ORIGINAL PAPER

ANALYSIS OF TWIN PRIMES LESS THAN A TRILLION

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Abstract. *Twin primes are pairs of successive primes which differ by* 2*. They have been conjectured to be infinite in number but this awaits a proof. The consistent presence in higher number ranges is shown here. A detail analysis of twin primes, considering their starter partners, is done. For blocks of* $1–10^n$*,* $1 \le n \le 12$*, first and last twin prime pair starters are determined. Minimum number of twin primes in blocks, minimum occurrence frequency, first and last minimum twin prime container blocks are given. Similar analysis for maximum number of twin primes in blocks is also carried out.*

Keywords: *Primes; twin primes; twin prime density.*
2010 Matheamatics Subject Classification: 11A41, 11N05, 11N25

1. INTRODUCTION

Primes are well-known category of numbers. They are those positive integers, which have only two positive integral divisors, viz., 1 and themselves. Primes themselves are prone to lot of analysis [10] to [15] as they lack a simple formula [2].

Successive primes which differ by 02 are called twin primes whose few examples are

3 and 5, 5 and 7, 11 and 13, ⋯

There is a well known conjecture about the number of twin primes stating that there are infinitely many pairs of twin primes, but as yet it is unresolved.

2. TWIN PRIMES DENSITY

As is the case of all primes numbers, that their number gradually decreases, so is the matter of twin primes that their abundance decreases.

The number of primes less than or equal to a given positive value x is a function of x, denoted by $\pi(x)$. We introduce a notation ${}_2\pi(x)$ for the number of twin prime pairs less than or equal to a given positive value x. The second member of the last prime pair is also less than or equal to x.

We have computed the number of twin primes till 10^{12}, where the best algorithm, chosen from analysis in [3-9], was implemented in the Java Programming Language, the simple & lucid power of which is highlighted excellently in [1]. Higher values are from [16].

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Table 1. Number of Twin Primes in Various Ranges.

Sr.No.	Range 1-*x*	Ten Power (*x*)	Number of Twin Primes $_2\pi(x)$
1.	1-10	10^1	2
2.	1-100	10^2	8
3.	1-1,000	10^3	35
4.	1-10,000	10^4	205
5.	1-100,000	10^5	1,224
6.	1-1,000,000	10^6	8,169
7.	1-10,000,000	10^7	58,980
8.	1-100,000,000	10^8	440,312
9.	1-1,000,000,000	10^9	3,424,506
10.	1-10,000,000,000	10^{10}	27,412,679
11.	1-100,000,000,000	10^{11}	224,376,048
12.	1-1,000,000,000,000	10^{12}	1,870,585,220
13.	1-10,000,000,000,000	10^{13}	15,834,664,872
14.	1-100,000,000,000,000	10^{14}	135,780,321,665
15.	1-1,000,000,000,000,000	10^{15}	1,177,209,242,304
16.	1-10,000,000,000,000,000	10^{16}	10,304,195,697,298

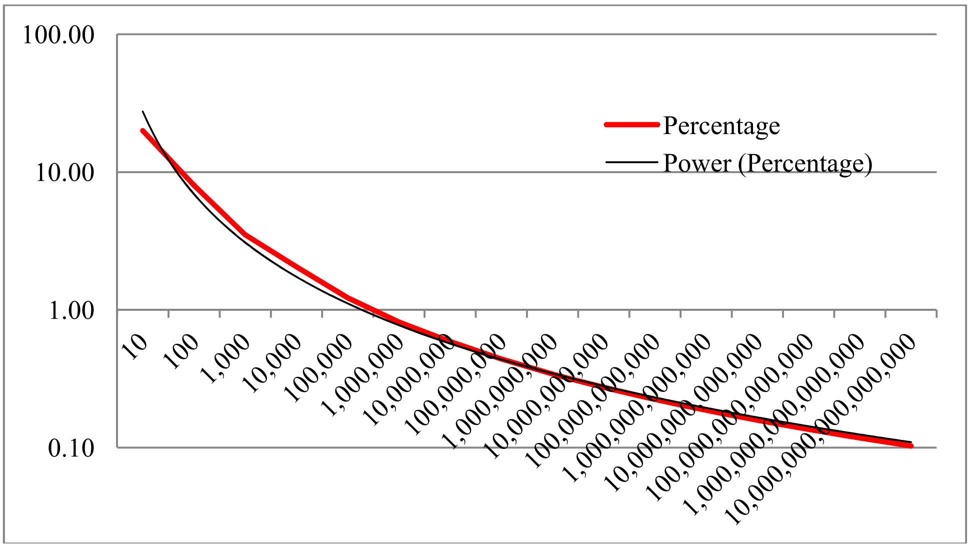


Figure 1. Percentage of Twin Primes.

