ORIGINAL PAPER

GEOMETRY OF STREAM LINES IN ROTATING FLUIDS

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Abstract. *Flow equations in a rotating reference frame are considered. Flow is with uniform angular velocity and absence of body forces. All the flow variables in the physical plane are transformed into orthogonal curvilinear coordinate system. Using the results of differential geometry, it is proved that the flow must satisfy* $\Delta \nu J + \mu_e \frac{\delta Q}{\delta \psi} - \rho \frac{\delta \xi}{\delta \phi} = 0$. *The theoretical part is illustrated by two applications. The geometry of streamlines is discussed in these two applications.*

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1. INTRODUCTION

The dynamics of rotating fluids is a fertile area to scout for general analytical solutions using differential geometry technique. Since earth is rotating uniformly about its own axis from west to east, a reference frame fixed on it is clearly a rotating frame of references. The effect of centrifugal force due to rotation of the earth reduce the effective value of g on its surface, also slightly changes its direction. At the same time, it is important to realize how much of engineering depends on a proper understanding of fluids; from flow of water through pipes, to studying effluent discharge into the sea; from motions of the atmosphere, the flow of lubricants in a car engine.

We know that Navier-Stokes equations are highly non linear partial differential equations in almost every real situation. Because of this, most problems are difficult or impossible to solve, and there doesn't exist a general method to solve them. Therefore, some special methods which can handle the above type of equations have gained considerable importance. One such technique is the Differential Geometry technique. Bagewadi etal [2] by using the fundamental magnitudes showed that in the Beltrami surface, streamlines are concentric circles and the magnetic permeability acts in the direction tangential to the Beltrami surface. Siddabasappa et al [4,5] by considering streamlines as parallel straight lines the geometry of magnetic lines and solutions to flow variables are discussed for viscous flows using stream function approach. In [5] Using magnetic flux function approach and assuming

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magnetic lines to be parallel straight lines the geometry of streamlines and solutions to flow variables are found.

In this part, we first consider the steady flow equations in a rotating frame of reference, which includes coriolis acceleration and centripetal acceleration with uniform angular velocity in the presence of magnetic field. Next all variables in the physical plane are transformed to an orthogonal curvilinear system. By using some of the results of differential geometry these equations can be solved and then two possible geometries of streamlines can be discussed

2. BASIC EQUATIONS

Consider the fluid flowing in the XY plane which is rotating about Z- field and in the absence of external body forces. We assume that viscosity and magnetic permeability of axis with uniform angular velocity. The flow is considered in the presence of magnetic the fluid is constant.

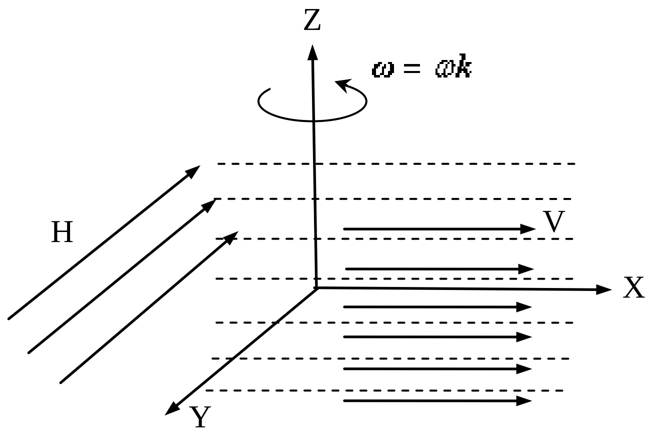

Figure 1. Schematic of flow pattern.

