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ANALYSIS THE ENTANGLEMENT IN THE ONE DIMENSIONAL KONDO NECKLACE MODEL

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Abstract. *In this paper, at first we review the history of Kondo models exactly. Then we investigate quantum entanglement in the one-dimensional Kondo Necklace model. In calculations we have got each of the matrices according to Hamiltonian section. These result shows that the ground state are included in two up and down spins.*

Keywords: *Quantum entanglement- Kondo Necklace model- Spin chain.*

1. INTRODUCTION

The topic of lattice quantum spin systems is a fascinating and by now well–established branch of theoretical physics. However, many important questions remain to be answered. Our paper was inspired by a set of papers and lectures on quantum spin systems. It will guide the reader through the foundations of the field. In particular, the solutions of the one-dimensional Kondo Neclace model. So firstly in Section 2 we discuss to the history of Kondo Necklace model exactly and secondly in Section 3 we introduce Hamiltonian of one Kondo Neclace model and related matrices and finally the calculate of eigen state and eigen values.

2. THE HISTORY OF KONDO NECKLACE

The Kondo models are paradigms to explore the quantum impurity problems. They identify a special class of physical systems whose macroscopic properties are dramatically influenced by the presence of few impurities with quantum internal degrees of freedom [1]. In its simplest formulation, the effective Hamiltonian describes a single impurity spin interacting with a band of free electrons. The many-body screening of the impurity spin provided by the electrons as a collective effect, leads the system from weak coupling to strong coupling regimes [2].

In the first studies of the entanglement the charge degrees of freedom of the electrons were considered frozen. The pairwise entanglement of spin degrees of freedom in the isotropic Kondo model was analyzed [3] within the variational formalism of Yosida where the Kondo singlet is described as

$$|\Psi_B\rangle = \frac{1}{2}\left(|\phi_\downarrow\rangle\,|\chi_\uparrow\rangle - |\phi_\uparrow\rangle\,|\chi_\downarrow\rangle\right)$$

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In the previous denote t|χ_σ> he impurity spin states; |ϕ_σ> represent the electronic states with an unbalanced spin σ [1]. In agreement with the common wisdom, the reduced density operator of the impurity is found to be maximally mixed, meaning that the Fermi sea and the impurity spin are in a maximally entangled state (the Kondo singlet). The impurity spin and a single electron are in a Werner state made of a superposition of the back ground and the Kondo singlets. Due to the entanglement monogamy (the electrons cooperatively form a singlet with the spin) two electrons cannot be entangled with each other within the Kondo cloud and the single-electron spin entanglement vanishes in the thermodynamical limit.

Pairs of electronic spin can be nevertheless entangled in a finite system through the scattering with the spin impurity; this effect might be used to manipulate the electron-electron entanglement by performing a projective measure on the impurity spin [4]. This suggests that some amount of electron-electron entanglement might be extracted even in the thermodynamical limit where it was demonstrated that the Kondo resonance is washed out by the measurement [5] effectively removing the constraint of the entanglement sharing. The two impurity Kondo model was studied as well. The new feature here is the Ruderman-Kittel-Kasuya-Yosida (RKKY) effective interaction between the impurity spins S_1 and S_2, that competes with the Kondo mechanism (favouring non magnetic states); it is ferromagnetic or antiferromagnetic depending on the distance between the impurities. Because of such interplay a quantum critical point emerges in the phase diagram separating the spin-spin interaction regime from the phase where the two spin are completely screened [6].

As for the single impurity, the two impurity spins are in a Werner state, for which the concurrence is characterized by single parameter ps, exhibiting a singlet type of entanglement between the two spin impurities. The concurrence is found to vanish at the critical point. For ferromagnetic RKKY interaction the concurrence between the impurity spin vanishes identically as the result of a S = 1 Kondo screening. It turns out that [7] the impurity spins can be entangled (with a finite concurrence) by the RKKY interaction only when certain amount of antiferromagnetic correlations fs = $<s_1 s_2>$ is established in the system; such value of the correlation function is that one reached at the quantum critical point. The entanglement between the conduction electrons and the Kondo impurities is quantified by a combined analysis of the Von Neumann entropies of the two impurities and of the single impurity (tracing out both the electronic spins and the remaining impurity). The latter quantity is maximized independently on fs, meaning that the impurity spin is completely screened either by the Kondo cloud or by the other impurity spin. In the regime where the Kondo mechanism dominates, the concurrence cannot be finite because of the entanglement sharing.

