_1^{14}(\xi) = {} & V\left(12C^2k^2 - 12kAC + 12k^2 + 12kA - 24Ck^2 + 8BC - 8B + A^2 + 1\right) + \frac{1}{V} \\
& + 12V\left(-2C^2k + 4Ck + AC - A - 2k\right) \\
& \times \left\{ k - \frac{1}{2(C-1)}\left(-A + \sqrt{-\Delta}\left(\tan(\sqrt{-\Delta}\xi) \pm \sec(\sqrt{-\Delta}\xi)\right)\right)\right\} \\
& + 12V(C-1)^2 \times \left\{ k - \frac{1}{2(C-1)}\left(-A + \sqrt{-\Delta}\left(\tan(\sqrt{-\Delta}\xi) \pm \sec(\sqrt{-\Delta}\xi)\right)\right)\right\}^2 .
\end{aligned} \tag{30}
$$

$$
\begin{aligned}
u_1^{15}(\xi) = {} & V\left(12C^2k^2 - 12kAC + 12k^2 + 12kA - 24Ck^2 + 8BC - 8B + A^2 + 1\right) + \frac{1}{V} \\
& + 12V\left(-2C^2k + 4Ck + AC - A - 2k\right) \\
& \times \left\{ k - \frac{1}{2(C-1)}\left(A + \sqrt{-\Delta}\left(\cot(\sqrt{-\Delta}\xi) \pm \csc h(\sqrt{-\Delta}\xi)\right)\right)\right\} \\
& + 12V(C-1)^2 \times \left\{ k - \frac{1}{2(C-1)}\left(A + \sqrt{-\Delta}\left(\cot(\sqrt{-\Delta}\xi) \pm \csc h(\sqrt{-\Delta}\xi)\right)\right)\right\}^2 .
\end{aligned} \tag{31}
$$

$$
\begin{aligned}
u_1^{16}(\xi) = {} & V\left(12C^2k^2 - 12kAC + 12k^2 + 12kA - 24Ck^2 + 8BC - 8B + A^2 + 1\right) + \frac{1}{V} \\
& + 12V\left(-2C^2k + 4Ck + AC - A - 2k\right) \\
& \times \left[k + \frac{1}{2(C-1)}\left\{ -A + \frac{\pm\sqrt{-\Delta(F^2 - H^2)} - F\sqrt{-\Delta}\cos(\sqrt{-\Delta}\xi)}{F\sin(\sqrt{-\Delta}\xi) + B} \right\}\right] \\
& + 12V(C-1)^2 \times \left[k + \frac{1}{2(C-1)}\left\{ -A + \frac{\pm\sqrt{-\Delta(F^2 - H^2)} - F\sqrt{-\Delta}\cos(\sqrt{-\Delta}\xi)}{F\sin(\sqrt{-\Delta}\xi) + B} \right\}\right]^2 .
\end{aligned} \tag{32}
$$

$$
\begin{aligned}
u_1^{17}(\xi) = {} & V\left(12C^2k^2 - 12kAC + 12k^2 + 12kA - 24Ck^2 + 8BC - 8B + A^2 + 1\right) + \frac{1}{V} \\
& + 12V\left(-2C^2k + 4Ck + AC - A - 2k\right) \\
& \times \left[k + \frac{1}{2(C-1)}\left\{ -A + \frac{\pm\sqrt{-\Delta(F^2 - H^2)} - F\sqrt{-\Delta}\cos(\sqrt{-\Delta}\xi)}{F\sin(\sqrt{-\Delta}\xi) + B} \right\}\right] \\
& + 12V(C-1)^2 \times \left[k + \frac{1}{2(C-1)}\left\{ -A + \frac{\pm\sqrt{-\Delta(F^2 - H^2)} - F\sqrt{-\Delta}\cos(\sqrt{-\Delta}\xi)}{F\sin(\sqrt{-\Delta}\xi) + B} \right\}\right]^2 .
\end{aligned} \tag{33}
$$