The declining curve of precentage of the twin primes against the numbers for increasing ranges is approximated by power trendline of function $y = 27.543x^{-1.993}$. The vertical axis in this figure is taken on logarithmic scale with base 10.

The reduction of number of twin primes can be estimated from the following data for the values in 10 blocks of 10^{11}:

Table 2. Number of Twin Primes in Blocks of 10^{11}.

Sr. No.	Range of numbers in Blocks of 10^{11}	Block Number of Size 10^{11}	Number of Twin Primes
1.	1-1,000,000,000	1	224,376,048
2.	1,000,000,001-2,000,000,000	2	199,708,605
3.	2,000,000,001-3,000,000,000	3	191,801,047
4.	3,000,000,001-4,000,000,000	4	186,932,018
5.	4,000,000,001-5,000,000,000	5	183,404,596
6.	5,000,000,001-6,000,000,000	6	180,694,619
7.	6,000,000,001-7,000,000,000	7	178,477,447
8.	7,000,000,001-8,000,000,000	8	176,604,059
9.	8,000,000,001-9,000,000,000	9	174,989,299
10.	9,000,000,001-10,000,000,000	10	173,597,482

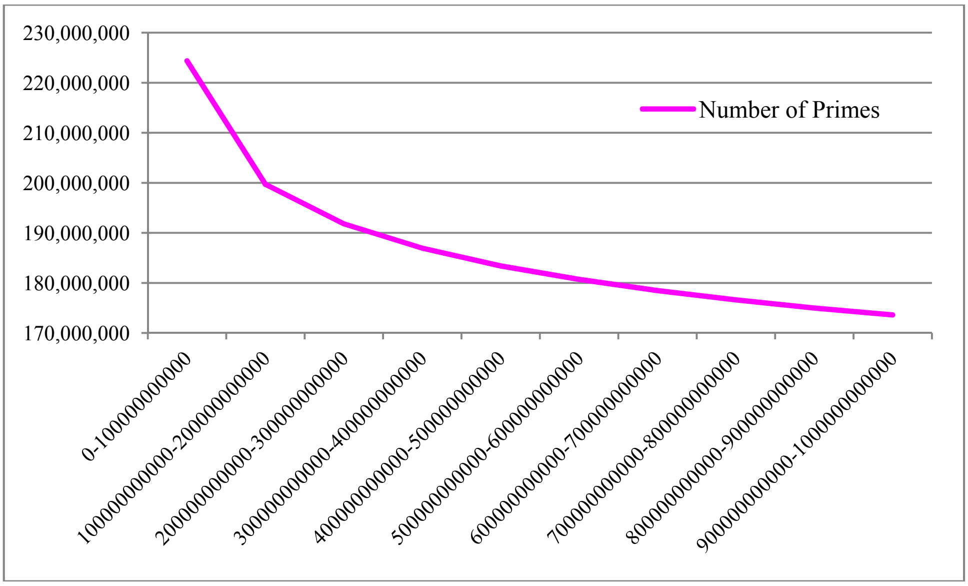


Figure 2. Number of Twin Primes in Blocks of 10^{11}.

3. ANALYSIS OF BLOCK-WISE DISTRIBUTION OF TWIN PRIMES

Since neither is there a formula for all primes, nor are the primes finite in number to consider them all together and so is the case of twin primes, to understand their random-looking distribution, we have adopted an approach of considering all primes up to a certain high limit, viz., one trillion (10^{12}) and dividing this complete number range under consideration in blocks of powers of 10 each

1-10, 11-20, 21-30, 31-40, ⋯
1-100, 101-200, 201-300, 301-400, ⋯
1-1000, 1001-2000, 2001-3000, 3001-4000, ⋯
⋮

A rigorous analysis has been performed on many fronts. As our range is 1-10^{12}, clearly there are 10^{12-i} number of blocks of 10^i size for each $1 \le i \le 12$.