Entanglement in the Kondo physics of double quantum dots in an external magnetic field was studied in [8]. The calculations were done numerically, both at zero and at finite temperature. The main phenomenology results to be consistent with the scenario depicted in [7] especially if the dots are arranged in series (each dot is coupled to the leads exclusively, resembling the configuration of the Kondo spins embedded in the electrons). The concurrence switches to finite values for a certain threshold of the inter-dot coupling (for which the assumptions of negligible charge fluctuations results still valid). The temperature weakens the entanglement between the qubits also at $T>T_k$. For the side- and parallel-coupled dots a more intense coupling amoung the qubits is required to entangle them. For the side-coupled dots this results because one has to win on the enhanced Kondo effect on the dot coupled to the leads (’two stage Kondo’ effect); therefore the critical inter-dots coupling is $\sim T_k$. For the parallel-coupled arrangement the concurrence is zero because the effective RKKY interaction turns out to be ferromagnetic up to a certain value that is the threshold to entangle the electrons. The RKKY interaction controls in an effective way the entanglement amoung the qubits also in the case of many impurity spins arranged as in the Kondo necklace model [9]. The Hamiltonian describes a Kondo lattice where the localized impurity spins, displaced in

every lattice site, interact with the (pseudo) spins of the electrons. It results that the additional on-site spin-spin interaction impose a 'selective' monogamy of the entanglement, depending on whether the Heisenberg interactions is ferromagnetic or antiferromagnetic. The effects of finite temperature and magnetic field were considered also in anisotropic models. It emerges that a critical field exists separating different patterns in the thermal entanglement between the eigenstates of the model. The effect of the fluctuations of the charge degrees of freedom of the electrons (frozen in the references cited above) in the RKKY mechanism was discussed quantifying the entanglement of particles in a small cluster described by the periodic Anderson model [11]. It was evidenced that the ground state of the system is characterized by a double occupancy of the electronic levels, whose entanlement can be only partially captured by assuming them as simple qubits. The spin's and charge's interplay was taken into account by considering the pairwise entanglement of particles. It is demonstrated that the phase diagram of the system can be reproduced from the analysis of the impurity-impurity and electron-impurity entanglement.

3. ONE – DIMENSIONAL KONDO NECKLACE MODEL

A standard Hamiltonian for this purpose is the Kondo lattice model. A simplified version of the Kondo lattice hamitonian was introduced by Doniach in [12]. This model, called Kondo Necklace (KN), replaces, into the Kondo interaction, the spins of the conduction electrons by a set of Pseudo-spins on a linear lattice, the charge degrees of freedom being frozen out. One-dimensional KN model is defined by the Hamiltonian

$$H = W\sum_{i=1}^{N}(\tau_i^x\tau_{i+1}^x + \tau_i^y\tau_i^y) + J\sum_{i=1}^{N}\vec{S}_i.\vec{\tau}_i$$

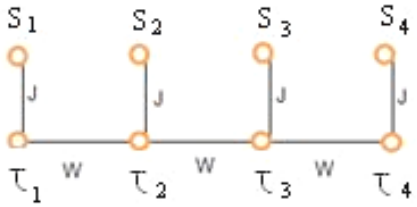

Fig. 1. One kondo necklace model system.

The one-dimensional anisotropic Kondo-Necklace model has been studid by several methods. Where τ_i^α and S_i^α are independent spin 1/2 operators acting on a site *i*, which are given by $\sigma^\alpha/2$, with σ^α (α = x; y; z) denoting the Pauli matrices. The spin operators τ_i^α and S_i^α are associated with the conductionelectrons and the innercore spins hanging from the τ-spin chain, respectively. We can write above hamoltonian as following:

$$H = 2W\ \left[(\tau_1^x\tau_2^x II) + (\tau_1^y\tau_2^y II)\right] + J\left[(S_1^x I\tau_1^x I + S_1^y I\tau_1^y I + S_1^z I\tau_1^z I) + (IS_2^x I\tau_2^x + IS_2^y I\tau_2^y + IS_2^z I\tau_2^z)\right]$$

$$\tau_1^x\tau_2^x II=\frac{1}{4}\begin{pmatrix}0&1\\1&0\end{pmatrix}\otimes\begin{pmatrix}0&1\\1&0\end{pmatrix}\otimes\begin{pmatrix}1&0\\0&1\end{pmatrix}\otimes\begin{pmatrix}1&0\\0&1\end{pmatrix}=$$

$$\frac{1}{4}\begin{pmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0
\end{pmatrix}$$

$$\Rightarrow 2W\left[(\tau_1^x\tau_2^x II)+(\tau_1^y\tau_2^y II)\right]\ =$$

$$\begin{pmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&W&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&W&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&W&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&W&0&0&0&0\\
0&0&0&0&W&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&W&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&W&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&W&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{pmatrix}$$

$$S^{x}I\tau^{x}I=\frac{1}{4}\begin{pmatrix}0&1\\1&0\end{pmatrix}\otimes\begin{pmatrix}1&0\\0&1\end{pmatrix}\otimes\begin{pmatrix}0&1\\1&0\end{pmatrix}\otimes\begin{pmatrix}1&0\\0&1\end{pmatrix}=$$

$$\frac{1}{4}\begin{pmatrix}
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0
\end{pmatrix}$$