$$u_1^{18}(\xi)=V\left(12C^2k^2-12kAC+12k^2+12kA-24Ck^2+8BC-8B+A^2+1\right)+\frac{1}{V}$$
$$+12V\left(-2C^2k+4Ck+AC-A-2k\right)$$
$$\times\left[k+\frac{1}{2(C-1)}\left\{-A+\frac{\pm\sqrt{-\Delta(F^2-H^2)}+F\sqrt{-\Delta}\cos(\sqrt{-\Delta}\xi)}{F\sin(\sqrt{-\Delta}\xi)+B}\right\}\right] \quad (34)$$
$$+12V(C-1)^2\times\left[k+\frac{1}{2(C-1)}\left\{-A+\frac{\pm\sqrt{-\Delta(F^2-H^2)}+F\sqrt{-\Delta}\cos(\sqrt{-\Delta}\xi)}{F\sin(\sqrt{-\Delta}\xi)+B}\right\}\right]^2,$$

where $F^2-H^2>0$.

$$u_1^{19}(\xi)=V\left(12C^2k^2-12kAC+12k^2+12kA-24Ck^2+8BC-8B+A^2+1\right)+\frac{1}{V}$$
$$+12V\left(-2C^2k+4Ck+AC-A-2k\right)\times\left\{k-\frac{2B\cos(\sqrt{-\Delta}\xi/2)}{\sqrt{-\Delta}\sin(\sqrt{-\Delta}\xi/2)+A\cos(\sqrt{-\Delta}\xi/2)}\right\} \quad (35)$$
$$+12V(C-1)^2\times\left\{k-\frac{2B\cos(\sqrt{-\Delta}\xi/2)}{\sqrt{-\Delta}\sin(\sqrt{-\Delta}\xi/2)+A\cos(\sqrt{-\Delta}\xi/2)}\right\}^2.$$

$$u_1^{20}(\xi)=V\left(12C^2k^2-12kAC+12k^2+12kA-24Ck^2+8BC-8B+A^2+1\right)+\frac{1}{V}$$
$$+12V\left(-2C^2k+4Ck+AC-A-2k\right)\times\left\{k+\frac{2B\sin(\sqrt{-\Delta}\xi/2)}{\sqrt{-\Delta}\cos(\sqrt{-\Delta}\xi/2)-A\sin(\sqrt{-\Delta}\xi/2)}\right\} \quad (36)$$
$$+12V(C-1)^2\times\left\{k+\frac{2B\sin(\sqrt{-\Delta}\xi/2)}{\sqrt{-\Delta}\cos(\sqrt{-\Delta}\xi/2)-A\sin(\sqrt{-\Delta}\xi/2)}\right\}^2.$$

$$u_1^{21}(\xi)=V\left(12C^2k^2-12kAC+12k^2+12kA-24Ck^2+8BC-8B+A^2+1\right)+\frac{1}{V}$$
$$+12V\left(-2C^2k+4Ck+AC-A-2k\right)\times\left\{k-\frac{2B\cos(\sqrt{-\Delta}\xi)}{\sqrt{-\Delta}\sin(\sqrt{-\Delta}\xi)+A\cos(\sqrt{-\Delta}\xi)\pm\sqrt{-\Delta}}\right\} \quad (37)$$
$$+12V(C-1)^2\times\left\{k-\frac{2B\cos(\sqrt{-\Delta}\xi)}{\sqrt{-\Delta}\sin(\sqrt{-\Delta}\xi)+A\cos(\sqrt{-\Delta}\xi)\pm\sqrt{-\Delta}}\right\}^2.$$

$$u_1^{22}(\xi)=V\left(12C^2k^2-12kAC+12k^2+12kA-24Ck^2+8BC-8B+A^2+1\right)+\frac{1}{V}$$
$$+12V\left(-2C^2k+4Ck+AC-A-2k\right)\times\left\{k+\frac{2B\sin(\sqrt{-\Delta}\xi/2)}{\sqrt{-\Delta}\cos(\sqrt{-\Delta}\xi/2)-A\sin(\sqrt{-\Delta}\xi/2)\pm\sqrt{-\Delta}}\right\} \quad (38)$$
$$+12V\left(C-1\right)^2\times\left\{k+\frac{2B\sin(\sqrt{-\Delta}\xi/2)}{\sqrt{-\Delta}\cos(\sqrt{-\Delta}\xi/2)-A\sin(\sqrt{-\Delta}\xi/2)\pm\sqrt{-\Delta}}\right\}^2.$$