3.1. THE FIRST & LAST TWIN PRIME PAIRS IN THE FIRST BLOCKS OF 10 POWERS

The inquiry of the first and the last twin prime pairs in each first block of (occurrence of) 10 powers till the range of 10^{12} under consideration is particularly interesting for the last twin prime pair, as the first twin prime pair of first power of 10 will naturally continue for all blocks ahead.

Table 3. First & Last Twin Prime Pair Starters in the First Blocks.

Sr. No.	Blocks of Size (of 10 Power)	First Twin Prime Pair Starter in the First Block	Last Twin Prime Pair Starter in the First Block
1.	10	3	5
2.	100	3	71
3.	1,000	3	881
4.	10,000	3	9,929
5.	100,000	3	99,989
6.	1,000,000	3	999,959
7.	10,000,000	3	9,999,971
8.	100,000,000	3	99,999,587
9.	1,000,000,000	3	999,999,191
10.	10,000,000,000	3	9,999,999,701
11.	100,000,000,000	3	99,999,999,761
12.	1,000,000,000,000	3	999,999,999,959

While the first twin prime pair in all the first blocks has respective fixed value, the last twin prime pair in the first blocks has natural rising; strengthening the famous conjecture about their infinitude.

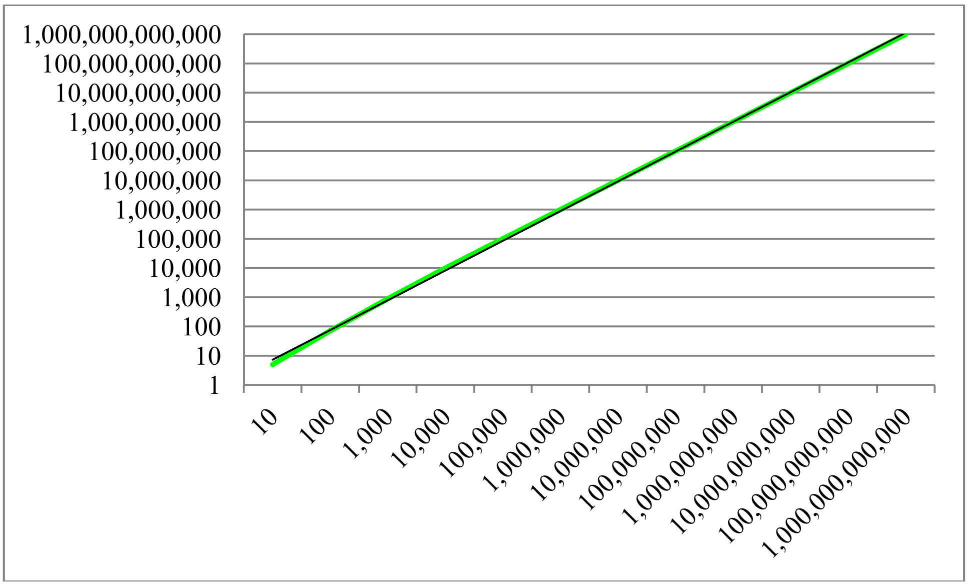


Figure 3. Last Twin Prime Pair Starter in First Block of 10 Powers.

The vertical axis is taken on logarithmic scale with base 10. This rising curve gets better approximated by exponential function $y = 0.696e^{2.343x}$, whose line (due to semivertical nature of graph) appears in the above graph overlapping closely with the curve.

3.2. MINIMUM NUMBER OF TWIN PRIME PAIRS IN BLOCKS OF 10 POWERS

Inspecting all blocks of each 10 power ranging from 10^1 to 10^{12} till 10^{12}, the minimum number of twin prime pairs found in each 10 power block has been determined rigorously. Here block 0 means first block and consequent numbers are for higher blocks. Like for 100, block 0 is 0-99, block 100 is 100 - 199 and so on.