With the above flow pattern the governing basic equations of steady, viscous, incompressible fluid in a rotating frame of reference having infinite electrical conductivity are

$$\mathrm{div}V = 0 \tag{1}$$

$$\rho\left(V\mathrm{grad}V + 2\omega\times V + \omega\times\left(\omega\times r\right)\right) = -\mathrm{grad}p + \nu\nabla^2 V + \mu_e \mathrm{curl}H\times H \tag{2}$$

$$\mathrm{curl}\left(V\times H\right) = 0 \tag{3}$$

$$\mathrm{div}H = 0 \tag{4}$$

where

V = Velocity vector of fluid
k = unit vector in z-direction
ρ = density of fluid
μ_e =magnetic permeability (constant)
H =magnetic field vector
$\omega \times (\omega \times r)$ =centripetal acceleration
$\omega = \underline{\omega\, k}$ is angular velocity vector (constant)
r = radius vector
p = pressure function
ν = kinematic viscosity
$\omega \times V$ = coriolis acceleration

Since it is assumed that the fluid is flowing in the XY plane, we now specialize the above equations for steady two dimensional flows.

Take **V** = (u, v, 0), **H** = (H_1, H_2, 0), where u and v are components of velocity vector and H_1, H_2 are components of magnetic field vector. Now we introduce vorticity function ξ, current density function Q, and energy function h. They are taken as

$$\xi(x,y) = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \tag{5}$$

$$Q(x,y) = \frac{\partial H_2}{\partial x} - \frac{\partial H_1}{\partial y} \tag{6}$$

$$h(x,y) = \frac{1}{2}\rho V^2 + P' \tag{7}$$

where

$$V^2 = u^2 + v^2 \quad \text{and} \quad P' = P - \frac{\rho}{2}|\omega \times r|^2$$

Here P' is the reduced pressure. Using equations (5), (6) and (7) in equation (2) we get equations in terms ξ, Q and h by equating real and imaginary parts on both sides on introducing $h = \frac{1}{2}\rho V^2 + P - \frac{1}{2}|\omega \times r|^2$ equations (1) to (4) reduces to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{8}$$

$$v\frac{\partial \xi}{\partial y} - \rho\xi v - 2\rho\omega v + \mu_e Q H_2 = -\frac{\partial h}{\partial x} \tag{9}$$

$$v\frac{\partial \xi}{\partial x} - \rho\xi v - 2\rho\omega v + \mu_e Q H_1 = \frac{\partial h}{\partial y} \tag{10}$$

$$\frac{\partial H_1}{\partial x} + \frac{\partial H_2}{\partial y} = 0 \tag{11}$$

$$uH_2 - vH_1 = c \tag{12}$$

where c is a constant.

Equations (8) to (12) is a system of first order five non-linear partial differential equations. M.H.Martin [2] has introduced such a method of reduction of order by the introduction of vorticity and energy functions given by (5) to (7) in the study of two dimensional steady viscous fluids. Next with the help of (8) and (11) we define stream function and magnetic flux function.

3. STREAM FUNCTION AND MAGNETIC FLUX FUNCTION

The incompressibility constraint (8) implies the existence of a stream function $\psi\ (x, y)$ and the solenoidal equation (11) implies the existence of a magnetic flux function $\varphi\ (x, y)$ such that

$$u = \frac{\partial \psi}{\partial y},\ v = -\frac{\partial \psi}{\partial x} \tag{13}$$

$$H_1 = -\frac{\partial \phi}{\partial y},\ H_2 = \frac{\partial \phi}{\partial x} \tag{14}$$

Using these we now define curvilinear net.

4. CURVILINEAR COORDINATE SYSTEM

Now we introduce a curvilinear coordinate system $(\varphi,\ \psi)$ in place of the coordinates x, y in the physical plane. We represent $\psi(x, y)$ = constant as family of streamlines and $\varphi\ (x, y)$ = constant as family of magnetic lines. These two families of curves form a curvilinear coordinate system.