When $B=0$ and $A(C-1)\neq 0$,

$$u_1^{23}(\xi)=V\left(12C^2k^2-12kAC+12k^2+12kA-24Ck^2+8BC-8B+A^2+1\right)+\frac{1}{V}$$
$$+12V\left(-2C^2k+4Ck+AC-A-2k\right)\times\left\{k-\frac{Ac_1}{(C-1)\{c_1+\cosh(A\xi)-\sinh(A\xi)\}}\right\} \quad (39)$$
$$+12V\left(C-1\right)^2\times\left\{k-\frac{Ac_1}{(C-1)\{c_1+\cosh(A\xi)-\sinh(A\xi)\}}\right\}^2.$$

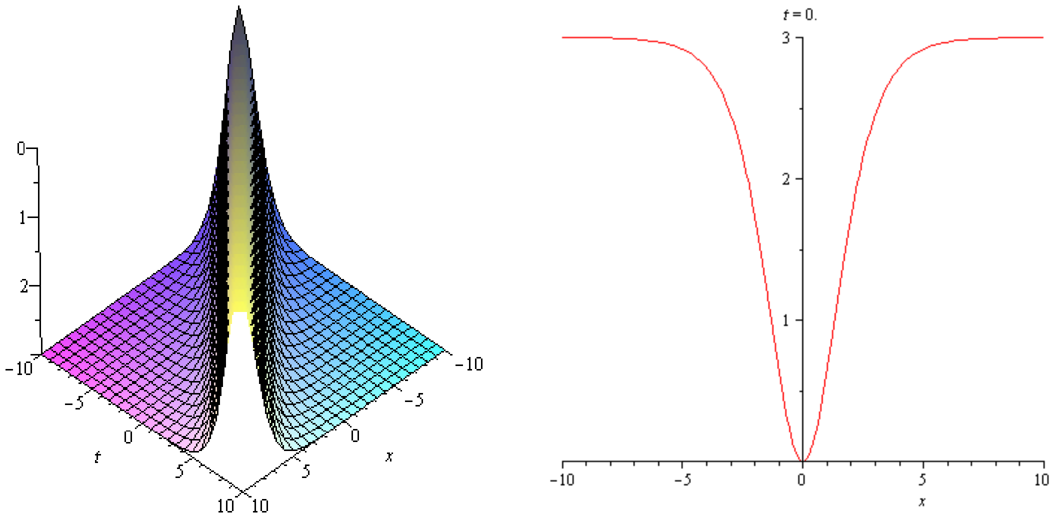

Figure 5. Soliton solution for Eq. (39).

$$u_1^{24}(\xi)=V\left(12C^2k^2-12kAC+12k^2+12kA-24Ck^2+8BC-8B+A^2+1\right)+\frac{1}{V}$$
$$+12V\left(-2C^2k+4Ck+AC-A-2k\right)\times\left\{k-\frac{A(\cosh(A\xi)+\sinh(A\xi))}{(C-1)\{c_1+\cosh(A\xi)+\sinh(A\xi)\}}\right\} \quad (40)$$
$$+12V\left(C-1\right)^2\times\left\{k-\frac{A(\cosh(A\xi)+\sinh(A\xi))}{(C-1)\{c_1+\cosh(A\xi)+\sinh(A\xi)\}}\right\}^2.$$

When $A=B=0$ and $(C-1)\neq 0$, the solution of Eq. (8) is

$$u_1^{25}(\xi) = V\left(12C^2k^2 - 12kAC + 12k^2 + 12kA - 24Ck^2 + 8BC - 8B + A^2 + 1\right) + \frac{1}{V}$$
$$+12V\left(-2C^2k + 4Ck + AC - A - 2k\right) \times \left\{k - \frac{1}{(C-1)\,\xi + c_2}\right\}$$
$$+12V\,(C-1)^2 \times \left\{k - \frac{1}{(C-1)\,\xi + c_2}\right\}^2, \qquad (41)$$