$$S^{y}I\tau^{y}I=\frac{1}{4}\begin{pmatrix}0&-i\\i&0\end{pmatrix}\otimes\begin{pmatrix}1&0\\0&1\end{pmatrix}\otimes\begin{pmatrix}0&-i\\i&0\end{pmatrix}\otimes\begin{pmatrix}1&0\\0&1\end{pmatrix}=$$

$$\frac{1}{4}\begin{pmatrix}
0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&-1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&-1&0&0&0&0&0&0&0&0&0&0
\end{pmatrix}$$

$$S^z I\tau^z I = \frac{1}{4}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\otimes\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\otimes\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\otimes\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=$$

$$\frac{1}{4}\begin{pmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&-1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&-1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&-1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&-1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1
\end{pmatrix}$$

$$S^x I\tau^x I + S^y I\tau^y I + S^z I\tau^z I =$$

$$\frac{1}{4}\begin{pmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&-1&0&0&0&0&0&2&0&0&0&0&0&0&0\\
0&0&0&-1&0&0&0&0&0&2&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&-1&0&0&0&0&0&2&0&0&0\\
0&0&0&0&0&0&0&-1&0&0&0&0&0&2&0&0\\
0&0&2&0&0&0&0&0&-1&0&0&0&0&0&0&0\\
0&0&0&2&0&0&0&0&0&-1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&2&0&0&0&0&0&-1&0&0&0\\
0&0&0&0&0&0&0&2&0&0&0&0&0&-1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1
\end{pmatrix}$$

$$IS^{y}I\tau^{y}=\frac{1}{4}\begin{pmatrix}1&0\\0&1\end{pmatrix}\otimes\begin{pmatrix}0&-i\\i&0\end{pmatrix}\otimes\begin{pmatrix}1&0\\0&1\end{pmatrix}\otimes\begin{pmatrix}0&-i\\i&0\end{pmatrix}=$$

$$\frac{1}{4}\begin{pmatrix}
0&0&0&0&0&-1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&-1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&-1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0
\end{pmatrix}$$

$$IS^{z}I\tau^{z}=\frac{1}{4}\begin{pmatrix}1&0\\0&1\end{pmatrix}\otimes\begin{pmatrix}1&0\\0&-1\end{pmatrix}\otimes\begin{pmatrix}1&0\\0&1\end{pmatrix}\otimes\begin{pmatrix}1&0\\0&-1\end{pmatrix}=$$

$$\begin{pmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&-1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&-1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&-1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1
\end{pmatrix}$$

$$IS^x I\tau^x + IS^y I\tau^y + IS^z I\tau^z =$$

$$\frac{1}{4}\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

So we conclude matrix of H as following:

$$H =$$

$$\begin{pmatrix}
\frac{J}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{J}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{J}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{J}{2} & 0 & 0 & \frac{J}{2} & 0 & 0 & \frac{J}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{J}{2} & 0 & 0 & 0 & 0 & 0 & 0 & W & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{J}{2} & 0 & 0 & 0 & W & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{J}{2} & 0 & 0 & -\frac{J}{2} & 0 & 0 & 0 & W & 0 & \frac{J}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & W & 0 & \frac{J}{2} & 0 & 0 \\
0 & 0 & \frac{J}{2} & 0 & W & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{J}{2} & 0 & W & 0 & 0 & 0 & -\frac{J}{2} & 0 & 0 & \frac{J}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & W & 0 & 0 & 0 & \frac{J}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & W & 0 & 0 & 0 & 0 & 0 & 0 & \frac{J}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{J}{2} & 0 & 0 & \frac{J}{2} & 0 & 0 & -\frac{J}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{J}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{J}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{J}{2}
\end{pmatrix}$$

Now we calculate above matrix straightforwardly by the $|H-\lambda I|=0$ [13]. Then we consider the minimum eigen value as ground eigen value. So we have

$$\in_{\circ}=-\frac{J}{6}-2\sqrt{Q}Cos\frac{\theta}{3}$$

where

$$\theta=arcCos(\frac{-J(9W^2-16J^2)}{54\sqrt{Q^3}})$$

$$Q=\frac{4J^2+3W^2}{9}$$

Eigen vector of the corresponding to eigen values can be obtain:

$$|\in_0\rangle=A[\ (|++--\rangle+|--++\rangle)+\alpha_1(|+--+\rangle+|-++-\rangle)+\alpha_2(|+-+-\rangle+|-+-+\rangle)\ ]$$

where

$$\alpha_1=\frac{J+2\in_0}{2J}$$

$$\alpha_2=\frac{\in_0^2+\in_0 J+\frac{3J^2}{4}}{WJ}$$

$$A=\frac{1}{\sqrt{2(1+\alpha_1^2+\alpha_2^2)}}$$

CONCLUSIONS

This paper discuss about Kondo models. Then a system with one dimenssional Kondo Necklace model was surveyed exactly. The ground state consist in Kets that they have two up and down spins.

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