Let

$$x = x(\varphi,\ \psi) \quad \text{and} \quad y = y(\varphi,\ \psi) \tag{15}$$

define the curvilinear net where the arc length dS is given by

$$dS^2 = Ed\phi^2 + 2Fd\phi d\psi + Gd\psi^2 \tag{16}$$

$$E = \left(\frac{\partial x}{\partial \phi}\right)^2 + \left(\frac{\partial y}{\partial \phi}\right)^2,\ F = \frac{\partial x}{\partial \phi}\frac{\partial x}{\partial \psi} + \frac{\partial y}{\partial \phi}\frac{\partial y}{\partial \psi},\ G = \left(\frac{\partial x}{\partial \psi}\right)^2 + \left(\frac{\partial y}{\partial \psi}\right)^2 \tag{17}$$

Equation (15) can be solved to determine φ and ψ as functions of x and y satisfying the following relations

$$\frac{\partial x}{\partial \phi}=J\frac{\partial \psi}{\partial y}, \quad \frac{\partial x}{\partial \psi}=-J\frac{\partial \phi}{\partial y}, \quad \frac{\partial y}{\partial \phi}=-J\frac{\partial \psi}{\partial x}, \quad \frac{\partial y}{\partial \psi}=J\frac{\partial \phi}{\partial x} \tag{18}$$

where J is the transformation Jacobian. After using (17), it is given by

$$J=\frac{\partial x}{\partial \phi}\frac{\partial y}{\partial \psi}-\frac{\partial x}{\partial \psi}\frac{\partial y}{\partial \phi}=\pm\sqrt{\left(EG-F^2\right)}=\pm W \tag{19}$$

with $0<\left|J\right|<\infty$. Now we list some useful results from Differential Geometry.

5. RESULTS FROM DIFFERENTIAL GEOMETRY

Denote α the local angle of inclination of the tangent to the coordinate line ψ = constant directed in the sense of increasing φ. We have from differential Geometry the following results.

$$\frac{\partial x}{\partial \phi}=\sqrt{E}\cos\alpha, \quad \frac{\partial y}{\partial \phi}=\sqrt{E}\sin\alpha$$

$$\frac{\partial x}{\partial \psi}=\frac{F}{\sqrt{E}}\cos\alpha-\frac{J}{\sqrt{E}}\sin\alpha, \quad \frac{\partial y}{\partial \psi}=\frac{F}{\sqrt{E}}\sin\alpha+\frac{J}{\sqrt{E}}\cos\alpha \tag{20}$$

$$\frac{\partial \alpha}{\partial \phi}=\frac{J}{E}\Gamma_{11}^{2}, \quad \frac{\partial \alpha}{\partial \psi}=\frac{J}{E}\Gamma_{12}^{2}$$

and the Gaussian curvature

$$K=\frac{1}{W}\left[\frac{\partial}{\partial \psi}\left(\frac{W}{E}\Gamma_{11}^{2}\right)-\frac{\partial}{\partial \phi}\left(\frac{W}{E}\Gamma_{12}^{2}\right)\right]=0 \tag{21}$$

Also we write

$$\frac{\partial V}{\partial \phi}=\frac{F\Gamma_{11}^{2}-E\Gamma_{12}^{2}}{\sqrt{E}W}, \quad \frac{\partial V}{\partial \psi}=\frac{F\Gamma_{12}^{2}-E\Gamma_{22}^{2}}{\sqrt{E}W} \tag{22}$$

$$\nabla^2\xi=\frac{1}{J}\left[\frac{\partial}{\partial \phi}\frac{\left(G\xi_\phi-F\xi_\psi\right)}{J}+\frac{\partial}{\partial \psi}\frac{\left(-F\xi_\phi+E\xi_\psi\right)}{J}\right] \tag{23}$$

where

$$\Gamma_{11}^{2}=\frac{-F\frac{\partial E}{\partial \phi}+2E\frac{\partial F}{\partial \phi}-E\frac{\partial E}{\partial \psi}}{2W^{2}},\quad \Gamma_{12}^{2}=\frac{-E\frac{\partial G}{\partial \phi}-F\frac{\partial E}{\partial \psi}}{2W^{2}},\quad \Gamma_{22}^{2}=\frac{F\frac{\partial G}{\partial \psi}-2F\frac{\partial F}{\partial \psi}+F\frac{\partial G}{\partial \phi}}{2W^{2}}$$

and

$$\xi_{\phi}=\frac{\partial \xi}{\partial \phi},\quad \xi_{\psi}=\frac{\partial \xi}{\partial \psi} \tag{24}$$

We now move to the system of equations (8) to (12) along with equations (5) to (7) and we write these equations in a new form in the new variables φ and ψ. We consider that the fluid flows towards higher or lower parameter values of φ and ψ so that $J = W > 0$. Now we reproduce the new forms of these equations obtained by M.H. Martin.