Table 4. Minimum Twin Prime Pairs in 10 Power Blocks

Sr. No.	Blocks of Size (of 10 Power)	Minimum No. of Twin Prime Pairs in Block	First Occurrence Block of Minimum No. of Twin Prime Pairs	Last Occurrence Block of Minimum No. of Twin Prime Pairs	No. of Blocks with Minimum Twin Prime Pairs
1.	10	0	20	999,999,999,990	98,761,323,303
2.	100	0	700	999,999,999,800	8,320,955,245
3.	1,000	0	2,498,000	999,999,994,000	147,412,743
4.	10,000	0	794,623,900,000	798,812,990,000	2
5.	100,000	116	913,964,200,000	913,964,200,000	1
6.	1,000,000	1,571	952,638,000,000	952,638,000,000	1
7.	10,000,000	16,890	954,460,000,000	954,460,000,000	1
8.	100,000,000	172,193	992,100,000,000	992,100,000,000	1
9.	1,000,000,000	1,726,678	996,000,000,000	996,000,000,000	1
10.	10,000,000,000	17,297,717	990,000,000,000	990,000,000,000	1
11.	100,000,000,000	173,597,482	900,000,000,000	900,000,000,000	1
12.	1,000,000,000,000	1,870,585,220	0	0	1

The minimum number of twin prime pairs keeps increasing with increasing inspection range.

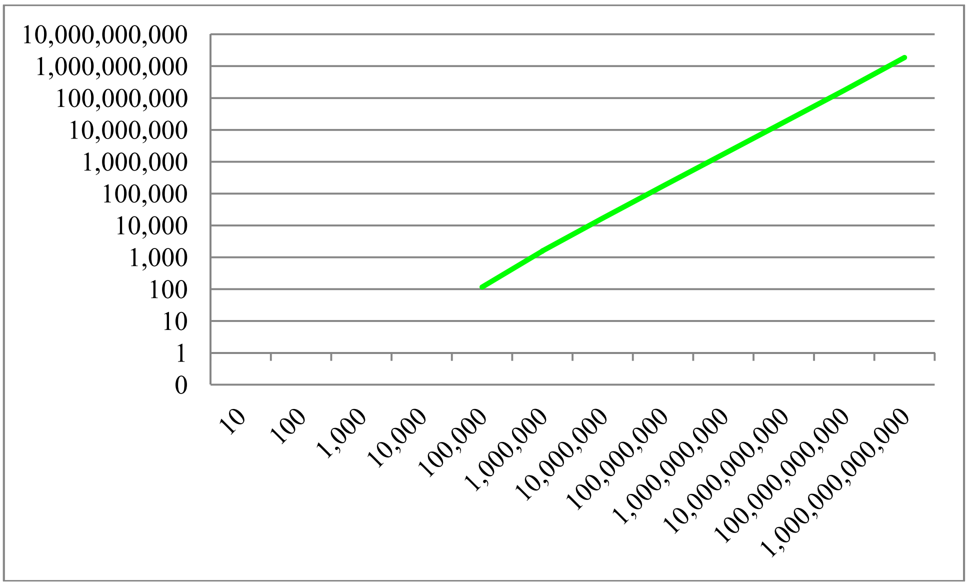


Figure 4. Minimum Number of Twin Prime Pairs in Blocks of 10 Powers.

The first and the last 10 power blocks of the occurrences of minimum number of twin prime pairs render following graphs.