Substituting value of equation (6) into equation (15) we acquire:
When $\Delta = A^2 - 4BC + 4B > 0$ and $A(C-1) \neq 0$ (or $B\,(C-1) \neq 0$),

$$u_2^1(\xi) = V\left(12C^2k^2 + 12k^2 + 12kA - 12kAC - 24Ck^2 + 8BC - 8B + 1 + A^2\right) + \frac{1}{V}$$
$$+12V\left(2Bk - 2C^2k^3 + BA - 2k^3 - 3Ak^2 - A^2k + 3ACk^2 + 4Ck^3 - 2BCk\right)$$
$$\times\left\{k - \frac{1}{2(C-1)}\left(A + \sqrt{\Delta}\tanh(\sqrt{\Delta}\xi/2)\right)\right\}^{-1} \qquad (42)$$
$$+12V\left(C^2k^4 - 2Bk^2 + B^2 + k^4 + 2Ak^3 + A^2k^2 - 2ACk^3 - 2Ck^4 - 2ABk + 2BCk^2\right)$$
$$\times\left\{k - \frac{1}{2(C-1)}\left(A + \sqrt{\Delta}\tanh(\sqrt{\Delta}\xi/2)\right)\right\}^{-2},$$

where $\xi = x - Vt$.

$$u_2^2(\xi) = V\left(12C^2k^2 + 12k^2 + 12kA - 12kAC - 24Ck^2 + 8BC - 8B + 1 + A^2\right) + \frac{1}{V}$$
$$+12V\left(2Bk - 2C^2k^3 + BA - 2k^3 - 3Ak^2 - A^2k + 3ACk^2 + 4Ck^3 - 2BCk\right)$$
$$\times\left\{k - \frac{1}{2(C-1)}\left(A + \sqrt{\Delta}\coth(\sqrt{\Delta}\xi/2)\right)\right\}^{-1} \qquad (43)$$
$$+12V\left(C^2k^4 - 2Bk^2 + B^2 + k^4 + 2Ak^3 + A^2k^2 - 2ACk^3 - 2Ck^4 - 2ABk + 2BCk^2\right)$$
$$\times\left\{k - \frac{1}{2(C-1)}\left(A + \sqrt{\Delta}\coth(\sqrt{\Delta}\xi/2)\right)\right\}^{-2}.$$

$$u_2^3(\xi) = V\left(12C^2k^2 + 12k^2 + 12kA - 12kAC - 24Ck^2 + 8BC - 8B + 1 + A^2\right) + \frac{1}{V}$$
$$+12V\left(2Bk - 2C^2k^3 + BA - 2k^3 - 3Ak^2 - A^2k + 3ACk^2 + 4Ck^3 - 2BCk\right)$$
$$\times\left\{k - \frac{1}{2(C-1)}\left(A + \sqrt{\Delta}(\tanh(\sqrt{\Delta}\xi) \pm i\,\mathrm{sec}\,h(\sqrt{\Delta}\xi))\right)\right\}^{-1} \qquad (44)$$
$$+12V\left(C^2k^4 - 2Bk^2 + B^2 + k^4 + 2Ak^3 + A^2k^2 - 2ACk^3 - 2Ck^4 - 2ABk + 2BCk^2\right)$$
$$\times\left\{k - \frac{1}{2(C-1)}\left(A + \sqrt{\Delta}(\tanh(\sqrt{\Delta}\xi) \pm i\,\mathrm{sec}\,h(\sqrt{\Delta}\xi))\right)\right\}^{-2}.$$

When $\Delta = A^2 - 4BC + 4B < 0$ and $A(C-1) \neq 0$ (or $B\,(C-1) \neq 0$),

$$\begin{aligned} u_2^{12}(\xi) &= V\left(12C^2k^2 + 12k^2 + 12kA - 12kAC - 24Ck^2 + 8BC - 8B + 1 + A^2\right) + \frac{1}{V} \\ &+ 12V\left(2Bk - 2C^2k^3 + BA - 2k^3 - 3Ak^2 - A^2k + 3ACk^2 + 4Ck^3 - 2BCk\right) \\ &\times \left\{ k + \frac{1}{2(C-1)}\left(-A + \sqrt{-\Delta}\tan(\sqrt{-\Delta}\xi/2)\right)\right\}^{-1} \\ &+ 12V\left(C^2k^4 - 2Bk^2 + B^2 + k^4 + 2Ak^3 + A^2k^2 - 2ACk^3 - 2Ck^4 - 2ABk + 2BCk^2\right) \\ &\times \left\{ k + \frac{1}{2(C-1)}\left(-A + \sqrt{-\Delta}\tan(\sqrt{-\Delta}\xi/2)\right)\right\}^{-2}. \end{aligned} \tag{45}$$