6. VORTICITY FUNCTION AND CURRENT DENSITY FUNCTION

We recall equation of continuity (8) and equation of vorticity (5) have the same form as in the theory of viscous fluid. The necessary and sufficient conditions for the flow of a fluid along the streamlines ψ = constant (towards higher or lower parameter values of φ accordingly as J is positive or negative) of a curvilinear coordinate system (15) with dS^2 given by (16) to satisfy the principle of conservation of mass is

$$WV=\sqrt{E} \tag{25}$$

$$u+iv=\frac{\sqrt{E}}{J}\exp(i\alpha) \tag{26}$$

$$\xi=\frac{1}{W}\left[\frac{\partial}{\partial \phi}\left(\frac{F}{W}\right)-\frac{\partial}{\partial \psi}\left(\frac{E}{W}\right)\right] \tag{27}$$

Also it is shown that the magnetic field acts along the magnetic lines towards higher or lower parameter values of ψ accordingly as J is positive or negative. Also

$$WH=\sqrt{G} \tag{28}$$

$$H_1+iH_2=\frac{\sqrt{G}}{J}\exp(i\beta) \tag{29}$$

$$Q=\frac{1}{W}\left[\frac{\partial}{\partial \phi}\left(\frac{G}{W}\right)-\frac{\partial}{\partial \psi}\left(\frac{F}{W}\right)\right] \tag{30}$$

where $i=\sqrt{-1}$ and β is the angle made by the tangent to the coordinate line φ = constant directed in the sense of increasing ψ with the x-axis.

7. MARTIN FORM OF MOMENTUM EQUATIONS

Now we write the momentum equations (9) and (10) in terms of φ and ψ using the results of differential Geometry as defined above. Eliminating u and v from equations (9) and (10) using (13), (14) with the use of transformation relations (18) the above equations become,

$$\frac{\partial h}{\partial \psi}\frac{\partial x}{\partial \phi}-\frac{\partial h}{\partial \phi}\frac{\partial x}{\partial \psi}=\nu\left(\frac{\partial \xi}{\partial \phi}\frac{\partial y}{\partial \psi}-\frac{\partial \xi}{\partial \psi}\frac{\partial y}{\partial \phi}\right)-\rho\xi\frac{\partial x}{\partial \phi}-2\rho\varpi\frac{\partial x}{\partial \phi}+\mu_e Q\frac{\partial x}{\partial \psi} \tag{31}$$

$$\frac{\partial h}{\partial \psi}\frac{\partial y}{\partial \phi}-\frac{\partial h}{\partial \phi}\frac{\partial y}{\partial \psi}=\nu\left(\frac{\partial \xi}{\partial \psi}\frac{\partial x}{\partial \phi}-\frac{\partial \xi}{\partial \phi}\frac{\partial x}{\partial \psi}\right)-\rho\xi\frac{\partial y}{\partial \phi}-2\rho\varpi\frac{\partial y}{\partial \phi}+\mu_e Q\frac{\partial y}{\partial \psi} \tag{32}$$

If we now multiply (31) by $\partial y/\partial \Phi$ and (32) by $\partial x/\partial \Phi$ and adding we get,

$$\nu J\frac{\partial \xi}{\partial \phi}=E\left(\rho\xi+2\rho\varpi+\frac{\partial h}{\partial \psi}\right)-F\left(\frac{\partial h}{\partial \phi}+\mu_e Q\right) \tag{33}$$

Similarly if we now multiply (31) by $\partial y/\partial \psi$ and (32) by $\partial x/\partial \psi$ and adding we get,

$$\nu J\frac{\partial \xi}{\partial \psi}=F\left(\rho\xi+2\rho\varpi+\frac{\partial h}{\partial \psi}\right)-G\left(\frac{\partial h}{\partial \phi}+\mu_e Q\right) \tag{34}$$