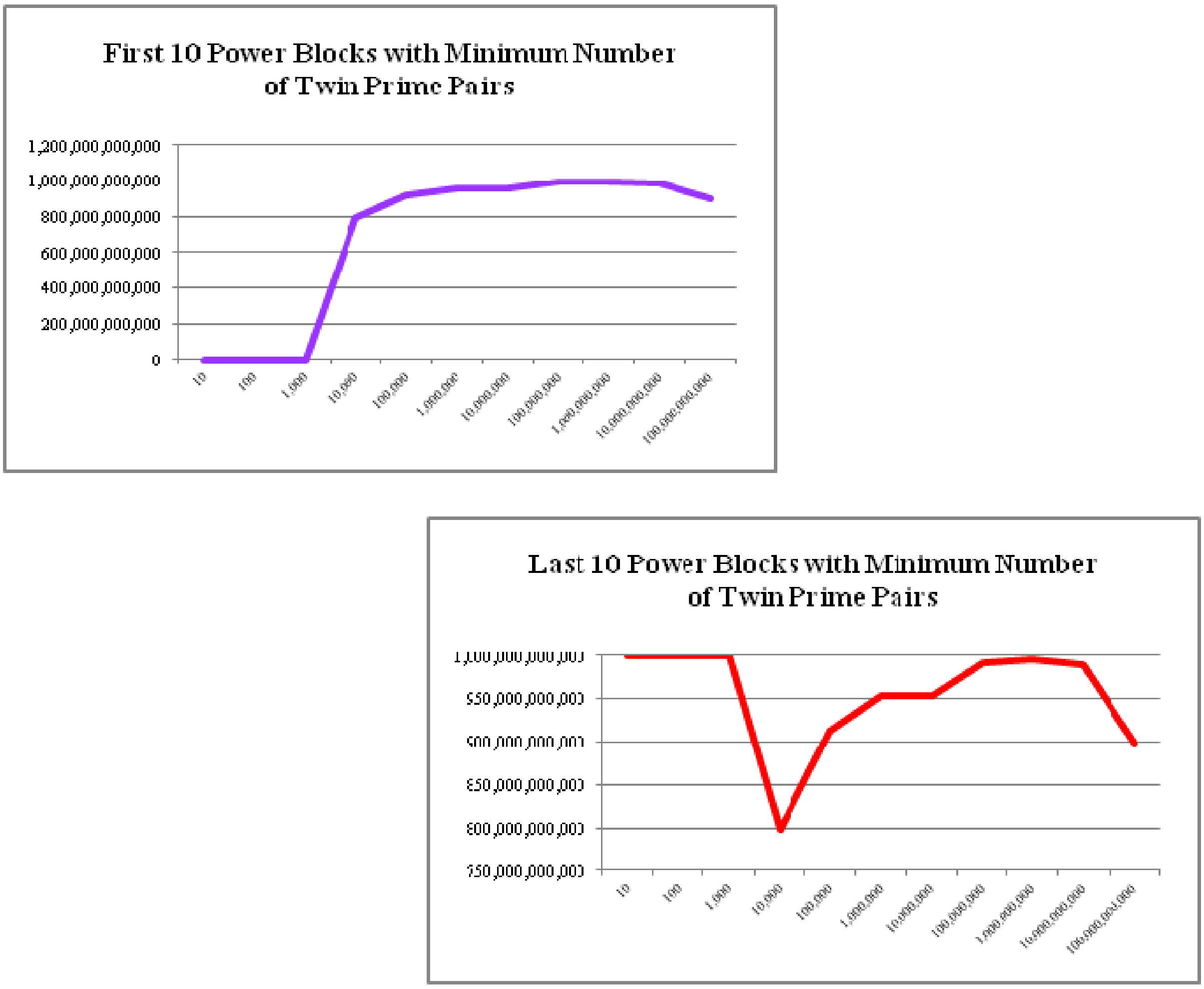


Figure 5. First & Last 10 Power Blocks with Minimum Number of Twin Prime Pairs.

The number of blocks containing the minimum number of twin prime pairs rapidly settles down to the unique value within our range of one trillion.

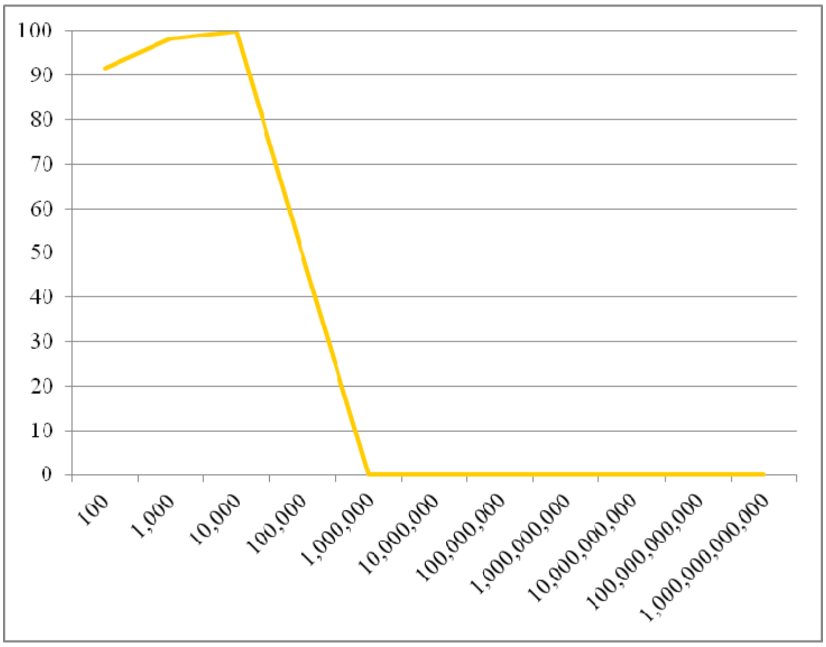


Figure 6. Percentage Decrease of Occurences of Minimum Number of Twin Primes in Blocks of 10 Powers.