$$\begin{aligned} u_2^{13}(\xi) &= V\left(12C^2k^2 + 12k^2 + 12kA - 12kAC - 24Ck^2 + 8BC - 8B + 1 + A^2\right) + \frac{1}{V} \\ &+ 12V\left(2Bk - 2C^2k^3 + BA - 2k^3 - 3Ak^2 - A^2k + 3ACk^2 + 4Ck^3 - 2BCk\right) \\ &\times \left\{ k - \frac{1}{2(C-1)}\left(A + \sqrt{-\Delta}\cot(\sqrt{-\Delta}\xi/2)\right)\right\}^{-1} \\ &+ 12V\left(C^2k^4 - 2Bk^2 + B^2 + k^4 + 2Ak^3 + A^2k^2 - 2ACk^3 - 2Ck^4 - 2ABk + 2BCk^2\right) \\ &\times \left\{ k - \frac{1}{2(C-1)}\left(A + \sqrt{-\Delta}\cot(\sqrt{-\Delta}\xi/2)\right)\right\}^{-2}. \end{aligned} \tag{46}$$

$$\begin{aligned} u_2^{14}(\xi) &= V\left(12C^2k^2 + 12k^2 + 12kA - 12kAC - 24Ck^2 + 8BC - 8B + 1 + A^2\right) + \frac{1}{V} \\ &+ 12V\left(2Bk - 2C^2k^3 + BA - 2k^3 - 3Ak^2 - A^2k + 3ACk^2 + 4Ck^3 - 2BCk\right) \\ &\times \left\{ k + \frac{1}{2(C-1)}\left(-A + \sqrt{-\Delta}(\tan(\sqrt{-\Delta}\xi) \pm \sec(\sqrt{-\Delta}\xi))\right)\right\}^{-1} \\ &+ 12V\left(C^2k^4 - 2Bk^2 + B^2 + k^4 + 2Ak^3 + A^2k^2 - 2ACk^3 - 2Ck^4 - 2ABk + 2BCk^2\right) \\ &\times \left\{ k + \frac{1}{2(C-1)}\left(-A + \sqrt{-\Delta}(\tan(\sqrt{-\Delta}\xi) \pm \sec(\sqrt{-\Delta}\xi))\right)\right\}^{-2}. \end{aligned} \tag{47}$$

When $A = B = 0$ and $(C-1) \neq 0$, the solution of Eq. (8) is

$$\begin{aligned} u_2^{25}(\xi) &= V\left(12C^2k^2 + 12k^2 + 12kA - 12kAC - 24Ck^2 + 8BC - 8B + 1 + A^2\right) + \frac{1}{V} \\ &+ 12V\left(2Bk - 2C^2k^3 + BA - 2k^3 - 3Ak^2 - A^2k + 3ACk^2 + 4Ck^3 - 2BCk\right) \\ &\times \left\{ k - \frac{1}{(C-1)\xi + c_3}\right\}^{-1} + 12V(C^2k^4 - 2Bk^2 + B^2 + k^4 + 2Ak^3 + A^2k^2 \\ &- 2ACk^3 - 2Ck^4 - 2ABk + 2BCk^2) \times \left\{ k - \frac{1}{(C-1)\xi + c_3}\right\}^{-2}. \end{aligned} \tag{48}$$

Substituting value of equation (6) into equation (16) we acquire:

When $\Delta = A^2 - 4BC + 4B > 0$ and $A(C-1) \neq 0$ (or $B\,(C-1) \neq 0$),

$$u_3^1(\xi) = \left(V(8BC - 2A^2 - 8B + 1) + \frac{1}{V} \right) + 6V\,(C-1)^2 \times \left(\frac{1}{2\,(C-1)} (\sqrt{\Delta}\tanh(\sqrt{\Delta}\,\xi/2)) \right)^2$$
$$+ \frac{3}{4\,(C-1)^2} \left(16B^2C^2 - 8BCA^2 - 32B^2C + 16B^2 + 8A^2B + A^4 \right) \tag{49}$$
$$\times \left(\frac{1}{2\,(C-1)} (\sqrt{\Delta}\tanh(\sqrt{\Delta}\,\xi/2)) \right)^{-2},$$

where $\xi = x - Vt$; A, B and C are arbitrary constants.