Equations (33) and (34) are the new forms for the momentum equations called Martin form of equations. Solving (33) and (34) for $\partial h/\partial \Phi$ and $\partial h/\partial \psi$, we have

$$\frac{\partial h}{\partial \phi}=\frac{\nu}{J}\left(F\frac{\partial \xi}{\partial \phi}-E\frac{\partial \xi}{\partial \psi}\right)-\mu_e Q \tag{35}$$

$$\frac{\partial h}{\partial \psi}=\frac{\nu}{J}\left(G\frac{\partial \xi}{\partial \phi}-F\frac{\partial \xi}{\partial \psi}\right)-\rho\xi-2\rho\varpi \tag{36}$$

Making use of the integrability condition $\frac{\partial^2 \alpha}{\partial\phi\partial\psi}=\frac{\partial^2 \alpha}{\partial\psi\partial\phi}$, we obtain

$$\frac{\partial}{\partial \psi}\left(\Gamma_{11}{}^{2}\frac{J}{E}\right)-\frac{\partial}{\partial \phi}\left(\Gamma_{12}{}^{2}\frac{J}{E}\right)=0 \tag{37}$$

Thus we state the following theorem:

Theorem: If the streamlines ψ (x, y) = constant and the magnetic lines φ (x, y) = constant generate a curvilinear net in the physical plane of a steady plane viscous incompressible rotating fluid, the system of equations (5) - (12) of 8 partial differential equations may be replaced by a system (25) to (30) and (35) to (37).

Differentiating (35) with respect to ψ, (36) with respect to φ and making use of integrability condition $\frac{\partial^2 h}{\partial\phi\partial\psi} = \frac{\partial^2 h}{\partial\psi\partial\phi}$, we have,

$$\Lambda \nu J + \mu_e \frac{\partial Q}{\partial \psi} - \rho \frac{\partial \xi}{\partial \phi} = 0 \tag{38}$$

where

$$J^2 = W^2 = EG - F^2 = \frac{1}{c^2}$$

$$\Lambda = \frac{1}{J}\left[\frac{\partial}{\partial \phi}\left\{\frac{1}{J}\left(G\frac{\partial \xi}{\partial \phi} - F\frac{\partial \xi}{\partial \psi}\right)\right\} + \frac{\partial}{\partial \psi}\left\{\frac{1}{J}\left(E\frac{\partial \xi}{\partial \psi} - F\frac{\partial \xi}{\partial \phi}\right)\right\}\right]$$

Hence we have the following

Theorem: If the streamlines ψ = constant and magnetic lines φ = constant are taken as curvilinear coordinate system (φ , ψ) in the physical plane of a steady, plane, viscous, incompressible, rotating MHD flows with uniform angular velocity, then

$$\Lambda \nu J + \mu_e \frac{\partial Q}{\partial \psi} - \rho \frac{\partial \xi}{\partial \phi} = 0$$

The above system can be determined in a number of ways. Here we may suppose coordinate lines φ = constant and ψ = constant are orthogonal trajectories of each other and therefore letting F = 0 we obtain

$$\Lambda \nu J + \mu_e \frac{\partial Q}{\partial \psi} - \rho \frac{\partial \xi}{\partial \phi} = 0 \tag{39}$$

where

$$\xi = -\frac{1}{W}\frac{\partial}{\partial \psi}\left(\frac{E}{W}\right) \qquad Q = \frac{1}{W}\frac{\partial}{\partial \phi}\left(\frac{G}{W}\right) \qquad J = \sqrt{EG} \tag{40}$$

$$\Lambda = \frac{1}{J}\left[\frac{\partial}{\partial \phi}\left(\frac{G}{J}\frac{\partial \xi}{\partial \phi}\right) + \frac{\partial}{\partial \psi}\left(\frac{E}{J}\frac{\partial \xi}{\partial \psi}\right)\right]$$

8. APPLICATIONS

In this part we shall discuss some of the applications of the above theoretical part. We study certain flows for which certain conditions are imposed on the coefficients of E, F, and G of the first fundamental magnitudes given by (16). By taking magnetic lines φ = constant to be orthogonal to the streamlines ψ = constant, i.e. when F=0 and assume streamline pattern when we study possible plane flows of a rotating fluid.