3.3. MAXIMUM NUMBER OF TWIN PRIME PAIRS IN BLOCKS OF 10 POWERS

For all blocks of each 10 power ranging from 10^1 to 10^{12}, the maximum number of twin prime pairs found in each 10 power block has also been determined rigorously.

Table 5. Maximum Twin Prime Pairs in 10 Power Blocks

Sr. No.	Blocks of Size (of 10 Power)	Maximum No. of Twin Prime Pairs in Block	First Occurrence Block of Maximum No. of Twin Prime Pairs	Last Occurrence Block of Maximum No. of Twin Prime Pairs	Number of Blocks with Maximum Twin Prime Pairs
1.	10	2	0	999,999,843,250	8,398,278
2.	100	8	0	0	1
3.	1,000	35	0	0	1
4.	10,000	205	0	0	1
5.	100,000	1,224	0	0	1
6.	1,000,000	8,169	0	0	1
7.	10,000,000	58,980	0	0	1
8.	100,000,000	440,312	0	0	1
9.	1,000,000,000	3,424,506	0	0	1
10.	10,000,000,000	27,412,679	0	0	1
11.	100,000,000,000	224,376,048	0	0	1
12.	1,000,000,000,000	1,870,585,220	0	0	1

The maximum number of twin prime pairs keeps increasing with increasing inspection range.

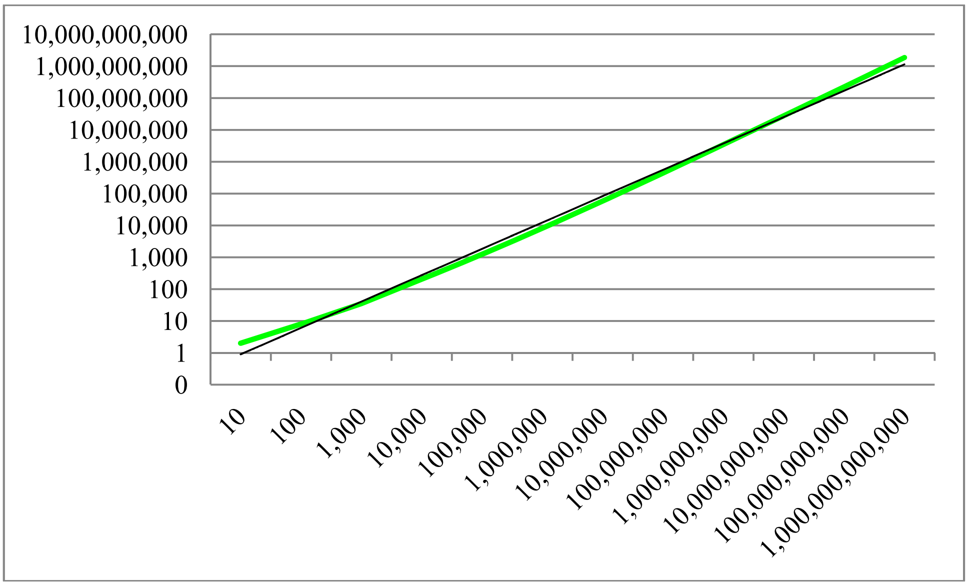
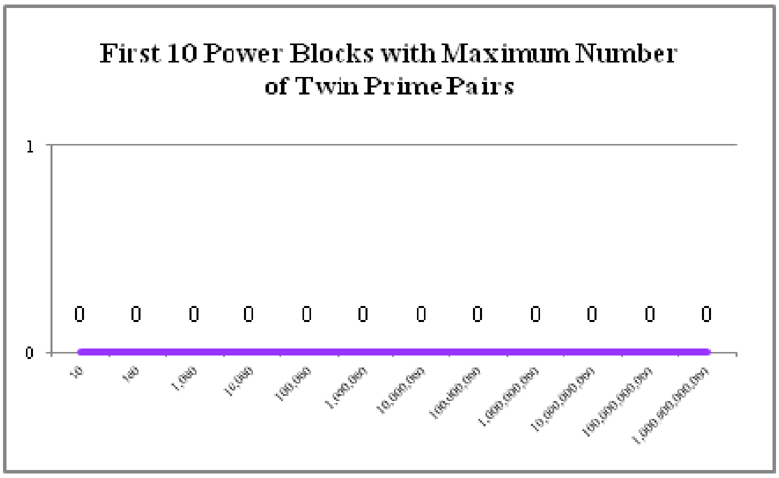


Figure 7. Maximum Number of Twin Prime Pairs in Blocks of 10 Powers.