$$u_3^2(\xi) = \left(V(8BC - 2A^2 - 8B + 1) + \frac{1}{V} \right) + 6V\,(C-1)^2 \times \left(\frac{1}{2\,(C-1)} (\sqrt{\Delta}\coth(\sqrt{\Delta}\,\xi/2)) \right)^2$$
$$+ \frac{3}{4\,(C-1)^2} \left(16B^2C^2 - 8BCA^2 - 32B^2C + 16B^2 + 8A^2B + A^4 \right) \tag{50}$$
$$\times \left(\frac{1}{2\,(C-1)} (\sqrt{\Delta}\coth(\sqrt{\Delta}\,\xi/2)) \right)^{-2}.$$

$$u_3^3(\xi) = \left(V(8BC - 2A^2 - 8B + 1) + \frac{1}{V} \right) + 6V\,(C-1)^2 \times \left(\frac{1}{2(C-1)} \left\{ \sqrt{\Delta}\,(\tanh(\sqrt{\Delta}\,\xi) \pm i\,\mathrm{sech}(\sqrt{\Delta}\,\xi) \right\} \right)^2$$
$$+ \frac{3}{4(C-1)^2} \left(16B^2C^2 - 8BCA^2 - 32B^2C + 16B^2 + 8A^2B + A^4 \right) \tag{51}$$
$$\times \left(\frac{1}{2(C-1)} \left\{ \sqrt{\Delta}\,(\tanh(\sqrt{\Delta}\,\xi) \pm i\,\mathrm{sech}(\sqrt{\Delta}\,\xi) \right\} \right)^{-2}.$$

When $\Delta = A^2 - 4BC + 4B < 0$ and $A(C-1) \neq 0$ (or $B\,(C-1) \neq 0$),

$$u_3^{12}(\xi) = \left(V(8BC - 2A^2 - 8B + 1) + \frac{1}{V} \right) + 6V\,(C-1)^2 \times \left(\frac{1}{2(C-1)} (\sqrt{-\Delta}\tan(\sqrt{-\Delta}\,\xi/2)) \right)^2$$
$$+ \frac{3}{4(C-1)^2} \left(16B^2C^2 - 8BCA^2 - 32B^2C + 16B^2 + 8A^2B + A^4 \right) \tag{52}$$
$$\times \left(\frac{1}{2(C-1)} (\sqrt{-\Delta}\tan(\sqrt{-\Delta}\,\xi/2)) \right)^{-2}.$$