8.1. STRAIGHT STREAMLINES

In this application we assume that streamlines are nonparallel straight lines but envelope a curve C. Further assume that tangent lines, to C and their orthogonal trajectories, the involutes of C determine an orthogonal curvilinear net for which the squared element of arc length is given by

$$dS^2 = dS_1^2 + dS_2^2 \tag{41}$$

where dS_1 and dS_2 are the elements of arc length of involutes and tangent respectively.

Let σ denote the arc length of C, η the angle formed by the tangent line to C with X-axis, J-the parameter, constant along each involute and $K = \frac{1}{R} = \eta'(\sigma)$ to be curvature of C, then (41) becomes

$$dS^2 = d\Im^2 + (\Im - \sigma(\eta))^2 d\eta^2 \tag{42}$$

where $\sigma = \sigma(\eta)$ denote the arc length. Clearly $\Im$ = constant are the involutes of C and η = constant are the tangent lines to C.

We now investigate the flows for which

$$\phi = \phi(\Im) \qquad \psi = \psi(\eta) \tag{43}$$

Using (43) in (16),

$$dS^2 = E\phi' d\Im^2 + G\psi'^2 d\eta^2 \tag{44}$$

Comparing (44) and (42) we have,

$$E = \frac{1}{\phi'^{2}}, \quad G = \left(\frac{\Im - \sigma(\eta)}{\psi'}\right)^{2}, \quad J = \frac{\xi - \sigma(\eta)}{\phi'\,\psi'} \tag{45}$$

Since F=0

$$\Gamma_{11}^{\ 2} = \left(\frac{\Im - \sigma(\eta)}{\psi'}\right)\phi', \qquad \Gamma_{12}^{\ 2} = 0 \tag{46}$$

Substituting (46) in Gauss equation, we see that it is identically satisfied and we have the following

$$\xi = -\frac{1}{(\Im - \sigma(\eta))^3}\left[\psi''(\Im - \sigma(\eta)) + \sigma'(\eta)\right], \quad Q = \phi'' + \frac{\phi'}{\Im - \sigma(\eta)} \tag{47}$$

On eliminating ξ and Q from (39) with the help of (47), we obtain

$$\mu_e \phi'^{V} \sigma'(\eta)(\Im - \sigma(\eta))^4 - \left(2\rho\psi'\psi'' + \nu\psi'^{V} + 4\nu\psi''\right)(\Im - \sigma(\eta))^3$$
$$-\left(9\nu\psi'\sigma'(\eta) + 4\nu\psi''\sigma''(\eta) + \nu\psi'\sigma'''(\eta) + 6\nu\psi'''\sigma'(\eta) + 3\rho\psi'^2\psi''\sigma''(\eta)\right)(\Im - \sigma(\eta))$$
$$-15\nu\psi'\sigma'^3(\eta) = 0 \qquad (48)$$

Since $\Im$ and η are independent variables, for the relation (48) to hold identically, all the coefficients of different powers of $(\Im - \sigma(\eta))$ vanish identically. Therefore we have

$$15\nu\psi'\sigma'^3(\eta) = 0$$

Now since $\nu \neq 0$ and ψ' cannot vanish identically, we find that $\sigma'(\eta) = 0$

But $\sigma' = \frac{1}{k} = 0$ implies that R = 0.

Therefore C has zero radius of curvature. Then the streamlines are concurrent. Hence we have the following theorem:

Theorem: If the streamlines are nonparallel straight lines and envelope a curve C, η=constant are tangent lines and $\Im$=constant are involutes of C are taken as orthogonal curvilinear coordinates, taken in a plane flow of a rotating fluid streamlines are concurrent.