This is gets approximated by exponential function $y = 0.134e^{1.904x}$ within our range.

The first and the last 10 power blocks of the occurrences of maximum number of twin prime pairs render following graphs.



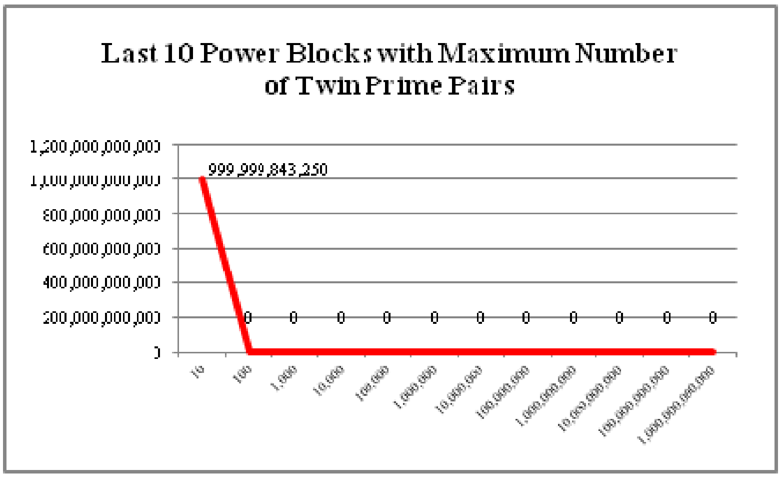


Figure 8. First & Last 10 Power Blocks with Maximum Number of Twin Prime Pairs.

The number of blocks containing the maximum number of twin prime pairs immediately settles down to the unique value within our range of one trillion.

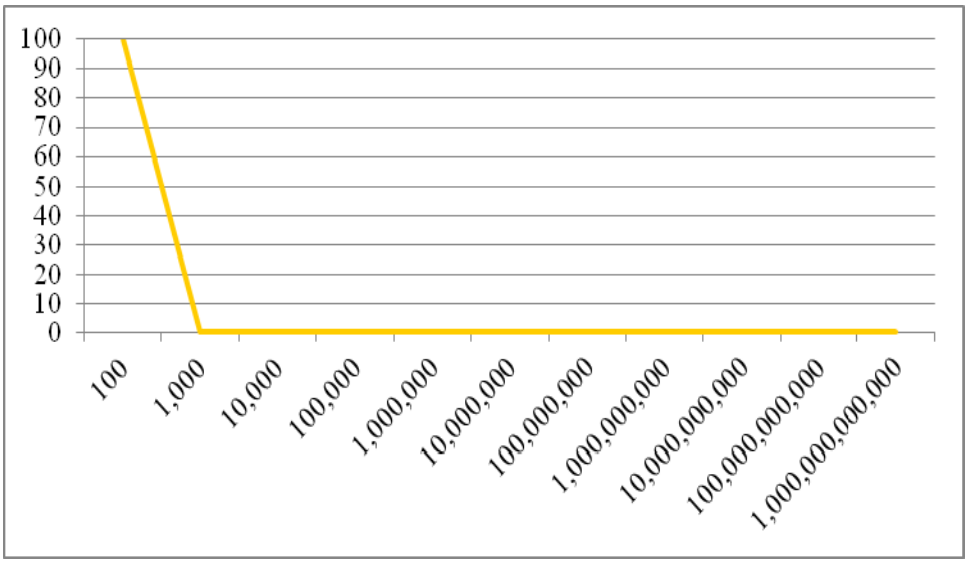


Figure 9. Percentage Decrease of Occurences of Maximum Number of Twin Primes in Blocks of 10 Powers.

The density of twin primes is destined to decrease as that of primes themselves does decrease. But like primes, they are expected to exhibit infinitude of existence. Some trends of their occurrences are presented here.

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