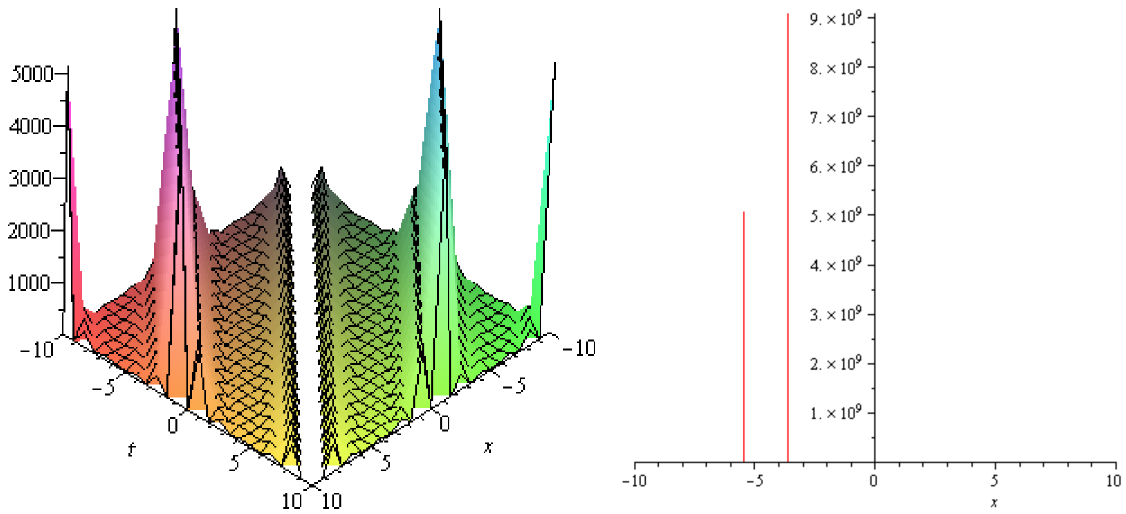

Figure 6. Singular soliton solution for Eq. (52).

$$u_3^{13}(\xi)=\left(V(8BC-2A^2-8B+1)+\frac{1}{V}\right)+6V(C-1)^2\times\left(\frac{1}{2(C-1)}(\sqrt{-\Delta}\cot(\sqrt{-\Delta}\,\xi/2))\right)^2$$
$$+\frac{3}{4(C-1)^2}\left(16B^2C^2-8BCA^2-32B^2C+16B^2+8A^2B+A^4\right) \quad (53)$$
$$\times\left(\frac{1}{2(C-1)}(\sqrt{-\Delta}\cot(\sqrt{-\Delta}\,\xi/2))\right)^{-2}.$$

$$u_3^{14}(\xi)=\left(V(8BC-2A^2-8B+1)+\frac{1}{V}\right)+6V(C-1)^2\times\left(\frac{1}{2(C-1)}\left\{\sqrt{-\Delta}\,(\tan(\sqrt{-\Delta}\,\xi)\pm\sec(\sqrt{-\Delta}\,\xi))\right\}\right)^2$$
$$+\frac{3}{4(C-1)^2}\left(16B^2C^2-8BCA^2-32B^2C+16B^2+8A^2B+A^4\right) \quad (54)$$
$$\times\left(\frac{1}{2(C-1)}\left\{\sqrt{-\Delta}\,(\tan(\sqrt{-\Delta}\,\xi)\pm\sec(\sqrt{-\Delta}\,\xi))\right\}\right)^{-2}.$$

When $(C-1)\neq 0$ and $A=B=0$, the solution of Eq. (8) is

$$u_3^{25}(\xi)=\left(V(8BC-2A^2-8B+1)+\frac{1}{V}\right)+6V(C-1)^2\times\left(\frac{A}{2(C-1)}-\frac{1}{(C-1)\xi+c_4}\right)^2+\frac{3}{4(C-1)^2}$$
$$\times\left(16B^2C^2-8BCA^2-32B^2C+16B^2+8A^2B+A^4\right)\times\left(\frac{A}{2(C-1)}-\frac{1}{(C-1)\xi+c_4}\right)^{-2}. \quad (55)$$

Other exact solutions of Eq. (8) are omitted for convenience.

4. RESULTS AND DISCUSSION

One of the important findings was that, if we put $\tanh^2\left(\frac{\sqrt{\Delta}}{2}\xi\right) = 1 - \sec h^2\left(\frac{\sqrt{\Delta}}{2}\xi\right)$, $A = c_2, B = C = k = 0$ and $V = -c$ in our solution u_1^1 and $\tan^2\left(\frac{\sqrt{\Delta}}{2}\xi\right) = \sec^2\left(\frac{\sqrt{\Delta}}{2}\xi\right) - 1$, $A = c_2, B = C = k = 0$ and $V = -c$ in our solution u_1^{12}, then Xu's [17] solutions (19) and (21) are identical to our solutions. In addition by using the suggested scheme we obtained more travelling wave solutions than Xu. Furthermore, it can also be shown that solutions obtained by Exp-function method are only special case of our results.

5. CONCLUSION

In this work, a new analytical technique is introduced and is implemented successfully on SRLW equation. As a result of which we have obtained copious exact solutions for the considered problem. The procedure of implementation is simple and is practically well suited for nonlinear evolution equations. One of the important observation is that methods such as (G'/G)-expansion method, generalized (G'/G)-expansion method and improved (G'/G)-expansion method are special cases of the developed method. So, the developed method can be considered as an authoritative method for nonlinear evolution equations.

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