8.2. STREAMLINES AS INVOLUTES OF A CURVE

In this application we consider involutes of a curve C as streamlines ψ=constant. We take curvilinear coordinate system ($\Im$, η), where now coordinate curves $\Im$=constant are the involutes and η = constant are tangent lines to the curve C.

Now for the flows

$$\Phi = \Phi(\eta) \text{ and } \psi = \psi(\Im) \qquad (49)$$

Using (49) in (16) we find that

$$dS^2 = \phi'^2 E d\eta^2 + 2F\phi'\psi' d\Im d\eta + G\psi'^2 d\Im^2 \qquad (50)$$

Comparing (50) and (42)

$$E = \left(\frac{\Im - \sigma(\eta)}{\phi'}\right)^2, \quad F = 0, \quad G = \frac{1}{\psi'^2}, \quad J = \frac{\Im - \sigma(\eta)}{\phi'\psi'}$$

Gauss equation (21) is automatically satisfied and as before we find

$$\xi = -\psi'' - \frac{\psi'}{\Im - \sigma(\eta)} \qquad Q = \frac{1}{(\Im - \sigma(\eta))^3}\left[\phi''(\Im - \sigma(\eta)) + \sigma'(\eta)\right] \qquad (51)$$

Eliminating ξ and Q from (39) using (51), simplifying and multiplying it by $(\Im - \sigma(\eta))^4$, we have

$$
\begin{aligned}
&(\Im - \sigma(\eta))^5 \psi'^{V} + (\Im - \sigma(\eta))^4 2\nu \psi''' + (\Im - \sigma(\eta))^3 \nu \psi'' \\
&+(\Im - \sigma(\eta))^2 \left[\nu \psi' + \rho \psi'^2 \sigma'(\eta) \right] + (\Im - \sigma(\eta)) \left[\psi' \nu \sigma''(\eta) + \mu_e \phi' \phi'' \right] \\
&+3 \psi' \sigma'^2(\eta) \nu + \mu_e \phi'^2 \sigma'(\eta) = 0
\end{aligned}
\tag{52}
$$

For the relation (52) to hold identically, it must hold on the curve $\Im = \sigma(\eta)$ and consequently

$$\sigma'(\eta)\left[3\psi' \sigma'(\eta) \nu + \mu_e \phi'^2 \right] = 0$$

implies (i) $\sigma'(\eta) = 0$ or $\sigma(\eta)$ = constant. Therefore, radius of curvature vanishes identically and C reduces to a point. We thus have the following theorem.

Theorem: The streamlines in a two dimensional steady flow of a rotating fluid of infinite electrical conductivity are involutes of a curve C only if C reduces to a point and the streamlines are circles concentric at this point. or (ii) the ratio of the magnetic permeability μ_e and kinematic viscosity ν of the fluid medium is given by $\frac{\mu e}{\nu} = -3 \frac{\sigma'(\eta)\psi'}{\phi'^2}$

9. CONCLUSIONS

An attempt to geometry of some particular flows is the basis of this paper. Here basic equations of a steady, viscous, incompressible rotating fluid are transformed into curvilinear coordinate system where ψ = constant and Φ = constant generate a curvilinear net. Then the fundamental magnitudes and vorticity function, current density function and energy function are found in terms of Φ and ψ. Two applications on the geometry of streamlines are discussed. In the first application it is proved that streamlines are concurrent, if the streamlines are nonparallel straight lines and envelope a curve C, η=constant are tangent lines and $\Im$=constant are involutes of C are taken as orthogonal curvilinear coordinates, taken in a plane flow of a rotating fluid. In the second application it is proved that the streamlines in a two dimensional steady flow of a rotating fluid of infinite electrical conductivity are involutes of a curve C only if C reduces to a point and the streamlines are circles concentric at this point and the ratio of the magnetic permeability μ_e and kinematic viscosity ν of the fluid medium is given by $\frac{\mu e}{\nu} = -3 \frac{\sigma'(\eta)\psi'}{\phi'^2}$.

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